Efficiency, access, and the mixed delivery of health care services[*]

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Abstract

Universal health systems often rely on both public provision and contracting arrangements with private hospital. This paper studies the optimal mix of public and private provision and how it depends on the available technology. We propose a model in which the regulator acts as a third-party payer, and aims to ensure universal access to treatment at minimal cost. Patients need one unit of medical service and differ in the severity of illness. A private and a public hospital are available. Under incomplete contracts, ownership affects the regulatory constraints and the power of managerial incentives. Only the private manager internalizes profits, and has incentives to reject costly patients and to exert effort in cost reduction. Contracting with the private hospital is optimal when managerial effort is relatively effective in reducing costs. By using the public hospital as a last resort provider, the regulator can ensure access, provide incentives to the private manager, and internalize part of the resulting cost savings. Imposing a no-dumping constraint on the private hospital reduces the power of incentives and is not always optimal.

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1   Introduction

In most countries, medical services are delivered by both public and private providers. In the last twenty years, public procurement arrangements with private hospitals have become common even in countries with a national health system, such as Italy and the UK. However, the health systems of these countries still rely on public hospitals. Typically, patients can get treated in either a public or a private hospitals (contracting with the government) for the same out-of-pocket fee. Despite the widespread use of mixed delivery in national health systems, the conditions under which mixed delivery is optimal have not been intensively studied.

One of the arguments justifying this trend towards procurement is that private hospitals are more efficient than their public counterparts. Private providers may face different costs and be able to adjust their inputs more promptly. Private managers may also be more sensitive to financial incentives because they internalize profits.[1] However, efficiency is not the only dimension to evaluate the delivery of health services. In many countries, governments have to ensure universal health coverage; this target has been put forward to justify the role of public hospitals.[2] Profit driven private hospitals might reject patients whose cost exceeds the reimbursement received from the government. While dumping is often forbidden by law, hospitals can cherry pick healthy patients in a number of ways: specializing in less complex treatments, offering a higher quality to low-cost patients or transferring already admitted patients to public facilities. Such behaviors are the result of a regulation failure; unobservability or contract incompleteness make it impossible to adapt the price to the severity of illness of each patient, within a certain diagnostic group. If such a problem is severe, the regulator might prefer to use public hospitals, where access can be ensured.

This paper studies the trade-off between cost efficiency and access to care and its effects on the optimal mix of public and private provision.[3] We model a simple health care market where the government acts as a third-party payer and can contract with a public and a private hospital. All patients are entitled to get the treatment for free and benefit from the treatment in the same way, but

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[1] The higher efficiency of private hospitals has yet to be shown empirically. Burgess and Wilson (1996), find contradicting results on the impact of ownership on efficiency and results depend on the measure of technical inefficiency used. Zuckerman et al. (1994) use US data and Chang et al. (2004) use Taiwan data. They both find that public hospitals are less efficient than private ones. For a review, slightly pointing towards a greater efficiency of public hospitals, see Hollingsworth (2003).

[2] In France, the role of the private sector in the provision of publicly funded health services was enhanced by the "Bachelot Law", introduced in 2009. The concern that the use of private hospitals would reduce the access to care was central in the fierce public debate that preceded the adoption of the law.

[3] While in this we only focus on efficiency and access, it has to be noted that other elements are likely affect the optimal mix of public and private provision. For instance, by running a public firm, the regulator might be able to acquire information about the production technology of private regulated firms.

differ in the severity of illness and thus in the cost of treatment. Because of contract incompleteness, ex ante the regulator is unable to design a contract such that hospital reimbursements depend on cost realizations; she is constrained to use a fixed wage for the manager of the public hospital. Furthermore, the regulator can only rely on a unitary price per treatment when contracting with the private hospital. The managers of both the public and the private hospital can choose which patients to treat and can exert effort to reduce variable costs. The incentives faced by the managers, however, depend on the ownership of the hospital. The private manager earns profits, and has an incentive both to turn down costly patients and to exert effort in cost reduction. The public manager receives a fixed salary, and has no incentive to reduce costs or to dump costly patients.

We show that it is optimal to use the public hospital as a last resort provider; the regulator should contract with the private hospital if and only if managerial effort leads to high cost reductions. This result is rather intuitive. Since all patients have to be treated, it is impossible to distort quantities in order to provide incentives to the last resort hospital. The public hospital, which does not dump patients, is the most suitable to play this role. Because of the presence of a last resort hospital, it is possible for the regulator to set a low price for the private hospital. If the efficiency of managerial effort is high, even such a low price elicits managerial effort and leads to a reduction in total health expenditures. The model predicts that mixed delivery is optimal for procedures such as elective surgery, for which an efficient management of the operating rooms might be sufficient to obtain considerable cost reductions. Procedures with smaller margins for cost reductions (for instance, emergency ones) should be provided entirely by public hospitals. These results may be of guidance to policymakers considering public procurement arrangements for health services.

The second contribution of the paper concerns the role of dumping. We analyze a no-dumping regulation compelling the private manager to report cost realizations and to treat all patients as long as profits are non-negative. If managerial effort leads to high cost reductions, the no-dumping regulation is effective in reducing total health expenditures. If the potential cost reduction is small, conversely, the manger has incentives to misreport costs in order to dump some patients. No reduction in government expenditures is expected from contracting with the private hospital. Thus, forbidding dumping is not always beneficial from a welfare prospective. This result stems from the trade-off between efficiency and access, and to our knowledge, is novel in the literature.

There is a prolific theoretical literature dealing with dumping in the health care sector. Ellis and McGuire (1986), Dranove (1987), Ma (1994), Ellis(1998), and Chalkley and Malcomson (2002), among

others, show that prospective reimbursement schemes may lead to undesired refusal of patients. In presence of dumping or cream-skimming, a mixed reimbursement scheme (including some cost reimbursement) enhances social welfare, even though it lowers the power of incentives in cost reduction.[4] In our study, the private manager internalizes profits, and has incentives to dump and to exert effort. On the contrary, the public hospital's profits are captured by the government; thus, in practice, the public manager has the same incentives as under a pure cost reimbursement scheme. Under incomplete contracts, ownership affects the regulatory constraints and the power of managerial incentives. We thus shift the focus of the analysis from the optimal mix of prospective and retrospective reimbursement to the optimal mix of public and private ownership.

The literature comparing public and private ownership considers incomplete contracts as a source of inefficiency in the public sector. Hart et al. (1997), Schmidt (1996) and Laffont and Tirole (1991), for instance, show that public managers invest too little in cost reduction, as the government can appropriate the returns of their investment. Because of contract incompleteness, the government/owner cannot commit to leave enough rent to the public manager. Our model borrows many elements from the incomplete contract approach. However, this literature always focuses on dichotomous solutions, where either public or private provision is optimal. We allow for the presence of mixed delivery systems.

Because incomplete contracts do not allow for pay-for-performance remuneration schemes, the public managers face a soft budget constraint (Kornai, 1980): public managers are not liable for their losses, and their surpluses can be diverted by the government. There is a lot of evidence suggesting that the health care sector, and in particular public providers, are affected by soft budget constraints.[5] A piece of evidence particularly relevant for our purposes is provided by Baicker and Staiger (2008). Using US data on the Medicaid Disproportionate Share Hospital program, they show that local governments make use of intergovernmental transfers in order to reap federal funds originally allocated to public hospitals. Not only public hospitals are not responsible for their losses, but local governments are also able to appropriate their revenues. Efforts to provide financial incentives to public managers are thus jeopardized by such an expropriation.

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[4]Ellis and McGuire (1995) present empirical evidence that hospitals facing a tight prospective reimbursement select low-cost patients. Meltzer et al. (2002) show that a prospective system reduces spending on expensive patients. Cooper et al. (2012) find evidence that, in the British NHS, private providers tend to attract healthier patients, so that public hospitals treat a more costly case mix of patients when they are located in areas with more private providers.

[5]Kornai (2007), in a qualitative study, concludes that soft budget constraints are pervasive in European hospital markets. In the case of the US hospital industry, Deily et al. (2000) show that the survival of public hospitals is less tied to inefficiency measures than the survival of private hospitals. Shen and Eggleston (2009) and Eggleston and Shen (2011), find that budget constraints are softer in areas where the presence of public hospitals is small, supporting the idea that the public hospital are safety nets. Using the same natural experiment of Baicker and Staiger (2008), Duggan (2000) finds that public managers do not respond to this financial incentives as much as their private counterparts (both for-profit and not-for-profit), concluding that public managers face a soft budget constraint.

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Even if a public and a private hospital coexist in our the model, the paper does not take the approach of the mixed-oligopoly literature, which assumes different objectives for the public and the private manager and the presence of competition (see Cremer et al., 1989, De Fraja and Delbono, 1989, and Jofre-Bonet, 2000). In our model the public manager maximizes his own rent, and the price is regulated.

A number of papers deals with the public-private mix of care in national health systems. Iversen (1997), Ma (2003), and Grassi and Ma (2011, 2012) examine the relationship between public rationing and price of private hospitals. In a similar framework, Marchand and Schroyen (1997) study the redistributional implication of a mixed system. In all these contributions, the public sector is free, while patients have to pay out-of-pocket if they visit a private provider. In our model the price of the private hospital is regulated and the government is the third-party payer. Barros and Martinez-Giralt (2005) propose a model where the public payer keeps some idle public capacity in order to gain bargaining power vis-à-vis private contractors. Differently from us, they do not impose universal access, so that private hospitals can privately treat patients if negotiation fails. Furthermore, the inefficiency of the public hospital is an assumption in their model, while it is endogenous in our analysis.

In Section 2 we present the main features of the model. In Section 3, we characterize the optimal mix of public and private provision. In Section 4 we study the optimality of a no-dumping regulation. We devote Section 5 to a discussion of our assumption and results, and Section 6 to concluding remarks. All proofs of lemmas and propositions, when not specified otherwise, are collected in the appendix.

2 The model

The regulator acts as an insurer for the population and must guarantee medical treatment to all patients. The mass of patients is normalized to one and each patient needs one unit of a homogeneous treatment. Patients all get the same benefit from the treatment, but they differ in the severity of their illness, which affects the cost of the treatment they receive. Elderly patients, for instance, may require a more care, implying higher costs. The severity of illness is denoted by c, with $c \sim U(0,1)$.[6] Patients do not observe the realization of c .

Two hospitals are available on the market; one is publicly owned, the other is privately owned.

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[6]Assuming a uniform distribution simplifies the exposition. In Section 5.3 we argue that the main qualitative results are robust to alternative distributions of c.

The government acts as a third payer, so that patients pay nothing irrespective of the hospital they patronize. We make the assumption that patients perceive that the private hospital offers a higher level of quality. The private hospital may provide greater comfort and amenities, for instance. We assume that there are no search or switching costs, and that patients can costlessly change hospital if rejected. Thus, all patients visit the private hospital first .

The public and the private hospitals have the same technology. Treating a patient with severity of illness c entails for hospital i (with $i = PU, \ PR$) a cost equal to $k_i + c$, where $k_i \in \{\underline{k}, \ \bar{k}\}$ is a hospital-specific parameter. Each hospital observes both k_i and c prior to the treatment; both are private information of the hospital. However, the regulator can observe the costs of the public hospital ex post. There are no fixed costs for the treatment. Without loss of generality, we normalize $\underline{k}$ to be equal to zero.

Managers can influence the hospital-specific cost parameter through an efficiency enhancing investment that can be made before production takes place. The investment costs S to the manager, and leads to a low hospital-specific cost ($\underline{k} = 0$) with probability $e \in (0, 1)$. If no investment is made, the hospital has a high hospital-specific cost ($\bar{k}$) with probability one. We assume that S is a private cost of the manager, so that the investment can be interpreted as managerial effort, which is unobservable by the regulator. As an example, consider the management of operating rooms. Hospital managers can allocate effort in reducing excess staffing, cancellation rates and turnaround times between surgical cases, for instance. The success of these strategies depends on corporate culture, on the degree if coordination of different hospital services, and on the ease of predicting the duration of individual cases.[7]

Ex ante, the regulator cannot write complete contracts, conditional on costs. Consequently, the public manager receives a fixed wage independent of his performance. Ex post, the profits of the public hospitals are appropriated by the government, while the private manager is the residual claimant of the profits he generates.[8] Furthermore, we assume that the only instrument available for the regulator is a price per treatment p. This is in line with the tariff system (based on Diagnosis Related Groups) that is in place in many countries.

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[7]We assume that the cost of managerial effort, S, is independent on the number of patients treated or on hospital size. For instance, process innovations aimed at reducing turnaround times entail a fixed cost independent on the number of operating rooms in the hospitals. Our results are robust to an alternative specification in which S is increasing in the expected supply of the hospital, under mild assumptions on the shape of S.

[8]Thus, the costs of the public hospital are observable by the regulator ex post. This is not incompatible with the assumption that cost realizations are not contractible ex ante. Schmidt (1996) explicitly assumes that the government observes the realization of costs of the public firm, but the public manager only gets a fixed salary. Similarly, Laffont and Tirole (1991) assume that the government appropriates the returns of the investments made by the public manager but contract incompleteness makes it impossible to compensate him for the incurred cost.

Both managers maximize their monetary benefits. Thus, the private manager maximizes profits, which he can appropriate. He can influence the average cost of treatment in different ways: he can dump costly patients, which reduces the average c, he can exert effort in cost reduction, which reduces k in expectation, or both. Conversely, the public manager receives a fixed salary. Whenever profits (or losses) occur in the public hospital, they add to the public sector balance sheet.Contract incompleteness rules out any claim on profits by the public manager, and the latter has no incentive to reject patients or to exert effort.

We can thus establish a correspondence between ownership and regulatory instruments. The costs of the private hospital are not observable nor contractible upon, thus private ownership only allows for prospective payments. The public hospital is also reimbursed through a prospective payment. However, the government appropriates profits ex post, so that the public manager faces *de facto* a full cost reimbursement scheme. This interpretation will allow us to interpret our results in light of the literature on the optimal regulation of health care providers.

The timing is as follows:

1- The regulator sets a price p.

2- The private and public managers choose whether to exert effort at a private cost S.

3- The realization of k occurs. The value of k is private information of the managers.

4- Patients seek care at the private hospital. The public manager decides which patients to treat. If dumped, patients patronize the public hospital.

5- Hospitals receive a price p for each patient treated. The profits of the public hospital are appropriated by the government.

As a benchmark, consider a single hospital controlled by the regulator. The regulator minimizes expenditures under the constraint that all patients are treated, and no patients are dumped ex post. Effort is optimal if and only if:

$$(1-e)\bar{k} + \frac{1}{2} + S \leq \bar{k} + \frac{1}{2},$$

The left hand side represents total costs when effort is exerted, while the right hand side represents total cost when no effort is exerted. Effort is optimal if and only $e\bar{k} \geq S$ i.e. the expected gains in efficiency exceed the private cost. To rule out cases in which managerial effort is never optimal, we will make the following assumption:

Assumption 1 $e\bar{k} \geq S$.

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Another assumption that will hold throughout the paper is:

Assumption 2 $\bar{k} \leq 1$.

This assumption ensures that the supports of the random unitary costs $\bar{k} + c$ and c intersect, so that the dumping behavior of the private hospital is not affected too severely by the hospital-specific cost realization.[9]

We will solve the problem by backward induction analyzing the behavior of managers first, and then turning to the problem of the regulator.

2.1 Private manager's behavior

The objective of the private manager is to maximize profits. Once the regulated price is set, the manager chooses whether to exert effort. Subsequently, all patients seek care at the private hospital. Given the realization of the hospital-specific cost parameter k_{PR}, patients are treated or dumped depending on their severity of illness. In the following, we proceed by backward induction to characterize the minimal regulated price that elicits the investment in cost reduction.

In Stage 4, all patients visit the private hospital. Given the regulated price p and the realization of k_{PR}, the hospital treats a patient with severity of illness c if and only if

$$p \geq k_{PR} + c.$$

For any given price p and realization of k_{PR}, the share of patients that receive the treatment is

$$d(p, \ k_{PR}) = \begin{cases} 0 & if \ \ p \leq k_{PR} \\ p - k_{PR} & if \ \ k_{PR} < p \leq k_{PR} + 1 \\ 1 & if \ \ p > k_{PR} + 1 \end{cases} \tag{1}$$

for $k_{PR} \in \{0, \ \bar{k}\}$. We define $\bar{d}(p) \equiv d(p, \ \bar{k})$ and $\underline{d}(p) \equiv d(p, \ 0)$. Assumption 2 implies that $0 < \bar{k} \leq 1 < \bar{k} + 1$. This rules out that, for any p, all patients are treated if the cost parameter k_{PR} is low, and no patients are treated if it is high. In other words, the gain in efficiency due to the effort $(\bar{k})$ is not too high.

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[9]This assumption facilitates the exposition and does not reduce the generality of the model, as we will discussed later.

In Stage 2, the manager decides whether to exert effort or not. If no effort is exerted, the expected profit is equal to

$$\Pi_{PR}(0,\ p) = \int_0^{\bar{d}(p)} (p - \bar{k} - c)dc.$$

Conversely, the expected profit when the effort is exerted is

$$\Pi_{PR}(e,\ p) = e\left(\int_0^{\underline{d}(p)} (p - c)dc\right) + (1 - e)\left(\int_0^{\bar{d}(p)} (p - \bar{k} - c)dc\right).$$

Let us define $\Delta\Pi(p) \equiv \Pi_{PR}(e,\ p) - \Pi_{PR}(0,\ p)$, the expected return of managerial effort. The private manager exerts effort if and only if $\Delta\Pi_{PR}(p) \geq S$. The following lemma establishes an important property of the function $\Delta\Pi(p)$.

Lemma 1 *The expected return of managerial effort $\Delta\Pi(p)$ is non-decreasing in the regulated price p.*

The incentives to exert effort increase in the regulated price. Thus, the manager exerts effort for any price $p \geq \tilde{p}$, where $\tilde{p}$ is such that

$$\Delta\Pi(\tilde{p}) = S. \tag{2}$$

The following proposition characterizes the minimal price eliciting effort, $\tilde{p}$, as a function of the cost of effort S.

Proposition 1 *The private hospital exerts effort if and only if the regulated price p is greater than $\tilde{p}$, where*

$$\tilde{p} = \begin{cases} \sqrt{\frac{2S}{e}} & \in\ ]0,\ \bar{k}] & if\ \ S \leq e(\bar{k})^2/2 \\ \frac{S}{e\bar{k}} + \frac{\bar{k}}{2} & \in\ ]\bar{k},\ 1] & if\ \ e(\bar{k})^2/2 < S \leq e\bar{k}\left(1 - \bar{k}/2\right) \\ 1 + \bar{k} - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)} & \in\ ]1,\ \bar{k} + 1] & if\ \ S > e\bar{k}\left(1 - \bar{k}/2\right) \end{cases} \tag{3}$$

The minimal price eliciting effort, $\tilde{p}$, increases in S and is decreases in e and in $\bar{k}$. This is intuitive: the higher the efficiency gain from managerial effort (or the lower its cost), the lower the monetary incentives necessary to elicit effort.

2.2   Public manager's behavior

The public manager cannot appropriate profits, and his compensation consists of a fixed wage. Irrespective of the realization of the hospital-specific cost parameter, he has no incentives to reject patients. Consequently, the public hospital treats all the patients that were dumped by the private hospital. The total supply of the public hospital depends on the initial demands and on the behavior of the private hospital, and is equal to $1 - d(p, \ k_{PR})$. The average severity of illness of public patients is equal to $\left(\int_{d(p, \ k_{PR})}^{1} cdc \right) / (1 - d(p, \ k_{PR})) = (1 + d(p, \ k_{PR})) / 2$. The higher the supply of the private hospital, the higher the expected severity of illness of public patients.

In Stage 2, the manager chooses whether to exert effort in cost reduction. Since the cost realization cannot be specified in the manager contract, it cannot affect his compensation. Even if a loss occurs ex post, it is internalized by the regulator. Faced with such a soft budget constraint, the public manager never exerts any effort. The cost realization at the public hospital is always $\bar{k}$.

Ex post, the total costs incurred by the public hospital is equal to

$$\left(\bar{k} + \frac{1}{2} \right) - d(p, \ k_{PR}) \left(\bar{k} + \frac{d(p, \ k_{PR})}{2} \right)$$

The first term of this expression represents the cost of the public hospital when it treats all patients. The second term represents the reduction in costs due to the fact that $d(p, \ k_{PR})$ patients are treated in the private hospital.

3   Optimal regulation

The regulator acts as an insurer that seeks to provide one unit of treatment to the entire population at minimal cost. In Stage 1, she chooses the price p that maximizes the welfare of the patients net of health expenditures, under the constraint that all patients get the treatment. Under universal access, the benefit from medical treatment is constant and equal to v. Thus, the problem of the social planner reduces to minimize health expenditures, which correspond to the sum of the expected revenue of the

private hospital and the expected costs in the public hospitals:[10]

$$\min_{p} \quad HE(p) = \begin{cases} d(p,\ \bar{k})p + \int_{d(p,\bar{k})}^{1}(c + \bar{k}) & if \ p < \tilde{p} \\ (1 - e)\left[d(p,\ \bar{k})p + \int_{d(p,\bar{k})}^{1}(c + \bar{k})\right] + e\left[d(p,\ 0)p + \int_{d(p,0)}^{1}(c + \bar{k})\right] & if \ p \geq \tilde{p}. \end{cases} \quad (4)$$

Importantly, the regulator does not take into account the perception of the patients about the relative quality of the public and the private hospitals. The private hospital may offer greater comfort to patients, but this is not an important dimension for the regulator.[11]

We consider first two benchmark scenarios. In the first one (PR), only a private hospital is available. In the second one (PU), only a public hospital is available. We then turn to the MIX scenario, in which the regulator can contract with both a public and a private hospital.

3.1 Government contracting with a single hospital

In the scenario PR the regulator has to provide the service to all patients and is constrained to contract with a private hospital. Ex post, the private hospital treats all patients if and only if $p \geq k_{PR} + 1$. Since the only instrument is the price, the regulator has to choose a price ex ante that is sufficient to cover costs in any state of the world. Thus, the optimal price, coinciding with the total health expenditures is equal to $p^{PR} = \bar{k} + 1$. As long as $e\bar{k} \geq S$, the manager has an incentive to exert effort. However, the returns to effort are completely captured by the manager.

In the scenario PU in which the regulator is constrained to contract with a public hospital, which treats all patients irrespective of the price. Total health expenditures equal the average cost $\bar{k} + 1/2$. Comparing this result with the previous one leads to the following proposition:

Proposition 2 *If the regulator is constrained to treat all patients and can rely on one hospital only, contracting with a public hospital dominates contracting with a private hospital.*

Proof. Straightforward ∎

The price has to cover the cost of treating all patients in any state of the world. It is not possible to threaten the private hospital with the prospect of smaller quantities if reported costs turn out to be

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[10]We assume that the profit of the private provider is not internalized by the regulator. Since the problem of the social planner is to minimize health expenditure, the cost of public funds can be ignored.

[11]This assumption is made in order to focus on the trade-off between efficiency and access, leaving out quality issues. If quality was taken into account, this would enhance the case for the private hospital.

high. Even if renegotiation was possible, ex post the private hospital would always claim high costs and get the highest possible price per patient. The price p^{PR} is thus renegotiation proof. Even the private hospital is in a situation where it faces a soft budget constraint.[12] The regulator prefers to contract with the public hospital, which entails health expenditures equal to the average cost. Quite naturally, the possibility to dump is a negative attribute of a hospital when the regulator needs to treat all patients.

This result is different from the ambiguous one of Schmidt (1996). In his model, the regulator can observe the cost realization in the public firm and choose the optimal output accordingly (the public firm is ex post efficient). Thus, the public manager enjoys no informational rent and has no incentive to exert effort in cost reduction. Conversely, the private manager exert effort (the private firm is ex ante efficient), but the output is distorted because of informational asymmetries. The comparison between public and private provision depends on the relative importance of ex ante and ex post efficiency. Also in our model the private manager exerts effort in cost reduction, but this does not lead to a reduction in health expenditures, because quantities are constrained and the private manager has always an incentive to report high costs.

3.2 Government contracting with a public and a private hospital

In this section, we consider the MIX scenario, in which the regulator can contract with both a public and a private hospital. Problem (4) can be rewritten as:

$$\min_{p} HE(p) = \begin{cases} \bar{k} + \frac{1}{2} + \bar{d}(p)\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right) & if \; p < \tilde{p} \\ \bar{k} + \frac{1}{2} + \left[e\underline{d}(p)\left(p - \bar{k} - \frac{\underline{d}(p)}{2}\right) + (1-e)\bar{d}(p)\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right)\right] & if \; p \geq \tilde{p}. \end{cases}$$

In the expression above, $\bar{k} + 1/2$ represents the expected health expenditures if the public hospital exclusively provides the service. The residual term in each row is the difference between the expected payment to the private hospital and the cost of treating private patients in the public hospital. Since the objective of the regulator is to minimize expenditures, it is optimal to contract with the private hospital if and only if this term has negative sign.

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[12]Kornai (2009), discussing soft budget constraints in the hospital market argues that "no institution in a monopoly position can be abolished however poor its economic performance may be" (p. 131).

We denote the optimal price by p^{MIX}. We will first establish a property.

Lemma 2 *Any price $p < \tilde{p}$ is dominated by $p = 0$, where $\tilde{p}$ is the minimal price eliciting effort defined in (3).*

There is no reason to contract with the private hospital (i.e. to offer a positive price) unless it is more efficient than the public one in expectation.

If the price is equal to zero only the public hospital provides the services, and $HE(0) = \bar{k} + 1/2$. A direct consequence of Lemma 2 is that the optimal price is positive if and only if

$$\bar{k} + \frac{1}{2} > \min_{p \geq \tilde{p}} \ HE(p) = \min_{p \geq \tilde{p}} \ \bar{k} + \frac{1}{2} + \left[e\underline{d}\left(p - \bar{k} - \frac{\underline{d}(p)}{2} \right) + (1-e)\bar{d}\left(p - \bar{k} - \frac{\bar{d}(p)}{2} \right) \right].$$

The following proposition characterizes the optimal regulated price.

Proposition 3 *If the regulator is constrained to treat all patients and can use both a public and a private hospital, the expenditure minimizing price depends on the efficiency of the managerial effort. Then,*

(i) if $S \leq e(\bar{k})^2/2$, then $p^{MIX} = \bar{k}$;

(ii) if $e(\bar{k})^2/2 < S \leq \bar{S}$, then $p^{MIX} = \tilde{p}$, where $\tilde{p}$ is the minimal price eliciting high effort defined in (3);

(iii) if $S > \bar{S}$, then $p^{MIX} = 0$,

where $\bar{S} \equiv min\left\{ e(\bar{k})^2\left(\sqrt{e} + 1/2\right), \ e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right) \right\}$.

To understand this result, first note that it is impossible to obtain cost reductions from the last resort hospital (quantities cannot be distorted). Consequently, it is always optimal to use the public hospital as a last resort: at the very least it does not dump patients. Moreover, contracting with the private hospital is optimal only if the price elicits managerial effort, i.e. is greater than $\tilde{p}$. If the effort in cost reduction is relatively efficient, eliciting effort is relatively cheap, and it is optimal to contract with the private provider. Moreover, in our setting, the regulator can benefit from the efficiency of the private hospital only because she can rely on a public provider. If not, under universal access, she would have to set a price ruling out dumping $(\bar{k} + 1)$, as we show in Section 3.1.[13]

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[13]As in Barro and Martinez-Giralt (2005), the presence of public capacity enhances the bargaining power of the regulator. In their model, public capacity affects the fallback value of private providers while here it affects the one of the regulator.

If the cost of effort is very low with respect to its benefit (i), the price offered to the private hospital is equal to $\bar{k}$. The private hospital treats some patients only if the hospital-specific cost realization is low. If costs turn out to be high, all patients are treated by the public hospital. Thus, the private hospital only produces if it is more efficient than the public one, which is efficient ex post. In this range of parameters $\tilde{p} \leq \bar{k}$, so that p^{MIX} elicits effort.

For an intermediate efficiency of managerial effort (ii), the regulator still finds it optimal to contract with the private hospital, but the minimal price eliciting effort is higher than $\bar{k}$. The private hospital earns a positive profit even if its costs turn out to be high. This is suboptimal ex post, since exclusive public provision would be cheaper. However, ex ante, the expected gains in efficiency justify the extra rent awarded to the private manager.

Finally, if the effort in cost reduction is very costly with respect to its benefit (iii) eliciting effort is too costly for the regulator. The optimal price is equal to zero.

Proposition 3 suggests that the regulator should contract with private hospitals for services involving high margins for cost reduction. This might be the case of elective surgery, where the hospital managers can engage in relatively cheap and reliable efficiency-enhancing activity, such as operating rooms management. For treatments involving high effort and low efficiency gains, production should be concentrated in public facilities. Emergency services are good candidates for exclusive public provision: it is impossible to plan operations in advance, and processes are hard to standardize.

Our result can be interpreted in terms of prospective versus retrospective payments. If efficiency gains are low, there is no reason to contract with a private hospital, and the system is purely public. If efficiency gains are higher, the optimal system is mixed, including a public hospital ultimately facing a pure cost reimbursement scheme and a private hospital receiving a prospective payment. This is in line with the literature on the optimal reimbursement of health providers engaging in dumping.[14] Thus, mixed delivery is a way to trade-off efficiency and access when the costs of private providers are not verifiable ex post and mixed reimbursements are not feasible .

Note that Assumption 2 plays a role in the results. If $\bar{k}$ was bigger than one, this would enhance the role of the private hospital. By setting a price in the interval $]1, \bar{k}]$, the regulator could elicit effort without any inefficiency ex post: the private hospital would only produce if $k_{PR} = 0$.

The model predicts that expected health expenditures increase in S. Conversely, the regulated price exhibits a non monotonic relationship with the cost of managerial effort. If S is low, the price is

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[14]For a discussion, see Ma, 1994, Section 4.

constant and equal to $\bar{k}$. For intermediate values of S, eliciting effort is still optimal, but the regulated price has to be higher than $\bar{k}$ and increasing in S. Finally, for very high levels of S, the optimal price equals zero and the private hospital is excluded from production. Also the expected profit of the private hospital is non monotonic in S. If S is small, the expected profit is constant. It increases for intermediate values of S, since the price increases in S. Finally, if the cost of managerial effort is very high, the expected profit is equal to zero. The expected amount of services provided follows a similar path.

4   No-dumping regulation.

Our previous results depend on the hypothesis that the private hospital can freely dump the patients whose cost exceeds the regulated price. We now turn to the case when the private hospital complies with a no-dumping regulation, stating that the private hospital can reject patients only if treating them translates into negative profits.[15] The timing is the same as above. In period one, the regulator can also decide whether to enforce or not a no-dumping regulation. Under a no-dumping regulation, the private manager has to report the hospital-specific cost realization (in Stage 3). Depending on the reported costs, the hospital has to treat the maximal amount of patients, under a non-negative-profit constraint. In many health care systems, this might be a relevant setting. The price (the DRG reimbursement) is often set ex ante by law. Ex post, the government may contract on quantities with private hospitals, and enforce a no-dumping regulation.[16] We show that a no-dumping regulation is not always in the best interest of the regulator: by reducing the rent of the private manager, a no-dumping regulation makes it difficult to elicit effort and truthful revelation from the private manager.

4.1   Private hospital's behavior when dumping is banned

In this section, we assume that the regulator can contract with both hospitals and can oblige the private hospital not to dump. We will call this regime MND.

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[15]We thus impose an ex post participation constraint, assuming that the private firm cannot be forced into negative profits. If the relevant participation constraint was ex ante, we would fall under the benchmark case of Section 2. The hospital would bear the risk of having high costs, and any price would elicit effort. A price $p = \bar{k}(1-e)+1/2+S < \bar{k}+1/2$ would minimize health expenditures.

[16]In Italy, for instance, the DRG reimbursement is set by law. Regions contract with private hospitals on the maximum amount of services to be provided, given this reimbursement. See Francese and Romanelli (2010).

All patients visit the private hospital first. The hospital-specific cost parameter k_{PR} is private information of the hospital. Once the realization of k_{PR} occurs, the private hospital reports k', and has to treat the maximum amount of patients under the constraint of non-negative profits.

For any price p offered to the private hospital and for any reported k', the private hospital has to treat $\hat{d}$ patients, where $\hat{d}$ satisfies:

$$\hat{d}(p,\,k') = \arg\max_{d}\ d$$
$$s.t.\ \int_0^d (p - k' - c)dc \geq 0.$$

Thus,

$$\hat{d}(p,\,k') = \begin{cases} 0 & if\ p \leq k' \\ 2(p - k') & if\ k' < p \leq k' + \frac{1}{2} \\ 1 & if\ p > k' + \frac{1}{2} \end{cases}$$

Ex post, the profit of the private manager is thus equal to:

$$\int_0^{\hat{d}(p,k')} (p - k_{PR} - c)\,dc,$$

where k' and k_{PR} are the reported cost parameter and the real cost parameter. The manager of a low-cost private hospital might have an incentive to mimic the high-cost type. On the one hand, truthfully reporting $k = 0$ permits to treat more patients, and get a higher revenue. On the other hand, mimicking the high cost type permits the hospital to get rid of patients with a high severity of illness and reduces total costs. The behavior of the manager depends on which of these effects dominates. The following proposition characterizes the behavior of the manager depending on the regulated price.

Proposition 4 *Consider the case when the private hospital is forbidden to dump.*

If $\bar{k} \leq 1/2$, the private manager exerts effort in cost reduction and truthfully reports the cost realization if and only if $p \geq \bar{k} + 1/2$.

If $\bar{k} > 1/2$, any price induces the truthful revelation of the hospital-specific cost parameter. For any price $p \geq \hat{p} = 1/2 + S/e$, the manager exerts effort.

The intuition for this result is very simple. If the private manager cannot freely dump patients, he might report a high hospital-specific cost parameter in order to be allowed to reject patients with

a high severity of illness. If the efficiency differential is low ($\bar{k} \leq 1/2$), $\hat{d}(p,\ 0) - \hat{d}(p,\ \bar{k})$ is quite low. Thus, if a manager mimics the high cost hospital, the loss in revenues is not too high. At the same time, the hospital can get rid of some high cost patients and reduce its total costs. For any $p < \bar{k}+1/2$, it can be shown that the reduction in costs exceeds the loss in revenues. Thus, mimicking is a dominant strategy. If $p \geq \bar{k} + 1/2$, there is no gain from mimicking, since $\hat{d}(p,\ 0) = \hat{d}(p,\ \bar{k}) = 1$.

If the efficiency differential is high ($\bar{k} > 1/2$), mimicking the high cost type is never profitable for the private manager, because it entails a loss in revenues larger than the reduction in total costs. The minimal price eliciting effort is always greater than $1/2$ and smaller than $\bar{k} + 1/2$, under Assumption 1.

4.2 Government contracting with a public and a private hospital when dumping is banned

We can use Proposition 4 to characterize the optimal price under a no-dumping regulation (p^{MND}). Consider first the case when the potential gain of managerial effort is low ($\bar{k} \leq 1/2$). If the price is higher than $\bar{k} + 1/2$, the private hospital treats all patients irrespective of the reported costs. Such a price is greater than the average expenditure in the public hospital and is clearly suboptimal. If the price is lower than $\bar{k} + 1/2$, the private manager always reports high costs. High severity patients are treated by the public hospital. The resulting health expenditures, $\bar{k} + 1/2$, could be obtained by contracting with the public hospital alone.

Consider now the case when $\bar{k} > 1/2$. If $p < \hat{p}$, no effort is exerted by the private manager, which behaves exactly as his public counterpart. Expected health expenditures are equal to $\bar{k} + 1/2$. If $p \geq \hat{p}$, the private manager exerts effort and treats all patients is the hospital-specific cost parameter is equal to zero. Health expenditures are given by $HE^{MND}(p > \hat{p}) = ep + (1 - e)\left(\bar{k} + 1/2\right)$. The expenditure minimizing price is thus $p^{MND} = \hat{p} = 1/2 + S/e$, leading to expected health expenditures equal to $\bar{k}(1 - e) + 1/2 + S$, which is smaller than $\bar{k} + 1/2$ under Assumption 1.

Summarizing, under a no-dumping regulation, it is impossible to reduce health expenditures by contracting with the private hospital if $\bar{k} \leq 1/2$. Conversely, if $\bar{k} > 1/2$, health expenditures can be reduced by offering to the private hospital a price $p^{MND} = \hat{p}$. Comparing the cost of provision when dumping is allowed and when a no-dumping regulation is enforced, it is possible to establish the following proposition.

Proposition 5 *Assume that the regulator can costlessly impose a no-dumping constraint on the private hospital.*

If $\bar{k} \leq 1/2$ and $S \leq \bar{S}$, it is optimal for the regulator not to enforce the constraint and to set $p = p^{MIX}$.

If $\bar{k} > 1/2$, it is optimal for the regulator to enforce the constraint and to set $p = p^{MND}$.

In all other cases, the regulator does not contract with the private hospital.

The regulator faces a trade-off between efficiency and access implying that a no-dumping regulation is not always optimal. For low values of $\bar{k}$, it is indeed optimal to let the private hospital dump patients. The no-dumping regulation implies that costs are over reported for any price smaller than $\bar{k} + 1/2$. The regulator is not able to appropriate the benefit of managerial effort. In such a case, allowing the private hospital to dump reduces expected health expenditures and the public hospital ensures that all patients are treated.

When $\bar{k}$ is larger, the private manager always reveals cost truthfully and the price $p^{MND} = \hat{p} > 1/2$ elicits effort. The private hospital treats all patients in case of low cost realizations, and its profit is equal to $\hat{p} - 1/2 > 0$. Eliciting managerial effort is not too costly for the regulator because the benefit of effort is high and internalized by the hospital through a positive profit. In this case, the model predicts that production should be concentrated in the private hospital if $k_{PR} = 0$. If the cost parameter is high, the private hospital may still serve a positive share of patients.

5   Discussion

5.1   Demand structure

In our model, patients differ in the severity of illness, but get the same benefit from the treatment. Under universal access, relaxing the assumption of uniform benefits does not affect the regulator's problem, as long as the benefit does not depend on the type of hospital patronized.

For sake of simplicity, we assumed that all patients prefer the private hospital on some dimensions not internalized by the regulator (amenities). However all our results would hold if the regulator endogenously set the quality of the public hospital. It is possible to show that the optimal quality of the public hospital should be slightly lower than the one of the private hospital. A small difference in

quality does not harm public patients too much, while pushing all patients to visit the private provider first. Intuitively, if it is optimal to contract with the private hospital, then *all* low-cost patients should be treated in the private hospital (expenditures for these patients are lower than in the public hospital). Thus, having all patients visit the private hospital first, and the private hospital dump high-cost ones actually ensures the optimal sorting of patients across hospitals.

An alternative way to model demand would be to introduce horizontal differentiation. In absence of quality competition, some patients would patronize the public hospital first. This would limit the role of the private hospital, since sorting would not be optimal any more. However, most of our qualitative results would be preserved.

5.2   More than two hospitals

In our two-hospitals setting, we have shown that, for a private hospital to accept to act as a last resort, its price must be set at the highest possible cost realization. Thus, the public hospital provides a safety net at a lower cost for the regulator. Consequently, a mixed system with one public and one private hospital dominates a pure private provision system with two private hospitals. Even if more than one private hospital is available, the last resort provider should be public.

We do not consider capacity constraints. In fact, contracting with private providers is often justified by the need to alleviate congestion problems in the public sector. However, the model could easily be extended to include capacity constraints by allowing for the presence of more than two hospitals. The regulator may choose which hospitals to contract with. This analysis is left for further research.

5.3   Alternative cost distributions

We assume that the severity of illness is uniformly distributed, which is unlikely in reality. If we relax this hypothesis the main results and intuitions are preserved. Assume that $c \in [0,1]$ is a stochastic variable with CDF $F(c)$. If $F(c)$ first order stochastically dominates the uniform distribution $U(c)$, then $F(c)$ is skewed towards the right, and the minimal price eliciting effort is greater under $F(c)$ then under $U(c)$.[17] In other words, if high severities of illness are relatively more common under $F(c)$, a greater fraction of patients gets dumped ex post by the private hospital, and it is harder to provide

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[17]This result follows form observing that, irrespective of the shape of $F(c)$, the private hospital treats a patient with cost c only if $c < p - k_{PR}$. Denoting by $\Delta\Pi^F(p)$ the difference between the private hospital expected profits with and without investment in cost reduction, after some manipulations one obtains $\Delta\Pi^F(p) = \int_{\underline{d}(p)}^{\overline{d}(p)} F(c)dc + \phi(p)$, where $\phi(p)$ is a term independent from the shape of $F(c)$. From the definition of first order stochastic dominance, $\Delta\Pi^F(p) \leq \Delta\Pi^U(p)$ for any level of p.

incentives to the private manager. Conversely, if $U(c)$ first order stochastically dominates $F(c)$, then it is easier to elicit effort from the private manager.

6   Conclusion

We provide conditions under which it is optimal to contract with both a public and a private hospital for the production of health services. We show that a mixed delivery system is optimal whenever a very efficient managerial effort in cost reduction is available. If not, only the public hospital should produce, despite its lower cost efficiency. Our result can be of guidance to policymakers in countries with a national health system: procurement to private hospitals should be considered for procedures whose costs are more easily affected by managerial effort (for instance, elective surgery).

Another finding of the paper is that imposing a no-dumping constraint on the private hospital is not always beneficial for the regulator, because it makes it more difficult to elicit effort and truthful revelation of the hospital-specific cost parameter. This result is somehow counter intuitive, but is economically justified by the presence of asymmetric information on costs leading to a trade-off between efficiency and access to care.

The analysis focuses on cost efficiency and access, leaving out quality competition. Endogenizing the quality of the public hospital does not change our results. However, the paper leaves open all questions related to the private hospital's quality choice, and the interaction of such choice with cost efficiency. Another important limitation is the absence of switching costs. Patients rejected by the private hospital might face negative consequences due to waiting times. Intuitively, in presence of switching costs contracting with the private hospital would be less attractive, and a no-dumping regulation more appealing. The model applies to cases where delays in treatment are not prohibitively costly.

An interesting extension would be to consider the role of physicians. González (2005) and Brekke and Sørgard (2007) study physicians' dual practice and its effect on public hospitals. Wright (2007) argues that using a salary to remunerate physicians in public hospitals might be motivated by the need to attract more fairness prone physicians in the public sector. Physicians' behavior is left for further investigation.

Public hospitals may have better incentives to keep reserve capacity to meet peak demands (Rodriguex-Alvarez et al., 2011). Our model could also be extended to study the optimal public/private hospital mix when the demand for health care services is stochastic.

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Appendix. Proofs of Lemmas and Propositions

Proof of Lemma 1

Since the supply of the private hospital is discontinuous in p, to prove the lemma we will analyze in turn all the possible cases that might arise.

- If $p \in \left]0,\ \bar{k}\right]$, some patients are treated if managerial effort is successful in reducing k. Otherwise, all patients are dumped. Then

$$\Delta\Pi(p) = \frac{ep^2}{2},$$

  which is increasing in p. Moreover, $\Delta\Pi(\bar{k}) = e(\bar{k})^2/2$.

- If $p \in \left]\bar{k},\ 1\right]$, the private hospital treats a positive share of patients whichever the realisation of k. Then

$$\Delta\Pi(p) = e\left(\frac{p^2}{2} - \frac{(p-\bar{k})^2}{2}\right),$$

  which is increasing in p . Moreover, $\lim_{p \to \bar{k}} \Delta\Pi(p) = e(\bar{k})^2/2$, and $\Delta\Pi(1) = e\left(\bar{k} - (\bar{k})^2/2\right)$.

- If $p \in \left]1,\ \bar{k}+1\right]$, the private hospital treats all patients if the hospital-specific cost parameter is equal to zero, and a positive share of patients if this parameter is equal to $\bar{k}$. Then,

$$\Delta\Pi(p) = e\left(p - \frac{1}{2} - \frac{(p-\bar{k})^2}{2}\right),$$

  which is increasing in p . Moreover, $\lim_{p \to 1} \Delta\Pi(p) = e\left(\bar{k} - (\bar{k})^2/2\right)$ and $\Delta\Pi(\bar{k}+1) = e\bar{k}$.

- Finally, if $p > \bar{k}+1$, the private hospital treats all the patients whichever the realisation of k. Then

$$\Delta\Pi(p) = e\bar{k},$$

  which is constant in p. Moreover $\lim_{p \to \bar{k}+1} \Delta\Pi(\bar{k}+1) = e\bar{k}$.

We have shown that the function $\Delta\Pi(p)$ is piecewise-defined and continuous in p. Furthermore, it is non-decreasing in any interval in which it is piecewise-defined. Thus, the function is monotonically non-decreasing.∎

Proof of Proposition 1

- Consider first the case in which $S < e(\bar{k})^2/2$. From the preceding proof we know that $\Delta\Pi(p) > S$ for any $p > \bar{k}$. Using the monotonicity of $\Delta\Pi(p)$, we can conclude that $\tilde{p} \in \left]0, \bar{k}\right]$. The value of $\tilde{p}$ is given by

$$e\frac{(\tilde{p})^2}{2} = S \iff \tilde{p} = \sqrt{\frac{2S}{e}}.$$

- Consider now the case in which $e(\bar{k})^2 \leq S < e\bar{k}\left(1 - \bar{k}/2\right)$. From the preceding proof, we know that $\Delta\Pi(p) < S$ for any $p < \bar{k}$ and that $\Delta\Pi(p) > S$ for any $p > 1$. Using the monotonicity of $\Delta\Pi(p)$, we can conclude that $\tilde{p} \in \left]\bar{k}, 1\right]$. The value of $\tilde{p}$ is given by

$$e\left(\frac{(\tilde{p})^2}{2} - \frac{(\tilde{p} - \bar{k})^2}{2}\right) = S \iff \tilde{p} = \frac{S}{e\bar{k}} + \frac{\bar{k}}{2}.$$

- Consider finally the case in which $S > \bar{e}\Delta k\left(1 - \Delta k/2\right)$. From the preceding proof, we know that $\Delta\Pi(p) < S$ for any $p < 1$ and $\Delta\Pi(p) > S$ for any $p > \bar{k} + 1$. Using the monotonicity of $\Delta\Pi(p)$, we can conclude that $\tilde{p} \in \left]1, \bar{k} + 1\right]$. The value of $\tilde{p}$ is given by

$$e\left(\tilde{p} - \frac{1}{2} - \frac{(\tilde{p} - \bar{k})^2}{2}\right) = S \iff \tilde{p} = 1 + \bar{k} - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)}.\blacksquare$$

Proof of Lemma 2

If $p = 0$ then only the public hospital produces. Then, $HE(0) = \bar{k} + 1/2$. Consider now a price $p' < \tilde{p}$. The private hospital does not exert any effort at this price. Expected health expenditures $HE(p')$ are then equal to $\bar{k} + 1/2 + (p - \bar{k})\bar{d}(p') + (\bar{d}(p'))^2/2$. Since $\bar{d}(p') \geq 0$, $HE(p') \geq HE(0)$ for any $p' < \tilde{p}.\blacksquare$

Proof of Proposition 3

If $S \leq \bar{e}(\bar{k})^2/2$ we know from Proposition 1 that $\tilde{p} \in \ ]0, \ \bar{k}]$. Expected health expenditures have the form

$$
\begin{cases}
\bar{k} + \frac{1}{2} + ep \left(\frac{p}{2} - \bar{k}\right) & if \ p \in \ ]\tilde{p}, \ \bar{k}] \\[2mm]
\bar{k} + \frac{1}{2} + \frac{(p-\bar{k})^2}{2} - e\frac{(\bar{k})^2}{2} & if \ p \in \ ]\bar{k}, \ 1] \\[2mm]
\bar{k} + \frac{1}{2} + e \left(p - \bar{k} - \frac{1}{2}\right) + (1-e)\frac{(p-\bar{k})^2}{2} & if \ p \in \ ]1, \ \bar{k}+1] \\[2mm]
p & if \ p > \bar{k}+1
\end{cases}
$$

It is easy to show that the function is decreasing in p for any $p \leq \bar{k}$, and increasing for any $p > \bar{k}$. Consequently, if $S \leq e(\bar{k})^2/2$, the price that minimizes expenditures conditional on eliciting effort is equal to $\bar{k}$. Such a price implies that a positive amount of patients is treated in the private hospital if the hospital-specific cost realisation is equal to zero. Otherwise, all patients are treated by the public hospital and the private hospital makes zero profits. Expected expenditures are equal to $\bar{k} + 1/2 - e(\bar{k})^2/2$, which is smaller than $HE(0)$. Thus, we can conclude that $p^{MIX} = \bar{k}$ whenever $S \leq e(\bar{k})^2/2$.

If $e(\bar{k})^2/2 < S \leq e\bar{k}\left(1 - \bar{k}/2\right)$, we know from Proposition 1 that $\tilde{p} \in \ ]\bar{k}, \ 1]$. Expected health expenditures have the form

$$
\begin{cases}
\bar{k} + \frac{1}{2} + \frac{(p-\bar{k})^2}{2} - e\frac{(\bar{k})^2}{2} & if \ p \in \ ]\tilde{p}, \ 1] \\[2mm]
\bar{k} + \frac{1}{2} + e \left(p - \bar{k} - \frac{1}{2}\right) + (1-e)\frac{(p-\bar{k})^2}{2} & if \ p \in \ ]1, \ \bar{k}+1] \\[2mm]
p & if \ p > \bar{k}+1
\end{cases}
$$

It is easy to show that this function is increasing for any $p \geq \tilde{p}$. Consequently, if $e(\bar{k})^2/2 < S \leq e\bar{k}\left(1 - \bar{k}/2\right)$, the price that minimize expenditures conditional on eliciting effort is equal to $\tilde{p} = S(e\bar{k}) - \bar{k}/2 + \bar{k}$. The private hospital treats a positive amount of patients for any cost realisation. Expected health expenditures are equal to $\bar{k} + 1/2 + \left(S - \bar{k}/2\right)^2 - e(\bar{k})^2/2$. They are smaller than $HE(0)$ if and only if

$$
\left(\frac{S}{e\bar{k}} - \frac{\bar{k}}{2}\right)^2 - e(\bar{k})^2 \leq 0 \iff S \leq e(\bar{k})^2 \left(\sqrt{e} + \frac{1}{2}\right).
$$

In conclusion, if $e(\bar{k})^2/2 < S \leq min\left\{e\bar{k}\left(1 - \bar{k}/2\right), \ e(\bar{k})^2\left(\sqrt{e} + 1/2\right)\right\}$, then the optimal price is the minimal price eliciting managerial effort, i.e. $p^{MIX} = \tilde{p}$.

If $S > e\bar{k}\left(1 - \bar{k}/2\right)$, we know from Proposition 1 that $\tilde{p} \in \ ]1, \ \bar{k}+1]$. Expected health expenditures

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have the form

$$\begin{cases} \bar{k} + \frac{1}{2} + e\left(p - \bar{k} - \frac{1}{2}\right) + (1 - e)\frac{(p-\bar{k})^2}{2} & if\ p \in \left]\tilde{p},\ \bar{k}+1\right] \\[2ex] p & if\ p > \bar{k}+1 \end{cases}$$

It is easy to show that this function is increasing for any $p \geq \tilde{p}$. Consequently, if $S > e\bar{k}\left(1 - \bar{k}/2\right)$, the price that minimize expenditures conditional on eliciting effort is equal to $\tilde{p} = 1 + \bar{k} - \sqrt{2\left(\bar{k} - S/e\right)}$. The private hospital treats all the patients if the cost realisation is low, and a positive share of patients if the cost realisation is high. Expected health expenditures are equal to

$$\bar{k} + \frac{1}{2} + \left[e\left(\frac{1}{2} - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)}\right) + (1-e)\frac{1}{2}\left(1 - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)}\right)^2\right].$$

They are smaller than $HE(0)$ if and only if

$$e\left(\frac{1}{2} - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)}\right) + (1-e)\frac{1}{2}\left(1 - \sqrt{2\left(\bar{k} - \frac{S}{e}\right)}\right)^2 \leq 0$$

$$\iff S \leq e\left(\bar{k} - \frac{(1 + \sqrt{e})^{-2}}{2}\right).$$

In conclusion, if $e\bar{k}\left(1 - \bar{k}/2\right) < S \leq e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right)$, then the optimal price is the minimal price eliciting managerial effort, i.e. $p^{MIX} = \tilde{p}$.

We have shown that

$$e(\bar{k})^2/2 \quad < \quad S \leq min\left\{e\bar{k}\left(1 - \frac{\bar{k}}{2}\right),\ e(\bar{k})^2\left(\sqrt{e} + \frac{1}{2}\right)\right\} \implies p^{MIX} = \tilde{p} \tag{5}$$

and

$$e\bar{k}\left(1 - \bar{k}/2\right) < S \leq e\left(\bar{k} - \frac{(1 + \sqrt{e})^{-2}}{2}\right) \implies p^{MIX} = \tilde{p} \tag{6}$$

Remark that $e(\bar{k})^2\left(\sqrt{e} + 1/2\right) \leq e\bar{k}\left(1 - \bar{k}/2\right) \iff \bar{k} \leq 1/(\sqrt{e}+1)$. In this case, $e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right) \leq e\bar{k}\left(1 - \bar{k}/2\right)$, and condition (6) cannot be satisfied. Thus, $p^{MIX} = \tilde{p}$ if and only if $e(\bar{k})^2/2 < S \leq e(\bar{k})^2\left(\sqrt{e} + 1/2\right)$, and $p^{MIX} = 0$ for all $S > e(\bar{k})^2\left(\sqrt{e} + 1/2\right)$.

Conversely, $e(\bar{k})^2\left(\sqrt{e} + 1/2\right) > e\bar{k}\left(1 - \bar{k}/2\right) \iff \bar{k} > 1/(\sqrt{e}+1)$. In this case, $e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right) > e\bar{k}\left(1 - \bar{k}/2\right)$ and condition (6) is satisfied. Then, $p^{MIX} = \tilde{p}$ if and only if $e(\bar{k})^2/2 < S \leq e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right)$, and $p^{MIX} = 0$ for all $S > e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right)$.

Summarising $p^{MIX} = \tilde{p}$ if and only if $e(\bar{k})^2/2 < S \leq min\left\{e(\bar{k})^2\left(\sqrt{e} + 1/2\right),\ e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right)\right\}$. ∎

Proof of Proposition 4

Consider an hospital whose cost realisation is $\underline{k}$.

Whenever $p < 1/2$, the profit of the hospital is equal to zero whenever the manager truthfully reports the low cost parameter. If $p \geq \bar{k}$, mimicking the high cost type leads to a positive production and a positive profits, and the manager has an incentive to misreport. If $p < \bar{k}$, mimicking the high cost type leads to a profit equal to zero, and the manager has no incentive to misreport. However, the expected profits are equal to zero, and the manager has no incentive to exert effort in the first place.

Whenever $p > 1/2$, different cases might arise depending on the level of $\bar{k}$. We consider separately the case when $\bar{k} \leq 1/2$ and the case when $\bar{k} > 1/2$

- If $\bar{k} \leq 1/2$, then $0 < \bar{k} \leq 1/2 < \bar{k} + 1/2$.

  - If $p \in \left]1/2, \bar{k} + 1/2\right]$, whenever the manager truthfully reports $k' = 0$, he gets a profit equal to $p - 1/2$; by mimicking the high cost type, the manager gets a profit equal to $2\bar{k}(p - \bar{k})$. Truthfully reporting the cost parameter is thus incentive compatible if and only if

  $$p - \frac{1}{2} - 2\bar{k}(p - \bar{k}) \geq 0,$$

  which is impossible if $p \in \left]1/2, \bar{k} + 1/2\right]$ and $\bar{k} \leq 1/2$. Thus, mimicking the high cost type is the best strategy for a low cost hospital.

  - If $p > \bar{k} + 1/2$, the hospital treats all patients irrespective of the reported cost parameter. There is no incentive to misreport. Ex ante, the manger exerts high effort if and only if

  $$p - (1 - e)\bar{k} - \frac{1}{2} - S \geq p - \bar{k} - \frac{1}{2} \iff S \leq e\bar{k},$$

  which is always true under Assumption 1.

- If $\bar{k} > 1/2$, then $1/2 < \bar{k} < \bar{k} + 1/2$.

  - If $p \in \left]1/2, \bar{k}\right]$ the private hospital does not treat any patient and profits are equal to zero whenever the manager reports a cost parameter $\bar{k}$. If the realisation of the cost parameter is $\underline{k}$ and the manager truthfully reports costs, all the patients have to be treated and profits are equal to $p - 1/2 \geq 0$. The manager has no incentives to misreports costs. Ex ante, the investment in

cost reduction is made if and only if

$$e\left(p - \frac{1}{2}\right) \geq S \iff p \geq \hat{p} = \frac{1}{2} + \frac{S}{e}.$$

- If $p \in \,]\bar{k},\, \bar{k} + 1/2]$, profits are equal to zero if the cost parameter turns out to be $\bar{k}$. In case of low costs, if the manager truthfully reports the cost realisation and profits are equal to $(p - 1/2)$. If the manager mimics the high cost type, the profit is equal to $2\bar{k}(p - \bar{k})$. Truthfully reporting the cost parameter is thus incentive compatible if and only if the difference between these two profits is positive:

$$p - \frac{1}{2} - 2\bar{k}(p - \bar{k}) \geq 0.$$

which is always true if $\bar{k} > 1/2$. Ex ante, the hospital exerts effort if and only if

$$e\left(p - \frac{1}{2}\right) \geq S \iff p \geq \hat{p} = \frac{1}{2} + \frac{S}{e}.$$

The minimal price necessary to exert effort is lower than $\bar{k} + 1/2$ for any $S \leq e\bar{k}$.

- Finally, if $p > \bar{k} + 1/2$ the private hospital will always treat all patients. Thus, the private manager does not have any incentive to misreport the realisation of costs. Ex ante, the hospital exerts effort if and only if

$$p - (1 - e)\bar{k} - \frac{1}{2} - S \geq p - \frac{1}{2} \iff S \leq e\bar{k},$$

which is always the case under assumption 1. Thus, the minimal price eliciting effort is always smaller than $\bar{k} + 1/2$.

Summarising, the private manager always reports truthfully. He exerts effort for any price greater or equal to $\hat{p} = 1/2 + S/e$. ∎

Proof of Proposition 6

To prove the proposition, we have to compare , $HE(p^{MIX})$ and $HE^{MND}(p^{MND})$.

• If $\bar{k} \leq 1/2$, we know from Proposition 5 that $p^{MND} = 0$ and expected health expenditures if the

no-dumping constraint is enforced are equal to $\bar{k} + 1/2$. Thus, letting the hospital free to reject patients is optimal whenever $p^{MIX} > 0$ This is true for any $S \leq \bar{S}$.

- If $\bar{k} > 1/2$ and the no-dumping constraint is enforced, expected health expenditure are equal to $HE^{MND}(p^{MND}) = \bar{k}(1-e) + 1/2 + S$. $HE(p^{MIX})$ depends on the efficiency of managerial effort.

  - If $S > \bar{S}$, then $p^{MIX} = 0$ and $HE(p^{MIX}) = \bar{k} + 1/2 \geq HE^{MND}(p^{MND})$.

  - If $S \leq \bar{e}(\bar{k})^2/2$, then $p^{MIX} = \bar{k}$ and

$$HE(p^{MIX}) = \bar{k} + \frac{1}{2} - e\frac{(\bar{k})^2}{2},$$

which is greater than $HE^{MND}(p^{MND})$ for any $S \leq \bar{e}(\bar{k})^2/2$.

  - If $e(\bar{k})^2/2 < S \leq e(\bar{k})^2 \left(\sqrt{\bar{e}} + 1/2\right) \leq e(\bar{k} - (\bar{k})^2/2)$, then $p^{MIX} = \tilde{p}$ and

$$HE(p^{MIX}) = \left(\frac{S}{e\bar{k}} - \frac{\bar{k}}{2}\right)^2 - \frac{e(\bar{k})^2}{2} + \bar{k} + \frac{1}{2}.$$

which is greater than $HE^{MND}(p^{MND})$ for any $S \leq e(\bar{k} - (\bar{k})^2/2)$.

  - Finally, if $e\left(\bar{k} - (\bar{k})^2/2\right) < S \leq e\left(\bar{k} - (1 + \sqrt{e})^{-2}/2\right)$, then $p^{MIX} = \tilde{p}$ and

$$HE(p^{MIX}) = 1 + \bar{k} - \sqrt{2\left(\frac{e\bar{k} - S}{e}\right)} + (1 - \bar{e})\left(\frac{e\bar{k} - S}{e}\right).$$

This expression is greater than $HE^{MND}(p^{MND})$ if and only if $S \geq e\bar{k} - e/2$. This condition is always satisfied when $S > e\left(\bar{k} - (\bar{k})^2/2\right)$ and $\bar{k} < 1$.∎