Labor Supply, consumption and domestic production: a new method to estimate labor supply elasticities

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Abstract

This article derives a new approach for estimating the elasticity of labor supply using Becker's allocation of time framework and household choices concerning time and monetary expenditures. Labor supply elasticities with respect to market wages and the opportunity cost of non-market time are calculated using data from France and Canada. They are found to be coherent with those obtained by classic methods on individual data while avoiding usual econometric problems. The estimates of the elasticity of labor supply fare lower for couples with children as expected.

JEL Codes: J22, D11, D13

Introduction

Estimation of the elasticity of the labor supply with respect to market wages has produced abundant literature in many countries (see Bargain and Peichl, 2013), with results varying according to the methods and the datasets (country, macro or micro data) used. Among the econometric problems encountered in these estimations, endogeneity and observability of wages and unearned income are among the most prominent, while explicit functional form assumptions or structural models are also sometimes employed.

An alternative approach involves recognizing that market labor supply is the complement of domestic production, their sum being equal to total time available (after deducting time devoted to sleep and other activities necessary to survive). This suggests that it should be possible to estimate labor supply elasticities indirectly, through the estimation of time use functions for home production and consumption.

This paper exploits the framework developed by Gardes (2014), which combines monetary expenditures (observable in data such as Family Expenditure Surveys) with time use data in a model that generates distinct values for time devoted to market work (supposed to depend on, but not be equal to, the household's wage rate net of taxes w) and time used in domestic production. This model is used in the first section to derive the opportunity cost of time and formulas for elasticities of monetary expenditures and time use with respect to the "full" prices of goods and to the opportunity cost of time. The second section presents the derivation

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This article uses the French dataset prepared in collaboration with C. Starzec (Gardes, Starzec, Sayadi, 2015).

of the labor supply function while the third section presents the dataset. These are used in section 4 to estimate the elasticity of the labor supply with respect to the opportunity cost of time and to the wage rate. Section 5 concludes.

1. Domestic production and consumption

In Gardes (2014), the household is supposed to divide its monetary income and total disposable domestic time (excluding its necessary time for sleeping and the working time) between a set of activities (final goods) i which are supposed to be produced by the consumption of x_i units of an aggregate market good (with unit price p_i) and an amount of time equal to t_i=$\tau_i x_i$, where τ_i is the time it takes to covert x_i units of the market good into a unit of final good i[3]. The "full" price π_i for one unit of the final good (domestic activity) i is written: $p_i x_i + \omega t_i = (p_i + \omega\tau_i)x_i$, in which ω is the opportunity cost of time spent outside the labor market. The full price and income budget constraints, with T being total time and t_w being working time, can thus be written:

$$\sum_i (p_i\, x_i + \omega t_i) = y^f + (\omega - w)(T - t_w) = y^f + (\omega - w)\sum_i \tau_i x_i \qquad (1)$$

In order to simplify the derivation of the opportunity cost of time, a Cobb-Douglas structure is chosen both for the utility and the domestic production functions of the final goods Q_i which depend on the monetary and time inputs[4]. The optimization program is, according to the assumptions of section 1 (all variables correspond to a household whose index is omitted in the equations):

$$\max_{m_i,t_i} u(Q) = \prod_i a_i Q_i^{\gamma_i} \;\; with\; Q_i = m_i^{\alpha_i} t_i^{\beta_i} \qquad (2)$$

under the full income constraint (1).

Note that $T - t_w = \sum_i t_i$ and that both the market wage and the shadow wage (i.e. the opportunity cost of time ω) appear in the budget equation: the shadow wage corresponds to the valuation of time in domestic production, and differs from the market wage w whenever there exists imperfections on the labor market or if the disutility of labor is smaller for domestic production.

In order to estimate the opportunity cost for time, the utility function can be re-written:

$$u(Q_i) = \prod_i a_i Q_i^{\gamma_i} = \prod_i a_i \left[\prod_i m_i^{\alpha_i\gamma_i/\sum\alpha_i\gamma_i}\right]^{\sum\alpha_i\gamma_i} \left[\prod_i t_i^{\beta_i\gamma_i/\sum\beta_i\gamma_i}\right]^{\sum\beta_i\gamma_i}$$

$$= a m'^{\sum\alpha_i\gamma_i} t'^{\sum\beta_i\gamma_i} \quad (3)$$

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[3] The production of the final good i is modeled by a Leontief technology with respect to time, based on the amount of market good used in production (or equivalently based on the amount of the final good, as each unit of final good necessitates, assuming the absence of economy of scale, the same consumption of the market good).

[4] Their parameters will be estimated on each point (for each household in the dataset), so that this specification just supposes the constancy of each household's elasticities of the domestic productions in the utility, and the two factors in the production functions.

with m' and t' the geometric weighted means of the monetary and time inputs with weights $\alpha_i\gamma_i / \sum \alpha_i\gamma_i$ and $\beta_i\gamma_i / \sum \beta_i\gamma_i$. Deriving the utility over income Y and total available leisure and domestic production time T gives the opportunity cost of time:

$$\omega = \frac{\frac{\partial u}{\partial T}}{\frac{\partial u}{\partial Y}} = \frac{\frac{\partial u \partial t'}{\partial t' \partial T}}{\frac{\partial u}{\partial m'}\frac{\partial m'}{\partial Y}} = \frac{m' \sum \beta_i\gamma_i}{t' \sum \alpha_i\gamma_i} \frac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}} \tag{4}$$

All parameters of the utility function are estimated locally (for each household) so that the household's welfare will depend both on the set of parameters (α, β, γ) and on its monetary and time expenditures m_i and t_i. In order to calculate the parameters of the utility and domestic production functions, Gardes (2014) considers the substitutions between time and money resources for the production of each activity and between money expenditures (or equivalently time expenditures) for two different activities[5]. The substitution between time and money in the domestic production function of activity i generates the first order condition:

$$\alpha_i = \frac{m_i}{\omega t_i + m_i} \; and \; \beta_i = \frac{\omega t_{i_i}}{\omega t_i + m_i} \tag{5}$$

under the constraint of a constant returns to scale for each production function: $\alpha_i + \beta_i = 1$, while the substitution between times t_i and t_j in the domestic production of two different final goods i and j implies another condition between the parameters of the domestic production functions and the utility function:

$$\gamma_i = \gamma_1 \frac{t_i}{t_1} \frac{\alpha_1}{\alpha_i} \tag{6}$$

The estimation of parameters α, β, γ allows one to define both the opportunity cost of time ω and the welfare index $u(Q_i)$ at the household level.

Appendix A discusses two methods for the estimation of these parameters. The estimation is made on six activities (food consumption, dwelling expenditures (including imputed rent), clothing expenditures, leisure expenditures, transport expenditures and miscellaneous goods and services), excluding expenditures on health and education, which contain many zeros. The estimation of the opportunity cost of time is made for data grouped into cells (defined by household size, the education level and the age of the head) in order to obtain robust estimates of the parameters α, β and γ of the utility and domestic production functions. Using equation (4), an estimate of the opportunity cost of time is obtained for each household. It averages 6.72, with a range between 5.3 and 10.3. It is thus significantly smaller than the average net wage rate (9.64), with 73% of households having an opportunity cost more than 25 percent lower than its net wage rate.

The average estimate of the opportunity cost of non-market time is close to the minimum wage rate (6.92). It corresponds qualitatively to individuals' responses in direct surveys of

[5] Note that it is sufficient to examine the implication of two over these three types of substitution.

their substitution between time and money, which usually generate an opportunity cost of time lower than the agent's wage rate net of taxes. Moreover, ω is positively related to the household's net wage (with an elasticity of 0.85) and to income conditional to net wage (elasticity of 0.19). It is also positively related to the household *relative* income, defined by the ratio of the household's average income to the average income of a reference population, defined by the aggregation used in the estimation of the parameters of the utility and domestic production functions. It increases until the household's head is 45 years old, then decreases 15 years later (just as observed by Aguiar and Hurst, 2007, in their estimation of the opportunity cost of time for shopping activities). It also increases with family size, especially with respect to the number of adults, which may show that home production is more valued in large families because of economies of scale (i.e. production of public goods).

This estimate of the opportunity cost of time allows one to define the full price of each activity and to estimate full price elasticities for these expenditures. As the monetary price is not observed, the full cost for activity i is proxied by the ratio of the full expenditure over its monetary component (see Gardes, 2014, for a discussion):

$$\pi_i = \frac{(p_i+\omega_h\tau_{ih})x_{ih}}{p_i x_{ih}} = \frac{p_i+\omega_h\tau_{ih}}{p_i} = 1 + \frac{\omega_h\tau_{ih}}{p_i} \qquad (7)$$

Using the full price elasticities E_π estimated with these proxies, the elasticities with respect to the monetary price E_p, the time used for the consumption activity E_t and the opportunity cost of time E_ω are easily recovered (see for instance De Vany, 1974[6]) and can be calculated by means of the expression for full expenditures and its monetary and time components:

$$E_{x_i/p_i} = E^i_{\pi_i}\frac{p_i}{\pi_i} = E^i_{\pi_i}\frac{p_i x_i}{\pi_i x_i} = E^i_{\pi_i}\frac{m_i}{m_i+\omega t_i} \qquad (8)$$

$$E_{x_i/t_i} = E^i_{\pi_i}\frac{\omega\tau_i}{\pi_i} = E^i_{\pi_i}\frac{\omega t_i}{m_i+\omega t_i} \qquad (9)$$

$$E_{x_i/\omega} = \sum_j E^i_{\pi_j}\frac{\omega\tau_j}{\pi_j} = \sum_j E^i_{\pi_j}\frac{\omega t_j}{m_j+\omega t_j} \qquad (10)$$

2. Labor supply

The relation between the allocation of time for market work, home production, investment activities (education, health care…) and leisure can be written as:

$$\sum_i t_i = \sum_i \tau_i x_i = T - t_w \qquad (11)$$

Taking the derivative of (10) with respect to the opportunity cost of time gives:

$$-\frac{\partial t_w}{\partial\omega} = \frac{\partial\sum_i t_i}{\partial\omega} = \sum\left(\tau_i\frac{\partial x_i}{\partial\omega} + x_i\frac{\partial\tau_i}{\partial\omega}\right) \qquad (12)$$

which can be written in terms of elasticities as:

[6] Note an error in De Vany's formula (11) for the opportunity cost elasticity.

$$-E_{t_w/\omega} = \sum_i \frac{\tau_i x_i}{t_w}\left(E_{\frac{x_i}{\omega}} + E_{\frac{\tau_i}{\omega}}\right) \quad (13)$$

The elasticity of τ_i (which is the time used per unit of market good i) with respect to the opportunity cost of time can be calculated in terms of the elasticity of the market good and the elasticity of substitution between the monetary (market good) and time factors used to home produce the final good (activity) i:

$$\sigma_i = \frac{\partial \ln\left({}^{x_i}/_{t_i}\right)}{\partial \ln(\omega)} = E{x_i}_{/\omega} - E{t_i}_{/\omega}$$

so that $E{\tau_i}_{/\omega} = E{t_i}_{/\omega} - E{x_i}_{/\omega} = -\sigma_i$. This elasticity of substitution has been estimated by Canelas et al. (2014) as 0.276 ($\sigma = 0.031$) for Food and 0.584 ($\sigma = 0.023$) for other expenditures[7]. We obtain finally the formula for the elasticity of labor supply with respect to the opportunity cost of time:

$$-E_{t_w/\omega} = \sum_i \frac{\tau_i x_i}{t_w}\left(E_{x_i/\omega} - \sigma_i\right) \quad (14)$$

3. Dataset

We use a French dataset from INSEE which combines monetary and time expenditures at the individual level into a common, unique goods and services consumption structure by a statistical match of the information contained in two surveys: the Family Expenditure Survey (FES, INSEE BDF 2001) and the Family Time Budget (FTB, INSEE BDT 1999). We define 8 types of activities or time use types compatible with the available data both from the FES and FTB: *Eating and cooking time* (FTB) and food consumption (FES), *cleaning and home maintenance* and dwelling expenditures (including imputed rent), *clothing maintenance* and clothing expenditures, *education time* and education expenditures, *health care time* and health expenditures, *leisure time* and leisure expenditures, *transport time* and *transport expenditures, miscellaneous time use* and miscellaneous goods and services.

Two matching methods were used: first, by clustering (into 40 cells) the whole population in terms of age, education, and location as key variables, one obtains a good treatment of measurement errors and zero expenditure problems. For each activity and for each household we compute the corresponding time use and then take the cell weighted average[8]. This allows us to create comparable cells of base information for the full time and money expenditure nomenclature from both surveys. The addition of both will be possible once the individual time value is estimated. This method was used in Gardes et al. (2013) and was shown to allow for robust estimation of the full cost of a child (which is estimated to be greater than the monetary cost). However, it seems more appropriate to use an alternative method, calculating the time component of each activity at the household level rather than for a cell that groups

[7] The estimates for various expenditures are all greater than for food (from 0.287 for transport to 1.272 for Leisure) but seem to be less robust than the estimate for all expenditures less food, so we use the estimate for the aggregate in our calculation.

[8] Weighting was necessary to take into account the survey interview day (week-end or a work day). The weights are the proportions of the persons interviewed in the week (0.74) or during the week-end (0.26).

many different households. The second matching method, used in this article, is based on an individual matching by regression: time used in each activity is regressed on household characteristics (see Gardes, 2014). Time use equations for all selected activities are estimated on households characteristics for all observation units in the FTB survey and these estimations serve to predict the time spent on these activities in the corresponding units in the FES survey.

4. Results

Table 1 contains the estimates of the elasticities of monetary expenditures with respect to full prices and the opportunity cost of time[9], which are used to compute (table 2) the elasticity of the labor supply for the whole population and for various sub-populations (bachelors, couples without children, couples with children).

Table 1
Income and full price elasticities

	Monetary income	Full income*	Opportunity Cost	Full Own-price elasticities	Elasticity of substitution
Food	0.774 (0.0207)	0.210 (.0082)	0.875 (0.0170)	-0.975 (0.0065)	0.276 (0.031)
Housing	1 .023 (0.0167)	0.550 (.0135)	0.899 (0.0204)	-1.286 (.0156)	0.584 (0.023)
Clothing	1 .131 (0.0346)	0.532 (.0158)	0.876 (0.0327)	-0.973 (.0058)	0.584 (0.023)
Transport	1.021 (0.0240)	0.469 (.0151)	0.913 (0.0222)	-0.851 (.0057)	0.584 (0.023)
Leisure	0.920 (0.0280)	-0.463 (.0092)	1.213 (0.0144)	-1.104 (.0092)	0.584 (0.023)
Other	1.081 (0.0117)	0.531 (.0206)	0.958 (0.0182)	-1.796 (.0128)	0.584 (0.023)

Source: Gardes, 2014, tables 1 and 2. See Appendix B and C.
N.B. *Formula (12) in Gardes (2014). For the calculation of full own-price elasticities: quality effect corrected by Deaton's method, the monetary budget share and income are used, and the full price is calculated with the estimated opportunity cost of time.

[9] Appendix A presents detailed results of the estimation for the whole population. The market work time has been estimated in France as 1750 hours multiplied by the average number of workers in the sub-population. In Canada, it is directly observed in the micro dataset.

Table 2

Elasticity of the labor supply

	$\sum_i \frac{\tau_i x_i}{t_w}\left(E_{\frac{x_i}{\omega}}\right)$	$\sum_i \frac{\tau_i x_i}{t_w}\sigma_i$	$\boldsymbol{E_{t_w/\omega}}$	$\boldsymbol{E_{t_w/w}}$
Whole Population	**-0.116**	**0.977**	**0.861 (0.0038)**	**0.732 (0.0032)**
Singles without children	**0.216**	**1.227**	**1.443 (0.0107)**	**1.227 (0.0091)**
Singles with children	**-0.136**	**0.842**	**0.706 (0.0159)**	**0.600 (0.0135)**
Couple without children	**-0.225**	**1.268**	**1.043 (0.0096)**	**0.887 (0.0082)**
Couple with children	**-0.236**	**0.827**	**0.590 (0.0064)**	**0.502 (0.0054)**

Table 2 shows that, on average, the estimated elasticity of hours of labor supplied with respect to the opportunity cost of time is 0.861, while the elasticity with respect to wages is 0.732. This result derives mechanically from the relation between wages and the opportunity cost of non-market time (the latter elasticity being multiplied by a factor of 0.85). This relation makes intuitive sense as well, as the difference between the opportunity cost of time and wages derives from the other income sources conditional on wages, and increasing other available income allows the household to substitute purchased final goods for own-produced final goods, freeing up the corresponding time for more market work.

Another result worth noting is that labor supply elasticities are estimated to be systematically lower when children are present, regardless of whether the household is a single or a couple. If there is an incompressible minimum amount of time that must be spent on child care in households with children, this will necessarily leave less time available for market work, and this less possible variation in the amount of time spent working in response to any given variation in wages or the opportunity cost of time.

As a point of reference, Bargain and Piechl (2013) identified many studies (including five studies for France) that estimate the elasticity of labor supply with respect to wages. Using a meta-analysis to consolidate information across studies and controlling for a series of study specific variables[10], they find that average married women's hour elasticities with respect to wages are 0.454 and have been trending down by 0.012 per year. With an estimated standard error of 0.089, one cannot reject the hypothesis that their estimate is the same as our estimate for couples with children (0.502, with a standard error of 0.0054).

The same model has been estimated on a similar dataset in Canada (matching a Time Use survey with a Family Expenditure survey in 1998[11]). The elasticity of the labor supply

[10] Their meta-analysis includes controls for year, model type, whether the outcome was desired instead of actual hours, whether joint decision making was assumed, fixed costs and an indicator variable for a US-based study.
[11] Dataset prepared by P. Merrigan.

with respect to the opportunity cost of time is estimated as 0.17 (s.e. 0.019) for the whole population (which corresponds to an elasticity with respect to the market wage rate around 0.14 using the proportion between the two elasticities calibrated for France, a value coherent with the estimation proposed in the litterature). Contrary to the estimation on French data, the elasticity does not differ significantly between couples with one child and couples without children.

Conclusion

Labor supply is one use of time among many, and time allocation models provide a means of estimating the sensitivity of consumption and time allocation to wages and the opportunity cost of non-market time. Using the Gardes (2014) model, one can use time and monetary expenditures on consumption goods to derive indirectly the elasticity of labor supply with respect to the opportunity cost of non-market time and wages, which are shown to differ. This approach leads to estimates of wage and opportunity cost elasticities that, although slightly higher than those found elsewhere in the literature, are not significantly different from those found elsewhere. These elasticities, being derived at the household level, can be decomposed along many dimensions and highlight important behavioral substitutions in terms of time allocation that take place when the price of time changes.

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Appendix A
Estimation of the opportunity cost of time in the domestic production model

In order to calculate the parameters of the utility and domestic production functions, we consider the substitutions which are possible, first between time and money resources for the production of some activity, second between money expenditures (or equivalently time expenditures) concerning two different activities[12]. First, the substitution between time and money in the domestic production function of activity i generates the first order condition:

$$\frac{\frac{\partial u}{\partial t_i}}{\frac{\partial u}{\partial m_i}} = \omega \rightarrow \frac{\alpha_i}{\beta_i} = \frac{m_i}{\omega t_i} \text{ which implies: } \alpha_i = \frac{m_i}{\omega t_i + m_i}, \beta_i = \frac{\omega t_{i_i}}{\omega t_i + m_i} \quad (5)$$

under the constraint of a constant economy of scale for each production function: $\alpha_i + \beta_i = 1$. We suppose also that all marginal productivities are positive: $\alpha_i, \beta_i, \gamma_i \geq 0$ and we normalize the utility: $\sum \gamma_i = 1$.

Consider now the substitution between times t_i and t_j in the domestic production of two different final goods i and j: this substitution implies another condition between the parameters of the domestic production functions and the utility function:

$$\frac{\gamma_i}{\gamma_j} = \frac{\beta_j t_i}{\beta_i t_j} = \frac{\alpha_j m_i}{\alpha_i m_j} \text{ so that: } \gamma_i = \gamma_1 \frac{m_i}{m_1} \frac{\alpha_1}{\alpha_i} \text{ for all } i \neq 1 \quad (6)$$

All other substitutions between monetary and time resources devoted to different final goods can be derived from (9) and (10).

In order to estimate these parameters, a possible method (C) consists in the calibration of the opportunity cost of time in a first stage, for instance at the minimum wage rate which is constant over the population. Equations (5) thus gives an estimate of α_i and β_i for each household, which gives γ_i by equation (6). In the second step, an estimate of the opportunity cost of time ω is given by equation (4) which allows the computation of the individual values of the parameters α, β, γ for each household using equations (5) and (6). These values enter equation (4) to give for each household the second step estimate of ω. The estimations on the French data as well as simulations[13] tends to show that this procedure may not converge rapidly to the true value of the opportunity cost of time.

Another method can be directly based on equations (5) and (6) which imply for all activities i:

$$m_i \gamma_j = m_j \gamma_i + \omega \gamma_i t_j - \omega \gamma_j t_i \quad (15)$$

[12] See estimates of the elasticity of substitution between time and monetary expenditures in Gardes et al ., 2014.
[13] performed by J. Boelaert.

This can be estimated as a system of (n-1) equations for j=1, i=2 to n, calibrating γ_j at the full budget share for food[14] or under the homogeneity constraint of the utility function: $\sum \gamma_i = 1$. In this system, the opportunity cost of time is over-identified, as well as all $\gamma_j, j > 1$.

[14] Note that taking together equations 8, 9 and 11 to calculate the value of ω gives rise to a highly non- linear equation in γ_i and ω which cannot be solved algebraically.

Appendix B
Econometric methodology for the estimation of the demand system (Gardes, 2014)

The Almost Ideal Demand System is the most commonly used model to estimate demand elasticities. One of the main advantages of the model is that even if the model is nonlinear, one can use a Stone price index to approximate the AI model to its linear version LAIDS, so as to facilitate estimation. As pointed out by Pashardes (1993), the errors coming from that approximation can result in biased parameter estimates, as it can be seen as an omitted variable. The bias is bigger when the AI model is applied to micro-data, because in this case the expenditure effects are highly correlated with the demographic characteristics of the household and thus very heterogeneous between households. In order to correct this bias, Pashardes proposes a simple re-parameterization of the price parameter that circumvents the problem created by the Stone price index. In the estimations presented in table 3 (3rd column), the full price elasticities are computed in a linear Almost Ideal demand system specification on full budget shares (corrected from the income effect by formula A1 below with income proxied by total expenditure $\sum \pi_i x_i$) and full prices (defined by equation 13). The own-full price elasticity for consumption i writes:

$$E_{ii} = \frac{\gamma_{ii}}{w_i} + \beta_i - 1$$

with β_i the income coefficient of the AI demand system equation for consumption i and γ_{ii} the coefficient of the own logarithmic price $log\pi_i$.

Four specific problems appear in the estimation of this demand system on matched data: *first*, supposing that full expenditures follow an independent optimization scheme, based either on a utility function or a cost function, implies a total substitution between time and monetary household's expenditures. It is also plausible to suppose that two independent optimizations exist for monetary and for time allocations, but in this case the demand system for full expenditure cannot in general be similar to the equations for monetary and time expenditures. If for instance the cost functions for the monetary and the time expenditures are supposed to be Piglog, both demands are specified as an Almost Ideal demand system (with different parameters). But in that case, the budget share for full expenditures w_{if} depends on the monetary and time budget shares: $w_{if} = \frac{y_m w_{\Im} + y_t w_{it}}{y_m + y_t}$ and the resulting demand equation for full expenditure cannot be written under as Almost Ideal specification because of the non-linearity in the income variable.

Following this hypothesis of two separate optimizations for the components of the full expenditure, we can calculate the full income elasticity E_{if} in terms of the monetary and time elasticities E_{im} and E_{it}:

$$E_{if} = E_{im} \cdot \frac{w_{\Im}}{w_{if}} \cdot \frac{1}{1+k} + E_{it} \cdot \frac{w_{it}}{w_{if}} \cdot \frac{k}{1+k} \qquad \text{(B1)}$$

with k the derivative of the temporal income over the monetary income. The income coefficient is fixed by equation (A1) in the estimation of the full expenditures demand system in the second column of Table 3.

Second, quality effects are likely to exist in full price and expenditure data. Indeed, an increase (in the cross-section dimension i.e. between two households) of the full price for commodity (activity) i may result either from the difference (between the two agents) of the opportunity cost ω or from the difference of their time allocated to activity i. Both causes may increase the quality of this activity, by means of an increased productivity (which can be supposed to be positively related to ω) or of the time devoted to i. This endogenous quality appears in the same form as in Deaton's technique to estimate price-elasticities on local prices after removing the quality incorporated in unit values (which is the ratio of expenditures over quantities consumed). In our matched dataset, local prices are replaced by the individual full prices for each household.

Deaton (1988) shows that the elasticity of expenditures Q_i over its unit value V_i writes

$$\boldsymbol{\varepsilon_{ip}} = \frac{\partial lnQ_i}{\partial lnV_i} = \frac{E_{ip}}{1+\eta_i\frac{E_{ip}}{E_{iy}}}$$

with E_{ip} the true price-elasticity, E_{iy} the income-elasticity and η_i the income-elasticity of the unit value. This formula allows to calculate the true price-elasticity in terms of the other parameters. In order to estimate $\boldsymbol{\eta_i} = \frac{\partial lnV_i}{\partial lny}$, the two equations model *(Deaton's equations 14 and 15)* is written for household h in cluster *C*:

$$w_{hC} = \alpha_1 + \beta_1 lny_{hC} + Z_{hC}\gamma_1 + u_{1hC} \qquad \text{(B2)}$$
$$lnV_{hC} = \alpha_2 + \beta_2 lny_{hC} + Z_{hC}\gamma_2 + u_{2hC} \qquad \text{(B3)}$$

I define clusters as households which are supposed to have the same opportunity cost of time and the same domestic production (thus the same τ_i) for instance by means of a common age class, location and education of the head. I thus estimate $\eta = \beta_2$ by (B3) within clusters and $E_{ip}=\gamma_1$ and $\beta_1=E_y$ by (A2) with the full individual prices included in Z_{hc}.

The third problem concerns the *correction of variances* necessitated by the fact that budget times are generated regressors being predicted for each household of the Family Expenditures survey from the time budgets recorded in the Time Use survey. These estimated times are added to the household's monetary expenditures to form the household's full expenditures. These expenditures serve to calculate indices of scarcity which are used as full prices π in the estimation of the demand system $h(x, \beta, W, \pi)$ where W is the set of variables used in the first step to predict π, α and β and the parameters of the demand functions. Thus, the full prices are generated in a first step before the estimation of the demand system, which necessitate to correct the estimated variances. Murphy and Topel (1985) proposed a method adapted to this case. Their theorem states that the second step estimator β is consistent and asymptotically normal with an asymptotic covariance matrix (as stated by Greene, 2008):

$$V_{\beta^*} = \sigma^2 V_b + V_b[CV_cC' - CV_cR' - RV_cC']V_b$$

where V_b is the covariance matrix given by the second step of the estimation,
$C = n\,plim\frac{1}{n}\sum_{i=0}^{1} x_i^0 \widehat{\epsilon_i}^2 (\frac{\partial h(x,\beta,w,\pi)}{\partial \pi})$ and $R = n\,plim\frac{1}{n}\sum_{i=0}^{1} x_i^0 \widehat{\epsilon_i} (\frac{\partial g(w,\pi)}{\partial \pi})$ with g depending on the log-likehood function. In the case where π is predicted by a linear regression, C can be written:

$$C = d\sum_{i=1}^{n} {e_i}^2\, x_i w_i$$

with e_i the error term of the demand function in the second step and d the coefficients in the estimation of π in the first step. R is null if the regression disturbances of the regression in the first and second steps are uncorrelated, which is the case of our model since full prices are predicted from the Time Use surveys and used as a regressor on the family Expenditures survey. Finally, we obtain $C = \sum_{i,j} d_i d_j d_{ij}$ depending on the coefficients and covariance terms of the first step regression.

A simple bootstrap procedure is an alternative to the Murphy-Topel method. Consider that the full prices are estimated using the activity times predicted by the actual times observed in the Time use survey and can be considered as instrumented values of the actual full prices. The usual method to correct the variances in case of instrumentation compares the residuals $\hat{\varepsilon}_1 = y - X\hat{\beta} - ln\pi^{IV}\hat{\gamma}^{IV}$ estimated in the second step regression using the logarithmic prices defined by the predicted times with the residuals $\hat{\varepsilon}_2$ computed with the parameters $\hat{\beta}$, $\hat{\gamma}^{IV}$ issued from instrumentation and the *actual* value for full prices lnπ:$\hat{\varepsilon}_2 = y - X\hat{\beta} - ln\pi\hat{\gamma}^{IV}$. In our case, actual full prices π_i are not observed in the monetary statistics. We can however simulate them knowing the distribution for the set of prices for household h issued from the prediction of prices: $ln\pi_{ih} \sim N(\log[\pi_i^{IV}(h), V_i]$ supposed to be normal (as indicated by the distribution of the logarithmic full prices) with the estimated price as the mean of the distribution. The variance $V_i = V(log\pi_i)$ of the logarithmic price for activity i is estimated by the empirical variance of the logarithmic full prices $ln\pi_i{}^{IV}$ computed on the monetary expenditures survey. Therefore, this residual writes for household h:

$$\hat{\varepsilon}_2(h) = \hat{\varepsilon}_1(h) + \sum \hat{\gamma}_i\left[ln\pi_i^{IV}(h) - ln\pi_i(h)\right]$$

so that: $V(\hat{\varepsilon}_2) = V(\hat{\varepsilon}_1) + \sum \hat{\gamma}_i^2 V_i$.

This procedure will be preferred because of its simplicity and the fact that it takes fully into account the non-linear nature of predicted logarithmic full prices. Both procedures give similar inflation of standard errors (for instance 156% and 165% for housing expenditures, 119% and 152% for transportation).

Finally, endogeneity may appear in the full demand equations because the opportunity cost of time (and the unit time for activity τ_i) appear both in the full expenditure for i, in the full total expenditure and in the vector of full prices for all commodities. This problem exists because full prices are endogenous, depending on the household type and characteristics (in classic demand systems, prices are on the contrary pre-determined and generally supposed to be constant across the population). This possible endogeneity bias can be taken into account by instrumentation of full prices and full total expenditure or GMM. Also, it is possible to calibrate the full income elasticity by formula (B1), using monetary and time elasticities estimated by two demand systems written respectively on monetary and time expenditures. In our estimations, we check that defining prices by an alternative valuation of time than the opportunity cost of time used to define the household's expenditures and full income (for instance, full prices are defined using the minimum wage rate as the opportunity cost of time while full expenditures are computed with an econometric estimate of the household's opportunity cost) gives close estimates (up to a 20% difference) to those using the opportunity cost to value all variables. Another way to estimate full price elasticities consists in estimating the demand system on monetary expenditures and full prices. It gives price elasticities similar to those obtained by the estimation on full expenditures which we use in the empirical application (Table 3, 3th column).

Appendix C

Table C1 includes the price elasticities estimated by LAIDS on monetary budget shares with quality correction (see Gardes, 2014). They are compared to the parameters estimated under separability restrictions by the Frisch method (Theil et al.,1987, Selvanathan, 1993). The price elasticity estimates under strong separability are estimated for two calibration of the inverse of the income flexibility at -0.5 and -1.18 which is the estimated value for this dataset obtained by the estimation of a Rotterdam model (GArdes, 2014, section III).

Full price elasticities estimated on full budget share by a LAIDS system (calibrating the full income elasticity by formula B1 in Appendix B) are very similar to those obtained on the equation using monetary expenditures. The estimation of price elasticities over the index of scarcity defined by equation (7) is robust to the specification of the demand system, either on full expenditures or on their monetary components and does not differ much according to the method used to compute the opportunity cost.

Table C1 (table 2, Gardes, 2014)
Income, price and opportunity cost Elasticities

	Monetary income	Full income*	Full price**	Monetary price	Time	Opportunity Cost	Mon. price : Grouping method ***	Mon. price : Strong separability**** $\breve{\omega}^{-1}$ -0.5 -1.18
Food	0.774 (0.0154)	0.875 (0.0170)	-0.977 (0.033)	-0.350 (0.012)	-0.627 (0.021)	0.210 (.0062)	-0.810 (0.169)	-0.459 -0.924 (.0111) (.0218)
Housing	1 .023 (0.0129)	0.899 (0.0204)	-0.537 (0.440)	-0.368 (0.301)	-0.169 (0.139)	0.550 (.0104)	-0.383 (0.150)	-0.680 -1.136 (.0082) (.0118)
Clothing	1 .131 (0.0242)	0.876 (0.0327)	-0.565 (0.183)	-0.363 (0.117)	-0.202 (0.066)	0.532 (.0110)	-0.527 (0.066)	-0.635 -1.281 (.0170) (.0369)
Transport	1.021 (0.0174)	0.913 (0.0222)	-0.526 (0.055)	-0.282 (0.030)	-0.244 (.025)	0.469 (.0109)	-0.549 (0.010)	-0.543 -1.191 (.0234) (.0234)
Leisure	0.920 (0.0190)	1.213 (0.0144)	-0.795 (0.031)	-0.218 (0.008)	-0.577 (.023)	-0.463 (.0062)	-1.306 (0.032)	-0.532 -1.074 (.0144) (.0278)
Other	1.081 (0.0082)	0.958 (0.0182)	-0.990 (0.049)	-0.562 (0.028)	-0.428 (0.021)	0.531 (.0206)	-0.953 (0.142)	-0.592 – 1.245 (.0178) (.0366)

* combining the monetary income and time elasticities, equation (A1) in Appendix.
** LAIDS specification with full budget share; income parameter calibrated by formula A1; full price calculated with the estimated opportunity cost of time $\hat{\omega}$ (method D, estimation on 40 cells, table 1). Quality effect corrected by Deaton's method (see Appendix).
***Hicks-Lewbel grouping method (Ruiz, 2006).
**** Frisch formulas for the own-price and cross-price elasticities E_{ii} and E_{ij} ($\breve{\omega}^{-1}$=-0.5 or -1.18; E_i =income elasticity; $\delta_{ij} = 0, i \neq j; \delta_{ii} = 1$): $E_{ij} = \breve{\omega}^{-1}\delta_{ij} - w_j E_i(1 + \breve{\omega}^{-1}E_j)$; see a discussion of the calibration of the income flexibility $\breve{\omega}$ in section III.

Some important results come out from these estimations: first of all, we observe that all the (compensated) monetary own-price elasticities are significantly negative. The estimates range from -0.6 and -0.2 and average -0.35, perhaps a little larger than those of the macroeconomics estimates that oscillate often between -0.1 and -0.3 for semi-aggregate commodities. As we already pointed out, elasticities derived from macroeconomic data face measurement errors and aggregation bias.

Second, we observe that the correction of quality decreases by 24% in average the magnitudes of the elasticities estimates, for both full price elasticities and monetary elasticities. This is consistent with the theory as the quality is included in the price, so once the quality effect is corrected the elasticity is smaller.

Third, regarding the estimation under a strong separability assumption on utility, we observe a significant distance with the LAIDS estimations. The price elasticity parameters under strong separability (for $\breve{\omega}^{-1} = -0.5$) have a larger magnitude than those estimated without the latter restriction for (among the 6 commodities. By the way, the estimation under separability depends heavily over the calibration of the income flexibility. We can therefore strongly suspect this hypothesis of strong separability[15]. The monetary price elasticities estimated by our method differ also from those indicated by the grouping method[16].

Fourth, all time elasticities are negative, and their magnitudes are not related to the income elasticity nor to price elasticities. Their definition (equations 7 to 9) shows that this is caused by the absence of systematic relationship between the monetary and time component of the full expenditures (as shown by Table C1). The elasticity of expenditures as regards the opportunity cost of time are positive for all items except leisure expenditures: this is probably explained by the fact that time plays a prominent role for these expenditures (the proportion of time in the full expenditures for leisure is equal to 81% compared to 57% for all other expenditures).

Fifth, price and time elasticities change significantly between different types of households, for instance between bachelors, couples without children and families with children: all price effect (as well as the elasticities over time or against the opportunity cost for time) increase with the family size. The larger sensibility for prices for large families can be related to the notion that of household's needs increase (conditional to income) with their size.

[15] Note however that the Pigou's law (which relates the direct price elasticity with the income elasticity $|E_{ii}| = 0.5E_i$) applies rather well to the average estimate of this ratio (column 4 over column 1): 0.57, very close to the average Pigou's ratio (0.60) for the calibration of $\breve{\omega}^{-1}$ at -0.5

[16] Individual prices for detailed items (such as bread, suggar…) are aggregated into semi-aggregates (food) using the houshold's budget-shares. The corresponding price index (for food) thus changes between households (see a critical view on this method, which appear as not robust, for instance in the recent application by Ruiz, 2006, in Gardes, 2014).

Appendix D
Estimations for the whole population in France

Final good	Elasticity of Expenditure Over the O.C.	Average Time Use by Expenditure
Food	0.1379*** (0.0084)	1060.33 (424.57)
Housing	0.5993*** (0.0191)	566.29 (282.44)
Clothing	0.5756*** (0.0305)	170.63 (100.03)
Transport	0.4591*** (0.0190)	561.58 (124.77)
Leisure	-0.4434*** (0.0091)	1671.00 (567.12)
Other	0.3955*** (0.0279)	333.63 (86.32)
Number of observations	6582	6582

Standard Errors and Standard Deviations in Parentheses