Would Letting People Vote for Several Candidates Yield Policy Moderation?[1]

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We investigate whether letting people vote for several of the candidates would yield policy moderation compared to Plurality Voting. We do so in a setting that takes three key features of elections into account, namely, strategic voting, strategic candidacy and policy motivation on the part of the candidates. We consider two classes of voting rules. One class consists of the voting rules where each voter casts several equally-weighed votes for the different candidates. The other class consists of the voting rules where each voter rank-orders the different candidates. We then identify conditions under which these voting procedures yield policy moderation compared to Plurality Voting. We also show that replacing Plurality Voting with one of these voting procedures may yield policy extremism if any of these conditions is not satisfied! Finally, we find that amongst these voting procedures the extent of policy moderation is maximal under Approval Voting and the Borda Count. Which of these two voting procedures yields the most moderate policy outcomes depends on (1) the symmetry of policy preferences and (2) the presence of spoiler candidates.

Key Words: Voting rules; Electoral competition; Policy moderation; Citizen-candidate model.
Subject Classification: C72, D72, D78.

1. INTRODUCTION

Several countries, such as the U.S. and Canada, elect their policy-makers by means of Plurality Voting (hereafter PV) that is, the voting procedure under which each citizen can vote for exactly one candidate and the candidate who gets the most votes wins the election. Several scholars of electoral systems have claimed that replacing PV with a voting procedure that would let people vote for several of the candidates would yield policy moderation. We define policy moderation as the adoption of policies that lie closer to the median citizen's ideal policy. Key to the argument underlying this claim is the so-called *wasting-the-vote effect* of PV, whereby the fear of wasting one's vote on an underdog induces a citizen to vote for the candidate he prefers *among the serious contenders*.[2] Now the wasting-

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[2] For example, in the 2004 U.S. Presidential election some of Nader's supporters voted for Kerry or Bush instead of voting for Nader. This is because they anticipated that Nader had no chance of winning the election and, therefore, that a vote for Nader would be wasted. In contrast, there was a (tiny) chance that a vote for Kerry or Bush would be pivotal. Consequently, a supporter of Nader who preferred, say, Kerry to Bush had an incentive to cast his vote for Kerry.

the-vote effect creates a barrier to entry by new candidates. This is because in countries where political elections are held under PV, citizens tend to anticipate that a new candidate has no chance of winning the election and, therefore, that a vote for a new candidate would be wasted. The argument then goes that if citizens were given several votes to cast for the different candidates, they would no longer fear wasting a vote on a new candidate. This would eliminate the barriers to entry by new candidates, especially by centrists who a majority of citizens would arguably consider as acceptable policy-makers. In other words, this would improve the electoral prospects of the centrists which, in turn, would yield policy moderation compared to PV.

The present paper investigates whether replacing PV with a voting procedure that would let people vote for several of the candidates would indeed yield policy moderation. For this purpose we consider a community that must elect a representative to choose a policy such as a tax rate or the level of public spending. We model political competition following the citizen-candidate approach. Under this approach, the policy-making process is modeled as a three-stage game. At the first stage, policy-motivated citizens decide simultaneously whether to stand for election. At the second stage, citizens decide strategically for whom to vote among the self-declared candidates. At the third stage, the winner of the election assumes office and chooses the policy to implement.

By adopting the citizen-candidate approach to political competition, we have been able to include three key features of elections into the analysis. These features are strategic voting, endogenous candidacy and policy motivation on the part of the candidates. This contrasts with the previous contributions that have looked at the same question. Note that there exist empirical and theoretical support for taking each of these three features into account.[3] Note also that the conclusions that are reached depend on whether these features are included into the analysis or not.

There exist countless voting procedures that let people vote for several candidates. In this paper, we consider the scoring rules. This class of voting rules encompasses many of the voting rules that are commonly used and studied. Under a scoring rule, a voter's ballot takes the form of a vector of points that are cast for the different candidates. The candidate who gets the most points wins the election. We shall focus on two broad classes of scoring rules. The first class contains the scoring rules where each vote is weighed equally. We shall call these scoring rules Multiple Voting rules (hereafter MVs). An example of MV is Dual Voting under which each citizen is given two votes to cast for the different candidates. Another example is Approval Voting under which a citizen can cast a vote for as many candidates as he wishes.[4] The second class of scoring rules to be considered contains

[3]Concerning strategic voting, we know that any non-dictatorial voting procedure is open to strategic voting whenever there are at least three alternatives (Gibbard 1973, Satterthwaite 1975). Moreover, empirical and experimental evidence suggest that a sizable number of voters vote strategically (e.g. Forsythe et al. 1996, Cox 1997). Concerning endogenous candidacy, we know that any non-dictatorial and unanimous voting procedure is subject to strategic candidacy (Dutta et al. 2001). Moreover, empirical evidence suggests that different voting procedures provide different incentives for candidate entry (e.g. Persson and Tabellini 2003). Concerning the policy-motivation of candidates, we know that policy moderation in PV elections depends critically on whether candidates are policy-motivated and, therefore, whether they can credibly commit to a policy (Alesina 1988). Moreover, empirical evidence suggests that the policy preferences of a policy-maker matter for his policy decisions (e.g. Levitt 1996).

[4]Approval Voting is a voting procedure that had been popularized by Brams and Fishburn (1978). It is currently used by several professional and academic associations to elect their office-bearers. Several scholars of electoral systems have recommended that Approval Voting be used

the scoring rules where voters rank-order the candidates and where the score that goes to a candidate increases with his position in the ranking. We shall call these scoring rules the Ordinal Voting rules (hereafter OVs). An example of OV is the Borda Count under which the candidate who is ranked last gets no points, the second lowest candidate gets one point, the next-ranked candidate gets two points, and so on.

The present analysis yields a number of interesting results. Our first set of results concerns serious equilibria, i.e., equilibria where each candidate is elected with a positive probability. In this context, we find that a MV always yields policy moderation compared to PV if and only if: (1) every citizen is allowed to truncate his ballot – i.e., cast as many of his votes as he wishes –, so that the equilibrium set under any MV is a subset of the equilibrium set under PV; (2) each citizen has 'enough' votes to cast, so that the wasting-the-vote effect is eliminated everywhere; and (3) policy preferences are symmetric, so that there is an equal number of candidates standing at each position. We also show that replacing PV with a MV may yield policy extremism instead of policy moderation if one (or several) of these three conditions does not hold. This is because all three conditions are necessary to deal with the fact that MVs can accomodate multiple similar candidacies. Key to observe is that Approval Voting satisfies conditions (1) and (2). Our analysis thus suggests that in the context of serious equilibria, whether we can replace PV with a MV that always yields policy moderation boils down to the question of whether policy preferences are symmetric or not.

Still in the context of serious equilibria, we find that an OV always yields policy moderation compared to PV. Importantly, no condition is required for an OV to yield policy moderation. This is because, in contrast to MVs, OVs cannot accomodate multiple similar candidacies. We also show that while all OVs always yield policy moderation compared to PV, the extent of policy moderation under an OV depends on (1) its second-place score in a three-way race and (2) whether it admits truncated ballots or not. Among all OVs, the extent of policy moderation is maximal under the Borda Count with only completely-filled ballots admissible.

Our second set of results concerns spoiler equilibria, i.e., equilibria where some of the candidates – called spoilers – are elected with probability zero. In this context, we find that the extent of policy moderation (if any) need not be substantial. This is because the presence of spoilers helps support self-fulfilling prophecies that create barriers to entry by new candidates and barriers to exit by incumbent candidates.

Our third set of results compares policy moderation under the different MVs and OVs. Focusing on serious equilibria, we find that the Borda Count (with only completely-filled ballots admissible) yields the most moderate policy outcomes. This is because the Borda Count can neither deter nor accomodate multiple similar candidacies. However, this result does not carry to spoiler equilibria. Indeed, when we consider spoiler equilibria, we find that the Borda Count can yield policy outcomes that are (1) as extreme as under PV and (2) more extreme than under Approval Voting.

The present work is related to a number of papers that adopt the Downsian approach to political competition and study the effect of alternative electoral systems on policy moderation.[5] Under the prototypical Downsian approach, a fixed

for political elections. For more details see, for example, Brams (2008).

[5] The Downsian framework has also been used to study, for example, the effect of different electoral systems on reducing government corruption (e.g. Myerson 1993a, 2006), on the provision of public goods (e.g. Lizzeri and Persico 2001, Milesi-Ferretti et al. 2002) or on the incentives to

set of purely office-motivated candidates compete by choosing a platform along a one-dimensional policy space. Cox (1987, 1990) compares policy moderation under alternative voting procedures. Assuming that citizens vote sincerely, he finds that increasing the number of votes cast by a citizen produces centripetal incentives that lead candidates to advocate moderate policies. Myerson and Weber (1993) relax the sincere voting assumption. They adopt instead the pivotal-voter approach, whereby a citizen conditions his vote on his anticipation of a close race between the different candidates. They find that PV imposes few restrictions on the location of the serious contenders. By contrast, in equilibrium all serious contenders are standing at the median citizen's ideal policy when the election is held under Approval Voting. The present paper goes further by relaxing the assumptions that candidacy is exogenous and that candidates are purely office-motivated. In Section 4, we discuss which of these assumptions matter for the conclusions. We do so by comparing the present analysis with the Downsian analysis of Cox (1990).

The present paper is also related to the citizen-candidate literature that was initiated by Osborne and Slivinski (1996) and Besley and Coate (1997). In contrast to the canonical Downsian framework, the citizen-candidate approach assumes that candidacy is endogenous and that candidates are policy-motivated. This approach to political competition yields polarization under PV, even in two-candidate races.[6] This is because entry is costly and candidates are policy-motivated, the latter preventing candidates from credibly committing to the policy they will implement if elected. Previous works in this literature are focused on PV. Notable exceptions are Osborne and Slivinski (1996) who compare policy outcomes under PV and Plurality Runoff, Hamlin and Hjortlund (2000) and Morelli (2004) who study elections under Proportional Representation, and Dellis and Oak (2006) who compare PV and Approval Voting. The present contribution extends this literature by comparing policy moderation under the different MVs and OVs.

At a more general level, the present paper is related to the formal literature on scoring rules. We have already discussed the contributions of Cox (1987, 1990). Myerson (2002) considers a large Poisson voting game with three candidates and studies which scoring rules elect universally liked/disliked candidates. Dellis (2007a) characterizes the set of weakly undominated voting strategies under the different scoring rules. Also, Buenrostro et al. (2007) study the dominance solvability of scoring rule voting games. We shall make use of these results since we require voting strategies to be weakly undominated. Finally, Dellis (2007b) studies whether replacing PV with another scoring rule would obliterate the two-party system that characterizes PV elections. This contrasts with the present paper that studies *where* the winning candidates are located, not *how many* winning positions there are. In particular, while candidacy is endogenous in the present paper, it is exogenous in Dellis (2007b).

favor minority interests (e.g. Myerson 1993b, Lizzeri and Persico 2005).

[6] This contrasts with the canonical Downsian approach where in two-candidate PV elections, both candidates choose to stand at the median citizen's ideal policy. Observe however that we get polarization in the Downsian framework if one of the following assumptions is made: (1) candidates are policy-motivated and uncertain about the distribution of preferences in the electorate (Wittman 1983, Calvert 1985, Roemer 1997), (2) candidates are policy-motivated and unable to credibly commit to a policy if elected (Alesina 1988), (3) the government is divided (Ortuno-Ortin 1997, Alesina and Rosenthal 2000), (4) one of the two candidates is an incumbent (Bernhardt and Ingberman 1985), (5) candidates cater to special interests in order to raise money for their campaign (Baron 1994), (6) voting is costly and voters can abstain (Feddersen 1992), or (7) the two candidates compete against each other under the threat of entry by a third candidate (Palfrey 1984, Weber 1992 and 1998, Callander 2005).

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies policy moderation. Section 4 discusses how the differences in assumption with the Downsian approach matter for our conclusions. Section 5 concludes. All proofs are in the Appendix.

2. THE MODEL

Consider a community that must elect a policy-maker to choose a policy such as a tax rate or the level of public spending. The community is made up of N citizens, indexed by $\ell \in \mathcal{N} \equiv \{1, ..., N\}$. The set of policy alternatives is $X = \mathbb{R}$.[7]

Citizens' preferences over the set of policy alternatives are represented by a utility function $u^{\ell} : X \to \mathbb{R}$, which is assumed to be strictly concave.[8] Let $x_{\ell} \equiv \arg\max_{x \in X} u^{\ell}(x)$ denote citizen ℓ's ideal policy. We assume throughout the analysis that citizens differ only in their ideal policy – that is, $u^{\ell}(x) \equiv u(x_{\ell} - x)$ for all citizen ℓ – with several citizens sharing the same ideal policy.[9] Let $u^{\ell}(x_{\ell}) = 0$ and denote the utility citizen ℓ derives from citizen i's ideal policy, x_i, by $v_i^{\ell} \equiv u^{\ell}(x_i)$. Furthermore, let m denote the median citizen's ideal policy and M the number of citizens who share this ideal policy.[10] Finally, let there be $\frac{N-M}{2} \geq 6$ citizens on either side of m and assume $M \leq \frac{N-4}{3}$.[11]

There are three stages to the policy-making process. At the first stage, each citizen decides whether to stand for office. Decisions are made simultaneously. At the second stage, an election is held where each citizen decides for whom to vote among the self-declared candidates. The candidate who gets the highest vote total is elected. If several candidates tie for first place, each is elected with an equal probability. At the third stage, the elected policy-maker chooses policy. We assume that a candidate cannot credibly pre-commit to implementing a policy different from his ideal one. Lee et al. (2004) provide empirical justification for this assumption. We now analyze each of these stages in a reverse order.

Policy-making stage. Because this stage is the last one, the elected policy-maker chooses to implement his ideal policy. In case no one runs for election, the status-

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[7] The unidimensionality of the policy space facilitates comparisons with the related literature which for the most part, has developed under this assumption. Besides, empirical works have shown that citizens' preferences over the different issues tend to be highly correlated and, therefore, that assuming a unidimensional policy space is relevant (e.g. Poole and Rosenthal 1991, Hinich and Munger 1994).

[8] The strict concavity assumption eliminates the knife-edge PV equilibria where three serious contenders are running for election, each standing on a different platform. This is not a very restrictive assumption however in the sense that these equilibria are non-generic. Besides, these equilibria run counter to empirical as well as experimental evidence (e.g. Cox 1997, Forsythe et al. 1996).

[9] The assumption that citizens differ only in their ideal policy is made to simplify the analysis. The assumption that several citizens share the same ideal policy is not key either. It is only made to avoid corner solutions where more citizens than there are would be willing to stand on a platform. Relaxing this assumption would complicate the statements of the results without adding any new insight.

[10] In the remainder of the paper, we shall abuse notation and call the median citizen m. Also, note that the median citizen's ideal policy is the Condorcet winner – that is, the policy that defeats any other in a pairwise contest.

[11] Assuming there are at least six citizens on either side of the median is made to rule out situations where too few votes are cast. Assuming less than a third of the electorate is at the median is made to rule out situations where the citizens at the median would be able to elect any candidate they wish without the support of other citizens.

quo policy $x_0 \in X$ is assumed to be kept.

Election stage. Suppose the election is held under V, an arbitrary MV or OV. For a given non-empty set of candidates $\mathcal{C} \subseteq \mathcal{N}$, let $A_V(\mathcal{C})$ denote the set of admissible voting strategies under V and let $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ be citizen ℓ's voting strategy, where $\alpha_i^\ell(\mathcal{C})$ is the score citizen ℓ gives to candidate i.[12] We denote the profile of voting strategies by $\alpha(\mathcal{C}) = \left(\alpha^1(\mathcal{C}), ..., \alpha^N(\mathcal{C})\right)$. We shall call the *winning set*, $W(\mathcal{C}, \alpha)$, the set of candidates with the highest score when $\alpha(\mathcal{C})$ is the voting profile; that is,

$$W(\mathcal{C}, \alpha) \equiv \left\{ i \in \mathcal{C} : \sum_{\ell \in \mathcal{N}} \alpha_i^\ell(\mathcal{C}) \geq \sum_{\ell \in \mathcal{N}} \alpha_j^\ell(\mathcal{C}) \text{ for any } j \in \mathcal{C} \right\}.$$

Letting $\#W(\mathcal{C}, \alpha)$ denote the number of winning candidates, citizen i's probability of becoming the policy-maker is equal to $\frac{1}{\#W(\mathcal{C},\alpha)}$ if $i \in W(\mathcal{C}, \alpha)$ and is equal to zero otherwise. Citizen ℓ's expected utility is thus $V^\ell(\mathcal{C}, \alpha) \equiv \frac{1}{\#W(\mathcal{C},\alpha)} \sum_{i \in W(\mathcal{C},\alpha)} v_i^\ell$.

We are now ready to define a voting equilibrium.

Definition 1 (Voting Equilibrium). Given a non-empty set of candidates $\mathcal{C}$, a strategy profile $\alpha^*(\mathcal{C})$ is a voting equilibrium if for any citizen ℓ, $\alpha^{\ell*}(\mathcal{C}) \in A_V(\mathcal{C})$ is such that
(1) $V^\ell\left(\mathcal{C}; \alpha^{\ell*}, \alpha^{-\ell*}\right) \geq V^\ell\left(\mathcal{C}; \alpha^\ell, \alpha^{-\ell*}\right)$ for all $\alpha^\ell \in A_V(\mathcal{C})$;
(2) $\alpha^{\ell*}(\mathcal{C}) = (0, ..., 0)$ whenever $v_i^\ell = v_j^\ell$ for all two candidates i and j; and
(3) $\alpha^{\ell*}(\mathcal{C})$ is weakly undominated. ‖

The first condition says that a citizen chooses a voting strategy that maximizes his expected utility, given others' voting strategies and his anticipation of the policy each candidate will implement if elected. The second condition states that a citizen who is indifferent between all candidates abstains from voting.[13] Finally, the third condition is a common refinement of the voting behavior. The following lemma – adapted from Dellis (2007a) – characterizes the set of (pure) voting strategies that are weakly undominated.

Lemma 1 (Dellis 2007a). *Let $\mathcal{C}$ be a non-empty set of candidates. Suppose that citizen ℓ is not indifferent between all the candidates in $\mathcal{C}$. Citizen ℓ's voting strategy $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ is weakly undominated if and only if there is no other voting strategy $\widetilde{\alpha}^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ such that for all pairs of candidates i and j, $v_i^\ell \geq v_j^\ell$ implies $\left[\widetilde{\alpha}_i^\ell(\mathcal{C}) - \alpha_i^\ell(\mathcal{C})\right] \geq \left[\widetilde{\alpha}_j^\ell(\mathcal{C}) - \alpha_j^\ell(\mathcal{C})\right]$.* ‖

Thus, a citizen's voting strategy is weakly undominated if and only if there does not exist another admissible voting strategy such that the ordering of score differences between the two voting strategies coincides with the citizen's preference ordering over the set of candidates.[14]

[12] For example, if the election is held under PV, then $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ if $\alpha^\ell(\mathcal{C}) = (0, ..., 0)$ – that is, citizen ℓ abstains from voting – or $\alpha^\ell(\mathcal{C})$ is any permutation of $(1, 0, ..., 0)$.

[13] We emphasize that this condition is imposed only to simplify the analysis. Neither of our results depends on it.

[14] For example, a voting strategy is weakly undominated under PV if and only if the citizen votes for a candidate other than his least-preferred candidate(s). Instead, a voting strategy is weakly undominated under Approval Voting if and only if the citizen votes for his most-preferred candidate(s) and does not vote for his least-preferred candidate(s). Finally, a voting strategy is weakly undominated under the Borda Count if and only if the citizen ranks his most-preferred candidate(s) above his least-preferred candidate(s).

Candidacy stage. Citizens decide simultaneously whether to stand for election at a utility cost $\delta > 0$. We assume that all citizens anticipate the same voting equilibrium when making their candidacy decision. Denote citizen ℓ's candidacy decision by $e^{\ell} \in \{0, 1\}$, where $e^{\ell} = 1$ if citizen ℓ chooses to stand for election and $e^{\ell} = 0$ otherwise. Let $e = \left(e^1, ..., e^N\right)$ be the profile of candidacy decisions and define $\mathcal{C}(e) \equiv \left\{\ell \in \mathcal{N} : e^{\ell} = 1\right\}$ as the set of candidates. For a given profile of entry decisions e, citizen ℓ's expected utility is $U^{\ell}(\mathcal{C}(e), \alpha) \equiv \left[V^{\ell}(\mathcal{C}(e), \alpha) - \delta e^{\ell}\right]$ if $\mathcal{C}(e) \neq \emptyset$ and $U^{\ell}(\mathcal{C}(e), \alpha) \equiv u^{\ell}(x_0)$ otherwise.

We are now ready to define a candidacy equilibrium.

Definition 2 (Candidacy Equilibrium). A profile of candidacy strategies e^* is a candidacy equilibrium if for each citizen ℓ, $e^{\ell *}$ is a best response to $e^{-\ell *}$ given the profile of voting strategies. ‖

Political equilibrium. A political equilibrium (hereafter an equilibrium) is a Subgame Perfect Nash Equilibrium of this game. It consists of a pair $(e^*, \alpha^*(.))$ where: (1) $\alpha^*(\mathcal{C})$ is an equilibrium profile of voting strategies for any non-empty set of candidates $\mathcal{C}$; and (2) e^* is an equilibrium profile of candidacy strategies, given $\alpha^*(.)$.[15]

Definition 3. Let E and $\widetilde{E}$ be any two equilibria. Denote the probability that alternative $x \in X$ is implemented in E ($\widetilde{E}$, resp.) by p_x ($\widetilde{p}_x$, resp.). For all alternative x, write $x \in E$ ($\widetilde{E}$, resp.) if and only if $p_x > 0$ ($\widetilde{p}_x > 0$, resp.). Then,

(1) E and $\widetilde{E}$ are *equivalent* if and only if $p_x = \widetilde{p}_x$ for all alternative x.

(2) E is *moderate* compared to $\widetilde{E}$ ($\neq E$) if and only if $v_x^m \geq v_y^m \geq v_z^m$ for all $x \in \left(E \backslash \widetilde{E}\right)$, $y \in \left(E \cap \widetilde{E}\right)$ and $z \in \left(\widetilde{E} \backslash E\right)$, with at least one inequality strict.

(3) E is *extreme* compared to $\widetilde{E}$ ($\neq E$) if and only if $\widetilde{E}$ is moderate compared to E. ‖

Thus, two equilibria are said to be equivalent if they yield the same probability distribution over policy outcomes. Also, an equilibrium E is said to be moderate compared to another equilibrium $\widetilde{E}$ if the median citizen (i) likes every policy outcome in E at least as much as all the policy outcomes that are specific to $\widetilde{E}$ and (ii) likes every policy outcome specific to E at least as much as all the policy outcomes in $\widetilde{E}$.

We can now define the notion of policy moderation.

Definition 4. A voting procedure V yields *policy moderation* compared to another voting procedure $\widetilde{V}$ if and only if:

(1) every equilibrium under V either has an equivalent equilibrium under $\widetilde{V}$ or is moderate compared to all equilibria under $\widetilde{V}$; and

(2) every equilibrium under $\widetilde{V}$ either has an equivalent equilibrium under V or is extreme compared to all equilibria under V.

A voting procedure V yields *policy extremism* compared to another voting procedure $\widetilde{V}$ if and only if $\widetilde{V}$ yields policy moderation compared to V. ‖

Intuitively, if V yields policy moderation compared to V, then the median citizen prefers ex ante that the election is held under V than under $\widetilde{V}$.

[15] Following previous contributions in the citizen-candidate literature (e.g. Besley and Coate 1997, Dhillon and Lockwood 2002), we shall focus on equilibria in pure strategies.

3. POLICY MODERATION

In this section, we study whether letting people vote for several of the candidates yields policy moderation compared to PV. We begin our analysis by considering the case of serious equilibria, i.e., equilibria where all candidates are in the winning set. In this context, we study the Multiple Voting rules, followed by the Ordinal Voting rules. We then add spoiler equilibria to our analysis, i.e., equilibria where some candidates are not in the winning set. We complete our analysis by comparing the Multiple Voting rules with the Ordinal Voting rules.

3.1. Serious Equilibria under the Multiple Voting Rules

We begin our analysis by considering serious equilibria in elections that are held under Multiple Voting rules (hereafter MVs). A MV is a scoring rule under which each citizen has $q \in \mathbb{N}$ votes to cast – or $(c-1)$ if the number of candidates c does not exceed q – and each vote is weighed equally. Formally, the vector of points a voter can cast $(s_1, s_2, ..., s_c)$ is given by

$$\begin{cases} 1 = s_1 = ... = s_q > s_{q+1} = ... = s_c = 0 & \text{if } c > q \\ 1 = s_1 = ... = s_{c-1} > s_c = 0 & \text{if } c \leq q \end{cases}.$$

Examples of MVs are PV ($q = 1$), Dual Voting ($q = 2$) and Negative Voting ($q = +\infty$).

The following result can be established.

Proposition 1. *Let V be an arbitrary MV and denote the maximum number of votes a citizen can cast by $q \in \mathbb{N}$. Consider the set of serious equilibria. Then, V always yields policy moderation compared to PV if and only if:*

(1) *truncated ballots are admissible – i.e., a voter can cast as many of his votes as he wishes;*

(2) *each citizen has enough votes to cast – i.e., $q > \overline{q}$ for some $\overline{q} \in \mathbb{Z}_+$ finite; and*

(3) *policy preferences are symmetric – i.e., $u^{\ell}(x) \equiv u(|x_{\ell} - x|)$ for all citizen ℓ.* ‖

Thus, Proposition 1 provides a (conditional) theoretical underpinning for the claim that letting people vote for several of the candidates would yield policy moderation compared to PV. However, as we shall argue below, replacing PV with another MV may yield policy extremism instead of policy moderation if one (or several) of the three conditions stated in Proposition 1 is not satisfied.

To understand this result, note first that when the election is held under PV, only two types of serious equilibria exist, namely, the one- and the two-position serious equilibria (Besley and Coate 1997). In a one-position serious equilibrium, only one citizen enters the race and is elected by acclamation. (Note that this holds true for all voting procedures, implying that the one-position serious equilibria are equivalent under all voting procedures.) In a two-position serious equilibrium, exactly two citizens enter the race, one on the left of the median citizen's ideal policy and another one on the right. Key to observe is that in all serious equilibria under PV, no two candidates are standing at the same position. This is because each citizen has only one vote to cast for the different candidates. By contrast, the other MVs can accomodate several serious contenders standing at the same position. This is because under these voting procedures, each citizen has several votes to cast for the different candidates. The role of the three conditions stated in Proposition 1 is to deal with these multiple similar candidacies. To make this clear, we now discuss each of the three conditions in turn.

Admissibility of truncated ballots. When truncated ballots are admissible, the serious equilibrium set under any MV is a *subset* of the serious equilibrium set under PV. The reverse holds true however if truncated ballots are not admissible, i.e., the serious equilibrium set under any MV is then a *superset* of the serious equilibrium set under PV.[16] This is because giving citizens several votes to cast for the different candidates and forcing voters to cast all their votes for their ballot to be valid triggers stronger incentives for several citizens at the same position to enter the race. The presence of multiple similar candidates can then bring the wasting-the-vote effect back, thereby preserving the barriers to entry by new candidates and yielding an expansion of the serious equilibrium set compared to PV. Replacing PV with another MV that does not admit truncated ballots may thus yield policy extremism if these 'extra' equilibria support policy outcomes that are more extreme compared to the equilibrium policy outcomes under PV. To appreciate this argument, consider the following example.

Example 1. Consider a community that must elect a representative to choose a tax rate. Suppose there are 39 citizens whose ideal tax rates are distributed as follows

Ideal tax rates	0	1	2	3	4	5	6	7	8	9	10
Number of citizens	2	2	3	4	5	7	5	4	3	2	2

i.e., two citizens prefer no tax, two citizens have 1 as an ideal tax rate, three citizens have 2 as an ideal tax rate, and so on. Assume that the preferences of a citizen ℓ can be represented by a quadratic loss utility function, i.e., $u^{\ell}(x) = -(x_{\ell} - x)^2$. Condition (3) of Proposition 1 is thus satisfied and the median citizen's ideal tax rate is 5. Finally, let the candidacy cost $\delta = 3$ and the status-quo tax rate $x_0 = 0$ (i.e., the community considers introducing a new tax).[17]

In this context, the two-position serious equilibrium set under PV consists of $\{3,7\}$, $\{2,8\}$, $\{1,9\}$ and $\{0,10\}$, where $\{x,y\}$ denotes an equilibrium that implements x and y.[18] In these equilibria, one citizen at each of the two positions enters the race, all the citizens at $0,...,4$ ($6,...,10$, resp.) vote for the left (right, resp.) candidate, and the citizens at 5 abstain from voting since they are indifferent between the two candidates. Each candidate thus receives 16 votes and ties for first place. It is easy to check that neither citizen has an incentive to deviate from his voting strategy and that neither of the two candidates can be better off by staying out of the race. Moreover, no other citizen wants to enter the race if he (correctly) anticipates that neither of the citizens at $0,...,4,6,...,10$ would vote for him because of the fear of wasting their vote on a new candidate.

[16] It is worth mentioning that neither the subset nor the superset need be proper. We would have liked to provide conditions that ensure properness. Unfortunately, it has not been possible. This is because the size of the serious equilibrium set depends on at least two factors, namely, the candidacy cost and the distribution of ideal policies. The difficulty with the candidacy cost comes from its non-monotonic relationship with the size of the serious equilibrium set. The difficulty with the distribution of ideal policies comes from its finiteness. Equilibrium policy outcomes that are specific to some MVs need not correspond to actual ideal policies, in which case these equilibria do not exist and the actual equilibrium sets may end up being equivalent.

[17] We emphasize that the choice of the status quo is without loss of generality for the comparison of equilibrium policy outcomes under the different scoring rules.

[18] In addition to the two-position serious equilibria, three one-position serious equilibria exist, namely, $\{4\}$, $\{5\}$ and $\{6\}$. For brevity, we shall ignore these equilibria since, in any case, they are equivalent under all voting procedures and are the most moderate serious equilibria.

Let us suppose now that the election is held under Dual Voting and that only completely-filled ballots are admissible. Then, $\{3,7\}$, $\{2,8\}$, $\{1,9\}$ and $\{0,10\}$ are still two-position serious equilibria.[19] To see this, suppose that one candidate is standing at 3 and another one is standing at 7. Both candidates thus tie for first place (as under PV). However, in contrast to PV, a second citizen at 3 cannot be deterred from entering the race. This is because he (correctly) anticipates that following his entry in the race, all the citizens at $0,...,4$ will cast their two votes for the two candidates at 3, the citizens at 5 will still abstain from voting, and the citizens at $6,...,10$ will cast one vote for the candidate at 7 and will have no other choice than casting their second vote for one of the candidates at 3. Consequently, one of the candidates at 3 will be elected the policy-maker. The probability that 3 is implemented thus increases from 1/2 to 1. A second citizen at 3 is therefore willing to enter the race since his expected utility gain (equal to 8) exceeds the candidacy cost. Likewise, a second citizen at 7 will enter the race. Once two candidates are standing at each position, the wasting-the-vote effect is back. No other citizen is then willing to enter the race if he (correctly) anticipates that the citizens at $0,...,4,6,...,10$ will fear wasting any of their two votes on a new candidate. $\{3,7\}$ can thus be supported as a two-position serious equilibrium. (The same argument applies for $\{2,8\}$, $\{1,9\}$ and $\{0,10\}$.)

Suppose that the election is still held under Dual Voting, but that truncated ballots are now admissible. In this case, $\{3,7\}$ cannot be supported as a two-position serious equilibrium. To see this, suppose again that there is one candidate standing at 3 and another candidate standing at 7. A second citizen at 3 can now be deterred from entering the race. This is because he (correctly) anticipates that all the citizens at $6,...,10$ will cast a vote only for the candidate at 7. Each candidate will thus receive 16 votes and tie for first place. Consequently, the probability that 3 is implemented increases from 1/2 to only 2/3. It follows that a second citizen at 3 does not want to enter the race since his expected utility gain (equal to 8/3) is lower than the candidacy cost. Likewise, a second citizen at 7 can be deterred from entering the race. Since there is only one candidate standing at each position, the wasting-the-vote effect is eliminated and the barriers to entry by moderates are removed. It is easy to check that a citizen at 5 will then enter the race and win the election outright. Thus, $\{3,7\}$ cannot be supported as a two-position serious equilibrium.[20] □

Throughout the remainder of this section, we shall maintain the assumption that truncated ballots are admissible. Consequently, only one- and two-position serious equilibria exist under all MVs. Recall from above that the one-position serious equilibria are (1) equivalent under all MVs and (2) moderate compared to the two-position serious equilibria (Lemma 2 in the Appendix). Consequently, we can restrict our attention to the two-position serious equilibria since they are the

[19] It is worth mentioning that Dual Voting with only completely-filled ballots admissible supports also serious equilibria where candidates are standing at three different positions. Since these equilibria do not have equivalent serious equilibria under PV, it follows that the serious equilibrium set under Dual Voting with only completely-filled ballots admissible is a superset of the serious equilibrium set under PV.

[20] Since no serious equilibrium with candidates standing at three or more positions exist when truncated ballots are admissible (Proposition 2 in Dellis 2007b), we get that the serious equilibrium set under Dual Voting with truncated ballots admissible is a subset of the serious equilibrium set under PV.

ones that determine whether a MV yields policy moderation compared to PV.[21]

Number of votes. Whether a MV always yields policy moderation compared to PV depends on whether the barriers to entry by moderates are removed *everywhere*. In turn, this depends on the number of votes a citizen is allowed to cast. To see this, let $\{x_L, x_R\}$ be a two-position serious equilibrium under PV. Recall from above that only one candidate is standing at x_L (x_R, resp.). This is because of the so-called *splitting-the-vote effect* of PV, whereby several candidates standing at one position would be splitting votes. By contrast, other MVs can accomodate up to q serious contenders standing at x_L (x_R, resp.). This is because each citizen can vote for up to q candidates, meaning that up to q candidates can be standing at a position without splitting votes. Now, the number of citizens at x_L (x_R, resp.) who actually want to stand for election increases with the distance between x_L and x_R. This is because for a citizen at x_L (x_R, resp.) his utility gain from having x_L instead of x_R (x_R instead of x_L, resp.) implemented increases with the distance between x_L and x_R. Letting $\{x_L, x_R\}$ be the most extreme two-position serious equilibrium under PV, define $\overline{q}$ to be the number of citizens at x_L or x_R – the largest of the two numbers – who would be willing to stand for election if there was no risk of splitting votes. Thus, $\overline{q}$ is the maximum number of candidates standing at the same position over all two-position serious equilibria and under all MVs.

Suppose that PV is replaced with a MV where $q > \overline{q}$. Then, less than q citizens at x_L (x_R, resp.) enter the race. This means that the wasting-the-vote effect is eliminated everywhere and the barriers to entry by moderates are removed. Now, whether a moderate actually enters the race – and the equilibrium is therefore eliminated – depends on how polarized x_L and x_R are. Indeed, the bigger the distance between x_L and x_R is, (1) the bigger the utility gain for a moderate to implement his ideal policy instead of having a lottery over x_L and x_R is and (2) the better the electoral prospects of a moderate are. Consequently, only the most moderate two-position serious equilibria under PV have equivalent serious equilibria under a MV where $q > \overline{q}$. Hence the possibility of policy moderation.

Suppose instead that PV is replaced with a MV where $q \leq \overline{q}$. Then, q citizens at x_L (x_R, resp.) will enter the race if $\{x_L, x_R\}$ is an extreme two-position serious equilibrium. In this case, the wasting-the-vote effect is back and the barriers to entry by moderates are kept unmoved. An equivalent serious equilibrium thus exists under a MV where $q \leq \overline{q}$. By contrast, less than q candidates will be standing at x_L (x_R, resp.) if $\{x_L, x_R\}$ is a moderate two-position serious equilibrium. In this case, the wasting-the-vote effect is eliminated and the barriers to entry by moderates are removed. An equivalent serious equilibrium need not therefore exist. In other words, a MV where $q \leq \overline{q}$ eliminates the wasting-the-vote effect only when x_L and x_R are sufficiently moderate. Consequently, only the most extreme two-position serious equilibria under PV may have equivalent serious equilibria under a MV where $q \leq \overline{q}$. Hence the possibility of policy extremism.[22]

[21] In an earlier version of the paper we also considered the admissibility of cumulated ballots, i.e., voters were allowed to cumulate their scores behind fewer candidates. It turns out that when cumulated ballots are admissible, the serious equilibrium set is equivalent under all voting procedures. This is because a voter has then an incentive to stack all his scores on the serious contender he prefers. Compared to PV, the vote total of every candidate is thus scaled up by the same factor, thereby leaving the vote shares unchanged.

[22] To appreciate this discussion, consider the community described in Example 1. From the proof of Proposition 1, we obtain $\overline{q} = 8$. It is easy to check that if the election is held under Dual Voting (i.e., $q = 2$), then only $\{2, 8\}$, $\{1, 9\}$ and $\{0, 10\}$ are two-position serious equilibria – with two

Symmetry of policy preferences. Whether the elimination of the barriers to entry by moderates is sufficient for a MV to yield policy moderation compared to PV depends on the symmetry of policy preferences. To see this, let $\{x_L, x_R\}$ be a two-position serious equilibrium under PV and suppose that PV is replaced with another MV that admits truncated ballots – call it V. Now, a citizen at x_L (x_R, resp.) is willing to enter the race if and only if the expected utility gain of doing so exceeds the candidacy cost. If policy preferences are symmetric, then the expected utility gain is the same for a citizen at x_L as for a citizen at x_R. Consequently, the number of candidates at x_L is the same as the number of candidates at x_R. Whether a serious equilibrium equivalent to $\{x_L, x_R\}$ exists under V therefore depends on whether the wasting-the-vote effect is eliminated and the barriers to entry by moderates are removed. This, in turn, depends on whether $q > \overline{q}$.

If instead policy preferences are not symmetric, then the expected utility gain from entering the race can, for example, be higher for a citizen at x_L than for a citizen at x_R. This means that, everything else equal, citizens at x_L have stronger incentives to stand for election than the citizens at x_R. Moreover, note that the expected utility gain of a citizen at x_R decreases with the number of candidates standing at x_L (provided the number of candidates at x_L does not exceed q). This is because the probability of being elected the policy-maker decreases with the number of winning candidates. Now, under PV only one candidate stands at x_L, which implies that a candidate at x_R is elected with probability 1/2. By contrast, other MVs can accomodate several candidates standing at x_L. In this case, a candidate at x_R is elected with a probability less than 1/2. We can then have a situation where the candidacy cost is such that a citizen at x_R wants to enter the race if his probability of election is equal to 1/2, but does not want to enter the race if his probability of election is (much) lower than 1/2. No citizen at x_R will then enter the race and no equivalent serious equilibrium exists under V.[23] Key to note is that this can happen only for the most moderate two-position serious equilibria. This is because the utility gain for a citizen at x_R to have x_R instead of x_L implemented increases with the distance between x_L and x_R. Hence the possibility of policy extremism.[24] (See Dellis and Oak 2006 for an example under Approval Voting.)

Observe that while the latter condition is imposed on the environment, the other two conditions are imposed on the voting procedure. These two conditions can therefore be dealt with at the constitutional stage if MVs exist that satisfy these two conditions. It turns out that such MVs do indeed exist. The most famous among these MVs is Approval Voting. We then have the following corollary to Proposition 1.

Corollary 1. *Consider the set of serious equilibria and suppose that policy preferences are symmetric. Then, Approval Voting always yields policy moderation*

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candidates standing at each position. By contrast, if the election is held under Approval Voting (i.e., $q = +\infty$), then the only serious equilibria are the one-position serious equilibria (which we know to be the most moderate serious equilibria).

[23] We emphasize that this can happen whether the barriers to entry by moderates are removed or not. Moreover, asymmetry in policy preferences is necessary for several citizens at x_L to be willing to stand for election while no citizen at x_R wants to enter the race.

[24] Interestingly, both this condition and the condition on the number of votes deal with the issue of multiple similar candidacies. However, the condition on the number of votes is concerned with the consequences on the barriers to entry by moderates while this condition is concerned with the consequences on the barriers to entry at the winning positions.

compared to PV.[25] ‖

Thus, whether we can replace PV with another MV that always yields policy moderation boils down to the empirical question of whether policy preferences are symmetric or not.

3.2. Serious Equilibria under the Ordinal Voting Rules

We now consider elections that are held under an Ordinal Voting rule (hereafter OV). An OV is a scoring rule under which a voter must rank-order the candidates and where the candidate who is ranked h^{th} on the ballot gets s_h points, with $1 = s_1 > s_2 \geq ... \geq s_c = 0$. In case truncated ballots are admissible, a candidate who is not ranked gets no points. Examples of OVs are PV ($s_1 = 1$ and $s_2 = ... = s_c = 0$) and the Borda Count ($s_h = \frac{c-h}{c-1}$ for $h = 1, ..., c$).

The following result can be established.

Proposition 2. *Let V and $\widetilde{V}$ be any two OVs that admit the same type of ballot (i.e., truncated or completely-filled). Denote their second-place scores in three-way races by s and $\widetilde{s}$, respectively. Consider the set of serious equilibria. Then, V always yields policy moderation compared to $\widetilde{V}$ if and only if $s > \widetilde{s}$.* ‖

To appreciate this result, consider the special case where $\widetilde{V}$ is PV (i.e., $\widetilde{s} = 0$). We then have:

Corollary 2. *Consider the set of serious equilibria. Then, OVs (other than PV) always yield policy moderation compared to PV.* ‖

Thus, Corollary 2 provides a theoretical underpinning for the claim that letting people vote for several of the candidates would yield policy moderation compared to PV. Interestingly, no condition (other than the restriction on serious equilibria) is required for this to be true. In particular, this result is robust to an asymmetry in policy preferences. This contrasts with the MVs. The key factor driving this difference between MVs and OVs is that MVs (other than PV) can accomodate multiple similar candidacies, while OVs cannot. This is because a voter can cast up to q one-point votes under a MV, but only one under an OV (i.e., $s_1 > s_2$). Conditions dealing with the issue of multiple similar candidacies are therefore necessary in the case of MVs, but not in the case of OVs.

While all OVs yield policy moderation compared to PV, the extent of policy moderation varies from one OV to another. To be more precise, the extent of policy moderation depends on (1) the second-place score in a three-way race, s, and (2) the admissibility of truncated ballots. We start by studying the effect of the second-place score, s, on policy moderation. Proposition 1 in Dellis (2007b) shows that the serious equilibrium set under an OV contains only one- and two-position serious equilibria. Recall from above that the one-position serious equilibrium set is equivalent under all voting procedures and is moderate compared to the two-position serious equilibria. Two-position serious equilibria are therefore the (only) ones to determine whether an OV yields policy moderation compared to PV. Consider such an equilibrium and suppose that a moderate enters the race. For simplicity, assume

[25] It is worth mentioning that when policy preferences are symmetric, the serious equilibrium set (and, therefore, the extent of policy moderation) is equivalent under all MVs that satisfy conditions (1) and (2) of Proposition 1. A formal proof of this result is available from the author.

that truncated ballots are admissible.[26] The vote total of the moderate is at its lowest if: (1) all the citizens at the median citizen's ideal policy, m, rank the moderate first and then truncate their ballot; and (2) all the citizens on the candidate at x_L (x_R, resp.) of m rank the leftist (rightist, resp.) first and, if the moderate is their most-preferred candidate, they rank the moderate second (otherwise they truncate their ballot). An OV is thus more effective at removing the barriers to entry by moderates and yielding policy moderation, the higher its second-place score in a three-way race is. This is because the minimum vote share of the moderate (and, therefore, his willingness to enter the race) increases with s. The following example illustrates this argument.

Example 2. Consider the community described in Example 1 and assume for simplicity that truncated ballots are admissible.

Let us first suppose that the election is held under an OV with $s = 1/3$. Then, the serious equilibrium set is equivalent to the serious equilibrium set under PV. To see this, suppose that one citizen at 0 and one citizen at 10 stand for election. It is easy to check that neither of them can be better off staying out of the race and that the risk of vote-splitting deters any other citizen at 0 and 10 from entering the race. Moreover, no other citizen is willing to enter the race if he (correctly) anticipates that following his entry in the race the citizens at $0, ..., 4$ $(6, ..., 10$, resp.) will continue top-ranking the candidate at 0 (10, resp.), the citizens at 5 will rank him first and truncate their ballot, and all the other citizens for whom he is a most-preferred candidate will rank him second. He would then get at most 14 points while the candidates at 0 and 10 would get 16 points each. $\{0, 10\}$ can thus be supported as a two-position serious equilibrium. (A similar argument holds for $\{3, 7\}$, $\{2, 8\}$ and $\{1, 9\}$.)

By contrast, only $\{3, 7\}$ and $\{2, 8\}$ can be supported as two-position serious equilibria if the election is held under the Borda Count. To see why $\{0, 10\}$ can no longer be supported as a two-position serious equilibrium, observe that if a citizen at 4 enters the race against a candidate at 0 and another candidate at 10, he can expect to receive at least 17.5 points – one point from every citizen at 5 and half a point from every citizen at 2, 3, 4, 6 and 7. By contrast, a candidate at 0 (10, resp.) receives at most 16 points – one point from every citizen at $0, ..., 4$ $(6, ..., 10$, resp.). It is then easy to check that the citizen at 4 is willing to enter the race, which implies that $\{0, 10\}$ cannot be supported as a serious equilibrium. (A similar argument holds for $\{1, 9\}$.) □

We now study the effect of ballot truncation on policy moderation. For that purpose, we distinguish two types of OVs: those with $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$ that shall be referred as worst-punishing, and the others that shall be referred as best-rewarding.[27] The serious equilibrium set under a best-rewarding OV is more moderate when truncated ballots are admissible than not. By contrast, the serious equilibrium set under a worst-punishing OV (e.g. the Borda Count) is more moderate when only completely-filled ballots are admissible compared to when truncated ballots are ad-

[26] We emphasize that this assumption is made for expositional purposes only. A similar argument applies if only completely-filled ballots are admissible.

[27] We owe the distinction between worst-punishing and best-rewarding scoring rules to Cox (1987). Our distinction is however slightly different since it includes the term $\left(\frac{N-M-2}{N-M-1}\right)$ that is not present in Cox (1987). This term is due to strategic voting.

missible.[28] This differential effect of truncated ballots is related to the issue of multiple similar candidacies. Recall that an OV cannot accomodate multiple similar candidacies. But whether an OV can deter them depends on its second-place score in three-way races, s.

Worst-punishing OVs with only completely-filled ballots admissible cannot deter multiple similar candidacies. Since they cannot accomodate them either, no two-position serious equilibrium exists. The serious equilibrium set consists therefore of only one-position serious equilibria. Admitting truncated ballots cannot therefore yield more moderate policy outcomes since the one-position serious equilibrium set is equivalent under any voting procedure and is moderate compared to the two-position serious equilibrium set.[29] This argument is illustrated most simply for the Borda Count.

Example 3. Consider the community described in Example 1 and suppose that the election is held under the Borda Count. We know from Example 2 that $\{3,7\}$ and $\{2,8\}$ are all two-position serious equilibria when truncated ballots are admissible. This is no longer true however if only completely-filled ballots are admissible. To see this, suppose one citizen at 3 and one citizen at 7 stand for election, and let a second citizen at 3 enter the race. Then all the citizens at $0,...,4$ rank first and second the two candidates at 3, the citizens at 5 abstain from voting since they are indifferent between all three candidates, and all the citizens at $6,...,10$ rank the candidate at 7 first and one of the candidates at 3 second. Consequently, the candidate at 7 gets 16 points while the two candidates at 3 share 32 points. It is then easy to see that in any voting equilibrium, one of the candidates at 3 wins outright. Thus, the entry in the race of a second citizen at 3 increases the probability that 3 is implemented from 1/2 to 1. A second citizen at 3 is therefore willing to enter the race since his expected utility gain (equal to 8) exceeds the candidacy cost. Hence, $\{3,7\}$ cannot be supported as a two-position serious equilibrium. (The same argument applies to $\{2,8\}$.) □

By contrast, best-rewarding OVs can deter multiple similar candidacies. This is because their second-place score, s, is small enough. The extent of policy moderation therefore depends on the effectiveness at eliminating the barriers to entry by moderates. It turns out that OVs are more effective at doing so when truncated ballots are admissible than when only completely-filled ballots are admissible. To see this, consider an election that is held under a best-rewarding OV. Suppose that one citizen at x_L and one citizen at x_R are standing for election and that a moderate enters the race. Recall from above that if truncated ballots are admissible, then the vote total of the moderate is at its lowest if: (1) all the citizens at the median citizens' ideal policy, m, rank the moderate first and then truncate their ballot; and (2) all the citizens on the left (right, resp.) of m rank the candidate at x_L (x_R, resp.) first and, if the moderate is their most-preferred candidate, they rank the moderate second (otherwise they truncate their ballot). If instead only completely-filled ballots are admissible, then the citizen on the left (right, resp.) of

[28] A formal proof of these two results can be found in the Technical Appendix of the working paper.

[29] It is worth mentioning that a unique serious equilibrium exists if the candidacy cost, δ, is relatively small and the election is held under a worst-punishing OV with only completely-filled ballots admissible. In this equilibrium, the median citizen's ideal policy (and, therefore, the Condorcet winner) is implemented with probability one. This provides a theoretical underpinning for the claim sometimes made that the Borda Count elects the Condorcet winner more often than the other commonly-discussed voting rules.

m for whom the moderate is not a most-preferred candidate must add one candidate to their ranking. The vote total of the moderate is then kept at its minimum if these citizens rank the candidate at x_R (x_L, resp.) second. Key to observe is that the minimum vote total of the moderate is identical whether truncated ballots are admissible or not. By contrast, the vote totals of the candidates at x_L and x_R are larger when only completely-filled ballots are admissible. Thus, a moderate can be deterred from entering the race when truncated ballots are admissible only if he can also be deterred from entering the race when only completely-filled ballots are admissible. The reverse is not true however.

To sum up, the Borda Count with only completely-filled ballots admissible is one of the OVs which serious equilibrium set is the most moderate.

3.3. Spoiler Equilibria

We now extend our analysis to spoiler equilibria, i.e., equilibria where some citizens – called spoilers – enter the race (correctly) anticipating that they have no chance of winning the election but that their presence in the race will yield an electoral outcome they prefer to the electoral outcome that would arise otherwise.

The results in Propositions 1 and 2 do not carry to spoiler races. This is because the presence of spoilers in the race creates barriers to entry and barriers to exit by candidates. Intuitively, candidates are deterred from exiting the race and citizens are deterred from entering the race if they (correctly) anticipate that this would trigger a reaction on the part of voters that would yield an electoral outcome they like less. These self-fulfilling prophecies are easier to support in the context of spoiler races than in the context of serious races. This is because there are more candidates in the race and more positions at which candidates are standing. The following example illustrates this argument.

Example 4. Consider the community described in Example 1 and suppose that the election is held under Approval Voting. As mentioned in footnote 22, no two-position serious equilibrium exists. This is because a citizen at 5 would then enter the race, receive a vote from (at least) all the citizens at 4, 5 and 6 (since he would be their most-preferred candidate), and win the election outright. The serious equilibrium set thus consists of $\{4\}$, $\{5\}$ and $\{6\}$, and the extent of policy moderation is maximal.

The extent of policy moderation is however much less substantial in the context of spoiler races. This is because two-position spoiler equilibria do exist. For example, an equilibrium exists where four citizens enter the race, one at 0, 1, 9 and 10, respectively. All the citizens at $0, ..., 5$ ($5, ..., 10$, resp.) vote for the candidate at 1 (9, resp.) and the citizens at 0 (10, resp.) cast a second vote for the candidate at 0 (10, resp.) Thus, the candidates at 1 and 9 tie for first place and no citizen has an incentive to deviate from his voting strategy. Moreover, neither of the candidates at 0 and 1 (9 and 10, resp.) is willing to exit the race and no other citizen at $0, ..., 5$ ($6, ..., 10$, resp.) is willing to enter the race if he anticipates that all citizens would then vote for the candidate at 9 (1, resp.). □

3.4. Comparing Multiple and Ordinal Voting Rules

We start by comparing MVs and OVs in the context of serious races. Taken together, Propositions 1 and 2 imply that OVs always yield policy moderation compared to PV, while MVs do so only if (1) they admit truncated ballots and give citizens enough votes to cast and (2) policy preferences are symmetric. Our next result compares the extent of policy moderation under the different MVs and OVs.

PROPOSITION 3. *Consider the set of serious equilibria and suppose that policy preferences are symmetric. Let V be any MV that admits truncated ballots and for which $q > \overline{q}$, where $\overline{q}$ is defined as in Proposition 1. Then,*

$$WPOV_C \prec V \prec WPOV_T \prec BROV_T \prec BROV_C$$

where $Y \prec Z$ reads as "Y always yields policy moderation compared to Z", WPOV (BROV, resp.) stands for worst-punishing OV (best-rewarding OV, resp.), and the subscript T (C, resp.) stands for truncated (only completely-filled, resp.) ballots admissible. ‖

To understand this result, recall from our discussion of Proposition 2 that the extent of policy moderation in OV elections depends on (1) the second-place score in three-way races and (2) the admissibility of truncated ballots. The former implies that the worst-punishing OVs yield policy moderation compared to the best-rewarding OVs. The latter implies that policy outcomes are more moderate when truncated ballots are admissible if the OV is best-rewarding. The reverse holds true however if the OV is worst-punishing. This is because worst-punishing OVs with only completely-filled ballots admissible can neither deter nor accomodate multiple similar candidacies. Consequently, only one-position serious equilibria exist. The extent of policy moderation is therefore maximal since the one-position serious equilibrium set is equivalent under all voting procedures and is moderate compared to the two-position serious equilibrium set.

It remains to understand why when truncated ballots are admissible and policy preferences are symmetric, a MV with $q > \overline{q}$ yields policy moderation compared to an OV. When truncated ballots are admissible and policy preferences are symmetric, the extent of policy moderation depends on the elimination of the barriers to entry by moderates. MVs with $q > \overline{q}$ are more effective at eliminating these barriers than OVs. This is because under a MV with $q > \overline{q}$, a moderate entering the race receives a one-point vote from every citizen for whom he is a most-preferred candidate. Instead, under an OV, he may receive only a s-point vote from every such citizen. The minimum vote share of a moderate entering the race is thus lower under an OV than under a MV with $q > \overline{q}$. Consequently, a moderate can be deterred from entering the race under a MV with $q > \overline{q}$ only if he can also be deterred from entering the race under an OV. The reverse is not true however. It follows that when policy preferences are symmetric and truncated ballots are admissible, MVs yield policy moderation compared to OVs.

Recall from our earlier discussion that the Borda Count with only completely-filled ballots admissible (hereafter BC) and Approval Voting are, respectively, the OV and the MV that yield the most moderate policy outcomes. In turn, Proposition 3 shows that BC always yields policy moderation compared to Approval Voting. Interestingly, this result is robust to an asymmetry in policy preferences. This is because BC can neither deter nor accomodate multiple similar candidacies. Approval Voting cannot deter multiple similar candidacies either, but it can accomodate them. The serious equilibrium set under BC is therefore robust to an asymmetry in policy preferences since it contains only one-position serious equilibria. This is not true however for the serious equilibrium set under Approval Voting since it can contain two-position serious equilibria with several candidates standing at the same position. Approval Voting can therefore yield more extreme policy outcomes when policy preferences are asymmetric than when policy preferences are symmetric.

The relation between BC and Approval Voting is reversed when we also consider spoiler equilibria. More specifically, BC yields (spoiler equilibrium) policy outcomes that are (1) as extreme as PV and (2) more extreme than Approval Voting.[30] This reversal in the relation between BC and Approval Voting is due to the differential restrictions that weak undominance imposes on the voting behavior. Under Approval Voting, a citizen must cast a one-point vote to every most-preferred candidate. There is no such restriction under BC or PV. As a result, the presence of spoilers at the winning positions can deter moderates from entering the race under BC or PV, but not under Approval Voting. The following example illustrates this argument.

Example 5. Consider a community that must elect a representative to choose a tax rate. Suppose that there are 44 citizens whose ideal tax rates are distributed as follows

Ideal tax rates	0	2	3	4	5	6	7	8	10
Number of citizens	3	3	6	6	8	6	6	3	3

Let the candidacy cost $\delta = 15$. Otherwise, everything else is as in Example 1.

It is not difficult to check that $\{0, 10\}$ is a two-position serious equilibrium under PV and a two-position spoiler equilibrium under BC (with all the citizens at 0 and 10 standing for election). However, $\{0, 10\}$ cannot be supported as an equilibrium under Approval Voting. To see this, assume by way of contradiction that an equilibrium exists that implements 0 and 10, each with an equal probability.[31]

Note first that a candidate at 4, 5 or 6 would be better off exiting the race and saving on the candidacy cost, even if this must lead to the outright election of his least-preferred candidate. Hence, there is no candidate standing at 4, 5 or 6. Likewise, no candidate is standing at 2, 3, 7 or 8. This is because one such candidate would be the most-preferred candidate of a majority of the electorate and would defeat any candidate standing at 0 or 10. Thus, only citizens at 0 and 10 enter the race, which implies that the equilibrium must be serious. But then a citizen at 5 wants to enter the race. This is because he would be a most-preferred candidate for all the citizens at $3, ..., 7$ who would therefore vote for him. He would then get at least 32 votes while a candidate at 0 (10, resp.) would get at most 18 votes – one vote from all the citizens at $0, ..., 4$ $(6, ..., 10$, resp.). Anticipating that he would win outright, a citizen at 5 would then enter the race. Thus, $\{0, 10\}$ cannot be supported as an equilibrium under Approval Voting. □

In summary, the existence of spoiler equilibria is key to determine whether Approval Voting yields more moderate policy outcomes than BC, or vice versa. It is therefore natural to seek conditions under which spoiler equilibria would not exist. Unfortunately, we have not (and, to our knowledge, nobody else has) been able to identify such conditions. We are currently designing a laboratory experiment to test whether the existence of spoiler equilibria does matter for policy moderation or whether serious equilibria tend to be focal. This being said, empirical evidence suggests that spoilers are rather rare in actual political elections. Indeed, few losing candidates are actually standing for election because their candidacy generates

[30] A formal proof is available from the author.

[31] We emphasize that the equal probability assumption is only made for expositional purposes.

self-fulfilling prophecies that lead to an electoral outcome they prefer.[32] While this observation addresses (partially) the issue of spoiler equilibria, it raises another question, namely, how policy moderation depends on the presence of losing candidates with motives other than spoiling the election (e.g. building a base, bringing an issue to the forefront). Answering this question goes obviously beyond the scope of this paper. More complex models with repeated elections and boundedly-rational voters would be necessary to incorporate these other motives into the analysis.

4. A COMPARISON WITH THE DOWNSIAN FRAMEWORK

The present analysis differs from the standard Downsian analysis of Cox (1987, 1990). In this section, we highlight the differences between the two approaches and discuss their implications for our results. Our assumptions differ from those of Cox on three counts:

(1) Strategic candidacy: We endogenize the set of candidates, while Cox assumes an exogenous set of at least three candidates.

(2) Strategic voting: We assume citizens cast a ballot that maximizes their expected utility anticipating the probability each candidate wins the election, while Cox assumes that citizens vote sincerely.

(3) Policy commitment: We assume that candidates are policy-motivated and, therefore, cannot credibly commit to implementing a policy different from their ideal one, while Cox assumes that candidates are purely office-motivated and can therefore credibly commit to implementing any policy if elected.

These differences in assumptions yield important differences in conclusions. For example, Cox (1990) finds that in the context of MV elections, giving a citizen more votes to cast strengthens the centripetal forces of political competition when only completely-filled ballots are admissible, while allowing citizens to truncate their ballot dampens those forces. On the other hand, our model predicts that in the context of serious races, (1) MVs may yield policy extremism when only completely-filled ballots are admissible and (2) admitting truncated ballots yields policy moderation provided that each citizen has enough votes to cast and policy preferences are symmetric. We also find that the centripetal forces of political competition are dampened by the presence of spoilers in the race. The key factor driving this reversal of Cox's results is the differential incentives for candidate entry under alternative voting procedures. To see this, we shall focus on serious MV elections where only completely-filled ballots are admissible.

Replacing PV with another MV would yield policy extremism if candidates were able to credibly commit to the policy they will implement if elected (keeping unchanged our other assumptions). This is because candidates converge towards the median citizen's ideal policy under PV while they need not under the other MVs. To see this, note that under PV, only one candidate stands on a platform since the presence of a second candidate would run the risk of splitting votes, thereby helping the election of rival candidates. But with only one candidate running on each of at most two platforms, the forces of political competition induce each candidate to move closer to the median citizen's ideal policy since by doing so a candidate can capture some of his rival's votes without losing any of his extreme votes. (For a formal proof, see Dellis and Oak 2007.) On the other hand, there is no such

[32] For example, one can be doubtful that Ralph Nader ran for president so that Bush defeats Gore and Kerry.

risk of vote splitting if the election is held under another MV. This implies that in equilibrium, several candidates can be running on the same platform. Candidates can therefore be deterred from moving closer to the median citizen's ideal policy if they anticipate that by doing so they will lose some of their extreme votes. In other words, the possibility of multiple similar candidacies neutralizes the centripetal forces of political competition, thus yielding policy extremism compared to PV. (An example illustrating this discussion can be found in the Technical Appendix of the working paper.)[33]

Replacing PV with another MV would yield policy extremism as well if voting was sincere (keeping unchanged our other assumptions). To understand this result, note that when voting is assumed to be sincere, the key argument behind the claim that letting people vote for several candidates yields policy moderation is the so-called squeezing effect of PV, whereby a centrist candidate is 'squeezed' between a left and a right candidates whose presence in the race creates barriers that absorb the extreme votes. The argument then goes that if we keep the set of candidates fixed and increase the number of votes a citizen casts, then some of the extreme votes will eventually overflow those barriers and reach a centrist candidate. Consequently, increasing the number of votes a citizen casts would improve the electoral prospects of the centrists and yield policy moderation. This argument is no longer true however with an endogenous set of candidates. This is because if voting is sincere and the election is held under a q-MV, then in any two-position serious equilibrium, q candidates are standing at each position. Indeed, more than q candidates running on the same platform would split their votes while less than q candidates would let some extreme votes go to the candidates at the other position, thereby helping the election of rival candidates. Now following the entry by a moderate, the loss of centrist votes is more diluted for the left and right candidates the more such candidates there are. Consequently, increasing the number of votes expands the equilibrium set towards the extreme since the loss of centrist votes is bigger when the two platforms are more polarized. (A formal proof can be found in the Technical Appendix of the working paper.)

5. CONCLUSION

This paper studies whether letting citizens vote for several of the candidates would yield policy moderation compared to Plurality Voting. In contrast to previous contributions, we do so in a setting that takes into account three key features of elections, namely, strategic voting, strategic candidacy and policy-motivation on the part of the candidates. We investigate two ways to let people express their preferences for the different candidates. One way is to give each citizen several equally-weighed votes to cast for the different candidates. The other way is to ask citizens to rank-order the candidates. We call the former voting procedures Multiple Voting rules and the latter Ordinal Voting rules.

Focusing on serious races (i.e., elections where all candidates are serious contenders), we found that Ordinal Voting rules yield policy moderation compared to Plurality Voting. However, the extent of policy moderation depends on the second-place score in a three-way race and the admissibility of truncated ballots. On the other hand, we found that replacing Plurality Voting with a Multiple Voting rule

[33]Dellis and Oak (2007) present a similar argument for Approval Voting, suggesting that the argument is not specific to the case where only completely-filled ballots are admissible.

yields policy moderation provided that (1) truncated ballots are admissible, (2) each citizen has enough votes to cast, and (3) policy preferences are symmetric. However, we can get policy extremism instead of policy moderation if any one of those three conditions is not satisfied! These conditions are required for the Multiple Voting rules but not for the Ordinal Voting rules because the former can accomodate multiple similar candidacies while the latter cannot. Observe that Approval Voting is a Multiple Voting rule that satisfies the first two conditions. Consequently, the existence of a Multiple Voting rule yielding policy moderation compared to Plurality Voting depends on the symmetry of policy preferences. Finally, we found that the presence of spoilers in the race (i.e., citizens who stand for election not to win but because their presence in the race yields an electoral outcome they prefer) creates barriers to entry and exit by candidates, in which case policy moderation need no longer be substantial.

Comparing policy moderation under the different Multiple and Ordinal Voting rules, we found that when all candidates are serious contenders, the extent of policy moderation is maximal under the Borda Count with only completely-filled ballots admissible. However, this result is not robust to the presence of spoilers in the race. Indeed, in the context of spoiler races, the Borda Count may yield more extreme policy outcomes than Approval Voting.

The present analysis has several limitations that deserve further research. First of all, we have assumed that candidates are only policy-motivated. Allowing for a mix of policy- and office-motivation is of interest. However, this extension is not trivial. The complication lies in the discontinuity that comes from the fact that a candidate will no longer be indifferent if another candidate running on the same platform wins the election instead of him. A second shortcoming is the assumption that information is perfect and complete. Relaxing this assumption in the citizen-candidate framework has yet to be done. Moreover, it would be interesting to extend the analysis to multi-dimensional policy spaces and to other electoral systems like, for example, multi-seat or runoff elections.

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APPENDIX

Let us first introduce some extra notation. Define $\mathcal{N}_L \equiv \{\ell \in \mathcal{N} : x_\ell < m\}$ the set of leftists, and $\mathcal{N}_R \equiv \{\ell \in \mathcal{N} : x_\ell > m\}$ the set of rightists. Denote by $\mathcal{M} \equiv \{\ell \in \mathcal{N} : x_\ell = m\}$ the set of citizens who share the same ideal policy as the median citizen. Also, for any non-empty set of candidates $\mathcal{C}$ and every citizen ℓ, let $G^\ell(\mathcal{C}) \equiv \left\{i \in \mathcal{C} : v_i^\ell \geq v_j^\ell \text{ for all } j \in \mathcal{C}\right\}$ be the set of citizen ℓ's most-preferred candidates. Similarly, let $L^\ell(\mathcal{C}) \equiv \left\{i \in \mathcal{C} : v_i^\ell \leq v_j^\ell \text{ for all } j \in \mathcal{C}\right\}$ be the set of citizen ℓ's least-preferred candidates. Finally, let $n_h \equiv \#\{\ell \in \mathcal{N} : x_\ell = x_h\}$ and $c_h \equiv \#\{i \in \mathcal{C} : x_i = x_h\}$ be the numbers of citizens and candidates at x_h, respectively. Also, for simplicity I shall abuse notation and designate an equilibrium as $(\mathcal{C}, \alpha)$ instead of (e, α).

We now state and prove an additional lemma. It shows that when the election is held under an OV or vote truncation is allowed, the one-position serious equilibrium set is moderate compared to the two-position serious equilibrium set.

LEMMA 2. *Let the election be held under V, and suppose that V is either an arbitrary MV with truncated ballots admissible, or an arbitrary OV. Take $(\mathcal{C}_1, \alpha)$ a one-position serious equilibrium, and let x_1 denote the policy outcome. Take $(\mathcal{C}_2, \alpha)$ a two-position serious equilibrium, and let $\{x_L, x_R\}$ be the policy outcome. Then $u^m(x_1) > u^m(x_h)$ for $h = L, R$.* ‖

Proof of Lemma 2. W.l.o.g. suppose $x_1 \leq m$ and $x_L < x_R$. Observe that in any two-position serious equilibrium we must have $v_L^m = v_R^m$ and $x_L < m < x_R$. Assume by way of contradiction that $u^m(x_h) \geq u^m(x_1)$ for $h = L, R$. Hence $x_1 \leq x_L$. Since $(\mathcal{C}_2, \alpha)$ is an equilibrium, it must be that a candidate standing at x_R is better off running for election. It must then be that $-u^R(x_L) \geq \delta$. At the same time $(\mathcal{C}_1, \alpha)$ an equilibrium implies that this citizen would be worse off entering the race against x_1. This cannot be the case if $x_1 = x_L$ since $(\mathcal{C}_2, \alpha)$ is an equilibrium. Hence, it must be that $x_1 < x_L$. But then $u^m(x_R) > u^m(x_1)$, and our citizen at x_R would win outright if he enters the race. It must then be that $-u^R(x_1) < \delta$, which contradicts $-u^R(x_L) \geq \delta$ and $x_1 < x_L$. **Q.E.D.**

We now turn to proving the propositions stated in the text.[34]

Proof of Proposition 1. (Sufficiency) Suppose that the election is held under a MV V with $q \in \mathbb{N}$ and that truncated ballots are admissible.

I first show that the serious equilibrium set under V is a subset of the serious equilibrium set under PV. To do so, observe first that Proposition 2 in Dellis (2007b) implies that in any serious equilibrium under V either $\mathcal{C} = \{i\}$ with $-v_i^m < \delta$, or

[34] For the sake of brevity, some of the details are omitted (they are available from the author). Moreover, we shall maintain two assumptions. One assumption is that the number of citizens on either side of the median citizens' ideal policy—i.e., $\#\mathcal{N}_h$, $h = L, R$—is assumed to be even. The other assumption is that a citizen decides to stand for election when he is indifferent between entering the race or not. Both assumptions are w.l.o.g., and are made only to simplify notation.

$\mathcal{C} = \{i, j\}$ with $x_i < m < x_j$ and $v_i^m = v_j^m$. Moreover, the one-position serious equilibrium set is equivalent under any MV and is moderate compared to the two-position serious equilibrium set (the latter from Lemma 2). Consequently, we only need to show that for any two-position serious equilibrium under V, an equivalent serious equilibrium exists under PV. Pick $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. I am now going to construct $\left(\overline{\mathcal{C}}, \overline{\alpha}\right)$ a two-position serious equilibrium under PV. Suppose that the election is held under PV and construct $\overline{\mathcal{C}} = \{i, j\}$ for some $i, j \in \mathcal{C}$ with $x_i = x_L$ and $x_j = x_R$. Weak undominance, together with $\overline{c} = 2$, implies that in equilibrium, voting is sincere. Moreover, $(\mathcal{C}, \alpha)$ an equilibrium under V implies that $-\frac{c_R}{c\,(c-1)} v_j^i \geq \delta$ and $-\frac{c_L}{c\,(c-1)} v_i^j \geq \delta$; that is, neither candidate i, nor candidate j would be better off not running for election. Now, $\frac{1}{2} \geq \frac{c_h}{c\,(c-1)}$ (for $h = L, R$) and $\overline{c} = 2$ imply that the same holds true under PV when $\overline{\mathcal{C}}$ is the set of candidates. It remains to show that no other citizen wants to run for election. Pick $k \in \left(\mathcal{N} \backslash \overline{\mathcal{C}}\right)$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \left(\overline{\mathcal{C}} \cup \{k\}\right)$. Construct $\overline{\alpha}\left(\widetilde{\mathcal{C}}\right)$ such that: (1) $\overline{\alpha}_i^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_L$, (2) $\overline{\alpha}_j^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_R$, and (3)

$$\begin{cases} \overline{\alpha}_k^\ell\left(\widetilde{\mathcal{C}}\right) = 1 & \text{if } x_k \in (x_L, x_R) \\ \overline{\alpha}_h^\ell\left(\widetilde{\mathcal{C}}\right) = 1 & \text{if } x_k \notin [x_L, x_R]\text{, with } h \in L^k\left(\overline{\mathcal{C}}\right) \\ \overline{\alpha}^\ell\left(\widetilde{\mathcal{C}}\right) = (0, 0, 0) & \text{if } x_k \in \{x_L, x_R\} \end{cases}$$

for all $\ell \in \mathcal{M}$. Hence, $W\left(\widetilde{\mathcal{C}}, \overline{\alpha}\right) = \overline{\mathcal{C}}$ if $x_k \in [x_L, x_R]$ and $W\left(\widetilde{\mathcal{C}}, \overline{\alpha}\right) = \{h\}$ otherwise. This, together with $\delta > 0$, implies $U^k\left(\widetilde{\mathcal{C}}, \overline{\alpha}\right) < U^k\left(\overline{\mathcal{C}}, \overline{\alpha}\right)$. Thus, $\left(\overline{\mathcal{C}}, \overline{\alpha}\right)$ is a two-position serious equilibrium under PV with policy outcome $\{x_L, x_R\}$. Since we took $(\mathcal{C}, \alpha)$ arbitrarily, this holds true for any two-position serious equilibrium under V. Thus, the serious equilibrium set under V is a subset of the serious equilibrium set under PV.

For any x_L,$x_R \in X$, with $v_L^m = v_R^m$ and $n_L, n_R \geq 1$, define $q\,(x_L, x_R)$ as the integer that satisfies

$$\frac{1}{2}\left(-1 - \frac{v_R^L}{2\delta}\right) < q\,(x_L, x_R) \leq \frac{1}{2}\left(1 - \frac{v_R^L}{2\delta}\right)$$

and let $\overline{q}$ be the largest $q\,(x_L, x_R)$. Hence, $\overline{q}$ is the largest number of candidates who would be willing to run on the same platform in a two-position serious equilibrium.

Suppose that $q > \overline{q}$ and that $u^\ell\,(x) = u\,(|x_\ell - x|)$ for all citizen ℓ. I am now going to show that the serious equilibrium set under V is a moderate subset of the serious equilibrium set under PV. That the serious equilibrium set under V is a subset of the serious equilibrium set under PV, together with Lemma 2, implies it is sufficient to show that the two-position serious equilibrium set under V is moderate compared to the two-position serious equilibrium set under PV. To show this, pick $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. Suppose that a two-position serious equilibrium $\left(\overline{\mathcal{C}}, \overline{\alpha}\right)$ with policy outcome $\{\widetilde{x}_L, \widetilde{x}_R\}$ such that $[\widetilde{x}_L, \widetilde{x}_R] \subsetneq [x_L, x_R]$ exists under PV. (Observe that $\overline{c}_L = \overline{c}_R = 1$ and that $\widetilde{x}_L < m < \widetilde{x}_R$ with $\widetilde{v}_L^m = \widetilde{v}_R^m$, where $\widetilde{v}_h^m \equiv u^m\,(\widetilde{x}_h)$ for $h = L, R$. We must show that an equivalent serious equilibrium $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ exists under V. Construct $\widetilde{\mathcal{C}}$ such that (1) $x_i \in \{\widetilde{x}_L, \widetilde{x}_R\}$ for all $i \in \widetilde{\mathcal{C}}$, (2) $\overline{\mathcal{C}} \subseteq \widetilde{\mathcal{C}}$, (3) $-\frac{\widetilde{c}_R}{\widetilde{c}\,(\widetilde{c}-1)} \widetilde{v} \geq \delta$ for all $i \in \widetilde{\mathcal{C}}_L$, where $\widetilde{v} \equiv \widetilde{v}_R^L = \widetilde{v}_L^R$, and a similar condition holds for any $j \in \widetilde{\mathcal{C}}_R$, and (4) there does not exist $i \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ with $x_i = \widetilde{x}_L$ and $-\frac{\widetilde{c}_R}{\widetilde{c}\,(\widetilde{c}+1)}\widetilde{v} \geq \delta$, and a similar condition holds for $j \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ with $x_j = \widetilde{x}_R$. Note that $\widetilde{c}_L = \widetilde{c}_R < q$, the equality by symmetry of preferences and the inequality by $q > \overline{q}$. Moreover, note that for every citizen ℓ either $G^\ell\left(\widetilde{\mathcal{C}}\right) = L^\ell\left(\widetilde{\mathcal{C}}\right)$, or $L^\ell\left(\widetilde{\mathcal{C}}\right) \neq G^\ell\left(\widetilde{\mathcal{C}}\right)$ and $\widetilde{\mathcal{C}} = \left(L^\ell\left(\widetilde{\mathcal{C}}\right) \vee G^\ell\left(\widetilde{\mathcal{C}}\right)\right)$. The same is true for every set of candidates $\left(\widetilde{\mathcal{C}}\backslash\{k\}\right)$, $k \in \widetilde{\mathcal{C}}$. This, together with weak undominance, implies that voting on $\widetilde{\mathcal{C}}$ is sincere. Consequently, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \widetilde{\mathcal{C}}$ since $\widetilde{v}_L^m = \widetilde{v}_R^m$. This, together with condition (3), implies that neither candidate in $\widetilde{\mathcal{C}}$ would be better off not running for election under V. It remains to show that there exists a profile of voting strategies such that no other citizen wants to run for election. Pick $k \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ arbitrarily. Condition (4) implies that citizen k does not want to run for election if $x_k \in \{\widetilde{x}_L, \widetilde{x}_R\}$. Suppose instead that $x_k < \widetilde{x}_L$. Construct $\alpha\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ such that

$$\begin{cases} \alpha_i^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right) = 1 & \text{for all } \ell \in \mathcal{N}_L \text{ and } i \in \widetilde{\mathcal{C}}_L \\ \alpha_k^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right) = 1 & \text{for all } \ell \in \mathcal{N} \text{ with } k \in G^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right) \\ \alpha_j^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right) = 1 & \text{for all } \ell \in (\mathcal{M} \cup \mathcal{N}_R) \text{ and } j \in \widetilde{\mathcal{C}}_R \end{cases}$$

and $\alpha_h^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right) = 0$ otherwise. Hence $W\left(\widetilde{\mathcal{C}} \cup \{k\}, \alpha\right) = \widetilde{\mathcal{C}}_R$, and $U^k\left(\widetilde{\mathcal{C}} \cup \{k\}, \alpha\right) < U^k\left(\widetilde{\mathcal{C}}, \alpha\right)$. Construct a similar profile of voting strategies for $x_k > \widetilde{x}_R$. Finally, suppose $x_k \in (\widetilde{x}_L, \widetilde{x}_R)$. Given that $[\widetilde{x}_L, \widetilde{x}_R] \subsetneq [x_L, x_R]$ and $(\mathcal{C}, \alpha)$ is a two-position serious equilibrium under V, it must be that $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$ and/or there exists α such that $\pi_k(\mathcal{C} \cup \{k\}, \alpha) = \gamma^k < \left(\frac{N-M}{2} - 1\right)$, where $\gamma^k \equiv \#\left\{\ell \in \mathcal{N} : k \in G^\ell(\mathcal{C} \cup \{k\})\right\}$ and $\pi_k(.)$ denotes candidate k's vote total. Since $x_k > \widetilde{x}_L > x_L$ and $u^k(.)$ single-peaked, we have that $\widetilde{v}_L^k > v_L^k$. Similarly, $\widetilde{v}_R^k > v_R^k$. Hence $-\left(\frac{\widetilde{v}_L^k + \widetilde{v}_R^k}{2}\right) < -\left(\frac{v_L^k + v_R^k}{2}\right)$. Moreover, $\widetilde{\gamma}^k \leq \gamma^k$, where $\widetilde{\gamma}^k \equiv \#\left\{\ell \in \mathcal{N} : k \in G^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right)\right\}$. To see this, take a citizen ℓ such that $k \in G^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$. Then $x_\ell \in (\widetilde{x}_L, \widetilde{x}_R)$, and $\widetilde{v}_h^\ell > v_h^\ell$ (for $h = L, R$). Consequently, $\max\left\{\widetilde{v}_L^\ell, \widetilde{v}_R^\ell\right\} > \max\left\{v_L^\ell, v_R^\ell\right\}$. But $k \in G^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ implies that $v_k^\ell \geq \max\left\{\widetilde{v}_L^\ell, \widetilde{v}_R^\ell\right\}$, and thus $v_k^\ell > \max\left\{v_L^\ell, v_R^\ell\right\}$. Hence $G^\ell(\mathcal{C} \cup \{k\}) = \{k\}$. Since we took ℓ arbitrarily, this holds true for any citizen ℓ for whom $k \in G^\ell\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$, and thus $\widetilde{\gamma}^k \leq \gamma^k$. To sum up, $-\left(\frac{\widetilde{v}_L^k + \widetilde{v}_R^k}{2}\right) < \delta$ and/or $\widetilde{\gamma}^k < \left(\frac{N-M}{2} - 1\right)$, implying that there exists α such that $U^k\left(\widetilde{\mathcal{C}} \cup \{k\}, \alpha\right) < U^k(\mathcal{C} \cup \{k\}, \alpha)$. Since $U^k(\mathcal{C} \cup \{k\}, \alpha) \leq U^k(\mathcal{C}, \alpha)$—$(\mathcal{C}, \alpha)$ an equilibrium implies that citizen k does not want to enter the race when the set of candidates is $\mathcal{C}$—and $U^k(\mathcal{C}, \alpha) < U^k\left(\widetilde{\mathcal{C}}, \alpha\right)$—since $\widetilde{v}_h^k > v_h^k$ (for $h = L, R$) while $c_L = c_R$ and $\widetilde{c}_L = \widetilde{c}_R$, the latter by symmetry of preferences—, we have $U^k\left(\widetilde{\mathcal{C}} \cup \{k\}, \alpha\right) < U^k\left(\widetilde{\mathcal{C}}, \alpha\right)$.

Finally, pick α an equilibrium profile of voting strategies for any other set of candidates. Hence, $\left(\widetilde{\mathcal{C}}, \alpha\right)$ is a two-position serious equilibrium under V with policy outcome $\{\widetilde{x}_L, \widetilde{x}_R\}$. This, together with $\widetilde{c}_L = \widetilde{c}_R$, implies $\left(\widetilde{\mathcal{C}}, \alpha\right)$ is equivalent to

$(\overline{\mathcal{C}}, \overline{\alpha})$.

(Necessity) We now prove that each of the three conditions in the statement is necessary for a MV to always yield policy moderation compared to PV. This is done by constructing examples where only one condition does not hold and showing that any MV that does not satisfy this condition yields policy extremism compared to PV. For the sake of brevity, these examples have been omitted here but can be found in the Technical Appendix of the working paper. **Q.E.D.**

Proof of Proposition 2. (Sufficiency) Let V and $\widetilde{V}$ be two OVs and denote by s and $\widetilde{s}$ their respective second-place scores in a three-way race. Suppose that $s > \widetilde{s}$. We shall start by making a couple of observations. First, under an OV a citizen has a unique top-score vote to cast (i.e., $s_1 \neq s_2$). This, together with Proposition 1 in Dellis (2007b), implies that the equilibrium set contains only one- and two-position equilibria. Moreover, the one-position serious equilibrium set is equivalent under any OV. Finally, Lemma 2 shows that under an OV, the one-position serious equilibrium set is moderate compared to the two-position serious equilibrium set. To prove the result, it is therefore sufficient to show that the two-position serious equilibrium set under V is a subset of the two-position serious equilibrium set under $\widetilde{V}$ and is moderate in the sense that if a serious equilibrium with policy outcome $\{x_L, x_R\}$ exists under V, then for every serious equilibrium under $\widetilde{V}$ with policy outcome $\{\widetilde{x}_L, \widetilde{x}_R\}$, with $[\widetilde{x}_L, \widetilde{x}_R] \subsetneq [x_L, x_R]$, an equivalent serious equilibrium exists under V. To do so, we shall distinguish two cases.

Case 1: Only completely-filled ballots are admissible. Before proving the result, we need to characterize the two-position serious equilibrium sets under the different OVs. First, it must be that $\mathcal{C} = \{L, R\}$ with $x_L < m < x_R$ and $v_L^m = v_R^m$. Moreover, equilibrium voting strategies must be sincere. Also, it must be that: (1) neither of the two candidates would be better off not running for election; and (2) no other citizen would be better off entering the race. Necessary and sufficient conditions for the former to hold are $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$. The necessary and sufficient conditions for the latter to hold are given in the next three claims. The first one considers the citizens whose ideal policies are x_L and x_R.

Claim 2.1. *Let the election be held under an OV. There exists an equilibrium profile of voting strategies such that $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$ for any $k \in (\mathcal{N} \backslash \mathcal{C})$ with $x_k \in \{x_L, x_R\}$ if, and only if, $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$.* ‖

Proof of Claim 2.1. W.l.o.g. take $k \in (\mathcal{N} \backslash \mathcal{C})$ with $x_k = x_L$, and let $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$. Observe that Proposition 1 in Dellis (2007b) implies that in any voting equilibrium, $W\left(\widetilde{\mathcal{C}}, \alpha\right) \subsetneq \widetilde{\mathcal{C}}$.

(Necessity) Assume by way of contradiction that $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$ and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$ for some equilibrium profile of voting strategies. First note that $-\frac{v_R^L}{2} \geq \delta$ (i.e., citizen L is willing to stand for election) and $v_R^k = v_R^L$ (since $x_k = x_L$) imply $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$ only if $R \in W\left(\widetilde{\mathcal{C}}, \alpha\right)$, i.e., $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ or $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{h, R\}$ for some $h \in \{k, L\}$. Note also that $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{k, L\}$ ($\{R\}$, resp.) and $L^\ell\left(\widetilde{\mathcal{C}}\right) = \{R\}$ ($\{k, L\}$, resp.) for all $\ell \in \mathcal{N}_L$ ($\mathcal{N}_R$, resp.), and $L^\ell\left(\widetilde{\mathcal{C}}\right) = G^\ell\left(\widetilde{\mathcal{C}}\right)$ for all $\ell \in \mathcal{M}$. Hence, in any voting equilibrium the citizens at the median abstain, while the others vote sincerely. This means that $\pi_R\left(\widetilde{\mathcal{C}},\alpha\right) = \frac{N-M}{2}$ and $\pi_i\left(\widetilde{\mathcal{C}},\alpha\right) \geq \left(\frac{N-M}{2}\right)\left(\frac{1}{2}+s\right)$ for some $i \in \{k, L\}$. It follows that $W\left(\widetilde{\mathcal{C}},\alpha\right) = \{R\}$ only if $\pi_R\left(\widetilde{\mathcal{C}},\alpha\right) > \left[\pi_i\left(\widetilde{\mathcal{C}},\alpha\right) + (1-s)\right]$ for all $i \in \{k, L\}$, which cannot be true if $s \geq \frac{1}{2}\left(\frac{N-M-4}{N-M-2}\right)$. And for $W\left(\widetilde{\mathcal{C}},\alpha\right) = \{h, R\}$ it must be that for $i \in \{k, L\}$, $i \neq h$, $\alpha_i^\ell\left(\widetilde{\mathcal{C}}\right) = s$ for all $\ell \in (\mathcal{N}\backslash\mathcal{M})$—otherwise some voters would be better off permuting their ranking of the two left candidates—, and $\pi_h\left(\widetilde{\mathcal{C}},\alpha\right) = \pi_R\left(\widetilde{\mathcal{C}},\alpha\right) = \frac{N-M}{2}$ and $\pi_i\left(\widetilde{\mathcal{C}},\alpha\right) = (N-M)\,s$. This, together with $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$, implies that $\pi_i\left(\widetilde{\mathcal{C}},\alpha\right) + (1-s) > \pi_R\left(\widetilde{\mathcal{C}},\alpha\right)$. A leftist would then be better off permuting his ranking of the two left candidates. Hence, in any voting equilibrium, $R \notin W\left(\widetilde{\mathcal{C}},\alpha\right)$, and $U^k\left(\widetilde{\mathcal{C}},\alpha\right) \geq U^k(\mathcal{C},\alpha)$ a contradiction.

(Sufficiency) Suppose that $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ such that

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s,1,0) & \text{for all } \ell \in \mathcal{N}_L \\ (s,0,1) & \text{for all } \ell \in \mathcal{N}_R \\ (0,0,0) & \text{for all } \ell \in \mathcal{M} \end{cases}$$

where $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_k^\ell, \alpha_L^\ell, \alpha_R^\ell\right)$. Hence, $W\left(\widetilde{\mathcal{C}},\alpha\right) = W(\mathcal{C},\alpha)$ and neither citizen is willing to deviate from his voting strategy. This, together with $\delta > 0$, implies $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k(\mathcal{C},\alpha)$. ■

The next claim considers the entry by extremists.

Claim 2.2. *Let the election be held under an OV with* $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$. *Then for any citizen* k *with* $x_k \notin [x_L, x_R]$, *there exists an equilibrium profile of voting strategies such that* $U^k(\mathcal{C}\cup\{k\},\alpha) < U^k(\mathcal{C},\alpha)$. ‖

Proof of Claim 2.2. Pick a citizen k with $x_k \notin [x_L, x_R]$, and let $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$. W.l.o.g. suppose $x_k < x_L$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s,1,0) & \text{for all } \ell \in \mathcal{N}_L,\ \ell \neq k \\ (1,s,0) & \text{for } \ell = k \\ (0,s,1) & \text{for all } \ell \in \mathcal{M} \\ (s,0,1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

where $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_k^\ell, \alpha_L^\ell, \alpha_R^\ell\right)$. It is easy to check that $W\left(\widetilde{\mathcal{C}},\alpha\right) = \{R\}$ and that $\alpha\left(\widetilde{\mathcal{C}}\right)$ is a voting equilibrium. It follows that $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k(\mathcal{C},\alpha)$. ■

It remains to consider the entry by moderates. Before doing so, let us introduce some extra notation. For a candidate k define $\Gamma^k(\mathcal{C}) \equiv \left\{\ell \in \mathcal{N} : k \in G^\ell(\mathcal{C})\right\}$ the set of citizens for whom k is a most-preferred candidate, and $\gamma^k(\mathcal{C}) \equiv \#\Gamma^k(\mathcal{C})$ the number of such citizens. Similarly, define $\Gamma_h^k(\mathcal{C}) \equiv \left\{\ell \in \mathcal{N}_h : k \in G^\ell(\mathcal{C})\right\}$ and $\gamma_h^k(\mathcal{C}) \equiv \#\Gamma_h^k(\mathcal{C})$ for $h = L, R$.[35] The next claim provides necessary and sufficient

[35] In order to simplify notation I shall omit $\mathcal{C}$ whenever it is possible to do so without causing a confusion.

conditions for the existence of an equilibrium profile of voting strategies such that no moderate is willing to enter the race.[36]

Claim 2.3. *Let $\mathcal{C} = \{L, R\}$ with $x_L < m < x_R$ and $v_L^m = v_R^m$. Suppose that the election is held under* an OV *with* $s \in \left(0, \frac{1}{2}\frac{N-M-2}{N-M-1}\right]$. *Let k be a citizen with $x_k \in (x_L, m]$. There exists an equilibrium profile of voting strategies $\alpha(.)$ such that $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$:*
(1) if M is even and $s \leq \frac{N-3M-2}{2N-3M-2}$, with the latter inequality strict if there exists a citizen ℓ with $x_\ell \neq m$ and $G^\ell(\mathcal{C} \cup \{k\}) = \{k\}$;
(2) if $\left(\gamma_L^k - \gamma_R^k\right) \geq (M-1)$, where $\gamma_h^k \equiv \gamma_h^k(\mathcal{C} \cup \{k\})$ for $h = L, R$;
(3) and otherwise only if at least one of the following conditions hold: (i) $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$; (ii) $\frac{N-3M-2}{2} > s_R$ and $\left(\frac{v_L^k - v_R^k}{2}\right) < \delta$; or (iii) $\frac{N-3M-2}{2} > s_L$, where $s_h \equiv \left(2\gamma_h^k + \gamma_i^k - \frac{N+M-2}{2}\right) s$ for $h, i \in \{L, R\}$, $h \neq i$. Moreover condition (i) is sufficient. Condition (iii) is sufficient if in addition one of the following hold: (a) $\Gamma_R^k = \emptyset$; (b) $\frac{N-3M-4}{2} > \left(2\gamma_L^k + \gamma_R^k - \frac{N+M+2}{2}\right) s$; or (c) $\frac{1-s}{s} \geq \left(M + \gamma_R^k - \gamma_L^k\right)$ and $\frac{N-3M-2}{2} > \left(\gamma_L^k + 2\gamma_R^k - \frac{N-M+2}{2}\right) s$. Similar conditions can be formulated for condition (ii) if $(M-1) > \left(\gamma_R^k - \gamma_L^k\right)$.[37]

Similar conditions can be formulated for the case where $x_k \in (m, x_R)$. ‖

Proof of Claim 2.3. Pick a citizen k with $x_k \in (x_L, m]$. Define $\widetilde{\mathcal{C}} \equiv (\mathcal{C} \cup \{k\})$. Let $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_L^\ell, \alpha_k^\ell, \alpha_R^\ell\right)$ denote citizen ℓ's voting strategy.

(1) Suppose that M is even and $s \leq \frac{N-3M-2}{2N-3M-2}$, with a strict inequality if there exists a citizen ℓ with $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{k\}$ and $x_\ell \neq m$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (1, s, 0) & \text{for all } \ell \in \mathcal{N}_L \\ (0, s, 1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

and $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \in \{(s, 1, 0), (0, 1, s)\}$ for all $\ell \in \mathcal{M}$ with $\sum_{\ell \in \mathcal{M}} \alpha_h^\ell\left(\widetilde{\mathcal{C}}\right) = \frac{M}{2}s$ (for $h = L, R$). Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \mathcal{C}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies given $\widetilde{\mathcal{C}}$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$.

Suppose from now on that at least one of the conditions in (1) does not hold. It follows that in any voting equilibrium $\#W\left(\widetilde{\mathcal{C}}, \alpha\right) = 1$ (the proof is available from the author).

(2) Suppose $\left(\gamma_L^k - \gamma_R^k\right) \geq (M-1)$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (1, 0, s) & \text{for all } \ell \in \mathcal{N} \text{ with } G^\ell\left(\widetilde{\mathcal{C}}\right) = \{L\} \\ (1, s, 0) & \text{for all } \ell \in \Gamma_L^k \\ (0, 1, s) & \text{for all } \ell \in \mathcal{M} \\ (0, s, 1) & \text{for all } \ell \in \Gamma_R^k \end{cases}$$

[36] While Claim 2.3 does not consider PV (i.e., $s = 0$) it is worth mentioning that $M \leq \frac{N-4}{3}$ is a sufficient condition in that case.

[37] Note that the set of sufficient conditions stated here is not tight. However, if neither of those conditions holds, then in any other set of sufficient conditions either condition (b) must hold with an equality, or condition (c) must hold with the second inequality replaced with an equality (the proof is available from the author). In any case, the proof of Proposition 2 below would still hold if we include those conditions.

and let $\alpha^{\ell}\left(\widetilde{\mathcal{C}}\right)=(0,s,1)$ for $\left(\gamma_L^k-\gamma_R^k-M+2\right)$ of the citizens for whom $G^{\ell}\left(\widetilde{\mathcal{C}}\right)=\{R\}$ and $(s,0,1)$ for the other ones. Note that $\left[\pi_R\left(\widetilde{\mathcal{C}},\alpha\right)-\pi_L\left(\widetilde{\mathcal{C}},\alpha\right)\right]=2s$ and $\left[\pi_R\left(\widetilde{\mathcal{C}},\alpha\right)-\pi_k\left(\widetilde{\mathcal{C}},\alpha\right)\right]>(1+s)$, the latter following from $\left(3\gamma_L^k+3-\frac{N+3M}{2}\right)<\frac{N-3M-2}{2}$ (the proof is available from the author). Hence, $W\left(\widetilde{\mathcal{C}},\alpha\right)=\{R\}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies, and $U^k\left(\widetilde{\mathcal{C}},\alpha\right)<U^k(\mathcal{C},\alpha)$ since $R\in L^k(\mathcal{C})$ and $\delta>0$.

(3) Suppose $\left(\gamma_L^k-\gamma_R^k\right)<(M-1)$. I first prove the necessity part. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as above but with $\alpha^{\ell}\left(\widetilde{\mathcal{C}}\right)=(s,0,1)$ for all citizen ℓ for whom $G^{\ell}\left(\widetilde{\mathcal{C}}\right)=\{R\}$. Hence, $\alpha\left(\widetilde{\mathcal{C}}\right)$ is the profile of weakly undominated voting strategies that maximizes $\left[\pi_R\left(\widetilde{\mathcal{C}},\alpha\right)-\pi_k\left(\widetilde{\mathcal{C}},\alpha\right)\right]$. If condition (iii) is not satisfied this difference is lower than $(1+s)$, and either k wins outright or citizen L wants to deviate and cast a ballot $(s,1,0)$; that is, $W\left(\widetilde{\mathcal{C}},\alpha\right)\in\{\{k\},\{L\}\}$ in any voting equilibrium. Now, suppose that condition (ii) does not hold either. Either $\left(\frac{v_L^k-v_R^k}{2}\right)\geq\delta$, in which case condition (i) does not hold and $U^k\left(\widetilde{\mathcal{C}},\alpha\right)\geq U^k(\mathcal{C},\alpha)$. Or $s_R\geq\frac{N-3M-2}{2}$, in which case it cannot be that candidate L wins outright. To see why, construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as above, replacing $\alpha^{\ell}\left(\widetilde{\mathcal{C}}\right)=(0,1,s)$ with $(s,1,0)$ for all $\ell\in\mathcal{M}$. Hence, $\alpha\left(\widetilde{\mathcal{C}}\right)$ is the profile of weakly undominated voting strategies that maximizes $\left[\pi_L\left(\widetilde{\mathcal{C}},\alpha\right)-\pi_k\left(\widetilde{\mathcal{C}},\alpha\right)\right]$. But this difference is smaller than $(1+s)$ (given the assumption on s_R), and either k wins outright or citizen R wants to deviate and cast a ballot $(0,1,s)$ since then candidate k would either tie with or defeat candidate L. Hence, if neither condition (ii), nor condition (iii) holds, then in any voting equilibrium $W\left(\widetilde{\mathcal{C}},\alpha\right)=\{k\}$. And if condition (i) does not hold either, then $U^k\left(\widetilde{\mathcal{C}},\alpha\right)\geq U^k(\mathcal{C},\alpha)$.

Condition (i) implies that in any voting equilibrium $U^k\left(\widetilde{\mathcal{C}},\alpha\right)<U^k(\mathcal{C},\alpha)$, and is thus sufficient. Condition (iii) is sufficient as well if the additional conditions stated hold since no $\ell\in\mathcal{N}_R$ for whom $G^{\ell}(\mathcal{C})=\{k\}$ wants to deviate from his voting strategy, either because there is no such citizen—(a)—, or he is not pivotal for candidate k—(b)—, or candidate L would be winning—(c). Similarly for condition (ii) if $(M-1)>\left(\gamma_R^k-\gamma_L^k\right)$, the latter condition for candidate L to defeat candidate R.[38] ■

We are now ready to prove Proposition 2 for the case where only completely-filled ballots are admissible. From Claim 2.1 we know that if $s>\frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$ the set of two-position serious equilibria under V is empty, and the result holds trivially. Suppose from now on that $s\leq\frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$.

Let us first show that the two-position serious equilibrium set under V is a subset of the two-position serious equilibrium set under $\widetilde{V}$. Take $(\mathcal{C},\alpha)$ a two-

[38] Note that this condition is necessarily satisfied if condition (iii) does not hold.

position serious equilibrium under V, and call L and R the two candidates. I am going to construct $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ an equivalent serious equilibrium under $\widetilde{V}$. Let $\widetilde{\mathcal{C}} \equiv \mathcal{C}$ and $\widetilde{\alpha}\left(\widetilde{\mathcal{C}}\right) \equiv \alpha(\mathcal{C})$. Since $(\mathcal{C}, \alpha)$ is a two-position serious equilibrium under V, it must be that neither candidate would be better off not running; that is, $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$. Pick $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ arbitrarily, and let $\overline{\mathcal{C}} \equiv \{L, R, k\}$. We need to construct $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ an equilibrium profile of voting strategies such that $U^k\left(\overline{\mathcal{C}}, \widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$. Claims 2.1 and 2.2, together with $s > \widetilde{s}$, imply that such an equilibrium profile of voting strategies exists if $x_k \notin (x_L, x_R)$. Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C}, \alpha)$ is a serious equilibrium under V, there must exist an equilibrium profile of voting strategies $\alpha\left(\overline{\mathcal{C}}\right)$ such that $U^k\left(\overline{\mathcal{C}}, \alpha\right) < U^k\left(\widetilde{\mathcal{C}}, \alpha\right)$. And given that $\widetilde{s} < s$ and $M \leq \frac{N-4}{3}$ the same holds true under $\widetilde{V}$. Hence, for any $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ there exists $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ such that $U^k\left(\overline{\mathcal{C}}, \widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$. Finally, for any other set of candidates take any equilibrium profile of voting strategies. Hence, $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ is a two-position serious equilibrium under $\widetilde{V}$ and is equivalent to $(\mathcal{C}, \alpha)$.

Let us now show that the two-position serious equilibrium set under V is moderate compared to the two-position serious equilibrium set under $\widetilde{V}$. To do so, it is sufficient to show that the two-position serious equilibrium set under any OV is moderate compared to the two-position serious equilibrium set under PV. Let V be an OV and suppose $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. Assume that a two-position serious equilibrium $\left(\widehat{\mathcal{C}}, \widehat{\alpha}\right)$ exists under PV, with policy outcome $\{\widehat{x}_L, \widehat{x}_R\}$ such that $[\widehat{x}_L, \widehat{x}_R] \subsetneq [x_L, x_R]$. I am going to construct $\left(\widehat{\mathcal{C}}, \alpha\right)$ an equivalent serious equilibrium under V. Let $\alpha\left(\widehat{\mathcal{C}}\right) \equiv \widehat{\alpha}\left(\widehat{\mathcal{C}}\right)$. Since $\left(\widehat{\mathcal{C}}, \widehat{\alpha}\right)$ is an equilibrium, it must be that neither candidate would be better off not running that is, $-\frac{\widehat{v}_R^L}{2} \geq \delta$ and $-\frac{\widehat{v}_L^R}{2} \geq \delta$ where $\widehat{v}_R^L = u(\widehat{x}_L - \widehat{x}_R)$ and $\widehat{v}_L^R = u(\widehat{x}_R - \widehat{x}_L)$.

Pick $k \in \left(\mathcal{N} \backslash \widehat{\mathcal{C}}\right)$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \left(\widehat{\mathcal{C}} \cup \{k\}\right)$. Claims 2.1 and 2.2 imply that if $x_k \notin (\widehat{x}_L, \widehat{x}_R)$, then a voting equilibrium $\alpha\left(\widetilde{\mathcal{C}}\right)$ exists such that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\widehat{\mathcal{C}}, \alpha\right)$. Suppose instead that $x_k \in (\widehat{x}_L, \widehat{x}_R)$. W.l.o.g. let $x_k \in (\widehat{x}_L, m]$. Since $(\mathcal{C}, \alpha)$ is a two-position serious equilibrium under V, there must exist an equilibrium profile of voting strategies $\alpha(\mathcal{C} \cup \{k\})$ such that $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$. It can then be shown that the same holds true under $\widehat{\mathcal{C}}$ (the proof is available from the author). Hence, for any $k \in \left(\mathcal{N} \backslash \widehat{\mathcal{C}}\right)$ an equilibrium profile of voting strategies exists such that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\widehat{\mathcal{C}}, \alpha\right)$.

Finally, for any other set of candidates take any equilibrium profile of voting strategies.

Hence, $\left(\widehat{\mathcal{C}}, \alpha\right)$ is a two-position serious equilibrium under V which is equivalent to $\left(\widehat{\mathcal{C}}, \widehat{\alpha}\right)$.

Case 2: Truncated ballots are admissible. The characterization of the two-position serious equilibrium set is identical to the one where only completely-filled ballots are admissible, except that Claims 2.1-2.3 are replaced with the following claim.

Claim 2.4. *Let the election be held under an OV, and suppose that citizens are allowed to truncate their ballot. Then, an equilibrium profile of voting strategies exists such that* $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$ *for any* $k \in (\mathcal{N} \backslash \mathcal{C})$ *with* $x_k \notin (x_L, x_R)$*. The same holds true for any citizen* k *with* $x_k \in (x_L, x_R)$ *if, and only if, at least one of the following two conditions holds: (1)* $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$*; or (2)* $\left(\gamma_L^k + \gamma_R^k\right) s \leq \frac{N-3M-2}{2}$*.* ‖

Proof of Claim 2.4. Let $\mathcal{C} = \{L, R\}$ be a set of candidates with $x_L < m < x_R$ and $v_L^m = v_R^m$. Pick $k \in (\mathcal{N} \backslash \mathcal{C})$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$.

First consider the case where $x_k \notin [x_L, x_R]$. W.l.o.g. suppose $x_k < x_L$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s,1,0) & \text{for all } \ell \in \mathcal{N}_L \text{ with } k \in G^\ell\left(\widetilde{\mathcal{C}}\right) \\ (0,1,0) & \text{for all } \ell \in \mathcal{N}_L \text{ with } k \notin G^\ell\left(\widetilde{\mathcal{C}}\right) \\ (0,s,1) & \text{for all } \ell \in \mathcal{M} \\ (0,0,1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

where $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \left(\alpha_k^\ell, \alpha_L^\ell, \alpha_R^\ell\right)$. Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies. This means that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$.

Now consider the case where $x_k \in \{x_L, x_R\}$. W.l.o.g. suppose $x_k = x_L$. First note that in any voting equilibrium, $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = (0,0,0)$ for all $\ell \in \mathcal{M}$ since $G^\ell\left(\widetilde{\mathcal{C}}\right) = L^\ell\left(\widetilde{\mathcal{C}}\right)$. Moreover, $\alpha_R^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ and $\alpha_k^\ell\left(\widetilde{\mathcal{C}}\right) = \alpha_L^\ell\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in \mathcal{N}_R$ since $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{R\}$ and $L^\ell\left(\widetilde{\mathcal{C}}\right) = \{k, L\}$. Similarly, $\alpha_h^\ell\left(\widetilde{\mathcal{C}}\right) \in \{s, 1\}$ (for $h = k, L$) and $\alpha_R^\ell\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in \mathcal{N}_L$. This means that in any voting equilibrium, $\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) = \frac{N-M}{2}$, while $\pi_h\left(\widetilde{\mathcal{C}}, \alpha\right) \leq \frac{N-M}{2}$ for $h = k, L$, with the inequality strict for at least one of those two candidates. Hence, either $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$, or $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{h, R\}$ for some $h \in \{k, L\}$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$.

It remains to consider the case where $x_k \in (x_L, x_R)$. Let us first prove the necessity part. Assume by way of contradiction that there exists an equilibrium profile of voting strategies $\alpha\left(\widetilde{\mathcal{C}}\right)$ such that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$, but that neither of the two conditions hold. Weak undominance implies that $\alpha_L^\ell\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in (\mathcal{M} \cup \mathcal{N}_R)$, $\alpha_R^\ell\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in (\mathcal{M} \cup \mathcal{N}_L)$, and $\alpha_k^\ell\left(\widetilde{\mathcal{C}}\right) \in \{s, 1\}$ for all citizen ℓ for whom $k \in G^\ell\left(\widetilde{\mathcal{C}}\right)$ with $\alpha_h^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{M}$. Hence, in any voting equilibrium $\pi_k\left(\widetilde{\mathcal{C}}, \alpha\right) \geq \left[M + \left(\gamma_L^k + \gamma_R^k\right) s\right]$ and $\pi_h\left(\widetilde{\mathcal{C}}, \alpha\right) \leq \frac{N-M}{2}$, for $h = L, R$. Now, since condition (2) does not hold, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{k\}$ in any voting equilibrium. And since condition (1) does not hold, $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) \geq U^k(\mathcal{C}, \alpha)$, a contradiction.

Let us now prove the sufficiency part. Suppose condition (1) holds. Then,

$U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k(\mathcal{C},\alpha)$ even if $W\left(\widetilde{\mathcal{C}},\alpha\right) = \{k\}$. Suppose condition (2) holds, and construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows: (i) $\alpha_h^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_h$, for $h = L, R$; (ii) $\alpha_k^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{M}$; (iii) $\alpha_k^\ell\left(\widetilde{\mathcal{C}}\right) = s$ for all $\ell \in (\mathcal{N}\backslash\mathcal{M})$ for whom $k \in G^\ell\left(\widetilde{\mathcal{C}}\right)$; and (iv) $\alpha_j^\ell\left(\widetilde{\mathcal{C}}\right) = 0$ otherwise. Hence, $W\left(\widetilde{\mathcal{C}},\alpha\right) = W(\mathcal{C},\alpha)$, and $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k(\mathcal{C},\alpha)$. ■

We are now ready to prove Proposition 2 for the case where truncated ballots are admissible. Let V and $\widetilde{V}$ be any two OVs with $s > \widetilde{s}$. Take $(\mathcal{C},\alpha)$ a two-position serious equilibrium under V, and call L and R the two candidates. We are now going to construct $(\mathcal{C},\widetilde{\alpha})$ an equivalent serious equilibrium under $\widetilde{V}$. Let $\widetilde{\alpha}(\mathcal{C}) \equiv \alpha(\mathcal{C})$. Given that $\mathcal{C}$ is an equilibrium set of candidates under V, we have $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$—i.e., neither candidate is better off not entering the race. Obviously, those two conditions hold as well under $\widetilde{V}$. Take $k \in (\mathcal{N}\backslash\mathcal{C})$, and let $\overline{\mathcal{C}} \equiv \{k, L, R\}$. By Claim 2.4 we know that if $x_k \notin (x_L, x_R)$, then an equilibrium profile of voting strategies exists such that $U^k\left(\overline{\mathcal{C}},\widetilde{\alpha}\right) < U^k(\mathcal{C},\widetilde{\alpha})$. Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C},\alpha)$ is an equilibrium under V, condition (1) and/or condition (2) in Claim 2.4 must hold. If condition (1) holds, then we are done since this condition does not depend on s. If condition (2) holds, then so does it under $\widetilde{V}$ since $\widetilde{s} < s$. It follows that for any $k \in (\mathcal{N}\backslash\mathcal{C})$ an equilibrium profile of voting strategies $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ exists such that $U^k\left(\overline{\mathcal{C}},\widetilde{\alpha}\right) < U^k(\mathcal{C},\widetilde{\alpha})$. For any other set of candidates, take any equilibrium profile of voting strategies. Hence, $(\mathcal{C},\widetilde{\alpha})$ is a serious equilibrium under $\widetilde{V}$ and is equivalent to $(\mathcal{C},\alpha)$.

Given that condition (2) in Claim 2.4 holding under $\widetilde{V}$ does not imply that it holds under V as well, we have proved the subset part of the result. The moderation part follows directly from the fact that it is only for moderates that there may not exist an equilibrium profile of voting strategies that deters them from entering the race.

(Necessity) Suppose that V always yields policy moderation compared to $\widetilde{V}$. Assume by way of contradiction that $s \leq \widetilde{s}$. The sufficiency part implies a contradiction if $s < \widetilde{s}$. It is not difficult to check that the serious equilibrium set under V is *always* equivalent to the serious equilibrium set under $\widetilde{V}$ if $s = \widetilde{s}$, which yields a contradiction. **Q.E.D.**

Proof of Proposition 3. Given Propositions 1 and 2 and the discussion in the text, we only need to show that the serious equilibrium set under V is a moderate subset of the serious equilibrium set under an OV where truncated ballots are admissible. Take $(\mathcal{C},\alpha)$ a two-position serious equilibrium under V. Let $\widetilde{V}$ be an OV and suppose that truncated ballots are admissible. We are now going to construct $\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$ a serious equilibrium under $\widetilde{V}$ which is equivalent to $(\mathcal{C},\alpha)$. Pick $i, j \in \mathcal{C}$ with $x_i = x_L$ and $x_j = x_R$, and construct $\widetilde{\mathcal{C}} = \{i, j\}$. Given $\widetilde{\mathcal{C}}$ there is a unique voting equilibrium under $\widetilde{V}$, with $W\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right) = \widetilde{\mathcal{C}}$. Since $(\mathcal{C},\alpha)$ is an equilibrium under V and preferences are symmetric, then $c_L = c_R$ and $\frac{-v_j^i}{2(c-1)} \geq \delta$ and $\frac{-v_i^j}{2(c-1)} \geq \delta$—i.e., neither candidate in $\mathcal{C}$ is better off not running for election under V. Given that $c \geq 2$, the same holds true for candidates i and j under $\widetilde{V}$. It remains to show that for any $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ there exists an equilibrium profile of voting strategies $\widetilde{\alpha}\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ such that $U^k\left(\widetilde{\mathcal{C}} \cup \{k\}, \widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$. Pick $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ arbitrarily. If $x_k \notin (x_L, x_R)$, then we are done (by Claim 2.4). Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C}, \alpha)$ is a serious equilibrium and that $c_L = c_R < q$, either $-\left(\frac{v_i^k + v_j^k}{2}\right) < \delta$, or $\left(\gamma_L^k + \gamma_R^k\right) < \frac{N-3M-2}{2}$ (or both). But then one of the two conditions of Claim 2.4 holds, and we are done. Now for any $\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ pick one such $\widetilde{\alpha}\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$, and for any other set of candidates $\widehat{\mathcal{C}}$ let $\widetilde{\alpha}\left(\widehat{\mathcal{C}}\right)$ be any profile of voting strategies. It then follows that $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ is an equilibrium under $\widetilde{V}$, which is equivalent to $(\mathcal{C}, \alpha)$. Hence the subset part of the result.

It is only when there exists a citizen k with $x_k \in (x_L, x_R)$ for whom $\left(\gamma_L^k + \gamma_R^k\right) \geq \frac{N-3M-2}{2} \geq \left(\gamma_L^k + \gamma_R^k\right) s$ that there may not exist a serious equilibrium under V which is equivalent to a two-position serious equilibrium under $\widetilde{V}$. Hence the moderation result since $\left(\gamma_L^k + \gamma_R^k\right)$ increases with the distance between x_L and x_R. **Q.E.D.**