The Inflation Output Trade-Off Revisited*

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— very preliminary and incomplete —

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Abstract

This paper shows that at the zero bound of the short term nominal interest rate there is a strong trade-off between inflation and output, i.e., excess inflation has significant positive impact on output and employment. This remains true even if expectations have fully adjusted and inflation is anticipated. We show that even in a microfounded version of the well known "New Classical" example with a Phelps/Lucas/Kydland/Prescott style aggregate supply curve the trade-off between anticipated inflation and output is not zero as is sometimes suggested based upon the economic experience of the 1970's. Instead the model solution features a well defined trade-off that may even approach infinity at which point the model collapses – due to a "clash" between aggregate demand and supply – and no solution exists. We offer tentative interpretations of this phenomenon.

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*The views expressed herein are solely those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

> "What we know, or should know, from the past is that once inflation becomes anticipated and ingrained — as it eventually would — then the stimulating effects are lost." (NY Times, 9/19/11)

That monetary policy is neutral, once inflation is fully anticipated, is inconsistent with the demand side of standard models of monetary policy if the short-term interest rate is constrained by the zero lower bound. The two sides of the model – aggregate demand and aggregate supply – clash so that no equilibrium exists. That is one of the main results of this paper. How should this result be interpreted? We suggest an interpretation that lead us to revisit the trade-off between inflation and output which may in turn strengthens the case for inflation when the zero bound is binding. Increasing anticipated inflation increases output when the economy is demand constrained. Let us explain.

There is a tendency to claim, that the adjustment of expectations to current economic conditions undoes government policy, potentially making it irrelevant (or even harmful). Indeed, that appears to be the professional consensus summarized by Paul Volcker in the New York Times cited above. There, Mr. Volcker was arguing against any increase, even temporary, of the inflation goal of the Federal Reserve in response to financial crisis that started in 2008. A similar sentiment is expressed by John Cochrane (2012): "it's a rare Phillips curve in which raising expected inflation is a good thing. It just gives you more inflation, with if anything less output and employment."

In broad terms, this absence of trade-offs between inflation and output is sometimes what people often claim as the lesson of the rational expectation revolution of the 1970's and 1980's which arguable resulted in several Nobel prices. Here, we want to argue that this is not the lesson of models with rational expectations. It is not even the lesson of the particular type of models proposed, for example, by Phelps (1967), Lucas (1972), Sargent and Wallace (1975) and Kydland and Prescott (1977) cited by the Nobel committee. Not only is that wrong reading of history, we believe, but even more importantly, represents a wrong reading of a broad class of models. Within the context of the dual mandate of the Federal Reserve (full employment in the context of price stability) an implication of no trade-off would be that nothing is gained in terms of employment by accommodating some inflation today or in the future if it is anticipated. As we will see, this conclusion is not warranted. A logical implication is that since the Federal Reserve is legally mandated to trade the two off, it may want to tolerate deviation of one of its goals in favor getting the other one close to its ideal, to best achieve its objectives.

To illustrate the trade-off between inflation and output, we here analyze first a simple microfounded model that is to a linear approximation identical to the well known Kydland and Prescott (1977) example, or what Sargent (1999) has referred to as the "Phelps problem," before moving onto the now standard "New Keynesian" framework. We first review how that model can lead to excessive inflation due to discretionary policy without any gains in employment. This is the well known example of no inflation-output trade-off once expectations adjust. It is – presumably – what is behind the quote from Volcker above and to some extent Cochrane's discussion. We also suspect that it informs a large amount of policy discussion today.

We then extend the model to consider shocks which can have both short, medium and long-run effects. Moreover, we model the aggregate demand side of the model to explicitly specify how the central bank can implement a given rate of inflation via variations in the nominal interest rate. The central bank, however, is limited by the zero bound on the short-term nominal interest rates. And as we shall see, this will change things in important ways. In particular, it is no longer the case that anticipated inflation is of no help to improve economic outcomes. Instead, we will see a meaningful inflation output trade-off even when inflation is fully anticipated. Furthermore, this effect can be very large. A key conclusion of the paper is thus turning Volcker's statement above on his head: At zero interest rates, as inflation expectations become more anticipated, the benefits of higher inflation become *bigger*, to an extent that the model may even explode.

To be clear, unlike the traditional trade-off, this one involves trading off output gains today to *future inflation.* In fact, a key point of the paper is to clearly distinguish between two well defined trade-offs, a *static* trade-off between inflation today and output today, and an *intertemporal* trade-off between output today and inflation tomorrow (i.e., expected inflation). As we shall see the intertemporal trade-off is totally absent in the Phelps problem unless the zero bound is binding. Meanwhile, in the New Keynesian model the intertemporal trade-off is negative at positive interest rate – i.e. higher expected inflation reduces current output as suggested by Cochrane (2012) – but switches signs once the nominal interest rate is zero. This negative trade-off in normal times is also what often leads people to worry about the dangers of letting let inflation expectations rise and become "unanchored". That argument, simply put, is however not very persuasive at zero interest rate.

Wait (!), some readers may think, gains from expected inflation? This goes against everything we know! This reaction, perhaps, is guided by the now famous relationship from the original Phelps problem

$$\pi_t = \kappa \hat{Y}_t + E_{t-1}\pi_t \tag{1}$$

where π_t is inflation $\hat{Y}_t$ is output in deviation from steady state and $E_{t-1}\pi_t$ is the expectation of inflation at time t formed at a time at which either some prices or wages were set (we will be explicit about this in our microfounded example). If inflation is fully anticipated, then $\pi_t = E_{t-1}\pi_t$ and this equation says that $\hat{Y}_t = 0$, right? How can we claim, then, that there is any meaningful trade-off between inflation and output once inflation is anticipated? And perhaps more fundamentally, how could inflation have helped, in a country such as Japan since 1997, where the recession (the "lost decade") has lasted about 20 years, if a model economy has a similar structure? Does anybody seriously believe that expectations do not adjust at those horizons?

The intuition sketched out above is obtained only by looking at one equation, the "supply" side of the model, i.e., equation (1). Once you incorporate the "demand" side, it becomes clear that this intuition is only partial and can be misleading. The demand side that we outline in the paper is standard: Consumers choose a stream of consumption over the infinite horizon and policy can affect demand via variation in the nominal interest rate, so that inflation and output are determined in equilibrium. A key result is that if there are shocks so that the zero bound on the short-term nominal interest rate is binding then, given expectations, output is *demand determined.* For large enough shocks and a reasonable specification for policy, output demanded is then always *below* potential once the zero bound is binding, i.e. $\hat{Y}_t < 0$, according to aggregate demand. Moreover, even if inflation is perfectly anticipated, we show that the solution $\hat{Y}_t = 0$ is simply not consistent with the demand side equilibrium conditions of the model. Hence a key result of this paper is a non-existence result: That proposed solution of equation (1) suggested above, and implicitly assumed in Volcker's commentary, that if inflation is anticipated then $\hat{Y}_t = 0$ leads to a model collapse – a non-existence of equilibrium. In other words, there is a clash between the demand and the supply side of the model. How can this clash be resolved? What is the interpretation of the non-existence result?

One interpretation that we suggest can be illustrated by moving away from the assumption of perfectly anticipated inflation. By introducing a little uncertainty into the model, we show that an equilibrium exists, but that it leads to very "bad" outcomes: In this equilibrium the "clash of the two equations" is resolved in favor of the demand side and instead of non-existence we obtain a solution that features an output collapse and possibly deflation. This collapse in output and inflation converges to minus infinity — and ultimately non-existence once inflation becomes perfectly anticipated. We refer to this as a contractionary blackhole. Interestingly we show that in this equilibrium — as long as it exists — anticipated inflation is far from neutral. The benefits of anticipated inflation, in fact, become extremely large and approach infinity as the model converges towards the contractionary blackhole.

We should be clear that we don't think that the small deviation of the model considered — introducing some uncertainty in order to break the monetary neutrality in the simple Phelps problem — is the most natural way of resolving this issue. At heart of the non-existence problem here lies the fact, as in most modern macroeconomic general equilibrium models, that demand deficiencies can only occur very temporarily. Within the context of the Phelps problem, this short-run lasts only *one period*. But even in more dynamic models, such as the standard New Keynesian model, there is still a very serious constraint on how long the slump can last, as we show explicitly later in the paper. We will illustrate that the New Keynesian model, just like the New Classical model characterized by equation (1) will also "blow up" if the slump is long enough. Yet, the long slump in Japan and the current slump in the world seem to suggest that this fundamental "hard-wired" feature of the current generation of models needs to be revisited in some way. This is challenge to modern DSGE models. We offer some speculations on possible directions in the conclusions of the paper.

To put the paper in context, it belongs to a relatively large recent literature that has been generated about the zero bound on the short-term nominal interest rate, see e.g. Krugman (1998), Eggertsson and Woodford (2003). A key conclusion in this literature is that high anticipated inflation can eliminate the problem of the zero bound and eliminate large output losses. Yet that conclusion would seem to run contrary to a key precept of the monetary literature emerging from 1970's and the rational expectations revolution that once anticipated, inflation is neutral. A key conclusion of the paper is that these literatures cannot, in fact, be reconciled in a simple setting: this is our non-existence result, the clash between aggregate demand and supply. Making slight adjustments to the model, we show how the solution can be reconciled, and we see that the answer is consistent with the more recent literature on the zero bound. In other words, in the clash between aggregate demand and aggregate supply then demand wins.

2 The Phelps problem

Consider the classic Phelps problem where the government minimizes

$$\min_{\pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \lambda(\hat{Y}_t - \hat{Y}^*)^2 \right\} \tag{2}$$

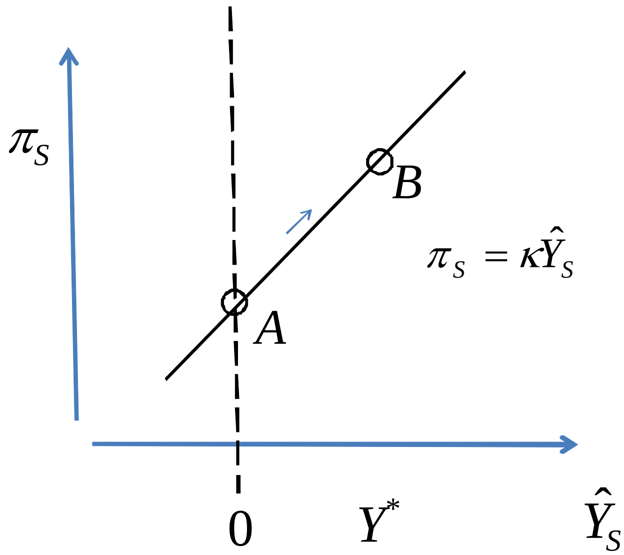

Figure 1: The tradeoff between inflation and output when inflation expectations are fixed.

subject to

$$\pi_t = \kappa \hat{Y}_t + E_{t-1}\pi_t \tag{3}$$

where π_t is inflation, $\hat{Y}_t$ is output in percent deviation from steady state, $\hat{Y}^*$ is the first-best level of output in percent deviation from steady state, and $E_{t-1}\pi_t$ is the expectation of inflation at time t formed at time $t-1$. Let us define the short run as a period $t = S$ in which inflation expectations have not adjusted, but are instead at zero so that $E_{S-1}\pi_S = 0$. Let us define a medium run as the period in which expectations have adjusted so that $\pi_M = E_{M-1}\pi_M = E_S\pi_M$. For now, there is no difference between long and medium run but we will make a sharper distinction between these shortly.

Figure 1 shows the output and interest rates in the model in the short run, given by the schedule $\pi_S = \kappa\hat{Y}_S$. We see that in the short-run the government can achieve higher output by creating (unexpected) inflation, and the trade-off between inflation and output is given by κ^{-1}, i.e. a one percentage point increase in inflation increases output by κ^{-1} percent. Hence in the short run the government has a menu of choices of inflation output pairs on the solid line dotted by A and B. For large enough inflation, the government can even achieve the first best output $\hat{Y}^*$ that minimizes the objective (2).

The key point of the literature from the 1970's is that this menu of choices is an illusion once expectations adjust. In particular, once inflation becomes fully anticipated, $\pi_t = E_{t-1}\pi_t$ and the AS equation (3) becomes $\pi_M = \kappa\hat{Y}_M + \pi_M$ which implies that $\hat{Y}_M = 0$. Once the policy is anticipated the government can only choose between different inflation rates on dashed curve from A to C in Figure 2 without any improvement in output. In particular, suppose the private sector anticipated that the government is going to try to achieve point B (where output is at potential). In this case $E_S\pi_M = \pi_M$ and the AS curve shifts

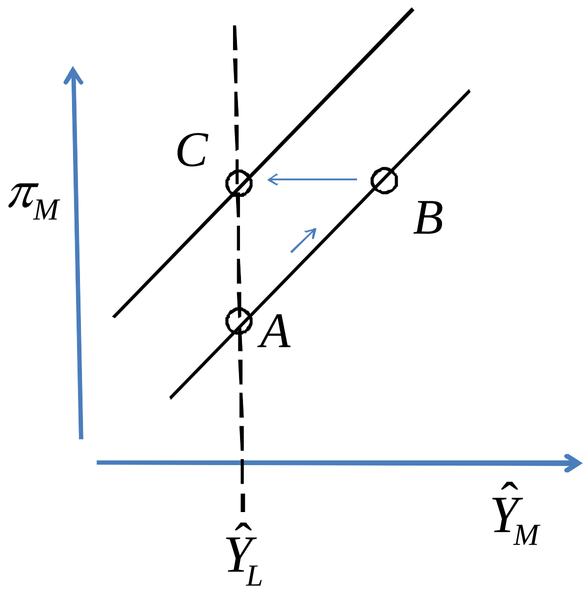

Figure 2: If inflation expectations adjust fully they eliminate any output gains from inflation.

as shown in the figure so that once the government chooses π_M there are no gains in output, only excessive inflation. This is at core the Volcker's remark "What we know, or should know, from the past is that once inflation becomes anticipated and ingrained — as it eventually would — then the stimulating effects are lost."

2.1 Introducing demand into the Phelps problem: The problem of non-existence

In the last section we reviewed a classic example of how an expectation augmented Phillips curve suggest that there is no medium run trade-off between inflation and output, where medium run is defined by the fact that expectation have adjusted. Missing in this picture, however, is a demand side. How does the government "select" an inflation rate? Does the modeling of how this is accomplished change the basic picture? In this section we extend the model to explicitly take account of the aggregate demand side, which then leads us to explore how the equilibrium is achieved. They key point is that once the demand side is introduced, then the benign solution described above, derived from the supply side with $\hat{Y}_t = 0$ (at any anticipated rate of inflation) can no longer be achieved under certain conditions. It clashes directly with the demand side of the model. Resolving the clash between the demand and supply sides of this model leads to some interesting findings.

2.2 Microfoundations

This subsection summarizes the microfoundations of the model, the impatient or experienced reader can move directly to the next section (the microfoundations spelled out are largely covered, e.g., in Woodford (2003)). There is a continuum of households of measure 1. The representative household maximizes

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \xi_T \left[u(C_T) - v(l_T) \right], \tag{4}$$

where $\beta \in (0,1)$ is a discount factor, C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods, $C_t \equiv \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ with an elasticity of substitution equal to $\theta > 1$, P_t is the Dixit-Stiglitz price index, $P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$, and l_t is an aggregate measuring the quantity of labor supplied. The disturbance ξ_t is a preference shock, and $u(.)$ is an increasing concave function while $v(.)$ is an increasing convex function. The period budget constraint can then be written as

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1} + \int_0^1 Z_t(j) dj + P_t W_t l_t - T_t, \tag{5}$$

where B_t denotes the quantity of one-period riskless bonds, i_t is the nominal interest rate, $Z_t(i)$ corresponds to profits that are distributed lump sum to the households, and W_t is the real wage rate. For the budget constraint to be well defined, and Ponzi schemes not possible, we require that[1]

$$(1 + i_t) B_t > - \sum_{T=t+1}^{\infty} E_{t+1} \left[Q_{t+1,T} \left(\int_0^1 Z_T(j) dj + P_T W_T l_T - T_T \right) \right] \tag{6}$$

and

$$\sum_{T=t}^{\infty} E_t \left[Q_{t,T} \left(\int_0^1 Z_T(j) dj + P_T W_T l_T - T_T \right) \right] < \infty, \tag{7}$$

where $Q_{t,T} \equiv \prod_{s=t+1}^{T} Q_{s-1,s}$ denotes the stochastic discount factor.

Households take prices and wages as given and maximize utility subject to the budget constraint. This gives rise to the first-order conditions for the optimal consumption allocation

$$u_c(C_t) \xi_t = (1 + i_t) \beta E_t \left[u_c(C_{t+1}) \xi_{t+1} \Pi_{t+1}^{-1} \right] \tag{8}$$

where we define $\Pi_t \equiv P_t / P_{t-1}$ and we note that $(1 + i_t)^{-1} = E_t Q_{t,t+1}$, and the condition for the optimal labor decision

$$W_t = \frac{v_l(l_t)}{u_c(C_t)}. \tag{9}$$

Furthermore we impose the constraint that the nominal interest rate has to be positive[2]

$$i_t \geq 0. \tag{10}$$

There is a continuum of firms of measure 1. A fraction $1-\gamma$ of the firms set prices one period in advance, while a fraction γ sets prices freely every period. Each firm sets its price and then hires the labor inputs

[1] See Woodford (2003, Chap 2.) for discussion.

[2] This constraint can be interpreted as a consequence of the household maximization problem if there exists money in the economy as a nominal store of value.

necessary to meet any demand that may be realized taking wages as given. A unit of labor produces one unit of output. The preferences of households imply a demand for good i of the form $y_t(j) = Y_t(\frac{p_t(j)}{P_t})^{-\theta}$, where $Y_t = C_t$ is aggregate output. We assume that all profits are paid out as dividends and that the firm seeks to maximize profits. Profits can be written as $Z_t(j) = p_t(j)Y_t(p_t(j)/P_t)^{-\theta} - W_t P_t Y_t(p_t(j)/P_t)^{-\theta}$, where j indexes the firm. The first-order condition of the firms pricing problem that sets its price freely every period is given by

$$\frac{p_t(1)}{P_t} = \frac{\theta}{\theta - 1} W_t \tag{11}$$

while the first-order condition of these firms that set their prices one period in advance is

$$E_{t-1}\left[u_c\left(Y_t\right) Y_t \left(\frac{p_t(2)}{P_t}\right)^{-\theta} \left(\frac{p_t(2)}{P_t} - \frac{\theta}{\theta - 1} W_t\right)\right] = 0 \tag{12}$$

where we have used

$$Y_t = l_t = C_t. \tag{13}$$

The aggregate price level then implies

$$1 = \gamma \frac{p_t(1)}{P_t} + (1 - \gamma) \frac{p_t(2)}{P_t}. \tag{14}$$

After using (13) to substitute for C_t and l_t in the above equations, we can define an equilibrium as a set of stochastic processes $\{\frac{p_t(1)}{P_t}, \frac{p_t(2)}{P_t}, \Pi_t, Y_t, W_t, i_t\}$ that satisfy equations (8)–(12), (14) for a given shock process $\{\xi_t\}$ and for given fiscal and monetary policies that satisfy equations (6)–(7).

2.3 A Linear Quadratic Approximation

The model just outlined can be approximated via log-linear-quadratic approximation around a deterministic steady state. We use this form of the model for simplicity and to connect it with the earlier literature. As we will show, however, nothing is lost with this simplified exposition; we show that the same basic insights carry to the nonlinear version of the model discussed in section 6.

The policy objective function (2) can be obtained as a quadratic approximation of the representative household's utility, under suitable assumptions. Similarly, a log-linear approximation of the firm's optimal pricing decisions delivers the aggregate supply equation (3) where $\kappa > 0$ and $\lambda > 0$ are functions of some of the underlying parameters of the model. In addition, a log-linear approximation of the demand side of the model ((8) using (13)) yields

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^e) \tag{15}$$

where $\hat{Y}_t \equiv \log Y_t/\bar{Y}$, $\pi_t \equiv \log P_t/P_{t-1}$, i_t now refers to $\log(1 + i_t)$ in terms of our previous notation, and $\sigma > 0$. The "natural" rate of interest $r_t^e \equiv \bar{r} + \log\left(\xi_t/\xi_{t+1}\right)$ is an exogenous variable that depends on $\bar{r} \equiv \log \beta^{-1} > 0$ and the preference shock ξ_t, and that reflects the desirability of consuming in the present relative to the future. The interest rate bound can once again be expressed as

$$i_t \geq 0.$$

Reviewing the objective (2) and equations (3) and (15), it should be clear why previous authors have often abstracted from the demand side. Since the nominal interest rate does not directly appear in equation

(3), there is no loss of generality in assuming that instead of choosing the nominal interest rate, i_t, the government directly chooses π_t. The nominal interest rate consistent with this inflation can then just be backed out of (15).[3] To be more specific we can model monetary policy as

$$i_t = \max\{0, r_t^e + \pi_t^* + \phi_\pi(\pi_t - \pi_t^*)\} \tag{16}$$

where π_t^* is the inflation target of the government and where we assume that $\phi_\pi > 1$ so that the so-called "Taylor principle" applies, whereby the nominal interest rate is raised more than one-for-one with inflation around the inflation target.[4] If inflation is perfectly anticipated, then $\pi_t = \pi_t^* = E_{t-1}\pi_t$. Moreover, if the zero bound is not binding then (16) will implement the inflation target in every period (this result is special to the fact that r_t^e is the only shock in the model).

In the case we considered in section 2, as inflation becomes anticipated, the equilibrium moves from point B to C: Higher inflation leads to no output gains once fully anticipated. We see from equation 15 that the net effect of this is simply an increase in the nominal interest rate, i.e., $i_S = \pi_M$. This is arguably – in broad terms – what happened during the "great inflation" of the 1970's: as inflation expectations rose, nominal rates rose as well, while output remained depressed. It would seem, then, at least when studying the 1970's, that the demand side is only relevant to "back out" the nominal interest rate implied by the equilibrium.

It is worth noting, however, that the demand side of the model may affect the equilibrium in the case in which the zero bound becomes binding. If the nominal interest rate drops to zero then equation (15) is no longer just a pricing equation for the nominal interest rate. Instead, it starts playing a role in determining the *overall number good demanded* which will have critical effects on the equilibrium determination. This is the case that we now turn to.

2.4 Short, medium, long run and non-existence

We now consider the case in which the zero bound can be binding, and explore how that changes the basic picture. To do so we assume that there is an unexpected negative shock $r_t^e < \bar{r}$ in period zero that we call the "short run". The short run is defined by the fact that at that time expectations have not adjusted in the model; they remain at "steady state" so that $E_{-1}\pi_S = \pi_L^*$. We then assume that the shock stays at its negative level in the next period (which we call the medium run) and then reverts back to normal in the third period which we call the "long run". Expectations have fully adjusted in the medium and the long run so that the only difference between the medium and the long run is the absence of the shock in the long run. Similarly the only difference between the medium and the short run is that expectations have adjusted in the medium run, but not in the short run. To summarize:

A1 Consider the three periods $t = S, M, L$. In period $t = S$ there is an *unexpected* shock $r_t^e = r_S^e < \bar{r}$. In period $t = M$ the shock is still $r_t^e = r_S^e$. In period $t \geq L$ the shock is back at steady state $r_t^e = r_L = \bar{r}$. While the shock is unexpected in period $t = S$, so that $E_{S-1}\pi_S = \pi^*$, there is perfect

[3] A similar argument can also be made for the money supply. We can add money in the utility function into our framework, so that the government's choice of the nominal interest rate is then modeled via its choice of the money supply. We omit this detail here.

[4] We could alternatively write the policy rule as $i_t = \max(0, r_t^e + \pi_t + \phi_\pi(\pi_t - \pi_t^*))$. All the results presented below remain valid with appropriate modifications to some of the parameters. Note that in this case, the Taylor principle would require $\phi_\pi > 0$.

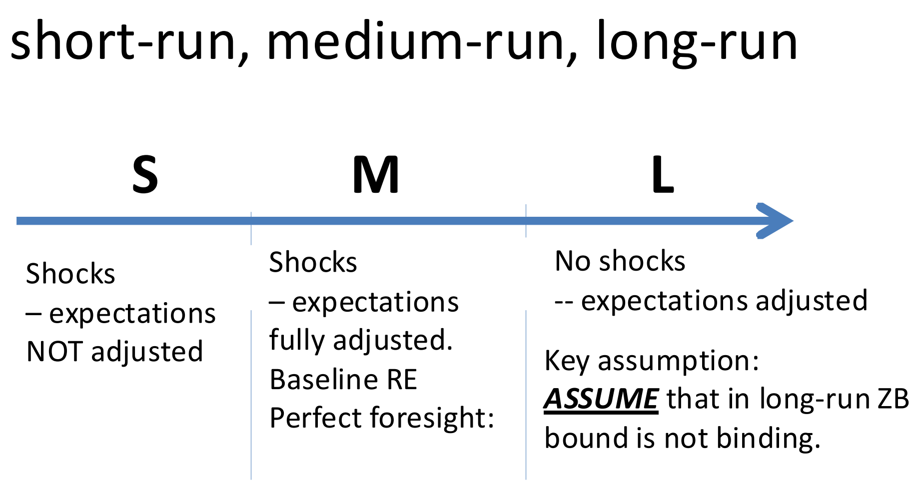

Figure 3: Time protocol according to Assumption 1.

foresight between S, M, and L. The shocks in periods $t > L$ are identical to L, i.e. there are no shock and perfect foresight.

Let us start with some preliminaries, namely proving that in the long run, $t \geq L$, with the policy rule (16), there is a unique bounded solution in which $\pi_t = \pi^*_L = \pi^*$. In proving this proposition, we *assume* that the zero bound is not binding in the long run and thus we exclude the possibility of self-fulfilling liquidity traps which is a subject of another branch of the literature.[5]

Proposition 1 *Suppose A1, that $\bar{r} > -\pi^*_L$, $\phi_\pi > 1$ and that the nominal interest rate is always positive in the long run $t \geq L$. Then the model (3), (15)–16) implies a unique bounded long-run equilibrium $\{\pi_L, \hat{Y}_L, i_L, E_L\pi_{L+1}, E_L\hat{Y}_{L+1}\}$ given by $\pi_L = E_L\pi_{L+1} = \pi^*_L$, $i_L = \bar{r} + \pi^*_L$ and $\hat{Y}_L = E_L\hat{Y}_{L+1} = 0$.*

Proof. Given Assumption A1, perfect foresight between periods M and L implies that $E_{L-1}\pi_L = E_M\pi_L = \pi_L$. It follows from equation (3) that for all $t \geq L$

$$\pi_t = \kappa\hat{Y}_t + \pi_t$$

or simply that that $\hat{Y}_t = 0$ for $t \geq L$. This implies that for any $t \geq L$ (15) simplifies to

$$i_t = E_t\pi_{t+1} + \bar{r}. \tag{17}$$

[5]Eggertsson and Woodford (2003), for instance, show how those types of equilibria can be excluded with the appropriate fiscal commitment.

It follows from (16) and (17) that

$$E_t\pi_{t+1} + \bar{r} = \bar{r} + \pi_L^* + \phi_\pi(\pi_t - \pi_L^*)$$

or equivalently that

$$\pi_t = \phi_\pi^{-1}E_t\pi_{t+1} + (1-\phi_\pi^{-1})\pi_L^*.$$

Iterating this forward yields

$$\pi_t = \phi_\pi^{-T}E_t\pi_{T+1} + (1-\phi_\pi^{-1})\pi_L^*\sum_{s=0}^{T}\phi_\pi^{-s} = \pi_L^*$$

for any bounded $\{\pi_t\}_{t\geq L}$ since $\phi_\pi > 1$ and $\lim_{T\longrightarrow\infty}\phi_\pi^{-T}E_t\pi_{T+1} = 0$. ■

Our focus, first, is on the equilibrium determination in the medium run given our specification for policy, and we then move to the short-run. Consider the aggregate supply (AS) equation (3) in the medium run. It is given by a vertical line

$$\hat{Y}_M = \kappa^{-1}(\pi_M - E_S\pi_M) = \kappa^{-1}(\pi_M - \pi_M) = 0$$

where the last equation follows from the fact that we assume perfect foresight between the short and medium run in A1.

The aggregate demand (AD) equation is obtained by combining (16) with (15) which yields

$$\hat{Y}_M = \begin{cases} -\sigma\phi_\pi(\pi_M - \pi_M^*) + \sigma(\pi_L^* - \pi_M^*) & \text{if } r_M^e + \pi_M^* + \phi_\pi(\pi_M - \pi_M^*) > 0 \\ \sigma r_M^e + \sigma\pi_L^* & \text{if } r_M^e + \pi_M^* + \phi_\pi(\pi_M - \pi_M^*) \leq 0 \end{cases} \tag{18}$$

The two curves are plotted up in Figure 4 for a realization of the shock r_M^e satisfying $r_M^e + \pi_M^* + \phi_\pi(\pi_M - \pi_M^*) > 0$. Since inflation is perfectly anticipated in the medium run, we see that that the AS curve is vertical. Meanwhile, the AD curve is downward sloping in inflation. This is due to the fact that $\phi_\pi > 1$ so that the central bank will reduce the nominal interest rate more than one-to-one with a fall in inflation (and vice versa) thus stimulating spending as inflation drops as shown in the first row of (18). Yet, there is a limit to how much the central bank can stimulate spending by cutting the nominal interest rate. If medium term inflation, π_M, is low enough so that the zero bound is binding then $\hat{Y}_M$ is given by the second row of (18), i.e. $\hat{Y}_M = \sigma r_M^e + \sigma\pi_L^*$. In this case the output demanded does not depend upon realized inflation (the ratio of prices today relative to yesterday). Instead demand only depends upon *expected inflation*, π_L, which determines the price of goods tomorrow relative to the price of today. Before this relative price was also affected through realized inflation because inflation affected the nominal interest rate setting of the central bank. This is no longer the case once the interest rate is pinned at zero. An equilibrium is determined by the intersection of the AS and AD equation. At positive interest rate we see that this equilibrium determination happens at $\pi_M = \pi_M^*$ and $\hat{Y}_M = 0$.

Figure 5 shows the effect of the shock r_M^e being more negative. We see that this will shift the AD curve leftward. If this shock is small enough, then the only thing that happens is that the nominal interest rate is reduced but inflation stays at π_M^* and output at potential.

There is nothing in the model, however, that prevents the AD curve to shift even further than in Figure 5. In particular consider the following shock: $r_M^e < -\pi_M^*$. This size of the shock put the AD curve to the left of the AS curve. Clearly the two curves do not intersect. In other words *there is no equilibrium.* To summarize

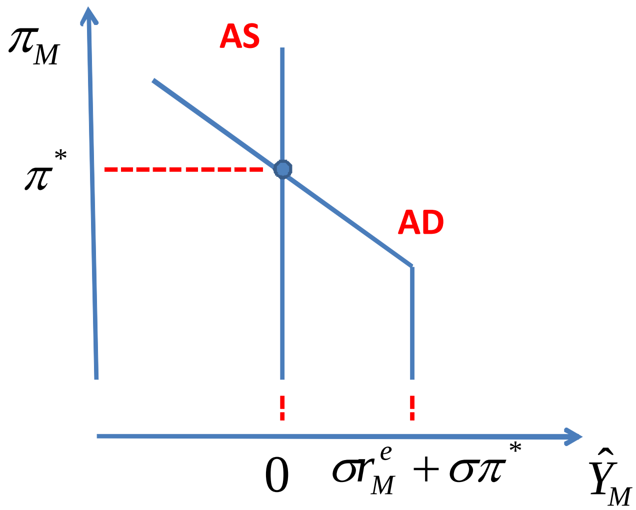

Figure 4: Equilibrium at positive interest rate.

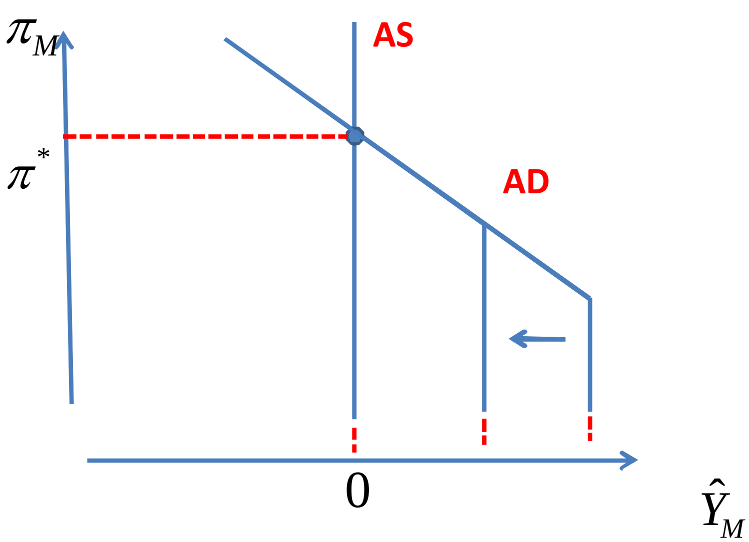

Figure 5: A shock to r_S^e has no effect on output or inflation as long as the zero bound is not binding.

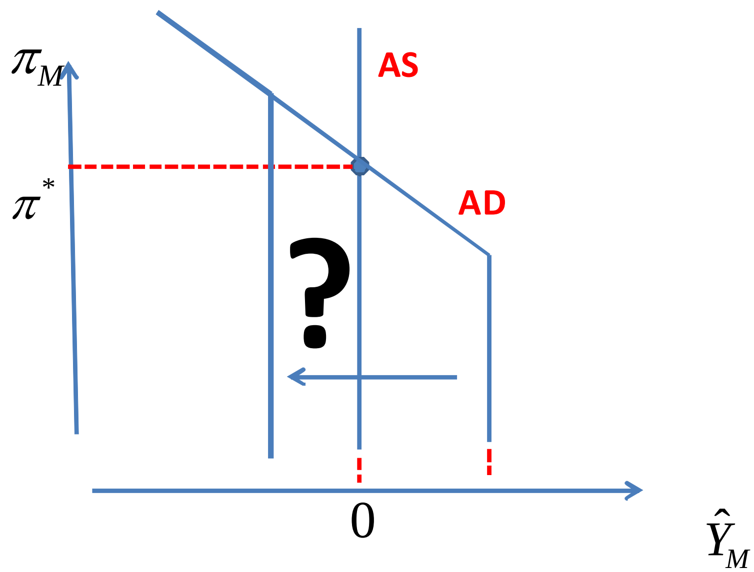

Figure 6: Aggregate demand and aggregate supply clash: No equilibrium.

Proposition 2 *Suppose A1, $r^e_M < -\pi^*_L$, $\bar{r} > -\pi^*_L$, and $\phi_\pi > 1$. Then there exists no bounded equilibrium in the medium run that satisfies equations (3) and (15)–(16).*

Proof. See Appendix. ■

How should this proposition, that no solution exist, be interpreted? That is the issue we now turn to.

3 Generating existence: Demand wins

The aggregate supply equation and the aggregate demand equation in Figure 5 are pointing in two different directions. One the one hand the aggregate supply equation simply imposes that $\hat{Y}_M = 0$. If everybody perfectly anticipates the future, then prices act *as if* they are perfectly flexible and output is thus at potential. Meanwhile, the aggregate demand equation imposes that demand must be below its steady state, i.e., $\hat{Y}_M < 0$. For households to be willing to buy all supplied goods, the real interest rate must be sufficiently negative, but this is not possible if expected inflation is below π^*_t without a negative nominal interest rate. Evidently demand and supply clash – no level of output and inflation satisfy both equations at the same time. What is particularly noteworthy here is that some of the policy discussion reviewed in the introduction – and professional consensus – seems to be driven by an intuition which is derived exactly when this clash occurs but by only using the AS equation.

At heart of the issue is the strong neutrality imposed by the AS equation in combination with a temporary shock to the AD equation that reduces demand. Once inflation is anticipated, then output cannot deviate from potential according to the AS equation thus hard-wiring no trade-off between inflation and

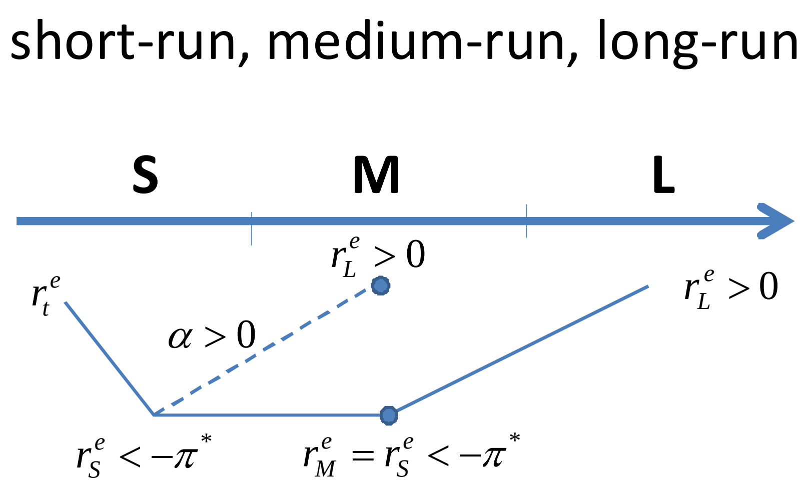

Figure 7: Introducing uncertainty.

output. This excludes – by assumption – the possibility of a long protracted demand driven slump. Given that we have some prices set one period in advance (so output *is* demand determined in equilibrium) a critical property of the model is *perfect foresight* by the agents, i.e. *no* uncertainty *and* rational expectations. Here we will deviate from this assumption in way that seems quite reasonable – while still maintaining rational expectations – and show how the equilibrium is determined. This, we argue, gives some insight how the non-existence result is best interpreted.

Recall our previous assumption that there is a shock in the short-run that stays "on" in the medium run and finally reverts back to steady state in the long run. Let us now deviate from perfect foresight by assuming that there is a probability, α, that the shock reverts back to steady state in the medium run rather than in the long run. This seemingly minor extension generates existence in the model. Moreover, it clarifies how the model should be interpreted which has important policy implications.

A2 Consider three periods $t = S, M, L$. In period $t = S$ there is an *unexpected* shock $r_t^e = r_S^e < \bar{r}$ In period $t = M$ the shock is still $r_t^e = r_S^e$ with probability $(1 - \alpha)$ and $r_t^e = r_L^e > 0$ with probability α. In period $t = L$ the shock is back at steady state $r_t^e = r_L^e = \bar{r}$. The shock is unexpected in period $t = S$ but people form rational expectations about the shock in period $t = M$ using the correct probability distribution of the model (α).

The long run is as before $\hat{Y}_L = 0$, $\pi_L = \pi_L^*$ and $i_L = r_L^e + \pi_L^*$. Moving to the medium run, we now have two possible states, i) that the shock reverts back to steady state $r_M^e = r_L^e > 0$ (which we call "high") and ii) that the shock remains at $r_M^e = r_S^e < -\pi_M^*$.The model then solves the following six equations in the medium run

$$\pi_M^j = \kappa \hat{Y}_M^j + \alpha \pi_M^{high} + (1-\alpha)\pi_M^{low}, \quad \text{for } j = \textit{ low} \text{ or } \textit{high} \tag{19}$$

$$\hat{Y}_M^{low} = \hat{Y}_L - \sigma(i_M^{low} - \pi_L^* - r_S^e) \tag{20}$$

$$\hat{Y}_M^{high} = \hat{Y}_L - \sigma(i_M^{high} - \pi_L^* - r_L^e) \tag{21}$$

$$i_M^{low} = 0 \quad \text{and} \quad i_M^{high} = r_L^e + \pi_M^* + \phi_\pi(\pi_M^{high} - \pi_M^*) \tag{22}$$

while in the short run it solves

$$\hat{Y}_S = \alpha \hat{Y}_M^{high} + (1-\alpha)\hat{Y}_M^{low} + \sigma\alpha\pi_M^{high} + \sigma(1-\alpha)\pi_M^{low} + \sigma r_S^e \tag{23}$$

$$\pi_S = \kappa \hat{Y}_S \tag{24}$$

which yields the following proposition:

Proposition 3 *Suppose A2 and $\phi_\pi > 1$. Then with $\alpha > 0$ there exists a unique bounded solution given by*

$$\begin{aligned}
\hat{Y}_M^{high} &= -\frac{1-\alpha}{\alpha}\sigma\left(r_S^e + \pi_L^*\right) \\
\pi_M^{high} &= \frac{1-\alpha}{\phi_\pi \alpha} r_S^e + \frac{1}{\alpha\phi_\pi}\pi_L^* + \frac{\phi_\pi - 1}{\phi_\pi}\pi_M^* \\
\hat{Y}_M^{low} &= \sigma\left(r_S^e + \pi_L^*\right) \\
\pi_M^{low} &= \frac{1-\alpha+\phi_\pi\kappa\sigma}{\phi_\pi\alpha} r_S^e + \frac{1+\phi_\pi\kappa\sigma}{\phi_\pi\alpha}\pi_L^* + \frac{\phi_\pi-1}{\phi_\pi}\pi_M^* \\
\hat{Y}_S &= \sigma\left(\frac{1-\alpha}{\alpha}\frac{1+\phi_\pi\kappa\sigma}{\phi_\pi} + 1\right) r_S^e + \sigma\frac{(1-\alpha)\phi_\pi\kappa\sigma + 1}{\phi_\pi\alpha}\pi_L^* + \sigma\frac{\phi_\pi - 1}{\phi_\pi}\pi_M^* \\
\pi_S &= \kappa \hat{Y}_S.
\end{aligned}$$

Proof. Is obtained by solving (19)–(24) (see Appendix for details). ■

What generates existence of the model equilibrium is that there is no longer perfect foresight in the medium run. This implies that if the shock is in the low state then the AS equation is given by

$$\pi_M^{low} = \kappa \hat{Y}_M^{low} + E_S \pi_M = \kappa \hat{Y}_M^{low} + \alpha\pi_M^{high} + (1-\alpha)\pi_M^{low}$$

or (with some manipulations)

$$\pi_M^{low} = \frac{1-\alpha+\phi_\pi\kappa\sigma}{\phi_\pi\alpha}\hat{Y}_M^{low} + \pi_L^* \tag{25}$$

Equation (25) shows that now the AS curve is no longer vertical in the medium run. Instead, it is upward sloping in the inflation output space as shown in Figure 8. This figure shows the AS and AD curves conditional on the shock remaining at its short-run value, i.e. $r_M^e = r_S^e$, hence the value of the shock is the same as in our previous exercise when no solution existed. What generates existence here is the fact that inflation is no longer perfectly anticipated, as there is no more than one state of the world in the medium run because there is a probability α to reach the "high" state in the medium run, where the

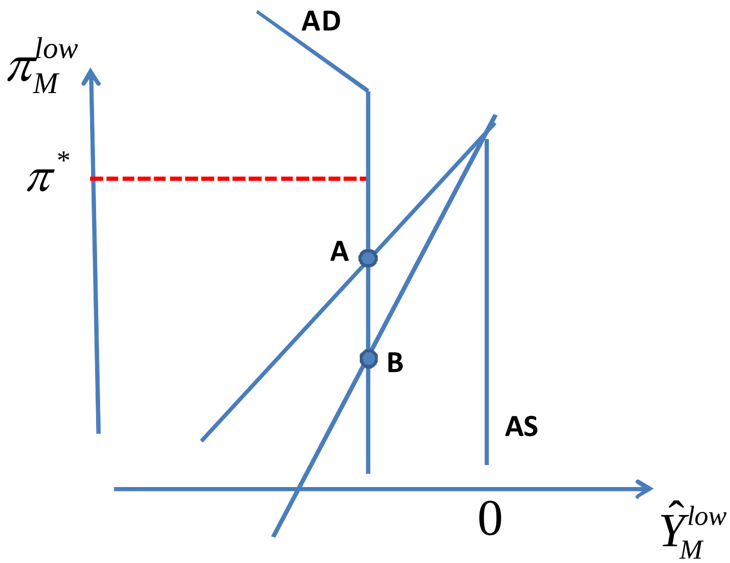

Figure 8: The clash between aggregate demand and aggregate supply resolved: Demand wins.

zero bound is no longer binding. As long as there is some uncertainty about outcomes, actual inflation is no longer equal to its anticipated level and the AS curve is *upward sloping* in the output-inflation space as shown in Figure 8. The reason is simple. In the microfounded model, the firms which have their prices fixed in advance are committed to *supply whatever is demanded by the consumers.* As people start demanding more goods and services, for some prices given, less labor is hired, which in turn depresses wages. Lower wages – then – reduce the firms' marginal costs, so that the firms that can freely adjust their prices choose to lower them. It follows that the fewer output is bought from the firms the greater the price deflation. This basic logic is only broken in the knife-edge case when all prices are perfectly anticipated. In that case demand does not have any bite, since the economy acts "as if" prices are perfectly flexible.

Plotting up the AD and AS curve we see that an upward sloping AS curve now implies the equilibrium determination illustrated in point A in Figure 8. As people start demanding fewer goods and services, for some while the AS curve determines the degree of deflation associated with that level of aggregate demand. In other words, in the clash between demand and supply in the medium run we see that aggregate demand wins.

4 Uncertainty and contractionary blackholes in the short run

While output is pinned down by aggregate demand in the medium run, the amount of deflation "needed" to clear the market depends on the AS curve. Thus deflation in the medium run – conditional on the shock being in the "low state" so that the zero bound is still binding – plays no role for medium run output determination as it is already pegged down by the vertical AD curve. As we have see in figure 8 the degree of deflation needed to clear the market in the medium run depends on the slope of the AS equation. This slope depends on α. In particular as the probability people assigned to being in the "low" state in the medium run goes up (i.e. $\alpha \to 0$) the needed deflation increases (see point B in Figure 8). Eventually the AS curve becomes vertical – the model is converging to perfect foresight – and no solution exists.

How much deflation is needed? That depends on the coefficient $\frac{1-\alpha+\phi_\pi \kappa\sigma}{\phi_\pi \alpha}$. As we have already indicated, this slope becomes steeper as $\alpha > 0$. Some uncertainty will therefore in general lead to a "static" trade-off between inflation and output (more on this in the next section). Beyond that an important determinant of this slope is the coefficient κ. Let us just emphasize two key determinant of this coefficient at that we decompose $\kappa \equiv \frac{\gamma}{1-\gamma}\psi$. First, we see that κ depends on the overall level of nominal rigidities – which is here indexed by the fraction of firms that set their prices one period in advance $1-\gamma$. If prices are very "sticky" then we see that the AS curve is very flat, hence very little drop in inflation beyond π_M^* is needed the equilibrate the market. Second, the degree of *real rigidities* ψ. This coefficient measures the strategic complementarities in the model, i.e., how strongly firms tend to cut their prices in response to variation in demand if other firms do not change theirs. To the extent that these rigidities are high, then even a large drop in output will generate only a modest decline in the price level. These points are worth making in the context of the recent world economic crisis, where many countries have seen downward drops in inflation pressures, yet those have been relatively contained. This can be viewed as evidence in favor of high degree of nominal and/or real rigidities in the context of this model.[6]

While the amount of deflation in the medium run has no effect on output in the medium run, it has important consequences for output in the short run. In particular, we will now see that the higher the degree of deflation, the bigger the drop in output in the short-run. What is interesting here is to note that what will in general trigger a great drop in inflation in the medium run are things one would typically associate with a stabilizing force, namely anything that makes the AS curve more vertical, be it real or nominal rigidity or a movement towards inflation becoming more anticipated (i.e. α is closer to 0). But now we will obtain precisely the opposite conclusion, higher real or nominal rigidities, or more anticipated inflation/deflation is destabilizing (this is reminiscent of the result documented in Eggertsson (2012), Christiano, Eichenbaum and Rebelo (2011) and Werning (2012)). Provided $\alpha > 0$, the solution in the short run is now well defined. Consider the solution in the short-run, taking the medium term solution derived in the last section as given. Given π_M and $\hat{Y}_M$, the solution is given by solving together (23) and (24), i.e.

$$\hat{Y}_S = E_S \hat{Y}_M + \sigma E_S \pi_M + \sigma r_S^e$$

$$\pi_S = \kappa \hat{Y}_S$$

that are plotted up in Figure 9.

We see that this figure is essentially the same as Figure 8. There is one important new element, however, lurking in the background. The AD curve will now shift not only due to the shock r_S^e. It will also shift due to expected deflation – given by $E_S \pi_M$ – which is no longer only given by the exogenous inflation target π_M^*. Instead, expected inflation/deflation is a function of the endogenous deflation in medium run which

[6] another reason for relatively modest drop in inflation is that the underlying shock that triggers the zero bound leads to a simultaneous "cost push" shock. They show how a debt deleveraging shock can have this effect.

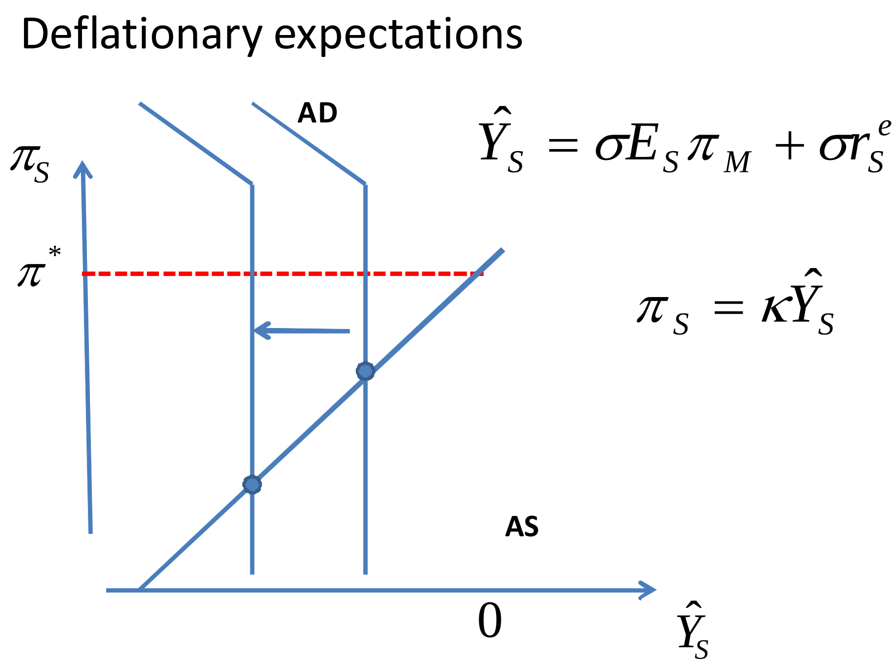

Figure 9: Deflation in the medium run reduces short run output since expected deflation increases the real interest rate and thus contracts demand.

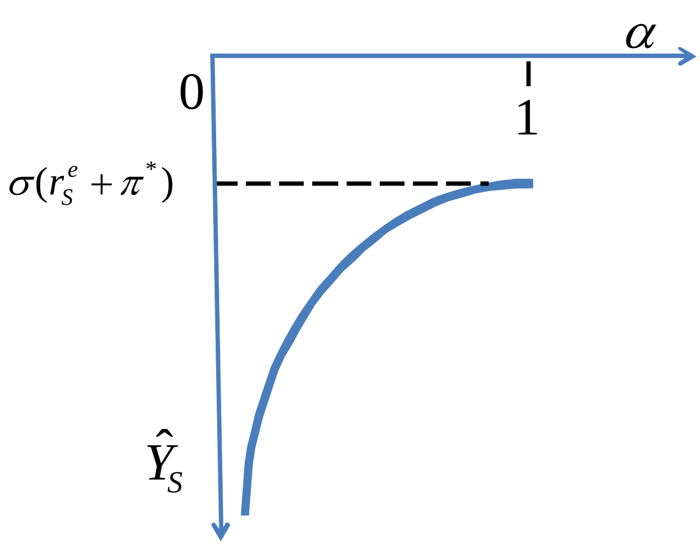

Figure 10: Output in the short run collapses as $\alpha \to 0$.

we saw in Figure 8. It can be expressed as

$$E_S\pi_M = \frac{(1-\alpha)(\kappa\sigma\phi_\pi + 1)}{\alpha\phi_\pi} r_S^e + \frac{\phi_\pi - 1}{\phi_\pi}\pi_M^* + \frac{(1-\alpha)\kappa\sigma\phi_\pi + 1}{\alpha\phi_\pi}\pi_L^* \tag{26}$$

illustrating that the backward shift in aggregate demand is now attenuated by the expected deflation in the medium run, resulting in further backward shift in aggregate demand as shown in Figure 9. Most importantly, perhaps, medium term inflation expectation are declining in the probability α. That is, the more likely the "low" state $1-\alpha$ then the higher the deflation in the low state, and thus the higher the expected deflation in the short run, this is captured by the first term in (26). The expected deflation increases without a bound, i.e. as$\alpha \to 0$ then $E_S\pi_M \to -\infty$ leading to an output collapse, i.e. $\hat{Y}_S \to -\infty$. Note that an increase in κ works in the same direction (e.g. due to higher real or nominal flexibility) .This is what we refer to as a contractionary black hole. To summarize

Proposition 4 *Assume A2, $r_S^e + \pi_L^* < 0$ and $\phi_\pi > 1$. As the probability of the medium-run recession increases,i.e. $\alpha \to 0$, the recession in the short run increases without a bound $\hat{Y}_S \to -\infty$.*

Proof. The expression for $\hat{Y}_S$ in proposition 3 can be rewritten as $\hat{Y}_S = \sigma\left(\frac{(1-\alpha)\phi_\pi\kappa\sigma+1}{\phi_\pi\alpha}\right)(r_S^e + \pi_L^*) + \sigma\left(1-\phi_\pi^{-1}\right)(r_S^e + \pi_M^*)$. Since $r_S^e + \pi_L^* < 0$ by assumption, $\lim_{\alpha\to 0}\hat{Y}_S = -\infty$. ■

The last proposition is illustrated in figure 10. We see that as the probability α converges to zero, the model explodes. Meanwhile as this probability approaches 1, output converges to $\sigma(r_S^e + \pi^*)$. This proposition above will have important implications for the inflation output trade-off, as we now will see.

5 The inflation output trade-off revisited

We now come to the heart of the paper. Let us start with the definition:

Definition 5 *The static trade-off between inflation and output is defined in the short, medium and long run as $\frac{d\hat{Y}_t}{d\pi_t}$ where $t = S, M, L$ and expectations may have adjusted or not. The intertemporal trade-off between expected future inflation and output is defined as $\frac{d\hat{Y}_S}{dE_S\pi_M}, \frac{d\hat{Y}_S}{dE_S\pi_L}$ in the short term and $\frac{d\hat{Y}_M^i}{dE_M\pi_L}$ ($i = high, low$) in the medium term, where expected inflation increases due to variations in π_M^* or π_L^* under rational expectations.*

The popular discussion of the inflation output trade-off focuses on the trade-off between current inflation and current output. We call this the *static* trade-off between inflation and output. As we saw already in section 2 the static trade-off between inflation and output is κ^{-1} in the short-run for given set of inflation expectation. If inflation expectations fully adjust, as is the case in perfect foresight, then there is no trade-off in the model considered thus far. This statistic has the interpretation of telling us how much output is gained in percentage terms for each percentage point increase in inflation.

There is another well defined trade-off between output and inflation in the model which becomes very important once the nominal interest rate hits zero. This is the trade-off between expected inflation in the future and output today. We call this the *intertemporal trade-off* between inflation and output. In our simple example, this trade-off is well defined; it is given by $\frac{d\hat{Y}_S}{dE_S\pi_M}$ and $\frac{d\hat{Y}_S}{dE_S\pi_L}$ in short run and by $\frac{d\hat{Y}_M^i}{dE_M\pi_L}$ in the medium run, where $i = high, low$ indexes the state of the world in the medium run depending on whether the shock has turned back to steady state ("*high*") or not ("*low*").

Let us first observe that in the absence of the zero bound, the intertemporal trade-off between inflation and output is not of principal importance, at least not in the medium run. To see why, observe first that under our policy rule (16), the central bank will fully offset r_t^e in the absence of the zero bound. Hence to the extent r_t^e is the only source of uncertainty, the model is deterministic in the medium and long run and $\hat{Y}_M = \hat{Y}_L$ for any value of π_L^*. The only thing that changes in the medium run with a different value of π_L^* is the nominal interest rate i_t that is needed to implement the equilibrium. Recall that in the medium run we have assumed that inflation expectations have adjusted to policy. In the short-run, however, we take expectations $E_0\pi_S = 0$ as given. This implies some intertemporal trade-off in the short-run as summarized below.

Proposition 6 *Suppose A2 and $r_S^e \geq 0$. Then the zero bound is not binding, the intertemporal trade-off between inflation and output in the medium run is zero, and it is given by $\frac{\sigma}{1+\sigma\phi_\pi\kappa}$ in the short run.*

Proof. It follows from the AS curve and our assumption about the medium run that $\hat{Y}_M = \hat{Y}_L = 0$. Furthermore $\pi_L = \pi_L^*$. Use this, (15) and (16) to back out the other equilibrium variables (see Appendix). ■

To obtain some further intuition for this result consider first the medium run. Note that the shock to the natural rate of interest are small enough so that the zero bound is not binding. This implies that the shock r_M^e is fully offset, so it makes no difference if it is in the "high" or "low" state – the equilibrium output and inflation is the same. It follows, given our policy rule, that

$$E_S\pi_M = \pi_M = \phi_\pi^{-1}\pi_L^* + (1-\phi_\pi^{-1})\pi_M^* \tag{27}$$

and $\hat{Y}_M = 0$, while $i_M^j = r_M^e(j) + \pi_M^* + \phi_\pi(\pi_M - \pi^*)$, $j = high, low$. This shows that increasing the long run inflation target has no effect on the medium-run output or inflation, it only increases the nominal interest rate in the medium run. This is the first part of the proposition which says there is no intertemporal trade-off in the medium run.

Consider now the effect of increasing the medium or long-term inflation target in the *short run* — this is the second part of the proposition — while taking the initial expectations as given, i.e. $E_0\pi_S = 0$. The solution then satisfies the two equations

$$\begin{aligned}\hat{Y}_S &= -\sigma i_S + \sigma E_S\pi_M + \sigma r_S^e = -\sigma\phi_\pi\pi_S + \sigma\pi_M + \sigma(\phi_\pi - 1)\pi_S^* \\ \pi_S &= \kappa\hat{Y}_S.\end{aligned}$$

The short-run equilibrium determination is plotted up in Figure 11. We see that in the first panel that any increase in medium-term inflation expectations — or long-term expectations of inflation which feed directly into the medium term via (27) — results in a shift out in demand and thus output. Note that the shift out in demand occurs due the fact that an increase in inflation expectations reduces the real interest rate and thus spending. This increase is offset to some extent via increase in the nominal interest rate and thus the increase in demand is less than the increase in inflation expectations depending on the value of ϕ_π which measures the sensitivity of the nominal interest rate to inflation. To make this clear the second panel shows the case when the central bank strictly targets inflation π_S^* which corresponds to the case $\phi_\pi \to \infty$. In this case we see that the increase in demand due to the increase in medium term inflation expectations is fully offset by the response of the central bank via the nominal interest rate, so that inflation expectations

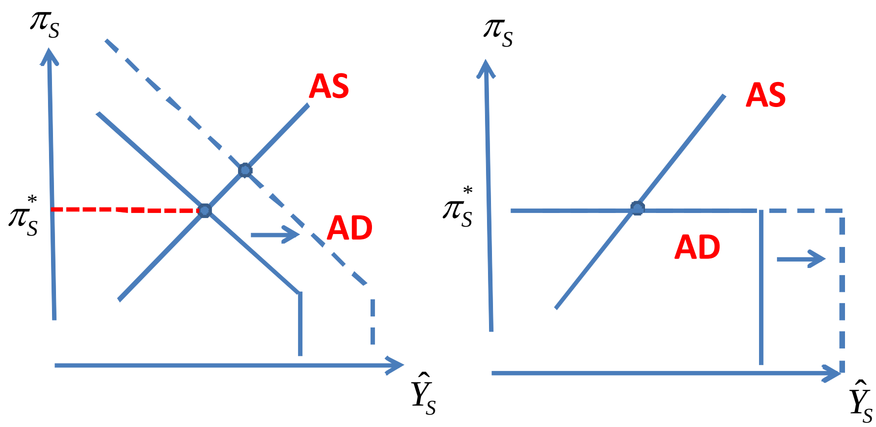

Figure 11: There is no intertemporal tradeoff between expected inflation and output if the central bank is responsive enough in the short run at positive interest rate.

don't change and hence the intertemporal trade-off disappears, i.e. $\lim_{\phi_\pi \to \infty} \frac{\sigma}{1+\sigma\phi_\pi\kappa} = 0$. To summarize, we see that while in the medium run there is no intertemporal trade-off between inflation and output, such trade-off does exist in the short-run but its size depends entirely on how much short-run inflation the central bank is willing to tolerate. If it targets a particular inflation rate in the short-run then the intertemporal trade-off also disappears.

We now consider the intertemporal trade-off when the zero bound is binding. In that case, not only is the intertemporal trade-off positive in the medium run. In the short-run, instead of possibly disappearing under some parameter configurations, it may become arbitrarily large. A key result of the paper is summarized next.

Proposition 7 *Suppose A2 and $r_S^e < 0$. Then the zero bound is binding and the intertemporal trade-off between medium term output and long term inflation $\frac{d\hat{Y}_M^i}{dE\pi_L^*}$, conditional on the zero bound being binding is σ ($i = low$), while the intertemporal trade-off between short term output and medium term inflation $\frac{d\hat{Y}_S}{dE_S\pi_M}$ is σ, and between short-run output and long-run inflation, $\frac{d\hat{Y}_S}{dE_S\pi_L}$, is $\sigma\frac{(1-\alpha)\kappa\sigma\phi_\pi+1}{\alpha\phi_\pi}$.*

Proof. See Appendix. ■

In sharp contrast with our previous proposition, we now see that long-term inflation increases medium-term output conditional on the zero bound being binding. This is because with the nominal interest rate pinned at zero in the medium term, output is demand determined and given by

$$\hat{Y}_M^{low} = \sigma E_M \pi_L + \sigma r_S^e.$$

This makes clear that the intertemporal trade-off in the medium run is given by σ. Meanwhile, the amount of deflation in the "low" state is given by

$$\pi_M^{low} = \frac{\kappa}{\alpha}\hat{Y}_M^{low} + \alpha\pi_M^{high} \tag{28}$$

where we used the fact that $E_S\pi_M = \alpha\pi_M^{high} + (1-\alpha)\pi_M^{low}$.

Let us now move to the short-run determination of output. It is given by

$$\hat{Y}_S = \sigma E_S\pi_M + \sigma r_S^e \tag{29}$$

and inflation is determined by $\pi_S = \kappa\hat{Y}_S$. From (29) we see immediately that the intertemporal trade-off between $\hat{Y}_S$ and $E_S\pi_M$ is again σ (note that expected inflation in the medium run can be completely controlled by the long run inflation target, i.e. $E_M\pi_L = \pi_L^*$). But what is perhaps even more interesting is the trade-off between short term output and long term inflation, i.e., $E_S\pi_L$. As the proposition shows this trade-off is given by $\sigma\frac{(1-\alpha)\kappa\sigma\phi_\pi+1}{\alpha\phi_\pi}$ so that as $\alpha \longrightarrow 0$ the trade-off explodes. This is the opposite of the contractionary. spiral we just saw in the last section, but here it works in a virtuous direction. The reason is that as α moves to 0, then a a greater deal of deflation is needed in the medium run by equation (28) in order to accommodate a given output gap $\hat{Y}_M^{low}$ (given by $\hat{Y}_M^{low} = \sigma r_S^e + \sigma E_M\pi_L$). The amount of deflation needed is greater the smaller is α, as we saw in Figure 8, which led do a collapse in the short-run. But this also implies that the benefit of increasing long run inflation (and thus $\hat{Y}_M^{low}$) is greater the lower is α as shown in Figure 12.

It is worth noting that even if we label the inflation once the shock has subsided "long run," one would not require inflation to increase permanently. Instead, what is important, is that inflation is increased in the period immediately after the shock has subsided; this is in fact how long-run is defined in our model.

6 Non-existence of the equilibrium in a non-linear version of the model

While our analysis has been conducted in the context of a linearized model, it is important to realize that our results are not merely a feature of the linear approximation. In fact, many of the results discussed above carry through in the non-linear model (6)–(30) described in Section 2.2. To show this, we briefly mention a few results, leaving the proofs in the Appendix.

The following proposition, which relates to Benhabib, Schmitt-Grohe and Uribe (2001), establishes the existence of multiple long-run steady states.

Proposition 8 *Consider the model given by (6)–(14) for all* $t \geq t_0$. *Suppose that monetary policy is given by an interest-rate reaction function*

$$i_t = \phi\left(\Pi_t, \Pi_t^*, \xi_t\right) \tag{30}$$

where $\Pi_t \equiv P_t/P_{t-1}$ *denotes inflation,* Π_t^* *is the inflation target, and that the policy rule* $\phi\left(\right)$ *is non-negative for all values of its arguments, is increasing in its first argument, and that* $\partial\phi\left(\Pi_t^*, \Pi_t^*, \xi_t\right)/\partial\Pi_t > 1$, *so that the "Taylor principle" applies around the inflation target. Suppose furthermore that agents know at some date* $t_1 \geq t_0$, *that the inflation target* Π_t^* *will remain constant at* $\Pi_t^* = \Pi^* > \beta$ *for all* $t \geq t_1$. *Then the model (6)–(30) admits at least two possible steady states characterized by constant values* $\bar{Y}_L, \bar{C}_L, \bar{l}_L, \bar{\imath}_L, \bar{\Pi}_L, \bar{W}_L$

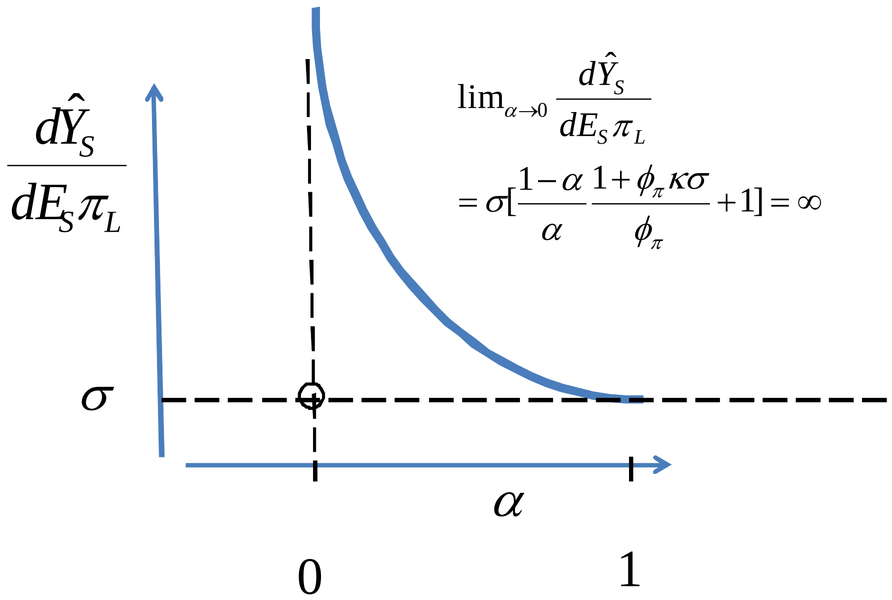

Figure 12: The intertemporal tradeoff between output and expected long term inflation goes to infinity as $\alpha \to 0$.

as well as paths for the price levels $\bar{p}_t(1), \bar{p}_t(2), \bar{P}_t$ *in all periods* $t \geq t_1$.
(i) In the first (regular) steady state, the nominal interest rate $\bar{\imath}_L$ *reaches a positive value* $\bar{\imath}_L = \phi(1, \bar{Y}_L, \bar{\xi}_L) = \Pi^*_L\beta^{-1} - 1 > 0$, *inflation is equal to the target rate* $\bar{\Pi}_L = \Pi^*_L$, *the price indices* $\bar{p}_t(1), \bar{p}_t(2), \bar{P}_t$ *grow at rate* Π^*_L, *and* $\bar{Y}_L = \bar{l}_L = \bar{C}_L$ *and* $\bar{W}_L$ *are given by*

$$\frac{v_l(\bar{Y}_L)}{u_c(\bar{Y}_L)} = \frac{\theta - 1}{\theta} = \bar{W}_L. \tag{31}$$

(ii) In the second (Friedman) steady state, the nominal interest rate is at the zero lower bound, $\bar{\imath}_L = 0$, *the steady-state value* $\bar{\Pi}_L$ *shows perpetual deflation at the rate of time preference,* $\bar{\Pi}_L = \beta$, *the price indices (in log) fall at rate* $\bar{\Pi}_L$, *and* $\bar{Y}_L = \bar{l}_L = \bar{C}_L$ *and* $\bar{W}_L$ *are again given by (31).*

Proof. See Appendix. ■

As stated, at least two long-run steady states are possible in this model: one with constant inflation at the central bank's target level and a positive nominal interest rate, and one with zero nominal interest rate (as in the Friedman rule) and perpetual deflation. Our earlier analysis took the first steady state as given. Importantly, however, the lack of existence of a medium run equilibrium does not depend on such an assumption. In fact, as established in the next proposition, if the preference shock takes a low enough value in the medium run, then no medium run equilibrium exists in the non-linear version of the model, under perfect foresight. We modify slightly Assumption A1 as follows:

A1' Consider the three periods $t = S, M, L$. In period $t = S$ there is an *unexpected* shock $\xi_S < \beta\bar{\Pi}_L^{-1}\bar{\xi}_L$, where $\bar{\xi}_L$ is the steady-state value of the preference shock and $\bar{\Pi}_L$ is the steady state inflation rate.

In period $t = M$ the shock is still $\xi_M = \xi_S$. In period $t \geq L$ the shock is back at its steady state $\bar{\xi}_L$. While the shock is unexpected in period $t = S$, there is perfect foresight between S, M, and L. The shocks in periods $t > L$ are identical to L, i.e. there are no shock and perfect foresight.

Proposition 9 *Under Assumption A1', for any given long-run equilibrium $\bar{Y}_L, \bar{C}_L, \bar{l}_L, \bar{\imath}_L, \bar{\Pi}_L, \bar{W}_L$ satisfying (6)–(30), there exists no medium-term equilibrium for small enough value of the exogenous disturbance ξ_M, i.e., if*

$$\xi_M < \beta \bar{\Pi}_L^{-1} \bar{\xi}_L. \tag{32}$$

Proof. See Appendix. ■

The condition (32) causing the absence of any medium-term equilibrium is analogous to the requirement that the natural rate of interest be low enough for the nominal rate to be constrained by the zero lower bound, as seen in our analysis above. It requires ξ_M to be low enough, so that the representative consumer finds it preferable to consume less in the present (i.e., in the medium term) than in the future (i.e., the long run), or equivalently, to save in the medium term. Doing so, reduces aggregate demand in the medium run, leading firms to lower their prices, which lowers inflation and increases the real interest rate. That increase in the medium-term real rate discourages households further from consuming, thereby leading to a collapse of the economy.

Note however that if we are in the regular steady state with $\bar{\Pi}_L = \Pi_L^*$, (and $\beta \bar{\Pi}_L^{*-1} < 1$) then the higher the long-run inflation target Π_L^*, the more difficult it is for the condition (32) to be satisfied, as ξ_M needs to fall possibly much below $\bar{\xi}_L$ for the medium run equilibrium to cease to exist. Instead, if we are in the "Friedman" steady state, where $\bar{\Pi}_L = \beta$, then the medium-run equilibrium ceases to exist as the economy collapses as soon as ξ_M falls below the long-run value of $\bar{\xi}_L$, regardless of the inflation target.

7 Inflation output trade-off in the New Keynesian model

So far we have only studied the inflation output trade-off in the model with New Classical Phillips curve. Part of the motivation for this, was that the consensus built from the stagflation experience was largely formed around this specification. More recently, an alternative, the so-called New Keynesian Phillips curve has become more popular, both in quantitative monetary models and in discussions of monetary policy (see, e.g., Woodford (2003), Christiano et al. (2005), Smets and Wouters (2007)). Such a relationship can again be obtained from a log-linear approximation to an optimal price-setting condition in the model described in section 2.2, but with a different assumption about price setting, e.g., when prices are staggered as in the Calvo model. The resulting Phillips curve can be expressed as

$$\pi_t = \kappa \hat{Y}_t + \beta E_t \pi_{t+1} \tag{33}$$

where the relevant expectations are now the expectations at date t of inflation at date $t + 1$.

Within that framework we confirm the two basic insights we have shown in the context of the previous model. First that inflation and output may contract dramatically if the nominal interest in constrained by the zero lower bound for a long time. Second, that at zero interest rate, even if the static trade-off between inflation and output is small, once expectations adjust, the intertemporal trade-off can be very large or even explosive.

We start by characterizing the long, medium, and short-term equilibrium in the New Keynesian model. Regarding the long-run equilibrium, we note from (33) that in the long run

$$\hat{Y}_L = \frac{1-\beta}{\kappa}\pi_L$$

so that an increase in long-run inflation is associated with a (bit) of an increase in long-run output. Note however that that effect is smaller, the closer β is to 1. As in the model considered before, though, with the output Euler equation (15) the policy rule (16), long-run inflation ends up being equal to the long-run target, $\pi_L = \pi_L^*$, and $i_l = r_L^e + \pi_L^*$ which is positive by assumption. Having characterized the long run, we can proceed with the medium-term equilibrium:

Proposition 10 *Suppose A2 and $\phi_\pi > 1$. Then for any $\alpha \in [0,1]$ there exists a unique bounded solution given by*

$$\begin{aligned}
\hat{Y}_M^{high} &= \frac{\frac{1-\beta}{\kappa} - \sigma(\beta\phi_\pi - 1)}{\kappa\sigma\phi_\pi + 1}\pi_L^* + \frac{\sigma(\phi_\pi - 1)}{\kappa\sigma\phi_\pi + 1}\pi_M^* \\
\pi_M^{high} &= \frac{1+\kappa\sigma}{\kappa\sigma\phi_\pi + 1}\pi_L^* + \frac{\kappa\sigma(\phi_\pi - 1)}{\kappa\sigma\phi_\pi + 1}\pi_M^* \\
\hat{Y}_M^{low} &= \left(\frac{1-\beta}{\kappa} + \sigma\right)\pi_L^* + \sigma r_S^e \\
\pi_M^{low} &= (1+\kappa\sigma)\pi_L^* + \kappa\sigma r_S^e.
\end{aligned}$$

Proof. See Appendix. ■

Interestingly in this case, and in contrast to the New Classical Phillips curve, the medium-term equilibrium does not depend on the specific value of α, the probability of exiting the lower bound in the medium term. This is because the Phillips curve now takes the form

$$\pi_M^j = \kappa\hat{Y}_M^j + \beta\pi_L^*, \qquad j = low, high$$

in the medium term, where inflation expectations are pinned down by the long-run inflation target, rather than being affected by α. So the medium-term equilibrium exists in this case even if $\alpha = 0$ in this model, that is, if the natural rate of interest remains in the low state in the medium run, $r_M^e = r_S^e$. The lower the natural rate r_S^e, the lower equilibrium output and inflation in the medium run. The effect is however, no as dramatic as in the previous model, since inflation is largely anchored by future inflation expectations, which are here assumed unaffected.

In the short run, however, the probability α does play a role in the equilibrium. But even if $\alpha = 0$, there is a well defined equilibrium given by

$$\begin{aligned}
\hat{Y}_S &= \kappa^{-1}\left((1+\kappa\sigma)^2 - \beta\right)\pi_L^* + \sigma(\kappa\sigma + 2)r_S^e \\
\pi_S &= \left((1+\kappa\sigma)^2 + \beta\kappa\sigma\right)\pi_L^* + \kappa\sigma(\beta + \kappa\sigma + 2)r_S^e.
\end{aligned}$$

These expressions make however clear that a negative value of r_S^e has even stronger adverse effects on short-run output and inflation than is the case in the medium run. Extending the model by adding more periods reveals that the short-term impact on inflation and output grows exponentially with the duration of the binding zero-lower-bound, as pointed out in Carlstrom, Fuerst and Paustian (2012). In fact, while the equilibrium exists in the basic version of the New Keynesian model, even in the face of a persistent drop in the natural rate of interest, a simple (and realistic) extension of the model that incorporates more lagged variables (e.g., due to inflation indexation) easily leads to exploding equilibria, so that the effect of persistent drop in the natural rate becomes unboundedly large. Again, increasing the inflation target temporarily may mitigate in the model the adverse impact the shock.

<To be continued>

8 Conclusion

<To be added>

References

[1] Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001), “The Perils of Taylor Rules,” *Journal of Economic Theory* 96, 40–69.

[2] Carlstrom, C. T., T. S. Fuerst, and M. Paustian (2012), “How Inflationary Is an Extended Period of Low Interest Rates?” Federal Reserve Bank of Cleveland, working paper no. 12-04.

[3] Christiano, L. , M. Eichenbaum, and C. Evans (2005), “Nominal Rigidities and the Dynamic Effect of a Shock to Monetary Policy,” *Journal of Political Economy*, 113(1): 1-45.

[4] Christiano, L., M. Eichenbaum, and S. Rebelo (2011), “When Is the Government Spending Multiplier Large?,” *Journal of Political Economy*, 119(1), 78–121.

[5] Cochrane, J., (2012), “Woodford and Jackson Hole,” at http://johnhcochrane.blogspot.com/2012/09/woodford-at-jackson-hole.html.

[6] Eggertsson, G. B., and M. Woodford (2003), "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34(2003-1), pp. 139–235.

[7] Eggertsson, G. B. (2012), “Was the New Deal Contractionary?” *American Economic Review*, 102(1).

[8] Krugman, P. (1998), “It’s Baaack! Japan’s Slump and the return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 2(1998).

[9] Kydland, F. E., and E. C. Prescott (1977). “Rules Rather than Discretion: The Inconsistency of Optimal Plans.” *Journal of Political Economy* 85, 473–491.

[10] Lucas, R.E. (1972), “Expectations and the Neutrality of Money,” *Journal of Economic Theory* 4, 103-124.

[11] Phelps, E. S. (1967), “Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time,” *Economica*, Vol. 34, 254-281.

[12] Sargent, T. and N. Wallace (1975), "Rational Expectations and the Theory of Economic Policy," *Journal of Monetary Economics* 2, pp. 169-83.

[13] Sargent, T. J. (1999), *The Conquest of American Inflation*. Princeton, NJ: Princeton University Press.

[14] Smets, F., and R. Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97(3): 586-606.

[15] Werning, I. (2012), "Managing a Liquidity Trap: Monetary and Fiscal Policy," Working Paper, MIT.

[16] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

A Appendix: Proof of Propositions

A.1 Proof of Proposition 2

Under the assumptions A1, $r^e_M < -\pi^*_L$, $\bar{r} > -\pi^*_L$, and $\phi_\pi > 1$, it follows from Proposition 1 that $\hat{Y}_L = 0$. Next, perfect foresight between periods S and M imply $E_S\pi_M = \pi_M$, so that equation (3) in period M implies $\hat{Y}_M = 0$. Suppose that $r^e_M + \gamma\pi^*_M + (1-\gamma)\,\pi_M + \phi_\pi(\pi_M - \pi^*_M) \leq 0$, so that $i_M = 0$ by (16). Since $r^e_M + \pi^*_L < 0$ by assumption, it follows from (15) that $\hat{Y}_M = \sigma\,(\pi^*_L + r^e_M) < 0$. This leads to contradiction with $\hat{Y}_M = 0$, so that $r^e_M + \gamma\pi^*_M + (1-\gamma)\,\pi_M + \phi_\pi(\pi_M - \pi^*_M)$ cannot be negative. Suppose instead that $r^e_M + \gamma\pi^*_M + (1-\gamma)\,\pi_M + \phi_\pi(\pi_M - \pi^*_M) > 0$, so that $i_M > 0$ by (16). In this case, (15) implies

$$\hat{Y}_M = -\sigma\,(i_M - \pi^*_L - r^e_M) = -\sigma i_M + \sigma\,(\pi^*_L + r^e_M) < 0$$

This leads again to a contradiction, so that $r^e_M + \gamma\pi^*_M + (1-\gamma)\,\pi_M + \phi_\pi(\pi_M - \pi^*_M)$ cannot be positive. Hence no medium run equilibrium $\left\{\pi_M, \hat{Y}_M, i_M\right\}$ satisfying (3)–(16) exists.

A.2 Proof of Proposition 3

Under the assumptions A2, $\bar{r} > -\pi^*_L$, and $\phi_\pi > 1$, it follows from Proposition 1 that $\hat{Y}_L = 0$. Using this, equations (19)–(24) can be written in matrix form as

$$\begin{bmatrix} \alpha & -\alpha & -\kappa & 0 & 0 & 0 & 0 & 0 \\ -(1-\alpha) & 1-\alpha & 0 & -\kappa & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\phi_\pi & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\kappa \\ -\sigma(1-\alpha) & -\sigma\alpha & -(1-\alpha) & -\alpha & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi^{low}_M \\ \pi^{high}_M \\ \hat{Y}^{low}_M \\ \hat{Y}^{high}_M \\ i^{low}_M \\ i^{high}_M \\ \pi_S \\ \hat{Y}_S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sigma(\pi^*_L + r^e_S) \\ \sigma(\pi^*_L + r^e_L) \\ 0 \\ r^e_L - (\phi_\pi - 1)\,\pi^*_M \\ 0 \\ \sigma r^e_S \end{bmatrix}$$

We note that the determinant of the matrix on the left is $\alpha\kappa\sigma\phi_\pi$. So, provided that $\alpha > 0$ and that $\kappa\sigma\phi_\pi > 0$, we can invert the matrix to obtain the solution expressed in the proposition.

B Appendix: Nonlinear Model

B.1 Model

The nonlinear model described in the text is repeated here for convenience using (13) to eliminate C_t and l_t. The household's optimal conditions for consumption and leisure are given by

$$u_c(Y_t)\xi_t = (1+i_t)\beta E_t\left[u_c(Y_{t+1})\xi_{t+1}\Pi^{-1}_{t+1}\right] \quad (34)$$

$$W_t = \frac{v_l(Y_t)}{u_c(Y_t)} \quad (35)$$

where

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \quad (36)$$

and the nominal interest rate satisfies the ZLB constraint

$$i_t \geq 0. \tag{37}$$

The firms' optimal conditions for pricing are given by

$$\frac{p_t(1)}{P_t} = \frac{\theta}{\theta - 1} W_t \tag{38}$$

and

$$E_{t-1}\left[u_c\left(Y_t\right) Y_t \left(\frac{p_t(2)}{P_t}\right)^{-\theta} \left(\frac{p_t(2)}{P_t} - \frac{\theta}{\theta-1} W_t\right)\right] = 0, \tag{39}$$

and the aggregate price level is given by

$$1 = \gamma \frac{p_t(1)}{P_t} + (1-\gamma)\frac{p_t(2)}{P_t}. \tag{40}$$

Finally government policy is given by an interest rate reaction function

$$i_t = \phi(\Pi_t, \Pi_t^*, \xi_t) \tag{41}$$

where and we assume that the function $\phi\left(\right)$ is non-negative for all values of its arguments (otherwise policy would not be feasible given the zero bound) and increasing in Π_t. An equilibrium can now be defined as a set of stochastic processes $\{\frac{p_t(1)}{P_t}, \frac{p_t(2)}{P_t}, \Pi_t, Y_t, i_t, W_t\}$, that satisfy equations (34)–(41) in all periods $t \geq t_0$,for given $\{\xi_t, \Pi_t^*\}$.

B.2 Long-run steady state: Proof of Proposition 8

Proof. Consider the long-run steady states given by constant values $\bar{Y}_L, \bar{\imath}_L, \bar{\Pi}_L, \bar{W}_L$, as well as paths for the price levels $\bar{p}_t(1), \bar{p}_t(2), \bar{P}_t$ for given exogenous variables $\bar{\xi}_L$, $\bar{\Pi}_L^*$ that satisfy the above equations in all periods $t \geq t_1$. Optimal pricing conditions are given for all $t \geq t_1$ by

$$\frac{\bar{p}_t(1)}{\bar{P}_t} = \frac{\theta}{\theta-1}\bar{W}_L$$

and

$$\frac{\bar{p}_t(2)}{\bar{P}_t} = \frac{\theta}{\theta-1}\bar{W}_L$$

given that $u_c\left(\bar{Y}_L\right)\bar{Y}_L\left(\bar{p}_t(2)/\bar{P}_t\right)^{-\theta} > 0$ by assumption. The two optimal pricing equations combined with the aggregate price level equation (40)

$$1 = \gamma\frac{\bar{p}_t(1)}{\bar{P}_t} + (1-\gamma)\frac{\bar{p}(2)}{\bar{P}},$$

imply

$$\bar{P}_t = \bar{p}_t(1) = \bar{p}_t(2)$$

and

$$\bar{W}_L = \frac{\theta-1}{\theta}.$$

Next, combining this with (35), steady-state output is determined by

$$\frac{v_l(\bar{Y}_L)}{u_c(\bar{Y}_L)} = \frac{\theta-1}{\theta}. \tag{42}$$

To determine steady-state inflation, we use the consumption Euler equation (34) at the steady state

$$u_c(\bar{Y}_L)\bar{\xi}_L = (1+\bar{\imath}_t)\beta u_c(\bar{Y}_L)\bar{\xi}_L\bar{\Pi}_{t+1}^{-1}$$

for any $t \geq t_1$. This simplifies to

$$\beta^{-1} = (1+\bar{\imath}_t)\bar{\Pi}_{t+1}^{-1}. \tag{43}$$

Combining this with the government's interest-rate reaction function

$$\bar{\imath}_t = \phi(\bar{\Pi}_t, \bar{\Pi}_t^*, \bar{\xi}_t)$$

we get

$$\bar{\Pi}_{t+1} = \beta(1+\phi(\bar{\Pi}_t, \bar{\Pi}_t^*, \bar{\xi}_t)), \tag{44}$$

for any $t \geq t_1$. As analyzed in Benhabib, Schmitt-Grohe and Uribe (2001), this equation admits at least two constant solutions. One solution $\bar{\Pi}_t = \bar{\Pi}_t^*$, and $\bar{\imath}_t = \phi(\bar{\Pi}_t^*, \bar{\Pi}_t^*, \bar{\xi}_t) = \beta^{-1}\bar{\Pi}_t^* - 1 > 0$, for all $t \geq t_1$. The other constant solution is $\bar{\Pi}_t = \beta < 1$ and $\bar{\imath}_t = \phi\left(\beta, \bar{\Pi}_t^*, \bar{\xi}_t\right) = 0$, for all $t \geq t_1$. As shown in Woodford (2003, chap. 2), for any initial inflation $\Pi_0 \in \left(0, \bar{\Pi}_L^*\right)$, one of these steady states will eventually be reached. In contrast, for any $\Pi_0 > \bar{\Pi}_L^*$, inflation will increase forever and get unboundedly large. Such an equilibrium is not consistent with a constant steady state inflation. ■

B.3 Non-existence of medium term equilibrium: Proof of Proposition 9

Proof. Given Assumption 1', there is perfect foresight between periods M and L, so that the optimal pricing conditions (11)–(12) simplify to

$$\frac{p_M(1)}{P_M} = \frac{\theta}{\theta-1}W_M$$

and

$$\frac{p_M(2)}{P_M} = \frac{\theta}{\theta-1}W_M.$$

given that $u_c\left(Y_M\right)Y_M\left(p_M(2)/P_M\right)^{-\theta} > 0$ by assumption. Combining this with (40) yields

$$\begin{aligned} P_M &= p_M(1) = p_M(2) \\ W_M &= \frac{\theta-1}{\theta}. \end{aligned}$$

Combining this with (35), we can determined medium-term output supplied using

$$\frac{v_l(Y_M)}{u_c(Y_M)} = \frac{\theta-1}{\theta},$$

so that

$$Y_M = \bar{Y}_L. \tag{45}$$

On the demand side, however, equilibrium output must satisfy (34) or

$$u_c(Y_M)\xi_M = (1+i_M)\beta u_c(\bar{Y}_L)\bar{\xi}_L\bar{\Pi}_L^{-1}.$$

Using, (45), this simplifies to

$$\xi_M = (1+i_M)\beta\bar{\xi}_L\bar{\Pi}_L^{-1}.$$

so that $i_M < 0$, if (32) holds. It follows that the zero-lower bound condition (37) is violated and hence that there is no medium-term equilibrium if (32) holds. ■