Explaining Cyclical Unemployment with Downward Sloped Labor Demand*

Michael W. L. Elsby
University of Michigan and NBER

Ryan Michaels
University of Michigan

October 10, 2007

Abstract

In this paper, we develop a novel generalization of the standard search model of the aggregate labor market which allows for short run downward–sloped labor demand and endogenous job destruction. We show that this generalized model is able to match the cyclicality of job finding and employment to unemployment flows observed in US data. Contrary to the standard search model, the generalized model can match the data whilst maintaining realistic surplus to employment relationships. In addition, we uncover a novel source of amplification of cyclical shocks that is generated by the interaction of firm heterogeneity and downward–sloped labor demand.

*Preliminary and incomplete. We are especially grateful to Gary Solon for his comments, support, and encouragement. We would also like to thank Shigeru Fujita, and William Hawkins, as well as seminar participants at the UM/MSU/UWO Labor Day, May 2007, the European Central Bank/Center for Financial Studies/Bundesbank Joint Lunchtime Seminar and the Kansas City Fed Workshop on "Microfoundations of Labor Markets and Implications for Aggregate Employment Dynamics" for helpful comments. All errors are our own. E–mail address for correspondence: elsby@umich.edu.

1 Introduction

This paper develops a novel generalization of the modern workhorse model of the aggregate labor market, the Mortensen-Pissarides (MP) search and matching model, and presents results which suggest that this aids the standard model's limited ability to explain the cyclical volatility of unemployment (Shimer, 2005a). The motivation for our generalization is based on two simple observations. First, the standard MP model makes the simplifying assumption that the demand for labor is infinitely elastic, even in the short run. This paper makes the simple point that, for the analysis of short run fluctuations in unemployment, the demand for labor is arguably downward-sloped, for the conventional reason that other production inputs, notably capital, are not fully flexible in the short run. Second, we incorporate endogenous job destruction into the model. This is informed by empirical evidence that a significant fraction of the cyclical upswing in unemployment in times of recession is accounted for by increased inflows into unemployment, especially layoffs (Perry, 1972; Marston, 1976; Blanchard & Diamond, 1990; Elsby, Michaels, & Solon, 2007; Fujita & Ramey, 2007; Pissarides, 2007; Yashiv, 2006).

Incorporating these two conventional features simultaneously is not a trivial exercise. We show, however, that it is also not a daunting one. In particular, downward sloped labor demand implies that firms face a non-linear production technology. This raises a number of theoretical issues. First, it complicates the wage bargaining solution in such models because the surplus generated by each of the employment relationships within a firm is not the same (e.g. "the" marginal worker generates less surplus than infra-marginal workers). We derive a very intuitive and explicit wage bargaining solution, something that has been considered challenging in recent research (see Acemoglu & Hawkins, 2006; Cooper, Haltiwanger & Willis, 2007; and Hobijn & Sahin, 2007). The simplicity of our solution is therefore a useful addition to the literature.[1]

[1] In particular, recent models with endogenous separations such as Cooper et al. (2007) and Hobijn &

Given our wage bargaining solution, we are then able to characterize the properties of the optimal labor demand policy of an individual firm in the presence of idiosyncratic firm heterogeneity. An interesting by-product of this exercise is that the optimal labor demand solution in the generalized model is analogous to that of a model of kinked hiring costs in the spirit of Bentolila & Bertola (1990), but where the hiring cost is endogenously determined by frictions in the labor market. Thus, the correspondence between the two major approaches to the economics of aggregate labor markets – search and matching models and employment adjustment cost models – sharpens in the process of generalizing the standard search model.

A second difficulty of incorporating a non-linear production technology in the presence of idiosyncratic heterogeneity is that aggregation of microeconomic behavior is not straightforward, because a representative firm interpretation of the model doesn't exist. We make progress on this by developing an analytical result for the aggregation of the behavior of individual firms that holds for a wide class of optimal labor demand policies at the microeconomic level. In particular, we are able to solve for the equilibrium distribution of employment across firms, which allows us to determine the level of aggregate employment in the model economy.

We then present illustrative simulation results which indicate that this generalized model can closely match the observed cyclical variation of both the job finding rate and the employment to unemployment transition rate in the US, a substantial improvement on the standard MP model. To uncover the processes underlying this result, we derive a simple method of approximating the decline in job creation following an adverse aggregate shock in the generalized model, analogous to the method first employed by Mortensen & Nagypal (2007). This exercise reveals two sources of amplification. The first generalizes a well-known result that

Sahin (2007) have set worker bargaining power to zero in order to derive wages. Acemoglu & Hawkins (2006) characterize wages in a model with exogenous separations, but with a time to hire aspect to job creation, which substantially complicates the solution. Our solution is analogous to the wage bargaining solution derived by Cahuc & Wasmer (2001) for the model with exogenous separations.

the standard MP model is consistent with observed unemployment cyclicality if the average flow surplus to employment relationships is sufficiently small.[2] We show that an analogous result occurs in the generalized model if a weighted average of the average and *marginal* flow surplus is sufficiently small. However, because downward sloped labor demand implies that the marginal surplus will be smaller than the average surplus, the generalized model can deliver amplification of the job creation response to aggregate shocks at the same time as preserving a sizeable average surplus from employment relationships. This is important because, as Mortensen & Nagypal (2007) show, the average surplus required for the standard model to match the observed cyclicality of the job finding rate is implausibly small, in the sense that the average worker obtains only a 2.3% surplus from employment.

Our results also suggest a second, more novel source of amplification that is generated by the interaction of heterogeneous firms and downward-sloped labor demand. We show that, in the same way that the standard MP model can be thought of in terms of a single representative firm, the generalized model can be thought of as comprising *two* firms for the purposes of determing the cyclical amplitude of job creation: A hiring firm and a shedding firm. Following a reduction in aggregate productivity, shedding firms wish to shed more workers, and hiring firms wish to hire fewer workers. Thus inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, causing unemployment to rise. To restore equilibrium in the model, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the labor market to slacken, so that unemployed workers become more abundant, and hiring (suitable) workers becomes less costly for firms. In the standard MP model with infinitely elastic labor demand, the representative firm responds fully by hiring more workers. With heterogeneous firms and downward-sloped labor demand, however, only a fraction of firms (hiring firms) will respond to these reduced hiring costs by hiring more workers. Thus, the

[2] See Costain & Reiter (2005), Hagedorn & Manovskii (2005), and Mortensen & Pissarides (1994).

labor market must slacken further, and unemployment must rise more, in order to return the economy back to equilibrium once again.

The remainder of this paper is organized as follows. Section 2 characterizes the microeconomic behavior of firms in the presence of idiosyncratic heterogeneity, kinked search costs, and surplus sharing. Section 3 then develops a general method of aggregating the microeconomic behavior of firms and derives the steady state aggregate employment stock and flows. Section 4 presents illustrative simulation results which suggest that the generalized model can account for the cyclicality of job finding and employment to unemployment flows. Section 5 examines the sources of amplification that account for the simulation results of section 4. Finally, section 6 concludes and discusses future research directions.

2 A Firm's Optimization Problem

In what follows we consider a model in which there is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L. In order to hire unemployed workers, firms must post vacancies. However, frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet. As is conventional in the search and matching literature, these frictions are embodied in a matching function, $M = M(U, V)$, that regulates the number of hires, M, that the economy can sustain given that there are V vacancies and U unemployed workers. We assume that $M(U, V)$ exhibits constant returns to scale.[3] Vacancies posted by firms are therefore filled with probability $q = M/V = M(U/V, 1)$ each period. Likewise, unemployed workers find jobs with probability $f = M/U = M(1, V/U)$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, is a sufficient statistic for the job filling (q) and job finding (f) probabilities in the model. Taking these flow probabilities as given, firms choose their optimal level of

[3] See Petrongolo & Pissarides (2001) for a summary of empirical evidence that suggests this is reasonable.

employment, to which we now turn.

2.1 Labor Demand

We consider a discrete time, infinite horizon model in which firms use labor, n, to produce output according to the production function, $y = pxF(n)$ where $F' > 0$ and $F'' \leq 0$. The latter is a key generalization of the standard MP model that we consider: When $F'' < 0$, the marginal product of labor will decline with firm employment, and thereby will generate a downward-sloped demand for labor at the firm level. p represents the state of aggregate labor demand, whereas x represents shocks that are idiosyncratic to an individual firm. We assume that the evolution of the latter idiosyncratic shocks is described by the c.d.f. $G(x'|x)$.

A typical firm's decision problem is completely analogous to that in Mortensen & Pissarides (1994), and is as follows. Firms observe the realization of their idiosyncratic shock, x, at the beginning of a period. Given this, they then make their employment decision. Specifically, they may choose to separate from part or all of their workforce, which we assume may be done at zero cost. Any such separated workers then join the unemployment pool in the subsequent period. Alternatively, firms may hire workers by posting vacancies, $v \geq 0$, at a flow cost of c per vacancy. If a firm posts vacancies, the matching process then matches these up with unemployed workers inherited from the previous period. After the matching process is complete, production and wage setting are performed simultaneously.

It follows that we can characterize the expected present discounted value of a firm's profits, $\Pi(n_{-1}, x)$, recursively as:[4]

$$\Pi(n_{-1}, x) = \max_{n} \left\{ pxF(n) - w(n, x)\, n - cv + \beta \int \Pi(n, x')\, dG(x'|x) \right\} \qquad (1)$$

[4] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

where $w(n,x)$ is the bargained wage in a firm of size n and productivity x. A typical firm seeks a level of employment that maximizes its profits subject to a dynamic constraint on the evolution of a firm's employment level. Specifically, firms face frictions that limit the rate at which vacancies may be filled: A vacancy posted in a given period will be filled with probability $q < 1$ prior to production. Thus, the number of hires an individual firm achieves is given by:

$$\Delta n \mathbf{1}^{+} = qv \tag{2}$$

where Δn is the change in employment, and $\mathbf{1}^{+}$ is an indicator that equals one when the firm is hiring, and zero otherwise. Substituting the constraint, (2), into the firm's value function, we obtain:

$$\Pi(n_{-1},x) = \max_{n} \left\{ pxF(n) - w(n,x)n - \frac{c}{q}\Delta n \mathbf{1}^{+} + \beta \int \Pi(n,x') \, dG(x'|x) \right\} \tag{3}$$

Note that the value function is not fully differentiable in n: There is a kink in the value function around $n = n_{-1}$. This reflects the (partial) irreversibility of separation decisions in the model. Whilst firms can shed workers costlessly, it is costly to reverse such a decision because hiring (posting vacancies) is costly. In this sense, the labor demand side is formally analogous to the kinked employment adjustment cost model of the form analyzed in Bentolila & Bertola (1990), except that the per–worker hiring cost, $c/q(\theta)$, is endogenously determined.

In order to determine the firm's optimal employment policy, we first take the first-order conditions for hires and separations (i.e. conditional on $\Delta n \neq 0$):

$$pxF'(n) - w(n,x) - w_n(n,x)n - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0, \text{ if } \Delta n \neq 0 \tag{4}$$

where $D(n,x) \equiv \int \Pi_n(n,x') \, dG(x'|x)$ reflects the marginal effect of current employment decisions on the future value of the firm. Equation (4) is quite intuitive. It states that the marginal product of labor ($pxF'(n)$) net of any hiring costs ($\frac{c}{q}\mathbf{1}^{+}$), plus the discounted expected future marginal benefits from an additional unit of labor ($\beta D(n,x)$) must equal the marginal cost of labor ($w(n,x)+w_n(n,x)\,n$). To provide a full characterization of the firm's optimal employment policy, it remains to characterize the future marginal benefits from current employment decisions, $D(n,x)$, and the wage bargaining solution, $w(n,x)$, to which we now turn.

2.2 Wage Setting

The existence of frictions in the labor market implies that it is costly for firms and workers to find alternative employment relationships. As a result, there exist quasi–rents over which the firm and its workers must bargain. The assumption of constant marginal product in the standard MP model has the tractable implication that these rents are the same for all workers within a given firm. It follows that firms can bargain with each of their workers independently, because the rents of each individual employment relationship are independent of the rents of all other employment relationships.

However, because we allow for the possibility of downward–sloped labor demand ($F'' < 0$), these rents will depend on the number of workers within a firm. Intuitively, the rent that a firm obtains from "the" marginal worker will be lower than the rent obtained on all infra–marginal hires due to diminishing marginal product. An implication of the latter is that the multilateral dimension of the firm's bargain with its many workers becomes important: The rents of each individual employment relationship within a firm are no longer independent.

To take this into account, we adopt the bargaining solution of Stole & Zwiebel (1996) which generalizes the Nash solution to a setting with downward–sloped labor demand.[5] Stole & Zwiebel present a game where the bargained wage is the same as the outcome of simple

[5] This approach was first used by Cahuc & Wasmer (2001) to generate a wage equation for the exogenous job destruction case.

Nash bargaining over the *marginal* surplus. The game that supports this simple result is one in which a firm negotiates with each of its workers in turn, and where the breakdown of a negotiation with any individual worker leads to the renegotiation of wages with all remaining workers. The intuition for the Stole & Zwiebel result is that, because breakdown of any individual negotiation leads to full renegotiation of all wages, the firm's surplus is the same for all individual negotiations and is equal to the marginal surplus.

In accordance with timing of decisions each period, wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage setting, and the marginal surplus, which we denote as $J(n,x)$, is equal to the marginal value of labor gross of the costs of hiring:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta D(n,x) \tag{5}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. The value of employment in a firm of size n and productivity x, $W(n,x)$, is given by:

$$W(n,x) = w(n,x) + \beta\mathbb{E}\left[sU' + (1-s)\,W(n',x')\,|n,x\right] \tag{6}$$

Whilst employed, a worker receives a flow payoff equal to the bargained wage, $w(n,x)$. She loses her job with (endogenous) probability s next period, upon which she flows into the unemployment pool and obtains the value of unemployment, U'. With probability $(1-s)$, she retains her job and obtains the expected payoff of continued employment in her current firm, $W(n',x')$. Likewise, the value of unemployment to a worker is given by:

$$U = b + \beta\mathbb{E}\left[(1-f)\,U' + fW(n',x')\right] \tag{7}$$

Unemployed workers receive flow payoff b, which represents unemployment benefits and/or the value of leisure to a worker. They find a job next period with probability f, upon which they obtain the expected payoff from employment, $W(n', x')$.

Wages are then the outcome of a Nash bargain between a firm and its workers over the marginal surplus, with worker bargaining power denoted as η:

$$(1-\eta)\left[W(n,x)-U\right]=\eta J(n,x) \tag{8}$$

Given this, we are able to derive a wage bargaining solution with the following simple structure:

Proposition 1 *The bargained wage, $w(n,x)$, solves the differential equation:*[6]

$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b \tag{9}$$

The intuition for (9) is quite straightforward. As in the MP model, wages are increasing in the worker's bargaining power, η, the marginal product of labor, $pxF'(n)$, workers' job finding probability, f, the marginal costs of hiring for a firm, c/q, and workers' flow value of leisure, b. There is an additional term, however, in $w_n(n,x)\,n$. To understand the intuition for this term, consider a firm's negotiations with a given worker. If these negotiations break down, the firm will have to pay its remaining workers a higher wage. The reason is that fewer workers imply that the marginal product of labor will be higher in the firm, which will partially spillover into higher wages ($w_n n < 0$). The more powerful this effect is (the more negative is $w_n n$), the more the firm loses from a given breakdown of negotiations with a worker, and the more workers can extract a higher wage from the bargain.

In what follows, we will adopt the simple assumption that the production function is of

[6] A nice feature of this solution is its similarity to the solution obtained by Cahuc & Wasmer (2001) for the exogenous job destruction model. It is also consistent with Acemoglu & Hawkins' (2006) Lemma 2, except that it holds both in and out of steady state.

the Cobb-Douglas form, $F(n) = n^{\alpha}$ with $\alpha \leq 1$. Given this, the differential equation for the wage function, (9), has the following simple solution:

$$w(n, x) = \eta \left[\frac{px\alpha n^{\alpha-1}}{1 - \eta(1-\alpha)} + \beta f \frac{c}{q} \right] + (1-\eta) b \tag{10}$$

2.3 A Firm's Optimal Employment Policy

Now that we have obtained a solution for the bargained wage at a given firm, we can combine this with the firm's first–order condition for employment and thereby characterize the firms optimal employment policy, which specifies the firm's optimal employment as a function of its state, $n(n_{-1}, x)$. Thus, combining (4) and (9) we obtain:

$$(1-\eta) \left[\frac{px\alpha n^{\alpha-1}}{1 - \eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} - \frac{c}{q} \mathbf{1}^{+} + \beta D(n, x) = 0 \tag{11}$$

Given (11) we are able to characterize the firm's optimal employment policy as follows:

Proposition 2 *The optimal employment policy of a firm is of the form:*

$$n(n_{-1}, x) = \begin{cases} R_v^{-1}(x) & \text{if} \quad x > R_v(n_{-1}) \\ n_{-1} & \text{if} \quad x \in [R(n_{-1}), R_v(n_{-1})] \\ R^{-1}(x) & \text{if} \quad x < R(n_{-1}) \end{cases} \tag{12}$$

where the functions $R_v(\cdot)$ *and* $R(\cdot)$ *satisfy:*

$$(1-\eta) \left[\frac{pR_v(n)\, \alpha n^{\alpha-1}}{1 - \eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} + \beta D(n, R_v(n)) \equiv \frac{c}{q} \tag{13}$$

$$(1-\eta) \left[\frac{pR(n)\, \alpha n^{\alpha-1}}{1 - \eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} + \beta D(n, R(n)) \equiv 0 \tag{14}$$

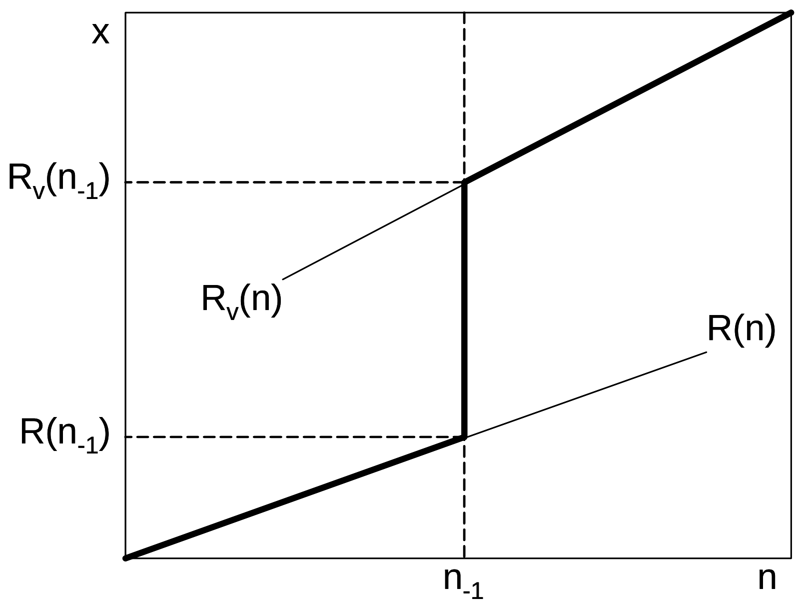

Figure 1: Optimal Employment Policy of a Firm

The firm's optimal employment policy will be similar to that depicted in Figure 1.[7] It is characterized by two reservation values for the firm's idiosyncratic shock, $R(n_{-1})$ and $R_v(n_{-1})$. Specifically, for sufficiently bad idiosyncratic shocks ($x < R(n_{-1})$ in the figure), firms will shed workers until the first-order condition in the separation regime, (14), is satisfied. Moreover, for sufficiently good idiosyncratic realizations ($x > R_v(n_{-1})$ in the figure), firms will post vacancies and hire workers until the first-order condition in the hiring regime, (13), is satisfied. Finally, for intermediate values of x, firms freeze employment so that $n = n_{-1}$. This occurs as a result of the kink in the firm's profits at $n = n_{-1}$, which arises because hiring is costly to firms, whilst separations are costless.

To complete our characterization of the firm's optimal employment policy, it remains to determine the marginal effect of current employment decisions on future profits of the firm, $D(n, x)$. It turns out that we can show that $D(n, x)$ has the following recursive structure:

[7] Note that the functions $R(n)$ and $R_v(n)$ need not be linear, as drawn in Figure 1.

Proposition 3 *The marginal effect of current employment on future profits, $D(n,x)$, is given by:*

$$D(n,x) = d(n,x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\, dG(x'|x) \tag{15}$$

where:

$$d(n,x) \equiv \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} \right\} dG(x'|x) + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x) \tag{16}$$

(15) is a contraction mapping in $D(n,\cdot)$, and therefore has a unique fixed point.

The intuition for this result is as follows. Because of the existence of kinked adjustment costs (costly hiring and costless separations) the firm's employment will be frozen next period with positive probability. In the event that the firm freezes employment next period ($x' \in [R(n), R_v(n)]$), the current employment level persists into the next period and so do the marginal effects of the firm's current employment choice. Proposition 3 shows that these marginal effects persist into the future in a *recursive* fashion. Propositions 2 and 3 thus summarize the microeconomic behavior of firms in the model.[8]

To get a sense for how the microeconomic behavior of the model works, we derive in the following result the response of an individual firm's employment policy function to changes in (exogenous) aggregate productivity, p, and the (endogenous) aggregate vacancy–unemployment ratio, θ in the case where idiosyncratic shocks are i.i.d.:

Proposition 4 *If idiosyncratic shocks, x, are i.i.d., then the effects of the aggregate state variables p and θ on a firm's optimal employment policy are as follows:*

$$\frac{\partial R_v}{\partial p} < 0; \frac{\partial R}{\partial p} < 0; \frac{\partial R_v}{\partial \theta} > 0; \text{ and } \frac{\partial R}{\partial \theta} > 0 \iff n \text{ is sufficiently large} \tag{17}$$

[8]It is straightforward to show that equations (10) to (16) reduce down to the discrete time analogue to the Mortensen & Pissarides (1994) model when $\alpha = 1$.

The intuition behind these marginal effects is quite simple. First, note that increases in aggregate productivity, p, shift a firm's employment policy function downwards in Figure 1. Thus, unsurprisingly, when labor is more productive, a firm of a given idiosyncratic productivity, x, is more likely to hire workers, and less likely to shed workers. Second, increases in the vacancy–unemployment ratio, θ, unambiguously reduce the likelihood that a firm of a given idiosyncratic productivity will hire workers (R_v increases for all n). The reason is that higher θ implies a lower job–filling probability, q, and thereby raises the marginal cost of hiring a worker, c/q. Moreover, higher θ implies a tighter labor market and therefore higher wages (from (9)) so that the marginal cost of labor rises as well. Both of these effects cause firms to cut back on hiring. Finally, increases in the vacancy–unemployment ratio, θ, will reduce the likelihood of shedding workers for small firms, but will raise it for large firms. This occurs because higher θ has countervailing effects on the separation decision of firms. On the one hand, higher θ reduces the job–filling probability, q, rendering separation decisions less reversible (since future hiring becomes more costly), so that firms become less likely to destroy jobs. On the other hand, higher θ implies a tighter labor market, higher wages, and thereby a higher marginal cost of labor, rendering firms more likely to shed workers. The former effect is dominant in small firms because the likelihood of their hiring in the future is high.

3 Aggregation and Steady State Equilibrium

3.1 Aggregation

Since we are ultimately interested in the equilibrium behavior of the aggregate unemployment rate, in this section we take on the task of aggregating up the microeconomic behavior of section 2 to the macroeconomic level. This exercise is non–trivial because each firm's employment is a non–linear function of the firm's lagged employment, n_{-1}, and its idiosyncratic shock realization, x. As a result, there is no representative firm interpretation that will aid aggregation of the model.

To this end, we are able to derive the following result which characterizes the steady state aggregate employment stock and flows in the model:

Proposition 5 *If idiosyncratic shocks, x, are i.i.d. with c.d.f. $G(x)$, the steady state c.d.f. of employment across firms is given by:*

$$H(n) = \frac{G[R(n)]}{1 - G[R_v(n)] + G[R(n)]} \tag{18}$$

Thus, the steady state aggregate employment stock is given by:

$$N = \int n dH(n) \tag{19}$$

and the steady state aggregate number of separations, S, and hires, M, is equal to:

$$S = \int [1 - H(n)] G[R(n)] dn = \int H(n)(1 - G[R_v(n)]) dn = M \tag{20}$$

Proposition 5 is useful because it provides a tight link between the solution for the microeconomic behavior of an individual firm and the macroeconomic outcomes of that behavior. Specifically, it shows that once we know the optimal employment policy function of an individual firm (that is, the functions $R(n)$ and $R_v(n)$) then we can directly obtain solutions for the aggregate employment stock and flows. An important feature to note about Proposition 5 is its generality. Specifically, it allows one to generate analytically the steady state aggregate employment stock and labor flows for any given employment policy function at the microeconomic level, not just that derived above.

The three components of Proposition 5 are also quite intuitive. The steady state distribution of employment across firms, (18), is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and since they are reducing their employment, it follows from (12) that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = G[R(n)]$. Thus, the inflow into $H(n)$ is equal to $[1 - H(n)]\, G[R(n)]$. Similarly, one can show that the outflow from the mass is equal to $H(n)(1 - G[R_v(n)])$. Setting inflows equal to outflows yields the expression for $H(n)$ in (18). Given this, the expression for aggregate employment, (19), follows directly.

The intuition for the final expression for aggregate flows in Proposition 5, (20), is as follows. Recall that the mass of firms whose employment switches from above some number n to below n is equal to $[1 - H(n)]\, G[R(n)]$. Equation (20) states that the aggregate number of separations in the economy is equal to the *cumulative* sum of these downward switches in employment over n. To get a sense for this, consider the following simple discrete example. Imagine an economy with two separating firms: one that switches from three employees to one, and another that switches from two employees to one. It follows that two firms have switched from > 2 employees to ≤ 2 employees, and one firm switched from > 1 to ≤ 1 employee. Thus, the cumulative sum of downward employment switches is three, which is also equal to the total number of separations in the economy.

An additional benefit of Proposition 5 is that it is very easily generalized to the case with serially correlated idiosyncratic shocks. In particular, the following corollary arises naturally:

Corollary 1 *If idiosyncratic shocks evolve according to the c.d.f.* $G(x'|x)$*, with steady state c.d.f.* $G^*(x)$*, the steady state c.d.f. of employment across firms is given by:*

$$H(n) = \int \frac{G[R(n)\,|x_{-1}]}{1 - G[R_v(n)\,|x_{-1}] + G[R(n)\,|x_{-1}]} dG^*(x_{-1})$$

with analogous expressions for the steady state employment stock and flows.

In the case of persistent idiosyncratic shocks, Proposition 5 holds for any given value of lagged productivity, x_{-1}. In steady state, lagged productivity will be distributed according to the steady state c.d.f., $G^*(x_{-1})$. Thus, taking expectations of each of the results in Proposition 5 over the steady state distribution $G^*(x_{-1})$ delivers the steady state aggregate employment stock and flows in the presence of serially correlated idiosyncratic shocks.

3.2 Steady State Equilibrium

Given (18), (19), and (20), the conditions for aggregate steady state equilibrium can be obtained as follows. First note that each firm's optimal policy function, summarized by the functions $R(n)$ and $R_v(n)$ in Proposition 2, depends on two aggregate variables: The (exogenous) state of aggregate productivity, p; and the (endogenous) ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, which uniquely determines the flow probabilities q and f.

In the light of Proposition 5, we can characterize the aggregate steady state of the economy for a given p in terms of two relationships. The first, the *job creation condition*, is simply equation (19), which we re–state here in terms of unemployment, making explicit its dependence on the aggregate vacancy–unemployment ratio, θ:

$$U(\theta)_{JC} = L - \int n dH(n;\theta) \tag{21}$$

(21) simply specifies the level of aggregate employment that is consistent with the inflows to (hires) and outflows from (separations) aggregate employment being equal as a function of θ. The second steady state condition is the *Beveridge Curve* relation. This is derived from the difference equation that governs the evolution of unemployment over time:

$$\Delta U' = S(\theta) - f(\theta) U \tag{22}$$

(22) simply states that the change in the unemployment stock over time, $\Delta U'$, is equal to the inflow into the unemployment pool – the number of separations, S – less the outflow from the unemployment pool – the job finding probability, f, times the stock of unemployed workers, U. In steady state, aggregate unemployment will be stationary, so that we obtain the steady state unemployment relation:

$$U(\theta)_{BC} = \frac{S(\theta)}{f(\theta)} \tag{23}$$

The steady state value of the vacancy–unemployment ratio, θ, is co–determined by (21) and (23).

4 An Illustrative Simulation

Shimer (2005a) demonstrated that the standard MP model could not explain the observed cyclical amplitude of the job finding rate in US data. A natural question is whether the generalized model analyzed here can alleviate this problem. To this end, we perform the following illustrative numerical exercise. We normalize aggregate productivity, p, to 1 and calculate the steady state response of the job finding rate, f, and the unemployment inflow rate, $s \equiv S/N$, to a 1% reduction in p. We examine the steady state response of the model as an approximation to the true dynamic response of the model based on the results of Mortensen & Nagypal (2007) and Rotemberg (2006), who show that such an approximation is very close in models of the aggregate labor market. To see why, note that we can rewrite the difference equation for the evolution of unemployment, (22), as:

$$\Delta U' = -f(U - U_{BC}) \tag{24}$$

where U_{BC} is defined in (23) and is the steady state unemployment level. In US data, the monthly outflow rate from unemployment is on the order of $f = 0.45$ (see Shimer, 2005b), implying that around half of the gap between actual and steady state unemployment is closed over the course of a month. That is, deviations of unemployment from steady state are very short-lived, and thus steady state responses to aggregate shocks are very good, and intuitive, approximations to the true dynamic response of the model.

In what follows, we take a time period to be equal to one week, which in practice acts as a good approximation to the continuous time nature of unemployment flows (see Hagedorn & Manovskii, 2005). We then assume that the matching function is of the conventional Cobb-Douglas form, $M = \mu U^{\phi} V^{1-\phi}$. We set $\phi = 0.6$ based on the estimates reported in Petrongolo & Pissarides (2001). We target a weekly unemployment outflow probability of $f = 0.1125$, to be approximately consistent with a monthly outflow hazard of 0.45. In addition, we follow Pissarides (2007) and target a mean value of the vacancy–unemployment ratio of $\theta = 0.72$. Noting from the matching function that $f = \mu\theta^{1-\phi}$, the latter implies that $\mu = 0.129$ on a weekly basis.

Idiosyncratic productivity shocks, x, are assumed to arrive with probability λ each period, and are drawn from the c.d.f. $\tilde{G}(x)$. Following Mortensen & Pissarides (1994), we assume for simplicity that the latter distribution is uniform on the interval $[\gamma, 1]$, so that $\tilde{G}(x) = \frac{x-\gamma}{1-\gamma}$. Given this setup, it is possible to solve for firms' optimal employment policy in closed form, the details of which are in Appendix A. We then use the aggregation results of Proposition 5 to derive numerically the aggregate job creation, (21), and Beveridge curve, (23), conditions, and thereby solve the model.

The empirical moments that we seek to match are summarized in Table 1. As mentioned above, we target a mean weekly outflow probability of $f = 0.1125$ to be consistent with a monthly outflow rate of 0.45 (Shimer, 2005b). In addition, we target a mean weekly

Table 1: Target Moments (Weekly)

Outcome	Level	Elasticity w.r.t. $\frac{Y}{N}$	Source
f	0.1125	2.95	Shimer (2005b)
s	0.0075	-2.48	Shimer (2005b)
$b/\left(\frac{Y}{N}\right)$	0.7	–	Mortensen & Nagypal (2007)
$\mathbb{E}$ [Wage of New Hires]	–	0.94	Haefke et al. (2007)

Notes: Following Mortensen & Nagypal (2007), elasticities are obtained by regressing the log deviation from trend of f and s on the log deviation from trend of non-farm business output per worker obtained from the Bureau of Labor Statistics. Following Shimer (2005a), series are detrended using a Hodrick–Prescott filter with smoothing parameter 10^5. The series for f and s are respectively the job-finding rate and the employment to unemployment transition rate derived in Shimer (2005b).

inflow probability of $s = 0.0075$ to be consistent with a mean unemployment rate of 6.25%. Following Mortensen & Nagypal (2007), we target the empirical elasticities of f and s with respect to aggregate output per worker, Y/N. We use Shimer's (2005b) estimate of the unemployment outflow rate as a measure of the job finding rate, f, and his estimate of the employment to unemployment transition rate to measure s.[9] Using these measures, we derive an elasticity of f of around 2.95 and elasticity of s of -2.48.[10] In addition to these, we also target the ratio of workers' opportunity cost of employment, b, to the average product of labor, Y/N, to be in the range of 0.7, as suggested by Mortensen & Nagypal (2007).

Finally, we target the elasticity of average wages of newly hired workers to be equal to 0.94, based on the results of Haefke et al. (2007). We target the elasticity of the average

[9] We are grateful to Robert Shimer for posting his estimates of flow transition rates among labor market states from the CPS Gross Flows data on his webpage.

[10] The elasticity for the job finding rate differs from the value of 2.34 reported in Mortensen & Nagypal (2005) because they base their calculations on Table 1 of Shimer (2005a) which reports summary statistics for the monthly job finding *probability*, rather than for the hazard, which is what matters for unemployment flows.

Table 2: Calibrated Parameters (Weekly)

Parameter	Meaning	Value	Reason
ϕ	Matching elasticity	0.6	Petrongolo & Pissarides (2001)
μ	Matching efficiency	0.129	Pissarides (2007)
α	$F(n) = n^{\alpha}$	0.67	Labor share = 2/3
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.4	Match target moments in Table 1
c	Flow vacancy cost	0.25	
η	Worker bargaining power	0.25	
L	Labor force	1.5	
λ	x: arrival rate	0.0375	
γ	x: lower support	0.4	

wages of newly hired workers rather than the elasticity of average wages of all workers for two reasons. First, it is well known empirically that the wages of workers in ongoing relationships are rigid (see *inter alia* Kahn, 1997; Card & Hyslop, 1997; Altonji & Devereux, 2000), which is at odds with the assumption of Nash wage setting that we employ here.[11] Second, it is also well known that it is the flexibility of wages of new hires, rather than of ongoing workers, that is relevant to the cyclicality of the job finding rate implied by search and matching models of the labor market such as the one studied here (Shimer, 2004; Hall, 2005).

We thus have six moments that we seek to match, and seven model parameters: α (production function, $F(n) = n^{\alpha}$), b (flow value of leisure), c (flow cost per vacancy), η (worker bargaining power), L (potential labor force), λ (arrival probability of idiosyncratic

[11] Indeed, in the calibration that follows, the Nash wage setting assumption implies an elasticity of average worker wages with respect to output per worker of approximately 1. This overstates the cyclicality of ongoing wages observed in the data, which display an elasticity with respect to output per worker of approximately 0.5 (see Pissarides, 2007).

Table 3: Model Outcomes (Weekly)

Outcome	Mean Level	Elasticity w.r.t. Y/N
f	0.1126	3.04
s	0.00775	-2.45
$b/$ [Output per Worker]	0.622	–
$\mathbb{E}$ [Wage of New Hires]	–	1.1

Parameter values: α=0.67; β=0.999; b=0.4; c=0.25; η=0.25; L=1.5; λ=0.0375; γ=0.4

shocks), and γ (lower support of idiosyncratic shock distribution). We therefore set α to be equal to the conventional 2/3,[12] and evaluate the steady state response of the model over a grid of values for the remaining six parameters. Given the results of this exercise, we pick the parameter values that most closely match the six target moments in Table 1.

The parameter results of this numerical exercise are reported in Table 2, and the implied model outcomes are in Table 3. The results are very encouraging: The model is able to match quite closely the target moments in Table 1. In particular, the ability of the model to match the empirical elasticity of the job finding rate makes substantial progress relative to the standard MP model. Shimer's (2005a) calibration of the standard MP model yields an elasticity of f equal to 0.48. Mortensen & Nagypal (2007) favor a different calibration of the standard MP model that yields an elasticity of f equal to 1.56 (see their section 3.2). Pissarides' (2007) calibration of the standard model with endogenous job destruction obtains an elasticity of f equal to 1.54. Thus, the standard MP model, with or without endogenous job destruction, appears to be able explain up to one half of the observed cyclicality in the job finding rate. The results of Table 3 suggest that the generalized model studied here can plausibly account for *all* of the cyclical variation in f.

[12] Strictly speaking, labor's share will be more than 2/3 in the model due to surplus sharing. We use a value of $\alpha = 2/3$ for simplicity.

Table 4: Measures of Flow Surplus to Employment Relationships

Flow Surplus Measure	Mortensen & Pissarides (1994) Hagedorn & Manovskii (2005)	Our Model
b/Average Product	0.925 – 0.955	0.622
$b/\mathbb{E}$ [Marginal Product]	0.925 – 0.955	0.928
$b/\mathbb{E}$ [Worker Wage]	$\approx$ 0.925 – 0.955	0.863

Parameter values: α=0.67; β=0.999; b=0.4; c=0.25; η=0.25; L=1.5; λ=0.0375; γ=0.4

It is well-known, however, that the standard MP model can generate realistic cyclicality in the job finding rate provided that the flow surplus to employment relationships, as measured by the distance between workers' flow valuation of leisure, b, and the productivity of a job, given by p in the standard $\alpha = 1$ case, is sufficiently small. In particular, the numerical analysis reported in the original Mortensen & Pissarides (1994) paper set $b/p = 0.925$. More recently, Hagedorn & Manovskii (2005) show that setting $b/p = 0.955$ reconciles the standard model with the data on the job finding rate. A criticism of this approach, however, has been that the flow surplus is implausibly small, in the sense that the average worker obtains only a 2.3% surplus from employment over unemployment (Mortensen & Nagypal, 2007).

To address this issue, Table 4 provides various measures of the flow surplus to employment relationships from the simulation in Table 2. It shows that, whilst the marginal flow surplus is relatively small in the simulation, the average flow surplus to employment relationships is nevertheless large relative to that in Hagedorn & Manovskii (2005) and Mortensen & Pissarides (1994) for the standard MP model. The reason that this can occur is, of course, because downward-sloped demand for labor can simultaneously allow small marginal surplus and large infra-marginal surplus to coexist. Moreover, in response to Mortensen & Nagypal's critique of the implied low worker surplus in previous studies, note that Table 4 reports that workers on average obtain a flow surplus from employment of $\frac{\mathbb{E}(w)-b}{b} = \frac{1}{0.86} - 1 = 16\%$.

This is much larger than the 2.3% implied by the Hagedorn & Manovskii analysis. Thus, the results of Tables 3 to 4 suggest that the model can deliver reasonable cyclicality in job finding rates and non–trivial rents to employment relationships simultaneously, something that the standard MP model cannot achieve.

5 Understanding Amplification

Figure 2 plots the response of the Job Creation, (21), and Beveridge Curve, (23), conditions to a 1% decline in p for the simulation detailed in Tables 2 to 4. The figure reveals that allowing for downward-sloped labor demand amplifies the response of the vacancy-unemployment ratio to aggregate disturbances primarily through movements in the Job Creation condition. This naturally leads to the question of why the JC condition moves so much. The following result provides a sense for where this amplification comes from by taking a log–linear approximation to a firm's marginal surplus around mean employment:

Proposition 6 *For small λ, the horizontal shift in the JC condition induced by a change in aggregate productivity, p, is given approximately by:*

$$\left.\frac{d\ln\theta}{d\ln p}\right|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)\left(\tilde{p}-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta} \tag{25}$$

where ω is the steady state employment share of hiring firms, and $\tilde{p} \equiv \rho\overline{apl} + (1-\rho)\,\overline{mpl}$ where $\overline{apl}$ and $\overline{mpl}$ are respectively the average and marginal product of labor of the average–sized firm, and $\rho \equiv \frac{\alpha\eta}{1-\eta(1-\alpha)}$.

Corollary 2 *The elasticity of the vacancy–unemployment ratio to aggregate productivity in the $\alpha = 1$ case (Mortensen & Pissarides, 1994) is approximately equal to:*

$$\frac{d\ln\theta}{d\ln p} \approx \frac{(1-\eta)\,p}{\phi\left[(1-\eta)\left(p-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta} \tag{26}$$

Equation (26) extends results presented in Mortensen & Nagypal (2007) for the standard MP model with exogenous job destruction to the endogenous job destruction case. It echoes Mortensen & Nagypal's results in that it shows that the cyclical response of the vacancy-unemployment ratio, θ, is amplified in the endogenous job destruction case when the average flow surplus to employment relationships, $p - b$, is small. Intuitively, when the flow surplus is small, small reductions in aggregate productivity, p, can easily exhaust that surplus and lead firms to cut back substantially on hiring. Thus, to incentivize firms to hire once more and thereby restore equilibrium, the model must allow the labor market to slacken, and labor market tightness to fall, substantially.

Equation (25) generalizes this result to the case of downward sloped labor demand. Inspection of (26) and (25) reveals that there are two ways that the addition of downward sloped labor demand can potentially yield amplification of the reponse of labor market tightness (and thereby of the job finding rate, $f = \mu\theta^{1-\phi}$) to changes in aggregate productivity. The first is that the effective surplus that matters for amplification is now given by by $\tilde{p} - b$, and this is smaller than the average flow surplus. The reason is that the effective flow surplus, $\tilde{p} - b$, is now a weighted average of the average and marginal flow surplus. When the demand for labor slopes downward, the marginal surplus will be smaller than the average surplus, because infra–marginal employment relationships have a higher marginal product. This provides a sense for why the numerical exercise above is able to generate greater volatility in θ even when the average flow surplus is relatively large: It is because the *marginal* flow surplus is relatively small in the simulation.

However, equation (25) suggests that there is an additional effect at work in the form of the variable ω, the steady state employment share of hiring firms. To understand the significance of this term, note that in the standard MP model with flat labor demand (the special case where $\alpha = 1$), ω is equal to 1: When $\alpha = 1$, a firm that reduces its employment

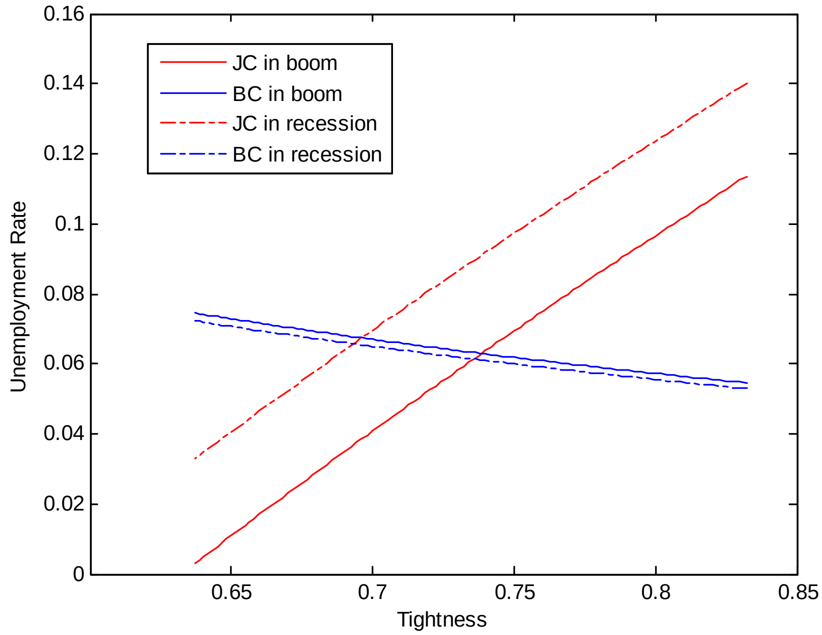

Figure 2: Job Creation and Beveridge Curve in the Simulated Model

will shed all of its workers since, if one worker is unprofitable at a firm, all workers are unprofitable. Thus shedding firms have zero employment, and all of steady state employment is accounted for by hiring firms. In particular, equation (26) is simply derived from total differentiation of the first–order condition of a hiring firm (equation (13) with $\alpha = 1$). Thus, for the purposes of the determining the elasticity of market tightness, θ, the standard MP model can be thought of as comprising one hiring firm that must be convinced to hire in equilibrium at all times.

The latter is a useful and intuitive point of contrast with the model with downward sloped labor demand. In particular, one interpretation of Proposition 6 is that the generalized model with $\alpha < 1$ can be thought of (approximately) as comprising *two* firms for the purposes of determing the amplification of θ: A hiring firm, that accounts for a share ω of employment, and a shedding firm, that accounts for $(1 - \omega)$ of employment. Because of downward sloped labor demand, shedding firms do not reduce their employment to zero because reducing employment replenishes the marginal product of labor in those firms. Hence ω will be less than unity, and inspection of (25) and (26) reveals that this will lead to greater amplification relative to the standard MP model.[13]

The intuition for this effect is subtle, but is clearly related to the interaction of downward-sloped labor demand and heterogeneous firms. Following a reduction in aggregate productivity, shedding firms wish to shed more workers, and hiring firms wish to hire fewer workers. Thus inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, and unemployment rises. To return the model to steady state, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the job filling probability, $q(\theta)$, to rise (and labor market tightness, θ, to fall) so that hiring becomes less costly for firms. In the standard MP model with $\alpha = 1$, the representative hiring firm responds by raising its steady state employment. In the $\alpha < 1$ case, however, part of steady state aggregate employment is accounted for by shedding firms ($\omega < 1$). Thus, the share of aggregate employment comprised of hiring firms is smaller, and this smaller fraction of firms must be convinced to hire enough workers from the unemployment pool. Therefore $q(\theta)$ must rise (and hence θ must fall) more to convince this smaller fraction of firms to increase hiring and return the economy back to steady state once more.

6 Summary and Discussion

The previous analysis has explored a novel generalization of the standard model of the aggregate labor market, the Mortensen–Pissarides model, that allows for the conventional notion of downward-sloped short run labor demand, as well as endogenous job destruction (informed

[13] The reader may worry whether the term $[(1-\eta)(\tilde{p}-b) - \eta\beta c\theta]$ is positive or not. To see that it is, note that we can rewrite the term as $\left[(1-\eta)\left(\frac{p\alpha x n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right) - \eta\beta c\theta\right]$, and observe from equations (13) and (14) that it is, in fact, the marginal flow surplus of a firm, and therefore must be positive.

by the evidence for countercyclical unemployment inflows cited in the introduction). It has characterized optimal labor demand at the firm level in the presence of kinked search costs, idiosyncratic heterogeneity, and surplus sharing. In addition, it has developed a very general technique for the aggregation of the employment behavior of individual firms to the macroeconomic level, and has used this to characterize the conditions for aggregate labor market equilibrium for this general model. Finally, this paper has provided simulation results that demonstrate that the model can generate realistic cyclicality in both the employment to unemployment transition rate, as well as in the job finding rate, something that has challenged the standard search model of the labor market (Shimer, 2005a).

A number of future extensions arise naturally from our analysis. First, until now we have ignored the cyclicality of vacancies generated by our generalized model. Readers of Shimer (2005a), however, will recall that the standard MP search and matching model also fails to match the observed cyclical volatility in the vacancy rate in the US. Specifically, the empirical elasticity of the vacancy rate with respect to output per worker in the US derived by Shimer is given by $0.364 \times 0.202/0.020 = 3.68$ (see Shimer, 2005a, Table 1). The implied elasticity from Shimer's calibration of the standard MP model is $0.995 \times 0.027/0.020 = 1.34$ (see Shimer, 2005a, Table 3). The analogous elasticity generated by our simulation of the generalized model studied here is 2.46. Although this is clearly a substantial improvement over the standard model, there remains a question of why the generalized model, which matches the cyclicality of job finding and employment to unemployment transition rates so well, cannot fully explain the cyclicality of vacancies.

We believe that the answer is that a complete understanding of the cyclical behavior of vacancies is intimately related to an understanding of the processes underlying job-to-job employment flows, a phenomenon that we abstract from in our analysis of unemployment flows. To see why, note the following identity that relates the job-filling rate, q, vacancies (or job openings), V, and the *numbers* (not the hazard rates) flowing from unemployment to employment, UE, and from job to job, EE:

$$qV = UE + EE \tag{27}$$

Log differentiation of this identity yields:

$$d \log q + d \log V = \varphi d \log UE + (1 - \varphi)\, d \log EE \tag{28}$$

where φ is the share of total hires that originates from unemployment. Recent research has shown that job-to-job flows (EE) are substantially procyclical and account for approximately 60% of total hires using Current Population Survey data from 1994 onwards (Fallick & Fleischman, 2004). Equation (28) shows that the procyclicality of EE flows must therefore contribute substantially to the procyclicality of vacancies observed in the data. For this reason, we feel that the elasticity of vacancies obtained from the generalized model without on-the-job search may in fact be quite reasonable: If we were able to match the empirical elasticity, we would implicitly be *over*-explaining the procyclicality of vacancies. For the same reason, however, we also feel that extending the model to account for job-to-job flows is an important task for future research.

A second natural extension of the analysis presented here relates to the generality of the aggregation results derived in Proposition 5. A key benefit of Proposition 5 is that it allows one to aggregate analytically the behavior of individual firms for a wide class of underlying labor demand models, not just the kinked adjustment cost model analyzed in this paper. This is useful because recent research has emphasized the importance of non–convex adjustment costs in explaining the empirical properties of labor demand at the micro level (see for example Caballero, Engel, & Haltiwanger, 1997, and Cooper, Haltiwanger, & Willis, 2004). More recently still, authors have sought to unify models of non–convex adjustment costs with the search and matching framework analyzed in this paper and elsewhere (see

Cooper, Haltiwanger, and Willis, 2007). To our knowledge, the solution of such models has typically been aided by numerical methods. Our hope is that the results of Proposition 5 will enable an understanding of the *analytics* of the economic processes underlying aggregate models with more general microeconomic adjustment cost structures. We hope that this will enable further progress toward a unified model of the aggregate labor market that distils the insights of both the literature on adjustment costs at the microeconomic level, and the search and matching literature.

7 References

Acemoglu, Daron, and William Hawkins, "Equilibrium Unemployment in a Generalized Search Model," mimeo, University of Rochester, 2006.

Altonji, Joseph & Paul Devereux, "The Extent and Consequences of Downward Nominal Wage Rigidity", *Research in Labor Economics*, Vol 19, Elsevier Science Inc. (2000): 383-431.

Blanchard, Olivier Jean and Peter Diamond, "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 1990(2): 85-143.

Bentolila, Samuel, and Giuseppe Bertola, "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *Review of Economic Studies*, Vol. 57, No. 3., pp. 381-402, July, 1990.

Caballero, R., E. Engel, and J. Haltiwanger, "Aggregate Employment Dynamics: Building from Microeconomic Evidence," *American Economic Review*, 87, 115–37, 1997.

Cahuc, Pierre and Etienne Wasmer "Does Intrafirm Bargaining Matter in the Large Firm's Matching Model?" *Macroeconomic Dynamics* (5) 2001, pp. 742-747.

Card, David & Dean Hyslop "Does Inflation 'Grease the Wheels of the Labor Market'?" in Christina D. Romer and David H. Romer, editors, *Reducing Inflation: Motivation and Strategy*. University of Chicago Press, 1997.

Cooper, Russell, John Haltiwanger, Jonathan L. Willis, "Dynamics of Labor Demand: Evidence from Plant-level Observations and Aggregate Implications," Working Paper No. 10297, National Bureau of Economic Research, 2004.

Cooper, Russell, John Haltiwanger, Jonathan L. Willis, "Implications of Search Frictions: Matching Aggregate and Establishment-level Observations," Working Paper No. 13115, National Bureau of Economic Research, 2007.

Costain, J.S., Reiter, M., "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," mimeo, Universidad Carlos III, 2005.

den Haan, Wouter J., Garey Ramey, and Joel Watson, "Job Destruction and Propagation of Shocks," *American Economic Review*, Vol. 90, No. 3 (Jun., 2000), pp. 482-498.

Elsby, Michael W.L., Ryan Michaels, and Gary Solon, "The Ins and Outs of Cyclical Unemployment," Working Paper No. 12853, National Bureau of Economic Research, 2007.

Fallick, Bruce and Charles Fleischman, "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows," Finance and Economics Discussion Series 2004-34. Washington: Board of Governors of the Federal Reserve System, 2004.

Fujita, Shigeru and Garey Ramey, "The Cyclicality of Job Loss and Hiring," mimeo, Federal Reserve Bank of Philadelphia and University of California at San Diego, 2006.

Haefke, C., M. Sonntag, and T. van Rens, "Wage Rigidity and Job Creation," mimeo, 2007.

Hagedorn, M., and I. Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," mimeo, University of Pennsylvania, 2005.

Hall, Robert E., "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, March 2005a, 95(1): 50-65.

Hall, Robert E., "Sources and Mechanisms of Cyclical Fluctuations in the Labor Market," mimeo, Stanford, 2006.

Hobijn, Bart, and Aysegul Sahin, "Firms and Flexibility," mimeo, Federal Reserve Bank of New York, 2007.

Kahn, Shulamit, "Evidence of Nominal Wage Stickiness from Microdata", *American Economic Review*, Vol. 87, No. 5., pp. 993-1008, December 1997.

Marston, Stephen T., "Employment Instability and High Unemployment Rates," *Brookings Papers on Economic Activity*, 1976(1): 169-203.

Mortensen, Dale T., and Eva Nagypal, "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics*, 10 (2007): 327–347.

Mortensen, Dale T., and Eva Nagypal, "More on Unemployment and Vacancy Fluctuations," Working Paper No. 11692, National Bureau of Economic Research, 2005.

Mortensen, Dale T. and Christopher A. Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, July 1994, 61(3): 397-415.

Perry, George L., "Unemployment Flows in the U.S. Labor Market," *Brookings Papers on Economic Activity*, 1972(2): 245-78.

Petrongolo, Barbara, and Christopher A. Pissarides, "Looking into the black box: A survey of the matching function," *Journal of Economic Literature* 39 (2), 390–431, 2001.

Pissarides, Christopher A., "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" mimeo, London School of Economics, 2007.

Rotemberg, Julio J., "Cyclical Wages in a Search-and-Bargaining Model with Large Firms," Working Paper No. 12415, National Bureau of Economic Research, 2006.

Shimer, Robert, "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association (Papers and Proceedings)*, 2: 469-479, 2004.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review* 95 (March), 25–49, 2005a.

Shimer, Robert, "Reassessing the Ins and Outs of Unemployment," mimeo, University of Chicago, 2005b.

Solon, Gary, Robert Barsky, and Jonathan A. Parker, "Measuring the Cyclicality of Real Wages: How Important is Composition Bias," *Quarterly Journal of Economics*, Vol. 109, No. 1 (Feb., 1994), pp. 1-25.

Stokey, Nancy L., and Robert E. Lucas Jr., *Recursive Methods in Economic Dynamics*, with Edward C. Prescott Cambridge, Mass. and London: Harvard University Press, 1989.

Stole, Lars A. and Jeffrey Zwiebel, "Intrafirm Bargaining under Non-binding Contracts," *Review of Economic Studies*, March 1996, vol 63, pp. 375-410.

Yashiv, Eran, "U.S. Labor Market Dynamics Revisited," mimeo, Tel Aviv University, 2006.

8 Appendix

A Solution of the Simulated Model

Optimal Employment Policy: We follow Mortensen & Pissarides (1994) and use the following process for idiosyncratic shocks:

$$x' = \begin{cases} x & \text{with probability} \quad 1-\lambda \\ \tilde{x} \overset{c.d.f.}{\sim} \tilde{G}(x) & \text{with probability} \quad \lambda \end{cases} \tag{29}$$

where $\tilde{x} \sim U[\gamma, 1]$ so that $\tilde{G}(x) = \frac{x-\gamma}{1-\gamma}$. In this case, we can rewrite the recursion for the function $D(n,x)$ in Proposition 3 as:

$$\begin{aligned} D(n,x) &= (1-\lambda)\chi(x) + \lambda \int_{R(n)}^{R_v(n)} \chi(x')\, d\tilde{G}(x') + \lambda \int_{R_v(n)}^{\infty} \frac{c}{q} d\tilde{G}(x') \\ &\quad + \beta(1-\lambda) D(n,x) + \beta\lambda \int_{R(n)}^{R_v(n)} D(n,x')\, d\tilde{G}(x') \end{aligned} \tag{30}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f \frac{c}{q}$. We then conjecture that the function $D(n,x)$ is of the form $D(n,x) = d_0 + d_1\chi(x)$. Substituting this into the latter, and equating coefficients, we obtain the following solution for $D(n,x)$:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) \\ &\quad + \frac{\lambda}{1-\beta(1-\lambda)} \frac{R_v - R}{[1-\beta(1-\lambda)](1-\gamma) - \beta\lambda(R_v - R)} \chi\left(\frac{R+R_v}{2}\right) \\ &\quad + \frac{1-R_v}{[1-\beta(1-\lambda)](1-\gamma) - \beta\lambda(R_v - R)} \lambda \frac{c}{q} \end{aligned} \tag{31}$$

Note also that differencing the first-order conditions (13) and (14) in this case implies that:

$$R_v(n) - R(n) \equiv \delta(n) = [1-\beta(1-\lambda)] \frac{c/q}{\psi p\alpha n^{\alpha-1}} \tag{32}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$. Using these two results, and after some tedious algebra, one obtains the following closed–form solution for $R_v(n)$:

$$R_v(n) = \frac{(1-\eta)b + \eta\beta c\frac{f}{q} + \left(1-\beta-\beta\lambda\frac{\gamma}{1-\gamma}\right)\frac{c}{q} - \beta\lambda[1-\beta(1-\lambda)]\frac{1}{1-\gamma}\frac{1}{2}\left(\frac{c}{q}\right)^2 \frac{1}{\psi p\alpha n^{\alpha-1}}}{\psi p\alpha n^{\alpha-1} - \beta\lambda\frac{1}{1-\gamma}\frac{c}{q}} \tag{33}$$

To derive $R(n)$, simply note that, by definition, $R(n) = R_v(n) - \delta(n)$.

Aggregate Employment Stock and Flows: The steady state distribution of employment across firms is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and it follows from (12) and the process for x, (29), that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = \lambda\tilde{G}[R(n)]$. Thus, the inflow into $H(n)$ is equal to $\lambda[1 - H(n)]\tilde{G}[R(n)]$. Similarly, the outflow from the mass is equal to $\lambda H(n)\left(1 - \tilde{G}[R_v(n)]\right)$. Setting inflows equal to outflows yields the steady state c.d.f. of employment:

$$H(n) = \frac{\tilde{G}[R(n)]}{1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]} \tag{34}$$

analogous to the i.i.d. result in Proposition 5. Thus the steady state employment stock is simply $N = \int n dH(n)$. To derive the steady state flows, recall that separations (which equal hires in steady state) are equal to the cumulative sum of downward employment switches. The mass of firms switching from an employment level above n to below n is equal to $\lambda\tilde{G}[R(n)][1 - H(n)]$. Thus, separations and hires are equal to:

$$S = \lambda\int[1 - H(n)]\tilde{G}[R(n)]dn = \lambda\int H(n)\left(1 - \tilde{G}[R_v(n)]\right)dn = M \tag{35}$$

Average Product and Average Marginal Product: The average product of labor implied by the model is given by $APL = \mathbb{E}[pxn^{\alpha-1}]$. Note that:

$$\mathbb{E}\left[xn^{\alpha-1}\right] = \int\left[\int xdG(x|n)\right]n^{\alpha-1}dH(n)$$

Moreover, the optimal employment policy implies that, given n, x must lie in the interval $[R(n), R_v(n)]$, but is otherwise independently distributed. Thus:

$$\int xdG(x|n) = \frac{\int_{R(n)}^{R_v(n)} xdG(x)}{G[R_v(n)] - G[R(n)]} = \frac{1}{2}[R(n) + R_v(n)] \tag{36}$$

where the last equality follows from the assumption of uniform idiosyncratic shocks in the simulation. Thus:

$$APL = \mathbb{E}\left[pxn^{\alpha-1}\right] = p\int\frac{1}{2}[R(n) + R_v(n)]n^{\alpha-1}dH(n) \tag{37}$$

Moreover, the average marginal product of labor is simply given by:

$$\mathbb{E}[MPL] = \mathbb{E}\left[px\alpha n^{\alpha-1}\right] = \alpha APL \tag{38}$$

Average Wages: It follows from equation (9) that the average wage across firms is given by:

$$\bar{w}_f = \frac{\eta}{1-\eta(1-\alpha)}\mathbb{E}[MPL] + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{39}$$

To obtain the average wage across workers, which we denote $\bar{w}_w$, note that $\bar{w}_w = \mathbb{E}\left[\frac{n}{\mathbb{E}(n)}w(n,x)\right]$ where $w(n,x)$ is the wage in a given firm defined in (9). That is, it is the employment-weighted average of wages across firms. Thus:

$$\bar{w}_w = \frac{\eta}{1-\eta(1-\alpha)}\frac{1}{\mathbb{E}(n)}\mathbb{E}[px\alpha n^{\alpha}] + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{40}$$

This has a very similar structure to the average wage across firms. It follows that:

$$\bar{w}_w = \frac{\eta p\alpha}{1-\eta(1-\alpha)}\frac{1}{\mathbb{E}(n)}\int\frac{1}{2}[R(n)+R_v(n)]n^{\alpha}dH(n) + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{41}$$

Finally, the average wage of new hires, which we denote $\bar{w}_m$, is equal to a hiring–weighted average of wages across hiring firms. Noting from (12) that idiosyncratic productivity of hiring firms is given by $x = R_v(n)$, we have that:

$$\begin{aligned}\bar{w}_m &= \mathbb{E}\left[\mathbb{E}\left(\frac{\Delta n\mathbf{1}^{+}}{M}w(n,x)\,|n>n_{-1},n_{-1}\right)\right]\\ &= \int\int_{n_{-1}}\frac{n-n_{-1}}{M}w(n,R_v(n))\,dG[R_v(n)]\,dH(n_{-1}) \end{aligned}\tag{42}$$

B Proofs

Conjecture 1 *The optimal employment policy function is of the form specified in (12).*

We will later verify that the Conjecture is consistent with the solution for the wage equation obtained in Proposition 1.

Proof of Proposition 1. Note first that, under the Conjecture, we can write the marginal surplus to a firm recursively as:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)n + \beta\int_{R_v(n)}^{\infty}\frac{c}{q}dG(x') + \beta\int_{R(n)}^{R_v(n)}J(n,x')\,dG(x') \tag{43}$$

In addition, we can write the value to a worker of unemployment as:

$$U = b + \beta\left\{(1-f)U' + f\int_0^{\infty}\int_{R_v(n)}^{\infty}W\left(R_v^{-1}(x'),x'\right)\frac{dG(x')}{1-G(R_v(n))}dH(n)\right\} \tag{44}$$

Upon finding a job, which occurs with probability f, the new job must be in a firm which is posting vacancies. This implies that the idiosyncratic productivity of the firm, $x' > R_v(n)$, and that the level of employment in the hiring firm, $n = R_v^{-1}(x')$. Moreover, since firms differ in size, there is a distribution of employment levels, $H(n)$, over which an unemployed worker will take expectations when evaluating the expected future benefits of being hired. It is useful to rewrite the worker's value of unemployment as:

$$U = b + \beta \left\{ U' + f \int_0^\infty \int_{R_v(n)}^\infty \left[W\left(R_v^{-1}(x'), x'\right) - U' \right] \frac{dG(x')}{1 - G(R_v(n))} dH(n) \right\} \tag{45}$$

Then note that, due to Nash sharing, the worker's surplus in an expanding firm, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R_v^{-1}(x'), x'\right)$, and moreover that, by the first-order condition for a hiring firm (see (4)), $J\left(R_v^{-1}(x'), x\right) = c/q$. Thus, we obtain the simple result:

$$U = b + \beta U' + \beta f \frac{\eta}{1-\eta} \frac{c}{q} \tag{46}$$

The value of employment to a worker can be written as:

$$W(n,x) = w(n,x) + \beta \left\{ \begin{array}{c} \int_0^{R(n)} \left[\tilde{s} U' + (1-\tilde{s}) W\left(R^{-1}(x'), x'\right) \right] dG(x') \\ + \int_{R(n)}^{R_v(n)} W(n,x') \, dG(x') \\ + \int_{R_v(n)}^\infty W\left(R_v^{-1}(x'), x'\right) dG(x') \end{array} \right\} \tag{47}$$

An employed worker's expected future payoff can be split into three regimes. If the firm sheds workers next period ($x' < R(n)$) then the worker may separate from the firm. We denote by $\tilde{s}$ the probability that a worker separates from a firm conditional on the firm shedding workers. If the worker separates, she transitions into unemployment and receives a payoff U'. Otherwise she continues to be employed in a firm of size $n' = R^{-1}(x')$. Note that Nash sharing implies that $W\left(R^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R^{-1}(x'), x'\right)$, and that, by the first-order condition, $J\left(R^{-1}(x'), x'\right) = 0$. Thus, $W\left(R^{-1}(x'), x'\right) = U'$. In the event that a firm freexes employment next period ($x' \in [R(n), R_v(n)]$) then Nash sharing implies that $W(n,x') - U' = \frac{\eta}{1-\eta} J(n,x')$. Finally, in the event that the firm hires next period, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} \frac{c}{q}$. Thus, we have that:

$$W(n,x) = w(n,x) + \beta U' + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^\infty \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x') \, dG(x') \tag{48}$$

Subtracting the value of unemployment to a worker from the latter, we obtain the following description of the worker's surplus:

$$W(n,x) - U = w(n,x) - b + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^\infty \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x') \, dG(x') - \beta f \frac{\eta}{1-\eta} \frac{c}{q} \tag{49}$$

Under Nash, this must be equal to $\frac{\eta}{1-\eta} J(n,x)$, where $J(n,x)$ is as derived in (43) so that we have:

$$w(n,x) = \eta \left[pxF'(n) - w_n(n,x)\, n + \beta f \frac{c}{q} \right] + (1-\eta) b \tag{50}$$

as required. ■

Proof of Proposition 2. Given the wage function in (9), it follows that the firm's objective, (3), is continuous in (n_{-1}, x) and concave in n. Thus, it follows from the Theorem of the Maximum that the firm's optimal employment policy function is continuous in (n_{-1}, x). Given this, it follows that the employment policy function must be of the form stated in Proposition 2. This verifies that the Conjecture stated at the beginning of the appendix holds. ■

Proof of Proposition 3. First, note that one can re-write the continuation value conditional on each of the three possible continuation regimes:

$$\Pi(n, x') = \begin{cases} \Pi^{-}(n, x') & \text{if} \quad x' < R(n) \\ \Pi^{0}(n, x') & \text{if} \quad x' \in [R(n), R_v(n)] \\ \Pi^{+}(n, x') & \text{if} \quad x' > R_v(n) \end{cases} \tag{51}$$

where superscripts $^{-/0/+}$ refer to whether their are separations, a hiring freeze, or hires tomorrow. Thus we can write[14]:

$$\int \Pi(n, x')\, dG(x'|x) = \int_0^{R(n)} \Pi^{-}(n, x')\, dG + \int_{R(n)}^{R_v(n)} \Pi^{0}(n, x')\, dG + \int_{R_v(n)}^{\infty} \Pi^{+}(n, x')\, dG \tag{52}$$

Taking derivatives with respect to n, recalling the definition of $D(\cdot)$, and noting that, since $\Pi(n, x')$ is continuous, it must be that $\Pi^{-}(n, R(n)) = \Pi^{0}(n, R(n))$ and $\Pi^{0}(n, R_v(n)) = \Pi^{+}(n, R_v(n))$, yields:

$$D(n, x) = \int_0^{R(n)} \Pi_n^{-}(n, x')\, dG + \int_{R(n)}^{R_v(n)} \Pi_n^{0}(n, x')\, dG + \int_{R_v(n)}^{\infty} \Pi_n^{+}(n, x')\, dG \tag{53}$$

Finally, using the Envelope conditions in Lemma 1 below, and substituting into (53) we obtain (15) and (16) in the main text:

$$\begin{aligned} D(n, x) &= \int_{R(n)}^{R_v(n)} \left\{ (1-\eta) \left[\frac{p x' \alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta \beta f \frac{c}{q} \right\} dG(x'|x) \\ &\quad + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x) + \beta \int_{R(n)}^{R_v(n)} D(n, x')\, dG(x'|x) \\ &\equiv (\mathbf{C}D)(n, x) \end{aligned} \tag{54}$$

To verify that $\mathbf{C}$ is a contraction mapping, we confirm that Blackwell's sufficient conditions for a contraction hold here (see Stokey & Lucas, 1989, p.54). To verify monotonicity, fix

[14] Henceforth, "dG" without further elaboration is to be taken as "$dG(x'|x)$".

$(n, x) = (\bar{n}, \bar{x})$, and take $\hat{D} \geq D$. Then note that:

$$\int_{R(\bar{n})}^{R_v(\bar{n})} \hat{D}(\bar{n}, x')\, dG(x'|\bar{x}) - \int_{R(\bar{n})}^{R_v(\bar{n})} D(\bar{n}, x')\, dG(x'|\bar{x}) = \int_{R(\bar{n})}^{R_v(\bar{n})} \left[\hat{D}(\bar{n}, x') - D(\bar{n}, x')\right] dG(x'|\bar{x}) \geq 0 \tag{55}$$

Since $(\bar{n}, \bar{x})$ were arbitrary, it thus follows that **C** is monotonic in D. To verify discounting, note that:

$$\begin{aligned} [\mathbf{C}(D + a)](n, x) &= (\mathbf{C}D)(n, x) + \beta a [G(R_v(n)|x) - G(R(n)|x)] \\ &\leq (\mathbf{C}D)(n, x) + \beta a \end{aligned} \tag{56}$$

Since $\beta < 1$ it follows that **C** is a contraction. It therefore follows from the Contraction Mapping Theorem that **C** has a unique fixed point. ■

Lemma 1 *The value function defined in (3) has the following properties:*

$$\begin{aligned} \Pi_n^-(n, x') &= 0 \\ \Pi_n^0(n, x') &= (1 - \eta)\left[\frac{px'\alpha n^{\alpha-1}}{1 - \eta(1 - \alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n, x') \\ \Pi_n^+(n, x') &= c/q \end{aligned} \tag{57}$$

Proof of Lemma 1. First, note that standard application of the Envelope Theorem implies that $\Pi_n^-(n, x') = 0$ and $\Pi_n^+(n, x') = c/q$. It is only slightly less obvious what happens when $\Delta n' = 0$, i.e. when the employment is frozen next period. In this case, $n' = n$ and this implies that:

$$\Pi^0(n, x') = px'F(n) - w(n, x')\,n + \beta \int \Pi(n, x'')\, dG(x''|x') \tag{58}$$

It therefore follows that:

$$\Pi_n^0(n, x') = px'F'(n) - w(n, x') - w_n(n, x')\,n + \beta \int \Pi_n(n, x'')\, dG(x''|x') \tag{59}$$

Since, by definition $D(n, x') \equiv \int \Pi_n(n, x'')\, dG(x''|x')$, the statement holds as required. ■

Proof of Proposition 4. First note that if x is i.i.d., then the function $D(n, x) = D(n)$ is independent of x and thus the LHS of the first–order conditions, (13) and (14) are increasing in x. Thus, to establish that $\partial R_v/\partial p < 0$ and $\partial R/\partial p < 0$, note that the function $D(n, x)$ defined in (15) and (16) is increasing in p and thus the LHS of (13) and (14) are also increasing in p.

To ascertain the marginal effects of θ we first need to establish the marginal effect of θ on the function $D(n)$. Rewriting $f/q = \theta$ and $q = q(\theta)$ in (15) and (16), differentiating with respect to θ, and using the first–order conditions, (13) and (14), to eliminate terms we obtain:

$$\begin{aligned} D_\theta(n) &= -\eta\beta c \frac{G(R_v(n)) - G(R(n))}{1-\beta[G(R_v(n)) - G(R(n))]} \\ &\quad -\frac{c}{q}\frac{q'(\theta)}{q}\frac{1 - G(R_v(n))}{1-\beta[G(R_v(n)) - G(R(n))]} \end{aligned} \tag{60}$$

Differentiating the first–order condition for a hiring firm, (13), with respect to θ we obtain:

$$\begin{aligned} -\eta\beta c + \frac{c}{q}\frac{q'(\theta)}{q} + \beta D_\theta(n) &= -\frac{\eta\beta c}{1-\beta[G(R_v(n)) - G(R(n))]} \\ &\quad + \frac{c}{q}\frac{q'(\theta)}{q}\frac{1-\beta[1 - G(R(n))]}{1-\beta[G(R_v(n)) - G(R(n))]} \\ &< 0 \end{aligned}$$

since $q'(\theta) < 0$. Thus it follows that $\partial R_v/\partial\theta > 0$. Likewise, differentiating the first–order condition for a shedding firm, (14), with respect to θ we obtain:

$$\begin{aligned} -\eta\beta c + \beta D_\theta(n) &= -\frac{\eta\beta c}{1-\beta[G(R_v(n)) - G(R(n))]} \\ &\quad -\beta\frac{c}{q}\frac{q'(\theta)}{q}\frac{1 - G(R_v(n))}{1-\beta[G(R_v(n)) - G(R(n))]} \end{aligned} \tag{61}$$

Thus, $\partial R/\partial\theta > 0 \iff n > R_v^{-1}G^{-1}\left(1 + \frac{\eta}{\varepsilon_{q\theta}}f\right)$ where $\varepsilon_{q\theta} \equiv \frac{d\ln q}{d\ln\theta}$. ■

Proof of Proposition 5. *Proof of (18) and (19):* See main text.

Proof of (20): First note that the number of separations in a firm that is shedding workers is equal to $[n_{-1} - R^{-1}(x)]$, since separating firms set employment, $n = R^{-1}(x)$. Now imagine, counterfactually, that all firms shared the same lagged employment level, n_{-1}. Then, the aggregate number of separations in the economy would equal:

$$S_{n_{-1}} = \int_{n_{\min}}^{R(n_{-1})} \left[n_{-1} - R^{-1}(x)\right] dG(x) \tag{62}$$

where $n_{\min}$ is the lower support of employment. Using the change of variables, $x = R(n)$, and integrating by parts:

$$\begin{aligned}
S_{n_{-1}} &= \int_{n_{\min}}^{n_{-1}} (n_{-1} - n) \frac{dG[R(n)]}{dn} dn \\
&= [(n_{-1} - n) G[R(n)]]_{n_{\min}}^{n_{-1}} + \int_{n_{\min}}^{n_{-1}} G[R(n)] dn \\
&= \int_{n_{\min}}^{n_{-1}} G[R(n)] dn \\
&\equiv \Lambda(n_{-1}) \qquad (63)
\end{aligned}$$

Now, of course, the true aggregate number of separations is equal to $S = \int \Lambda(n_{-1}) dH(n_{-1})$, where $H(\cdot)$ is the c.d.f. of employment. Denoting $n_{\max}$ as the upper support of $H(\cdot)$, further integration by parts reveals that:

$$\begin{aligned}
S &= [\Lambda(n_{-1}) H(n_{-1})]_{n_{\min}}^{n_{\max}} - \int \Lambda'(n_{-1}) H(n_{-1}) dn_{-1} \\
&= \Lambda(n_{\max}) - \int G[R(n_{-1})] H(n_{-1}) dn_{-1} \\
&= \int G[R(n)] dn - \int G[R(n_{-1})] H(n_{-1}) dn_{-1} \\
&= \int [1 - H(n)] G[R(n)] dn \qquad (64)
\end{aligned}$$

as required. A similar method reveals that the aggregate number of hires in the economy, $M = \int H(n)(1 - G[R_v(n)]) dn$. It follows from the steady state condition for the distribution for employment, (18), that separations, S, are equal to hires, M. ■

Lemma 2 *If idiosyncratic shocks evolve according to (29), and the matching function is of the form $M(U, V) = \mu U^{\phi} V^{1-\phi}$, then the marginal firm surplus defined in (43) is given by:*

$$\begin{aligned}
J &= \frac{\psi p \alpha n^{\alpha-1}}{1 - \beta(1-\lambda)} \left[x + \frac{\beta \lambda p^0}{1 - \beta(1-\lambda) - \beta\lambda p^0} \mathcal{E}(n) \right] \\
&\quad - \frac{(1-\eta) b}{1 - \beta(1-\lambda) - \beta\lambda p^0} - \beta \frac{c}{q} \frac{\eta f - \lambda p^+}{1 - \beta(1-\lambda) - \beta\lambda p^0} \qquad (65)
\end{aligned}$$

and the marginal effects of n, p and θ on J are given by:

$$\begin{aligned}
J_n &= -\frac{1}{n} \frac{(1-\alpha) \psi p \alpha n^{\alpha-1}}{1 - \beta(1-\lambda)} \left[x + \frac{\beta\lambda p^0}{1 - \beta(1-\lambda) - \beta\lambda p^0} \mathcal{E}(n) \right] \\
J_p &= \frac{\psi \alpha n^{\alpha-1}}{1 - \beta(1-\lambda)} \left[x + \frac{\beta\lambda p^0}{1 - \beta(1-\lambda) - \beta\lambda p^0} \mathcal{E}(n) \right] \\
J_\theta &= -\beta \frac{c}{q} \frac{1}{\theta} \frac{\eta f - \phi \lambda p^+}{1 - \beta(1-\lambda) - \beta\lambda p^0} \qquad (66)
\end{aligned}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$, $\mathcal{E}(n) \equiv \mathbb{E}(x'|x' \in [R(n), R_v(n)])$, $p^0 \equiv \tilde{G}[R_v(n)] - \tilde{G}[R(n)]$, *and* $p^+ \equiv 1 - \tilde{G}[R_v(n)]$.

Proof. Since firms only receive an idiosyncratic shock with probability λ each period, we can use the recursion for $J(n,x)$, (43), to write:

$$\begin{aligned} J(n,x) &= \psi p x \alpha n^{\alpha-1} - (1-\eta) b - \eta\beta c\theta + \beta\lambda \frac{c}{q}\int_{R_v(n)} d\tilde{G} \\ &\quad + \beta(1-\lambda) J(n,x) + \beta\lambda \int_{R(n)}^{R_v(n)} J(n,x')\, d\tilde{G} \end{aligned} \tag{67}$$

Solving for $J(n,x)$ we obtain:

$$\begin{aligned} J(n,x) &= \frac{1}{1-\beta(1-\lambda)}\left[\psi p x \alpha n^{\alpha-1} - (1-\eta) b - \eta\beta c\theta\right] \\ &\quad + \frac{\beta\lambda}{1-\beta(1-\lambda)}\frac{c}{q}\int_{R_v(n)} d\tilde{G} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J(n,x')\, d\tilde{G} \end{aligned} \tag{68}$$

We then conjecture that $J(n,x)$ is of the form $j_0 + j_1 x$. Substituting this assumption into the latter, and equating coefficients yields:

$$\begin{aligned} j_0 &= -\frac{(1-\eta) b}{1-\beta(1-\lambda)} - \beta\frac{c}{q}\frac{\eta f - \lambda p^+}{1-\beta(1-\lambda)} + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)}\left[j_0 + j_1 \mathcal{E}(n)\right] \\ j_1 &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \end{aligned} \tag{69}$$

Solving for j_0 we obtain the required solution for $J(n,x)$. Likewise, we can obtain recursions for the marginal effects of n and θ:

$$\begin{aligned} J_n(n,x) &= -\frac{1}{1-\beta(1-\lambda)}\frac{1-\alpha}{n}\psi p x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_n(n,x')\, dG \\ J_p(n,x) &= \frac{1}{1-\beta(1-\lambda)}\psi x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_p(n,x')\, d\tilde{G} \\ J_\theta(n,x) &= -\frac{\eta\beta c + \beta\lambda \frac{c}{q^2} q'(\theta)\int_{R_v(n)} dG}{1-\beta(1-\lambda)} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_\theta(n,x')\, dG \end{aligned} \tag{70}$$

Again using the method of undetermined coefficients, and noting that the Cobb Douglas matching function implies $q = \mu\theta^{-\phi} \implies \frac{c}{q^2}q'(\theta) = -\frac{c}{q}\frac{\phi}{\theta}$, yields the required solutions for J_n, J_p and J_θ. ■

Proof of Proposition 6. Total differentiation of the JC condition, $U(\theta) = L - \mathbb{E}(n)$, yields $\frac{d\theta}{dp} = -\frac{\mathbb{E}(\partial n/\partial p)}{\mathbb{E}(\partial n/\partial \theta)}$. In steady state, the probabilities of raising, freezing, and cutting employment will all be constants. Denoting these probabilities as $\mathbf{p}^+, \mathbf{p}^0$, and $\mathbf{p}^-$ respectively, it follows that we can write:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \mathbf{p}^{+}\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + \mathbf{p}^{0}\mathbb{E}\left(\frac{\partial n_{-1}}{\partial \xi}\right) + \mathbf{p}^{-}\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \tag{71}$$

for any variable ξ. Note further that in steady state $\mathbb{E}\left(\partial n/\partial \xi\right) = \mathbb{E}\left(\partial n_{-1}/\partial \xi\right)$ so that we obtain the result that:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \pi\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \tag{72}$$

where $\pi \equiv \frac{\mathbf{p}^{+}}{1-\mathbf{p}^{0}}$. Thus, we can rewrite the marginal effect of a change in p on θ as:

$$\frac{d\theta}{dp} = -\frac{\pi\mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n < 0\right)}{\pi\mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n < 0\right)} \tag{73}$$

It is immediate from Lemma 2 that $\frac{\partial n}{\partial p} = -\frac{J_p}{J_n} = \frac{1}{1-\alpha}\frac{n}{p}$ regardless of whether $\Delta n > 0$ or $\Delta n < 0$. Thus it remains to derive $\frac{\partial n}{\partial \theta}$ in each case. To this end, note that the first–order conditions for optimal labor demand set the marginal firm surplus, $J\left(n, x\right)$ as follows:

$$J\left(n,x\right) = \begin{cases} c/q\left(\theta\right) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{74}$$

Log–linearizing the function J around n, p, x, and θ, we obtain:

$$\log J \approx \varepsilon_{Jn}\log n + \varepsilon_{Jp}\left(\log p + \log x\right) + \varepsilon_{J\theta}\log\theta + const. \tag{75}$$

Using this and totally differentiating the first–order conditions for optimal labor demand with respect to n and θ, we obtain:

$$\varepsilon_{Jn} d\log n + \varepsilon_{J\theta} d\log\theta \approx \begin{cases} -d\log q\left(\theta\right) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{76}$$

Given the Cobb Douglas matching function assumption, $q\left(\theta\right) = \mu\theta^{-\phi}$, and it follows that $d\log q\left(\theta\right) = -\phi d\log\theta$. Thus:

$$\frac{\partial n}{\partial \theta} = \frac{\partial \log n}{\partial \log \theta}\frac{n}{\theta} \approx \begin{cases} \frac{\phi-\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n > 0 \\ -\frac{\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n < 0 \end{cases} \tag{77}$$

Substituting this into (73), we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx -\frac{1}{1-\alpha}\frac{\varepsilon_{Jn}}{\omega\phi - \varepsilon_{J\theta}} \tag{78}$$

where $\omega \equiv \frac{\pi\mathbb{E}(n|\Delta n>0)}{\mathbb{E}(n)}$ is the steady state share of employment in hiring firms. In what follows, we take the approximation to the marginal surplus around mean employment, $n = \mathbb{E}(n)$, and mean productivity conditional on mean employment, $x = \mathcal{E}(\mathbb{E}(n)) \equiv \mathbb{E}(x'|x' \in [R(\mathbb{E}(n)), R_v(\mathbb{E}(n))])$, . Thus, using the results of Lemma 2 it follows that we can write:

$$J_n = -\frac{1}{n}\frac{(1-\alpha)\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)-\beta\lambda p^0}\mathcal{E}(\mathbb{E}(n))$$

and:

$$J\left[1-\beta(1-\lambda)-\beta\lambda p^0\right] = \psi p \mathcal{E}(\mathbb{E}(n))\alpha n^{\alpha-1} - (1-\eta)b - \beta\frac{c}{q}\left[\eta f - \lambda p^+\right] \quad (79)$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$. Substituting back into our aggregate elasticity of θ with respect to p, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{\psi p \mathcal{E}(\mathbb{E}(n))\alpha n^{\alpha-1}}{\omega\phi\left[\psi p \mathcal{E}(\mathbb{E}(n))\alpha n^{\alpha-1} - (1-\eta)b - \eta\beta c\theta\right] + \eta\beta c\theta - (1-\omega)\phi\beta\frac{c}{q}\lambda p^+} \quad (80)$$

Noting that the marginal product of labor in the average–sized firm is equal to $p\mathcal{E}(n)\alpha n^{\alpha-1}$, and assuming λ is sufficiently small, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{(1-\eta)\tilde{p}}{\omega\phi\left[(1-\eta)(\tilde{p}-b) - \eta\beta c\theta\right] + \eta\beta c\theta} \quad (81)$$

where $\tilde{p} \equiv \rho p \mathcal{E}(\mathbb{E}(n)) n^{\alpha-1} + (1-\rho) p \mathcal{E}(\mathbb{E}(n))\alpha n^{\alpha-1}$ and $\rho \equiv \frac{\alpha\eta}{1-\alpha(1-\eta)}$, as required. ■