Uncertainty, Financial Frictions, and Irreversible Investment

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Abstract

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1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic returns such as labor income, business cash flows, and equity valuations is one of the stylized facts of business cycle fluctuations.[1] In macroeconomics, the framework of irreversible investment provides the traditional channel through which changes in uncertainty—by altering the "option value" of investing—affect economic activity; see, for example, Bernanke (1983); Bertola et al. (1994); Abel and Eberly (1996); Caballero and Pindyck (1996); and more recently, Bloom (2009) and Bloom et al. (2011).[2]

Financial market frictions, reflecting departures from the Modigliani-Miller paradigm of perfect capital markets, have the potential for providing an additional channel through which fluctuations in uncertainty can influence business cycle dynamics. In the canonical framework used to price corporate debt (Merton (1974)), the payoff structure of levered equity—under limited liability—resembles the payoff of a call option, while the bondholders face the payoff structure that is equivalent to that of an investor writing a put option. An increase in the riskiness of the firm's assets thus benefits equity holders at the expense of bondholders, implying a rise in the default-risk premium to compensate bondholders for heightened uncertainty. To the extent that default is costly and external funds command a premium over internal funds, an increase in uncertainty will raise the costs of capital, causing a decline in investment spending.

The aim of this paper is to analyze the interaction of uncertainty, irreversible investment, and financial frictions in a unified business cycle framework. To that purpose, we construct an analytically tractable bond-contracting model of the type analyzed by Bernanke et al. (1999), Cooley and Quadrini (2001), Hennessy and Whited (2007), and Philippon (2009). We embed this contracting framework into a capital accumulation problem, in which firms employ a production technology that is subject to a persistent idiosyncratic shock, the variance of which is allowed to vary over time according to a stochastic law of motion. In addition to time-varying uncertainty, the capital accumulation problem also encompasses the costly reversible investment framework of Abel and Eberly (1996), in which the resale price of capital is below its purchase price, reflecting the specificity of physical capital to the firm.

On the financial side, firms make investment decisions subject to a full range of choices regarding their capital structure—internal funds, debt, and equity financing—in an environment where external funds are costly because of agency problems in financial markets.[3] The costly reversible investment plays a double role in our model. First, the partial irreversibility creates the so-called option value of waiting—at a time of heightened economic uncertainty, the investment inaction

[1] See, for example, Campbell and Taksler (2003), Storesletten et al. (2004) Eisfeldt and Rampini (2006), and Bloom et al. (2011).

[2] Despite its intuitive appeal, the effect of uncertainty on investment in the presence of irreversibilities can be theoretically ambiguous. As shown, for example, by Abel (1983), Veracierto (2002), and Bachmann and Bayer (2009), the effect depends importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting.

[3] Cooley and Quadrini (2001), Hennessy and Whited (2007), and Philippon (2009) consider similar contracting frameworks, though only in partial equilibrium. Bernanke et al. (1999) do allow for general equilibrium feedback effects but consider only debt financing.

region expands, which will cause some firms to delay exercising their growth options. Second, the illiquidity of capital assets limits the firm's debt capacity because the implied liquidation cost lowers the recovery value of the firm in case of costly default.

The main contribution of our paper is to examine in a quantitative framework the implications of the interaction between the capital capacity and debt overhang problems for business cycle dynamics.[4] In particular, our framework allows us to quantify how much of the impact of uncertainty on investment dynamics reflects irreversibility and how much of it can be attributed to financial market frictions.

Importantly, our theoretical analysis is informed by a set of new empirical results that highlight the dynamic relationship between uncertainty, investment, and financial factors. First, we construct a novel proxy for time-varying idiosyncratic uncertainty using high-frequency firm-level stock market data. We use this measure of uncertainty to examine the interaction between economic activity, uncertainty, and credit spreads on corporate bonds—an indicator of financial market strains—within a structural vector autoregression (VAR) framework. Our empirical results indicate that conditions in the corporate debt markets are an important conduit through which fluctuations in uncertainty are propagated to the real economy. Unanticipated increases in uncertainty lead to a significant widening of credit spreads, a decline in output that is driven primarily by the protracted drop in the investment component of aggregate demand.

We complement this analysis by constructing a new firm-level panel data set that combines information on prices of individual corporate bonds trading in the secondary market with our estimates of firm-specific uncertainty and the issuers' income and balance sheet information. Results from this micro-level analysis confirm the aggregate time-series findings: Conditional on the firm's leverage, profitability, and other indicators of credit quality, our firm-specific measure of idiosyncratic uncertainty is an important determinant—both economically and statistically—of credit spreads on the firm's outstanding bonds. According to our results, an increase in uncertainty of 10 percentage points boosts credit spreads about 50 basis points, an economically substantial effect given the extent of the observed variation in idiosyncratic uncertainty.

We also find that conditional on investment fundamentals—that is, proxies for the marginal product of capital—the long-run elasticity of investment demand with respect to uncertainty lies in the range between -0.80 and -0.60, implying that an increase of 10 percentage points in idiosyncratic uncertainty is associated with a decline in the investment rate between 4.0 and 5.0 percentage points in the long run. However, once the information content of credit spreads is taken into account, the impact of uncertainty on investment is significantly attenuated. Capital formation, in contrast, is highly sensitive to the firm-specific financial conditions, with a 100 basis points rise in credit spreads leading to a drop in the investment rate of about 3.5 percentage points in the long run. All told, these aggregate and firm-level results strongly suggest that the interplay between uncertainty shocks and financial conditions significantly shapes business cycle dynamics.

[4]Theoretical models of the interaction between the capital capacity and debt overhang problems can be found in Shleifer and Vishny (1992) and Manso (2008).

2 Empirical Evidence

2.1 Measuring Time-Varying Economic Uncertainty

We utilize daily firm-level stock returns to construct our benchmark estimate of time-varying economic uncertainty. The advantage of using equity valuations to infer fluctuations in economic uncertainty is that asset prices should, in principle, encompass all aspects of the firm's environment that investors view as important. Specifically, from the Center for Research in Security Prices (CRSP) data base, we extracted daily stock returns for all U.S. nonfinancial corporations with at least 1,250 trading days (essentially five years) of data. This selection criterion yielded a panel of 11,159 firms over the period from July 1, 1963 (1963:Q3) to September 30, 2011 (2011:Q3).[5]

Our estimate of uncertainty is based on the following three-step procedure. First, we remove the systematic component of daily (excess) equity returns using the standard factor-model framework of asset returns:

$$(R_{it_d} - r^f_{t_d}) = \alpha_i + \boldsymbol{\beta}'_i \mathbf{f}_{t_d} + u_{it_d}, \tag{1}$$

where i indexes firms and $d = 1, \ldots, D_t$, indexes trading days in quarter t. In equation (1), R_{it_d} denotes the (total) daily return of firm i, $r^f_{t_d}$ is the risk-free rate, and $\mathbf{f}_{t_d}$ is a vector of observable risk factors, with the corresponding vector of factor loadings $\boldsymbol{\beta}_i$. In our empirical implementation, we employ a 4-factor model, namely the Fama and French (1992) 3-factor model, augmented with the momentum risk factor proposed by Carhart (1997).[6]

In the second step, we calculate the quarterly firm-specific standard deviation of daily idiosyncratic returns—denoted by σ_{it}—according to $\sigma_{it} = \left[D_t^{-1} \sum_{d=1}^{D_t} (\hat{u}_{it_d} - \bar{u}_{it})^2\right]^{1/2}$, where $\hat{u}_{it_t}$ denotes the OLS residual—the idiosyncratic return—from equation (1) and $\bar{u}_{it} = D_t^{-1} \sum_{d=1}^{D_t} \hat{u}_{it_d}$ is the sample mean of daily idiosyncratic returns in quarter t. Thus, σ_{it} is an estimate of time-varying equity volatility for firm i, a measure that abstracts from the common risk factors that drive differences in expected returns across firms.

In the final step, we assume that this quarterly estimate of idiosyncratic volatility follows an autoregressive process of the form:

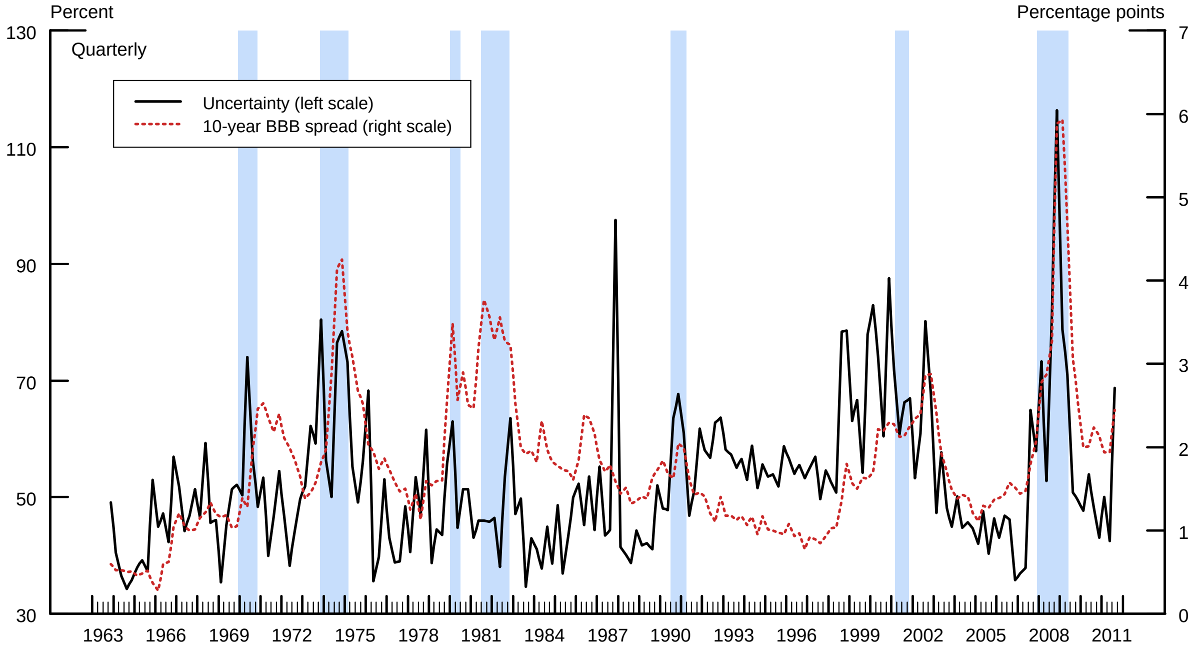
$$\log \sigma_{it} = \gamma_i + \delta_i t + \rho \log \sigma_{i,t-1} + v_t + \epsilon_{it}; \quad \epsilon_{it} \sim N(0, \omega^2_\epsilon), \tag{2}$$

where γ_i denotes a firm fixed effect intended to control for the cross-sectional heterogeneity in σ_{it},

[5] To ensure that our results were not driven by a small number of extreme observations, we eliminated all firm/day observations with a daily absolute return in excess of 50 percent.

[6] In their seminal contribution, Fama and French (1992) outline three risk factors that are empirically important in explaining security returns: (1) the market premium (the market excess return); (2) the size premium (the difference in returns between portfolios of small capitalization and big capitalization firms (SMB)); and (3) the book-to-market premium (the difference in returns between portfolios of high book-to-market and small book-to-market firms (HML)). We also include the momentum ("winners" minus "losers" (WML)) factor of Carhart (1997) because a number of empirical studies following Jegadeesh and Titman (1993) have shown that investment strategies that involve taking a long (short) position in well (poorly) performing stocks on the basis of past performance helps to resolve the momentum anomaly documented by Fama and French (1996). The daily risk factors and the risk free rate were obtained from the Kenneth R. French's website (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`).

Figure 1: Uncertainty and Credit Spreads



NOTE: Sample period: 1963:Q4–2011:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on firm-level equity returns (see text for details). The dotted line depicts the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield. The shaded vertical bars denote NBER-dated recessions.

whereas the firm-specific term $\delta_i t$ captures secular trends in the idiosyncratic risk of publicly-traded U.S. firms documented by Campbell et al. (2001). Our estimate of time-varying uncertainty corresponds to the sequence of time fixed effects v_t, $t = 1, \ldots, T$, which captures shocks to idiosyncratic volatility that are common to all firms. We estimate equation (2) by OLS, which yields an estimate of $\rho = 0.44$, an indication that idiosyncratic equity volatility tends to be fairly persistent.[7] The specification also fits the data quite well, explaining more than 75 percent of the variation in the dependent variable.

Figure 1 shows our benchmark estimate of time-varying uncertainty derived from the estimated time fixed effects in equation (2). The figure also plots the spread between the 10-year yield on BBB-rated corporate bonds and the 10-year Treasury yield, an indicator of conditions in the corporate debt markets. As evidenced by the correlation coefficient of 0.42, there is a substantial degree of comovement between the two series. Moreover, both series are clearly countercyclical, typically rising sharply before recessions.

[7]Because the average firm is in the panel for more than 60 quarters, the bias of the OLS estimator, owing to the presence of a lagged dependent variable and firm fixed effects, is negligible (see, for example, Arellano (2003)).

2.2 Aggregate Time-Series Evidence

In this section, we use a VAR framework to investigate the interaction between economic uncertainty, financial conditions, and the macroeconomy. In particular, we estimate a VAR consisting of the following eight endogenous variables: the logarithm of real business fixed investment (i_t); the logarithm of real personal consumption expenditures (PCE) on durable goods (c_t^D); the logarithm of real PCE on nodurable goods and services (c_t^N); the logarithm of real GDP (y_t); the logarithm of the GDP price deflator (p_t); the (nominal) effective federal funds rate (m_t) as an indicator of the stance of monetary policy; the 10-year BBB-Treasury credit spread (s_t); and our benchmark estimate of time-varying uncertainty (v_t). The VAR is estimated over the 1963:Q3–2011:Q3 period using four lags of each endogenous variable.

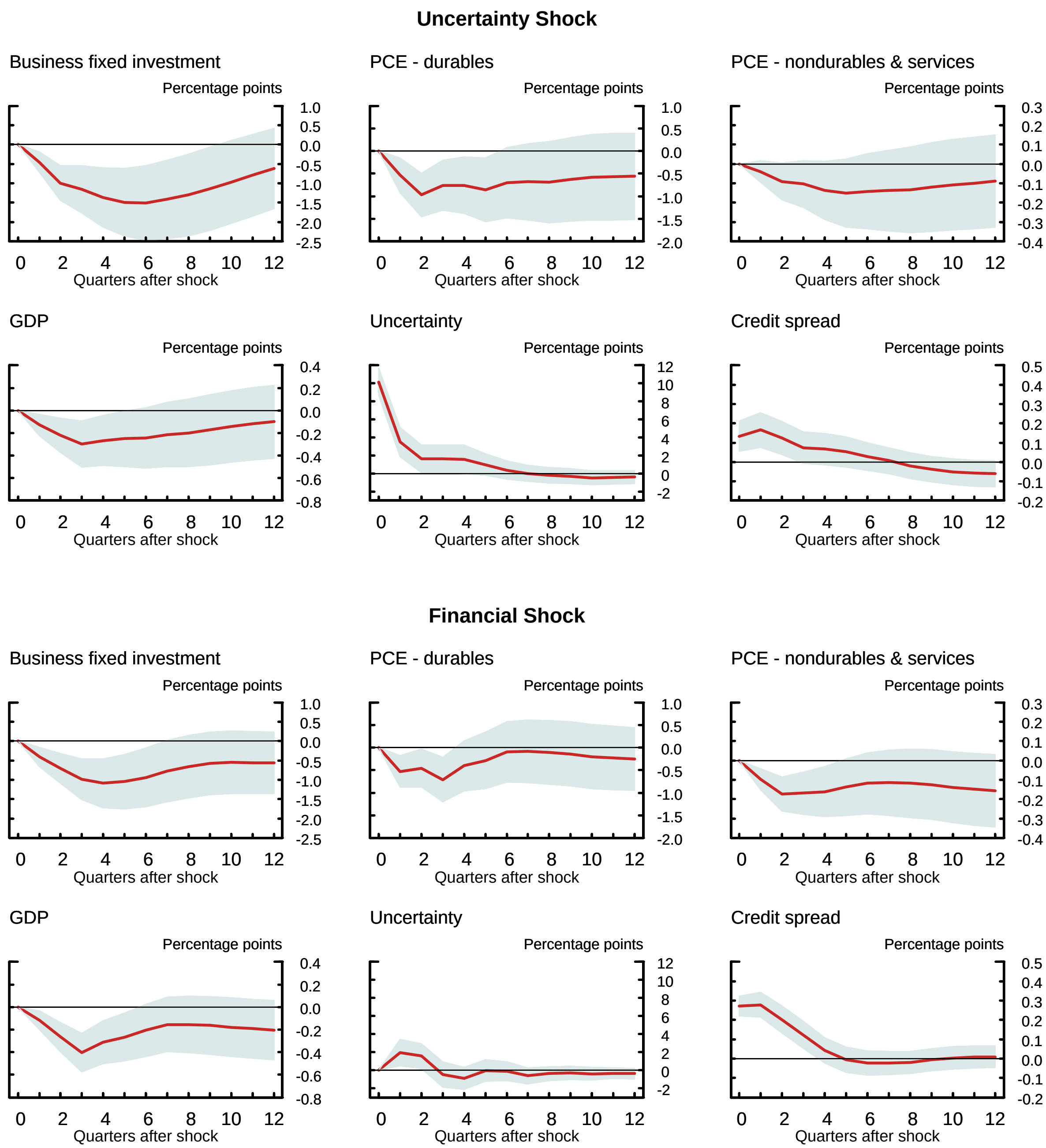
We focus on the implications of uncertainty shocks on credit spreads and economic activity. To identify these disturbances, we employ a standard recursive ordering technique, in which shocks to uncertainty have an immediate impact on credit spreads and short-term interest rates, but they affect economic activity and prices with a lag (Identification Scheme I); to provide a point of comparison, we rely on the same recursive ordering to examine the impact of shocks to credit spreads—that is, "financial shocks"—that are orthogonal to contemporaneous movements in uncertainty. We also consider a VAR specification that reverses this causal ordering, which allows us to examine the implications of uncertainty shocks conditional on the information contained in the current level of credit spreads (Identification Scheme II).

The top half of Figure 2 shows the impulse response functions of selected variables to an uncertainty shock, while the bottom half shows the responses of the same variables to a financial shock, where both shocks are orthogonalized using the first identification scheme. Given these identifying assumptions, an unanticipated increase in uncertainty is associated with an immediate widening of corporate bond spreads. Moreover, this uncertainty shock has significant adverse consequences for economic growth, as output declines almost immediately, reaching a trough about a year after the initial spike in uncertainty.

Consistent with the standard irreversibility argument, the impact of the uncertainty shock falls primarily on the investment component of aggregate demand: Business expenditures on fixed capital fall steadily, bottoming out 1.5 percentage points below the trend five quarters after the shock, while consumer spending on durable goods also declines significantly. A financial shock, which leads to an increase of about 25 basis points in the 10-year BBB-Treasury spread, similarly leads to a significant contraction in economic activity. This innovation to credit spreads, however, has only a small and short-lived effect on our measure of economic uncertainty.

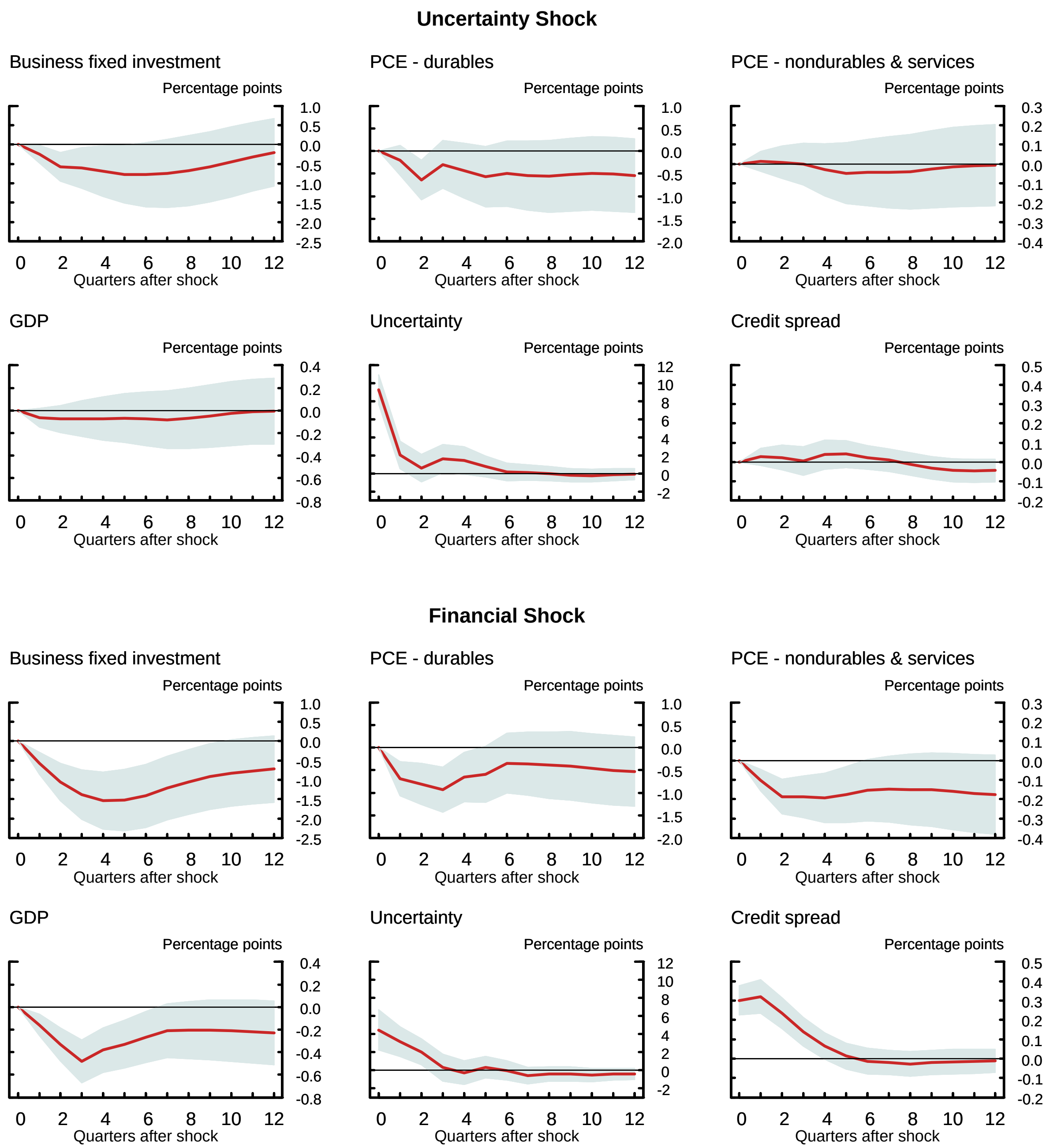
Figure 3 shows the implications of these two shocks orthogonalized using an alternative scheme, in which credit spreads are ordered before uncertainty. Under these identifying assumptions, an unanticipated increase in uncertainty has a notably less adverse effect on the real economy. Although both business fixed investment and consumer spending on durable goods fall in response to an increase in uncertainty, the declines are much smaller and less persistent compared with those in Figure 2. Indeed, the response of output to an uncertainty shock is economically and statistically

Figure 2: Macroeconomic Implications of Uncertainty and Financial Shocks (Identification Scheme I)



NOTE: The top six panels depict impulse response functions of selected macroeconomic and financial indicators to an orthogonalized 1-standard deviation shock to our estimate of idiosyncratic uncertainty; the bottom six panels depict the corresponding impulse response functions to an orthgonalized 1-standard deviation shock to the 10-year BBB-Treasury spread. Identification scheme I corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, s_t, \sigma_t, m_t)$; see text for details. Shaded bands represent 95-percent confidence intervals based on 1,000 bootstrap replications.

Figure 3: Macroeconomic Implications of Uncertainty and Financial Shocks
(Identification Scheme II)



Note: The top six panels depict impulse response functions of selected macroeconomic and financial indicators to an orthogonalized 1-standard deviation shock to our estimate of idiosyncratic uncertainty; the bottom six panels depict the corresponding impulse response functions to an orthgonalized 1-standard deviation shock to the 10-year BBB-Treasury spread. Identification scheme II corresponds to the following recursive ordering of the VAR system: $(i_t, c_t^D, c_t^N, y_t, p_t, \sigma_t, s_t, m_t)$; see text for details. Shaded bands represent 95-percent confidence intervals based on 1,000 bootstrap replications.

indistinguishable from zero.

Financial shocks, in contrast, have significant and long-lasting effects on economic activity. A one standard deviation shock to the 10-year BBB-Treasury spread is associated with an immediate jump in uncertainty, a substantial fall in real GDP and a protracted decline in business fixed investment, as well as in major categories of consumer spending. Indeed, the magnitude and the shape of the impulse response functions of output and its major components are very similar to those shown in Figure 2.

In summary, the time-series evidence presented above suggests that an increase in uncertainty leads to an economically and statistically significant widening of credit spreads, a drop in output, and a protracted decline in the major investment categories of aggregate demand. The evidence also suggests that changes in credit conditions—as measured by the movements in credit spreads—are an important part of the transmission mechanism propagating uncertainty shocks to the real economy. Indeed, our results indicate that once such innovations are orthogonalized with respect to the contemporaneous information from the corporate bond market, the impact of uncertainty shocks on economic activity is significantly attenuated.

2.3 Firm-Level Evidence

In this section, we utilize a new firm-level data set to provide additional evidence regarding the role of financial market frictions as a determinant of investment dynamics in response to fluctuations in economic uncertainty. As in Leahy and Whited (1996) and Bloom et al. (2007), our empirical strategy involves regressing business fixed investment on the firm-specific estimate of idiosyncratic uncertainty, while controlling for the fundamental determinants of investment spending.

Given the focus on the interaction between uncertainty and financial frictions, our regression specification also includes credit spreads at the level of an *individual* firm. To that purpose, we constructed a panel data set of almost 1,100 publicly-traded U.S. nonfinancial firms covered by CRSP and S&P's Compustat over the 1973–2011 period. The distinguishing characteristic of these large corporations is that a significant portion of their outstanding liabilities is in the form of long-term bonds that are actively traded in the secondary market. We use the secondary market prices of individual securities to construct firm-level credit spreads, which are then matched to the issuer's income and balance sheet data. (The description of the bond-level data set and the details regarding the construction of credit spreads are contained in Appendix A.)

2.3.1 Uncertainty and Credit Spreads

The first empirical exercise using our firm-level data examines the link between credit spreads and our measure of idiosyncratic uncertainty. To do so, we estimate the following credit-spread regression:

$$\log s_{it}[k] = \beta_1 \log \sigma_{it} + \beta_2 R^E_{it} + \beta_3 [\Pi/A]_{it} + \beta_4 \log[D/E]_{i,t-1} + \theta' \mathbf{X}_{it}[k] + \epsilon_{it}[k], \tag{3}$$

Table 1: Uncertainty and Credit Spreads

Explanatory Variable	(1)	(2)	(3)	(4)
$\log \sigma_{it}$	0.762	0.497	0.519	0.223
	(0.042)	(0.047)	(0.050)	(0.022)
R^E_{it}	-0.100	-0.115	-0.112	-0.053
	(0.027)	(0.025)	(0.025)	(0.009)
$[\Pi/A]_{it}$	-4.030	-1.802	-1.507	-1.362
	(0.680)	(0.507)	(0.476)	(0.388)
$\log[D/E]_{i,t-1}$	0.201	0.054	0.047	0.078
	(0.024)	(0.014)	(0.014)	(0.011)
Adjusted R^2	0.479	0.639	0.646	0.793
Credit Rating Effects[a]	-	0.000	0.000	0.000
Industry Effects[b]	-	-	0.000	0.000
Time Effects[c]	-	-	-	0.000

NOTE: Sample period: 1973:M1–2011:M9 at a quarterly frequency (No. of firms/bonds = 1,099/5,995; Obs. = 102,711). Dependent variable in all specification is $\log(s_{it}[k])$, the logarithm of the credit spread of bond k (issued by firm i) in month t. All specifications include a constant, a vector of bond-specific control variables $\mathbf{X}_{it}[k]$ (not reported) and are estimated by OLS. Robust asymptotic standard errors reported in parentheses are double-clustered in the firm (i) and time (t) dimension, according to Cameron et al. (2011).

[a] p-value for the test of the null hypothesis of the absence of fixed credit rating effects.
[b] p-value for the test of the null hypothesis of the absence of fixed industry effects.
[c] p-value for the test of the null hypothesis of the absence of fixed time effects.

where $s_{it}[k]$ denotes the credit spread of a bond issue k in period t, a security that is a liability of firm i.[8] In addition to our estimate of idiosyncratic uncertainty σ_{it}, credit spreads are allowed to depend on the firm's repayment prospects, as measured by the firm's realized quarterly return on equity R^E_{it} and the ratio of operating income to assets $[\Pi/A]_{it}$, while the ratio of the book value of total debt to the market value of the firm's equity—denoted by $[D/E]_{it}$—captures the strength of the firm's balance sheet. The vector $\mathbf{X}_{it}[k]$ contains variables capturing bond-specific characteristics that could influence bond yields through either liquidity or term premiums, including the bond's duration, the par amount outstanding, the bond's age, the bond's (fixed) coupon rate, and an indicator variable that equals one if the bond is callable and zero otherwise.[9]

Table 1 contains these estimation results. According to column 1, an increase in uncertainty leads to a significant widening of credit spreads—the elasticity estimate of 0.762 implies that an increase in uncertainty of 10 percentage points in quarter t is associated with an immediate jump

[8] Although our data on credit spreads are at a monthly frequency, the requisite income and balance sheet information from Compustat is available only at a quarterly frequency. In addition, the firms' fiscal years/quarters end at different months of the year. The timing of our firm-level data reflects these differences as our observations occur at different months but are spaced at regular quarterly (i.e., three-month) intervals.

[9] Specification (3) is similar to those used by Bharath and Shumway (2008) to predict credit default swap and corporate bond yield spreads. As in their paper, our main explanatory variables—volatility, expected profitability, and leverage—correspond to the "naïve" constituents of the distance-to-default, which, according to the Merton (1974) model, should be a sufficient statistic for default.

in credit spreads of about 50 basis points. The coefficients on the remaining key variables are also economically and statistically highly significant and have their expected signs: Strong profitability performance, as evidenced by a high realized return on equity or an increase in the ratio of operating income to assets, is associated with a narrowing of credit spreads, whereas an increase in the debt-to-equity ratio leads to a rise in credit spreads.

The results in Table 1 are robust to the inclusion of fixed credit rating effects (column 2) and to the inclusion of fixed industry effects (column 3). The specification in column 4 also controls for macroeconomic developments by adding a full set of time dummies to the regression. Although the magnitude of the coefficient on uncertainty diminishes noticeably, the impact of uncertainty on credit spreads remains statistically significant and economically important: A 10 percentage point increase in our measure of idiosyncratic uncertainty is associated with a rise in credit spreads of about 15 basis points, a magnitude that is remarkably consistent with the VAR results shown in Figure 2. These micro-level results provide further evidence that fluctuations in idiosyncratic uncertainty influence business financing conditions by significantly altering the level of credit spreads in the corporate bond market.

2.3.2 Uncertainty, Credit Spreads, and Investment

We now turn to the link between investment, uncertainty, and credit spreads. Our empirical investment equation is given by the following specification:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta \log Z_{it} + \eta_i + \lambda_t + \epsilon_{it}, \tag{4}$$

where $[I/K]_{it}$ denotes the investment rate of firm i in period t (i.e., the ratio of capital expenditures in period t to the capital stock at beginning of the period); σ_{it} is our measure of idiosyncratic uncertainty; s_{it} is the credit spread on the portfolio of bonds issued by firm i; and Z_{it} is a proxy for the marginal product of capital, a variable that measures firm i's future investment opportunities.[10] In addition to uncertainty, credit spreads, and investment fundamentals, the regression equation (4) includes a fixed firm effect η_i and a fixed time effect λ_t—the former controls for systematic differences in the average investment rate across firms, while the latter captures a common investment component reflecting macroeconomic factors, which can influence firm-level investment through movement in either output, interest rates, or aggregate uncertainty.[11]

We measure the investment fundamentals Z_{it} using either the current sales-to-capital ratio $[Y/K]_{it}$ or the operating-income-to-capital ratio $[\Pi/K]_{it}$. Taking logs of $[Y/K]_{it}$ is straightforward,

[10] The frequency of data on capital expenditures and capital stock is annual, but the data are recorded at different months of the year, reflecting the differences in the fiscal years across firms. As a result, the uncertainty measure σ_{it} in equation (4) is calculated using daily idiosyncratic returns over the 250 trading days of the firm's fiscal year, and the credit spread is the average of the monthly credit spreads calculated over the 12 months of the firm's fiscal year. For the firms that have more than one bond issue trading in the secondary market in a given period, we calculate the portfolio spread by computing an average of credit spreads on the firm's outstanding bonds.

[11] The log-log nature of regression (4) reflects the fact that the firm-level investment rates, uncertainty, and credit spreads are highly positively skewed, a feature of the data that is significantly ameliorated through the use of a logarithmic transformation.

Table 2: Uncertainty, Credit Spreads, and Investment
(*Static Panel Data Specification*)

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.173	-0.075	-0.154	-0.042	0.025	-0.061
	(0.037)	(0.034)	(0.035)	(0.036)	(0.033)	(0.035)
$\log s_{it}$	-	-	-	-0.197	-0.163	-0.146
				(0.020)	(0.020)	(0.021)
$\log[Y/K]_{it}$	0.558	-	-	0.535	-	-
	(0.044)			(0.044)		
$\log[\Pi/K]_{it}$	-	1.230	-	-	1.138	-
		(0.087)			(0.087)	
$\log Q_{i,t-1}$	-	-	0.720	-	-	0.652
			(0.041)			(0.042)
R^2 (within)	0.330	0.314	0.300	0.352	0.328	0.312

NOTE: Sample period: 1973:M1–2011:M9 at an annual frequency (No. of firms = 763; Obs. = 8,285). Dependent variable is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t. All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the within transformation, with the resulting specification estimated by OLS. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$; see text for details.

but because operating income may be negative, we use $\log(c + [\Pi/K]_{it})$—where c is chosen so that $(c + [\Pi/K]_{it}) > 0$ for all i and t—when relying on the operating income to measure the firm's investment opportunities.[12] As an alternative forward-looking measure of investment fundamentals, we also consider Tobin's Q, denoted by Q_{it}.

The result in columns 1–3 of Table 2 indicate a significant role for uncertainty in the capital accumulation process—the coefficient on uncertainty is statistically highly significant, regardless of the measure of investment fundamentals. The estimated elasticities of investment demand with respect to uncertainty lie in the range between -0.17 and -0.08, indicating than an increase in uncertainty of 10 percentage points is associated with the decline in the investment rate between 0.5 and 1.1 percentage points. However, once the credit spreads are included in the regression (columns 4–6), the effect of uncertainty on investment is significantly attenuated. Credit spreads, in contrast, are statistically and economically highly important determinants of investment spending, with a 100 basis points rise in credit spreads implying a drop in the investment rate between 1.4 and 1.8 percentage points.

A well-documented result from the empirical investment literature is the fact that lagged investment rate is economically an important determinant of current investment spending; see, for example, Gilchrist and Himmelberg (1995) and Eberly et al. (2011) for evidence and detailed dis-

[12] In principle, the estimated elasticities may depend on the constant c. In practice, however, reasonable variation in c has no effect on the estimated elasticities.

Table 3: Uncertainty, Credit Spreads, and Investment
(*Dynamic Panel Data Specification*)

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)
$\log \sigma_{it}$	-0.366	-0.275	-0.311	-0.127	-0.081	-0.137
	(0.074)	(0.075)	(0.072)	(0.067)	(0.066)	(0.072)
$\log s_{it}$	-	-	-	-0.154	-0.146	-0.180
				(0.043)	(0.045)	(0.046)
$\log[I/K]_{i,t-1}$	0.550	0.556	0.504	0.569	0.567	0.503
	(0.032)	(0.027)	(0.028)	(0.030)	(0.027)	(0.027)
$\log[Y/K]_{it}$	0.393	-	-	0.366	-	-
	(0.060)			(0.059)		
$\log[\Pi/K]_{it}$	-	0.897	-	-	0.805	-
		(0.156)			(0.158)	
$\log Q_{i,t-1}$	-	-	0.601	-	-	0.545
			(0.052)			(0.050)
L-R effect: uncertainty	-0.813	-0.620	-0.626	-0.295	-0.188	-0.276
	(0.156)	(0.158)	(0.138)	(0.155)	(0.152)	(0.145)
L-R effect: spreads	-	-	-	-0.358	-0.336	-0.362
				(0.083)	(0.097)	(0.086)

NOTE: Sample period: 1973:M1–2011:M9 at an annual frequency (No. of firms = 749; Obs. = 6,377). Dependent variable is $\log[I/K]_{it}$, the logarithm of investment rate of firm i in year t. All specifications include time fixed effects (not reported) and firm fixed effects, which are eliminated using the forward orthogonal deviations transformation. The resulting specification is estimated by GMM using a one-step weighting matrix; see Arellano and Bover (1995) for details. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. Parameter estimates for $\log[\Pi/K]_{it}$ and the associated standard errors are adjusted for the fact that $\log[\Pi/K]_{it}$ is computed as $\log(0.5 + [\Pi/K]_{it})$; see text for details.

cussion. As a robustness check, we also consider a dynamic investment specification of the form:

$$\log[I/K]_{it} = \beta_1 \log \sigma_{it} + \beta_2 \log s_{it} + \theta_1 \log Z_{it} + \theta_2 \log[I/K]_{i,t-1} + \eta_i + \lambda_t + \epsilon_{it}. \tag{5}$$

In this case, we eliminate fixed firm effects using the forward orthogonal deviations transformation of Arellano and Bover (1995) and estimate the resulting specification using GMM. Within this dynamic framework, both uncertainty and credit spreads are treated as endogenous and are instrumented with their own lagged values.

The results of this exercise are presented in Table 3. According to columns 1–3, fluctuations in uncertainty have economically and statistically significant effects on capital spending. Taking into account lagged investment dynamics, the estimated elasticities imply that an increase in uncertainty of 10 percentage points depresses the investment rate between 4.0 and 5.0 percentage points in the long run. However, the adverse effect of increased uncertainty on investment spending is again significantly attenuated once the information content of credit spreads is taken into account. Estimates of long-run elasticities in columns 4–6 imply that a same-sized increase in uncertainty lower the investment rate only between 1.0 and 2.0 percentage points, economically still a significant effect. In contrast, a jump in credit spreads of 100 basis points is estimated to shave off almost 3.5 percentage points from the rate of capital formation in the long run.

All told, our aggregate time-series and firm-level panel analysis suggests that the uncertainty-investment nexus is strongly influenced by conditions in the corporate bond market. In particular, increases in economic uncertainty are associated with a substantial widening of credit spreads, which, in turn, leads to a significant contraction in economic activity. To the extent that movements in credit spreads provide a useful barometer of the degree of frictions in the financial system, our empirical evidence indicates that such frictions may be an importnant part of the transmission channel through which shocks in economic uncertainty are propagated to the real economy.[13]

3 Structural Model

In this section, we present a general equilibrium model, in which fluctuations in economic uncertainty influence corporate bond prices and investment in a manner consistent with our empirical findings. The richness of the framework allows us to incorporate other well-documented sources of cyclical fluctuations, namely both aggregate and idiosyncratic productivity shocks and shocks to credit-supply conditions, where the latter effects matter for macroeconomic outcomes because of the presence of financial market fricitions.

The novel feature of our model is that the agency cost associated with external finance interact with the capital capacity overhang problem caused by irreversible investment. In particular, the illiquidity of the firm's capital stock limits its ability to take on additional debt, causing a further increase in the external finance premium. Our framework, therefore, allows us to quantify how much of the impact of uncertainty on investment dynamics reflects irreversibility and how much of it can be attributed to financial market frictions.

3.1 Preferences, Technology, and Shocks

We consider a real business cycle model with two types of economic agents: A representative household and a continuum of firms (indexed by j) producing a homogeneous output good. The representative household lives forever and maximizes the expected present-value of period-specific utilities. The household earns a competitive real market wage w by working h hours and saves by purchasing bonds and equity shares of the firms in the economy. The utility function of the representative household is strictly increasing in consumption (c), strictly decreasing in hours worked (h), and concave in both arguments. Specifically, we assume the following parametric form for the utility function:

$$u(c, c_{-1}, h) = \log(c - \tau c_{-1}) - \zeta h; \quad 0 \leq \tau < 1, \quad \zeta > 0. \tag{6}$$

Our baseline set of results is based on the specification in which $\tau = 0$, implying a logarithmic utility of consumption and a linear disutility of labor. We also consider an extension of the model

[13]See Gilchrist and Zakrajšek (2011) for evidence and detailed discussion of the role of credit spreads in business cycle fluctuations.

with $0 < \tau < 1$—the "catching up with Joneses" form of habit formation of Abel (1990)—in which the household's current utility also depends on the level of consumption from the previous period. In that case, as shown by Campbell and Cochrane (1999), the local curvature of the utility function (6) depends on the surplus consumption ratio $(c - \tau c_{-1})/c$, which generates a time-varying risk aversion.

Final good output, denoted by y and which can be used either for consumption or investment, is produced by heterogeneous firms that combine capital (k) and labor input (h) using a decreasing returns-to-scale (DRS) Cobb-Douglas technology. The production technology is subject to two type of shocks: (1) a persistent *idiosyncratic* productivity shock—denoted by z—which evolves according to

$$\log z' = \rho_z \log z + \log \epsilon'_z; \quad |\rho_z| < 1 \text{ and } \log \epsilon'_z \sim N(-0.5\sigma_z^2, \sigma_z^2); \tag{7}$$

and (2) a persistent *aggregate* productivity shock—denoted by a—which evolves according to

$$\log a' = \rho_a \log a + \log \epsilon'_a; \quad |\rho_a| < 1 \text{ and } \log \epsilon'_a \sim N(-0.5\sigma_a^2, \sigma_a^2). \tag{8}$$

We also assume that production requires firms to incur quasi-fixed operation costs $F > 0$ per unit of installed capital. The technological assumptions can be summarized by a net production function

$$y = (az)^{(1-\alpha)\chi}(k^\alpha h^{1-\alpha})^\chi - Fk; \quad 0 < \alpha, \chi < 1, \tag{9}$$

where α denotes the value-added share of capital and the parameter χ governs the degree of decreasing returns to scale in production. The normalization constant $(1-\alpha)\chi$ ensures that the firm's profit function is linear in z and a, which formally can be expressed as

$$\pi(a, z, w, k) = \max\left\{(az)^{(1-\alpha)\chi}(k^\alpha h^{1-\alpha})^\chi - Fk - wh\right\} = az\psi(w)k^\gamma - Fk, \tag{10}$$

where

$$\gamma \equiv \frac{\alpha\chi}{1-(1-\alpha)\chi} \quad \text{and} \quad \psi(w) \equiv [1-(1-\alpha)\chi]\left[\frac{(1-\alpha)\chi}{w}\right]^{\frac{(1-\alpha)\chi}{1-(1-\alpha)\chi}}.$$

The combination of the DRS technology and fixed operation costs implies that firms can earn positive (or negative) profits in equlibrium, a feature that can lead to complex entry/exit industry dynamics. To keep the model tractable, we do not explicitly model the firms' endogenous entry/exit decisions. Rather, as in Cooley and Quadrini (2001) and Veracierto (2002), we assume that a constant fraction $(1-\eta)$ of firms in the economy exogenously exits in each period and is replaced by new entrants within the same period. An additional benefit of this stochastic overlapping generation structure is that it provides a convenient way to motivate the use of leverage by firms in the steady state, thereby obviating the need to introduce a corporate income tax shield in the model.

To model time-varying economic uncertainty, we assume that the level of idiosyncratic uncertainty evolves over time according to a persistent Markov process. Specifically, we let the volatility of the idiosyncratic productivity shock—that is, σ_z in equation (7)—follow a continuous AR(1) process given by

$$\log \sigma_z' = (1-\rho_\sigma)\log \bar{\sigma}_z + \rho_\sigma \log \sigma_z + \log \epsilon_\sigma'; \quad |\rho_\sigma| < 1 \text{ and } \log \epsilon_\sigma' \sim N(-0.5\sigma_\sigma^2, \sigma_\sigma^2). \tag{11}$$

Note that according to this setup, a shock to the level of idiosyncratic uncertainty corresponds to an aggregate shock that alters the level of uncertainty faced by all firms in the economy. Because ϵ_z' is distributed log-normally with $E(\epsilon_z'|\sigma_z) = \exp[0.5\sigma_z^2 + E(\log \epsilon_z'|\sigma_z)] = 1$, fluctuations in uncertainty do not change the conditional expectation of the idiosyncratic productivity shock z; that is, an increase in uncertainty induces a mean-preserving spread to the conditional distribution of profits. As a result, uncertainty shocks in our model do not have any direct implication for investment dynamics under the standard neoclassical assumptions.

The capital accumulation problem in our model follows the costly reversible investment framework of Abel and Eberly (1996). Specifically, we assume that the resale price of capital p^- is strictly less than its purchase price p^+, with the difference reflecting the extent to which capital assets are firm specific. This asset-specificity assumption implies that capital stock can be liquidated, but only at a discount, which is given by the ratio p^-/p^+. In this environment, the cost of investment can be written as

$$p(k', k)[k' - (1-\delta)k] = [p^+ \cdot 1_{\{k' \geq (1-\delta)k\}} + p^- \cdot 1_{\{k' < (1-\delta)k\}}][k' - (1-\delta)k], \tag{12}$$

where $0 < \delta < 1$ denotes the rate of capital depreciation, and $1_{\{\cdot\}}$ is an indicator function that equals 1 if the expression $\{\cdot\}$ is true and zero otherwise. The asset-specificity assumption also provides a natural way to introduce a financial—that is, a liquidity—shock into the model. In particular , we allow the resale value of the capital p^- to vary over time, according to a continuous AR(1) process:

$$\log p^{-\prime} = (1-\rho_p)\log \bar{p}^- + \rho_p \log p^- + \epsilon_p'; \quad |\rho_p| < 1 \text{ and } \log \epsilon_p' \sim N(-0.5\sigma_p^2, \sigma_p^2). \tag{13}$$

With regards to timing, we assume that all economic agents in the model observe the realization of the idiosyncratic and aggregate productivity shocks, the level of uncertainty, and the resale value of capital at the beginning of each period. The timing convention adopted is such that the level of economic uncertainty in the current period σ_z determines the distribution of ϵ_z' in the subsequent period. From a perspective of agents in the economy, an increase in uncertainty today represents "news" about the distribution of profits tomorrow. To streamline notation, we let the vector $\mathbf{s} = [a, \sigma_z, p^-, \mu]$ denote the aggregate state of the economy, where μ represents the joint distribution of idiosyncratic technology, capital, and debt across heterogeneous firms.

3.2 The Firm's Problem

To finance investment projects, firms use a combination of internal and external funds, where the sources of external funds are debt and equity. Relative to internal funds, external funds command a premium, either because of the direct cost of issuing equity, or in the case of debt, because of the agency costs associated with default.

Debt finance in the model consists of a sequence of one-period, zero-coupon bonds. The debt contract specifies the par value of the issue b' and the price q, yielding the total amount of debt financing equal to qb' in each period. By combining the proceeds from debt issuance with other sources of funds, the firm purchases capital to be used in production. In the subsequent period—after observing the realization of shocks—the firm decides whether or not to fulfill its debt obligation. If the firm decides not to default, it pays the face value of the debt b' to the lender and makes its production and financial decisions for the next period. If the firm chooses to default, it enters a debt-renegotiation process with the investor. The renegotiation process is conducted under limited liability by assuming that there exists a lower bound to the net worth (n) of the firm—denoted by $\bar{n}$—below which the firm cannot promise to pay back any outstanding liability.[14]

The realized net worth next period is defined as the sum of net profits and the market value of undepreciated capital less the face value of debt:

$$n' = a'z'\psi(w(\mathbf{s}'))(k')^{\gamma} - Fk' + p^{-\prime}(1-\delta)k' - b'. \tag{14}$$

Note that the value of capital in (14) follows a stochastic process and entails a discount of $1-p^{-\prime}/p^{+}$. Combining the above definition of net worth with the default condition $n' \leq \bar{n}$, we can define a default trigger level of idiosyncratic technology, conditional on the next period's aggregate state $\mathbf{s}'$ and individual state (k', b'), according to

$$z^{D}(k', b'; \mathbf{s}') \equiv \frac{\bar{n} + b' + Fk' - p^{-\prime}(1-\delta)k'}{a'\psi(w(\mathbf{s}'))(k')^{\gamma}}. \tag{15}$$

For computational reasons, however, it is more useful to define the default trigger in terms of the innovation ϵ_z. Using equation (7), we can redefine the default trigger as

$$\epsilon_z^{D}(k', b', z; \mathbf{s}') \equiv \exp\left[\log z^{D}(k', b'; \mathbf{s}') - \rho_z \log z\right],$$

such that the firm defaults if and only if $\epsilon_z' < \epsilon_z^{D}(k', b', z; \mathbf{s}')$.

[14] This type of bond contract is similar to that of Merton (1974), Cooley and Quadrini (2001), and Hennessy and Whited (2007). However, in our setup, a default occurs when the net worth of the firm n hits the lower bound $\bar{n}$, whereas in the aforementioned papers, a default occurs when the value of the equity v hits the lower bound $\bar{v}$. If the technology shock follows an i.i.d. process and the analysis is conducted in a partial equilibrium, the two assumptions are equivalent. However, if the technology shock is persistent or the firm's value function has other arguments, such as aggregate state variables, the two assumptions are no longer equivalent. The decision to use a lower bound for the net worth to determine the default threshold is a simplifying assumption that allows us to avoid the computationally intensive task of inverting the value function to compute the default boundary $n(z)$ in each iteration of the dynamic programming routine.

Under limited liability, the new level of debt renegotiated by the firm and the investor—denoted by b^R—cannot exceed the amount of debt $\bar{b}(k', z'; \mathbf{s}')$ that is consistent with the lower bound of the net worth. Formally,

$$b^R \leq \bar{b}(k', z'; \mathbf{s}') \equiv a' z' \psi(w(\mathbf{s}'))(k')^\gamma - Fk' + p^{-\prime}(1-\delta)k'.$$

We assume that the firm does not have any bargaining power during the renegotiation process. Consequently, the renegotiated level of debt is set equal to the upper bound of the amount of debt that can be recovered in the event of bankruptcy—that is, $b^R = \bar{b}(k', z'; \mathbf{s}')$.

The default entails a dead-weight loss, captured by bankruptcy costs that are assumed to be proportional to the face value of the debt outstanding. Thus, the actual recovery in the case of default is given by $b^R - \xi(1-\delta)k'$, where the parameter $0 < \xi < 1$ governs the magnitude of the bankruptcy costs and hence the degree of frictions in debt markets. Under these assumptions, the recovery rate—denoted by $\mathcal{R}$—in the case of default is given by

$$\mathcal{R}(k', b', z'; \mathbf{s}') = \frac{\bar{b}(k', z'; \mathbf{s}')}{b'} - \xi(1-\delta)\frac{k'}{b'}.$$

Standard no-arbitrage arguments then imply the following bond-pricing formula:

$$q(k', b', z; \mathbf{s}) = \mathbb{E}\left[m(\mathbf{s}, \mathbf{s}') \left(1 + \int_{\epsilon_z' \leq \epsilon_z^D} \left[\mathcal{R}(k', b', z'; \mathbf{s}') - 1 \right] dH(\epsilon_z' | \sigma_z) \right) \Big| z, \mathbf{s} \right],$$

where $m(\mathbf{s}, \mathbf{s}') = \beta u_c(\mathbf{s}')/u_c(\mathbf{s})$ is the pricing kernel of the representative household ($0 < \beta < 1$ is the time discount factor), and $H(\cdot)$ denotes the CDF of the log-normal distribution. Our distributional assumptions allow us to derive an analytical expression for the bond price. First, we define a monotonic transformation of the default trigger—denoted by ω^D—such that $\omega^D(k', b', z; \mathbf{s}') = [\log \epsilon_z^D(k', b', z; \mathbf{s}') + 0.5\sigma_z^2]/\sigma_z$. Using the properties of the log-normal distribution, we can then express the bond price as

$$\begin{aligned} q(k', b', z; \mathbf{s}) \quad = \quad & \mathbb{E}\Bigg[m(\mathbf{s}, \mathbf{s}') \Bigg(1 - \Phi(\omega^D(k', b', z; \mathbf{s}')) \\ & + \Phi(\omega^D(k', b', z; \mathbf{s}') - \sigma_z) \left(\frac{a' z^{\rho_z} \psi(w(\mathbf{s}'))(k')^\gamma}{b'} \right) \\ & + \Phi(\omega^D(k', b', z; \mathbf{s}')) \left(\left[(p^{-\prime} - \xi)(1-\delta) - F \right] \frac{k'}{b'} - \frac{\bar{n}}{b'} \right) \Big| z, \mathbf{s} \Bigg) \Bigg], \end{aligned} \tag{16}$$

where $\Phi(\cdot)$ denotes the standard normal CDF.[15]

Because we have assumed that the exit shock occurs after the firm makes the decision of whether or not to default, the exogenous exit rate $(1-\eta)$ does not appear in the bond pricing formula—that is, the exit shock does not directly affect the returns of bond investors. In other words,

[15] The expression $\Phi(\omega^D(k', b', z; \mathbf{s}') - \sigma_z)$ is equivalent to $\int_{\epsilon_z' \leq \epsilon_z^D} \epsilon_z' dH(\epsilon_z' | \sigma_z)$; see Kotz et al. (2000) for details.

credit spreads that arise in our framework are not the direct result of the exogenous exit rate. The probability of exit is reflected in credit spreads only insofar as it affects the firm's choice of leverage. Because firms face a constant "death" probability, their effective discount factor in the steady state is equal to $\eta\beta$. If a firm has no debt on its balance sheet, its marginal borrowing rate will be close to $1/\beta$, as the default probability is essentially zero. This marginal borrowing rate is lower than the firm's internal discounting rate of $1/(\eta\beta)$, which represents the cost of equity finance in the case of zero equity issuance costs. As a result, firms are induced to hold a positive amount of debt in equilibrium.

We now turn to the equity financing problem. Dividends are defined as

$$d \equiv az\psi(w(\mathbf{s}))k^{\gamma} - Fk - p(k',k)[k' - (1-\delta)k] - b + q(k',b',z;\mathbf{s})b' + e, \tag{17}$$

where e represents new equity issuance when positive and share repurchase when negative. We posit that firms face a minimum dividend constraint of the form $d \geq \bar{d} \geq 0$. When the combination of the firm's internal funds and funds raised in debt markets falls short of its financing needs, the firm has to raise outside equity ($e > 0$) to satisfy the dividend constraint.[16]

In the absence of frictions in equity markets, the notional amount of equity issuance $e > 0$ reduces the value of existing shares by the same amount. However, we assume the existence of capital market imperfections, which make equity issuance costly in the sense that the value of existing shares is reduced by more than the amount of equity issuance. Following Bolton and Freixas (2000), Gomes (2001), Cooley and Quadrini (2001), and Hennessy and Whited (2007), the additional loss is proportional to the amount of equity issued—that is, $\varphi \max\{0,e\}$—where the parameter $\varphi > 0$ governs the degree of frictions in equity markets. Share repurchase, in contrast, are assumed to be frictioneless.

Given the financial environment, the firm's problem yields the following Bellman equation:

$$\begin{aligned} v(z,k,b;\mathbf{s}) \quad = \quad & \min_{\phi} \max_{\{d,e,k',b'\}} \Big\{ d + \phi(d - \underline{d}) - \bar{\varphi}(e) \\ & + \eta\mathbb{E}\Big[m(\mathbf{s},\mathbf{s}')\max\big\{v(z',k',b';\mathbf{s}'), v(z',k',b^R;\mathbf{s}')\big\}\big|z,\mathbf{s}\Big]\Big\}; \\ & \text{subject to } (16), (17), \text{ and } \mathbf{s}' = \Gamma(\mathbf{s}), \end{aligned} \tag{18}$$

where $\phi \geq 0$ is the Lagrange multiplier associated with the dividend constraint $d - \underline{d} \geq 0$; $\bar{\varphi}(e) \equiv \varphi \max\{0,e\}$; and $\mathbf{s}' = \Gamma(\mathbf{s})$ is the law of motion of the aggregate state vector $\mathbf{s}$, which will be described below. Note that the continuation value of the firm is bounded below by the default/renegotiation value $v(z',k',b^R;\mathbf{s}')$.

[16]Fama and French (2005) document that between 1973 and 2002, on average, almost 60 percent of each year's dividend-paying firms were also net issuers of equity. While this empirical evidence seems at odds with the assumption of frictionless financial markets, another possibility is that a significant fraction of firms faces some kind of implicit strictly positive lower bound on dividend payouts, an interpretation that is consistent with our assumption.

Differentiating (18) with respect to e, yields the first-order condition for equity issuance:

$$1+\phi=\bar{\varphi}(e)=1+\varphi\cdot 1_{\{e>0\}}(e). \tag{19}$$

Equation (19) has a straightforward interpretation: The firm will issue new shares if and only if the dividend constraint binds. If the lower bound on dividends $\underline{d}$ is strictly positive, it is possible that a firm will issue new shares while paying out dividends.

Similarly, differentiating (18) with respect to b' yields

$$(1+\phi)\Big[q(k',b',z;\mathbf{s})+q_b(k',b',z;\mathbf{s})b'\Big]=\eta\mathbb{E}\left[m(\mathbf{s},\mathbf{s}')\int_{\epsilon_z^D}^{\infty}v_b(z',k',b';\mathbf{s}')dH(\epsilon_z'|\sigma_z)\;\Big|z,\mathbf{s}\right].$$

The associated envelope condition is given by $v_b(z,k,b;\mathbf{s})=-(1+\phi)$, which implies that the firm is willing to take on an additional dollar of debt only if the resulting extra cashflow is greater than or equal to the shadow value of internal funds. Using the envelope condition, the first-order condition for debt issuance can be expressed as

$$\begin{aligned} q(k',b',z;\mathbf{s})+q_b(k',b',z;\mathbf{s})b' &= \eta\mathbb{E}\left[m(\mathbf{s},\mathbf{s}')\int_{\epsilon_z^D}^{\infty}\left[\frac{1+\phi'}{1+\phi}\right]dH(\epsilon_z'|\sigma_z)\;\Big|z,\mathbf{s}\right] \\ &= \eta\mathbb{E}\left[m(\mathbf{s},\mathbf{s}')\int_{\epsilon_z^D}^{\infty}\left[\frac{1+\varphi\cdot 1_{\{e'>0\}}(e')}{1+\varphi\cdot 1_{\{e>0\}}(e)}\right]dH(\epsilon_z'|\sigma_z)\;\Big|z,\mathbf{s}\right], \end{aligned} \tag{20}$$

where the second equality follows from (19). Intuitively, the expression on the left side of (20) represents the marginal cost of debt finance, while the expression on the right equals the expected present value of an additional dollar of debt on the firm's balance sheet. By borrowing, the firm improves its cashflow today but, at the same time, increases the chance of a liquidity shortfall in the future, thereby rasing the likelihood that it will have to issue costly new shares. Under limited liability, the firm cares about future cashflows only to the extent that it avoids default, an aspect of the firm's financial policy captured by the truncated integral in equation (20).

The implication of the costly reversible investment framework is that the price of capital $p(k',k)$ has a kink at $k'=(1-\delta)k$. As a result, the derivation of the first-order conditions characterizing the firm's optimal investment policy is somewhat more complicated. To deal with this issue, we reformulate the Bellman equation (18) as a discrete choice problem. Specifically, let

$$v(z,k,b;\mathbf{s})=\max\{v^{+}(z,k,b;\mathbf{s}),v^{-}(z,k,b;\mathbf{s})\}, \tag{21}$$

where v^{+} denotes the value of the firm that is expanding its capital stock, and v^{-} is the value of the firm that is contracting its productive capacity. In other words, v^{+} is the value of the firm in an *expansionary* capital regime, whereas v^{-} is the value of the firm committed to a *contractionary* capital regime.

To formalize these two notions, we introduce into the firm's problem an expansion constraint—

that is, $k' - (1-\delta)k \geq 0$—and a corresponding contraction constraint, namely, $k' - (1-\delta)k \leq 0$. Letting $\lambda^+ \geq 0$ and $\lambda^- \geq 0$ denote the Lagrange multipliers associated with the expansion and contraction constrains, respectively, allows us to express the auxiliary value functions corresponding to the expansionary and contractionary regimes as

$$\begin{aligned} v^+(z,k,b;\mathbf{s}) = \min_{\{\phi,\lambda^+\}} \max_{\{d^+,e^+,k^+,b^+\}} \Big\{ & d^+ + \phi(d^+ - \underline{d}) - \bar{\varphi}(e^+) + \lambda^+[k^+ - (1-\delta)k] \\ & + \eta\mathbb{E}\Big[m(\mathbf{s},\mathbf{s}') \max\big\{v(z',k^+,b^+;\mathbf{s}'), v(z',k^+,b^{R+};\mathbf{s}')\big\}\big|z,\mathbf{s}\Big]\Big\}; \end{aligned} \tag{22}$$

and

$$\begin{aligned} v^-(z,k,b;\mathbf{s}) = \min_{\{\phi,\lambda^-\}} \max_{\{d^-,e^-,k^-,b^-\}} \Big\{ & d^- + \phi(d^- - \underline{d}) - \bar{\varphi}(e^-) + \lambda^-[(1-\delta)k - k^-] \\ & + \eta\mathbb{E}\Big[m(\mathbf{s},\mathbf{s}') \max\big\{v(z',k^-,b^-;\mathbf{s}'), v(z',k^-,b^{R-};\mathbf{s}')\big\}\big|z,\mathbf{s}\Big]\Big\}, \end{aligned} \tag{23}$$

where

$$d^i = az\psi(w(\mathbf{s}))k^\gamma - Fk - p^i[k^i - (1-\delta)k] - b + q(k^i,b',z;\mathbf{s})b' + e, \quad \text{for } i = +,-.$$

Equation (22) expresses the value of the firm that chooses to commit itself to a nonnegative sequence of capital expenditures—regardless of the optimality of doing so—whereas equation (23) considers the opposite case, namely a firm whose capital spending plan includes only disinvestment (or no new capital expenditures). The optimality of the firm's actual investment policy in ensured by equation (21). The first-order conditions with respect to k^+ and k^- are given by the following two Euler equations (see Appendix B for details):

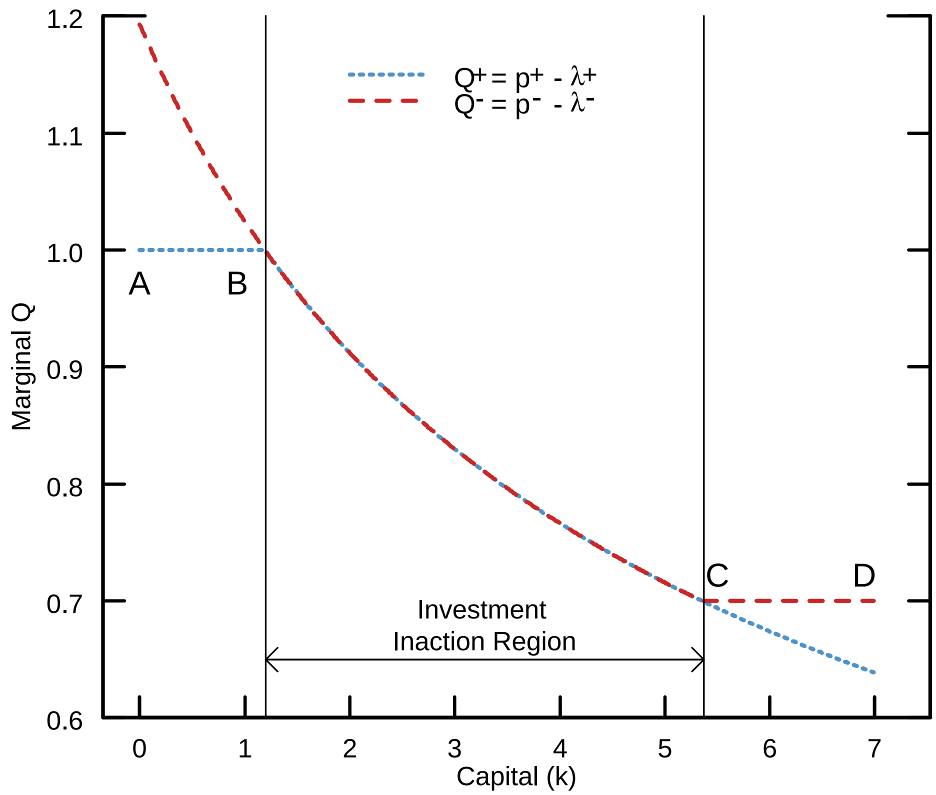
$$\begin{aligned} p^+ - \frac{\lambda^+}{1+\phi} = {} & \eta\mathbb{E}\left[\tilde{m}(\mathbf{s},\mathbf{s}')\left[\pi'_{k^+} + (1-\delta)\min\left\{p^+, \max\left\{p^{-\prime}, \left(p^+ - \frac{\lambda^{+\prime}}{1+\phi'}\right)\right\}\right\}\right] \Big| z,\mathbf{s}\right] \\ & + q_k(k^+,b^+,z;\mathbf{s})b^+ - \eta\mathbb{E}\left[\tilde{m}(\mathbf{s},\mathbf{s}')\int_0^{\epsilon_z^D}\left[\pi'_{k^+} + (1-\delta)p^{-\prime}\right]dH(\epsilon'_z|\sigma)\Big| z,\mathbf{s}\right]; \end{aligned} \tag{24}$$

and

$$\begin{aligned} p^- + \frac{\lambda^-}{1+\phi} = {} & \eta\mathbb{E}\left[\tilde{m}(\mathbf{s},\mathbf{s}')\left[\pi'_{k^-} + (1-\delta)\min\left\{p^+, \max\left\{p^{-\prime}, \left(p^- + \frac{\lambda^{-\prime}}{1+\phi'}\right)\right\}\right\}\right] \Big| z,\mathbf{s}\right] \\ & + q_k(k^-,b^-,z,\mathbf{s})b^- - \eta\mathbb{E}\left[\tilde{m}(\mathbf{s},\mathbf{s}')\int_0^{\epsilon_z^D}\left[\pi'_{k^-} + (1-\delta)p^{-\prime}\right]dH(\epsilon'_z|\sigma)\Big| z,\mathbf{s}\right], \end{aligned} \tag{25}$$

where the firm's stochastic discount factor $\tilde{m}(\mathbf{s},\mathbf{s}') = m(\mathbf{s},\mathbf{s}')(1+\phi')/(1+\phi)$ reflects the presence of financial market frictions.

Figure 4: Marginal Q and Irreversible Investment
(*Perfect Financial Markets*)



NOTE: The dotted line labeled $Q^+ = p^+ - \lambda^+$ depicts the relationship between marginal Q and capital in the expansionary regime, while the dashed line labeled $Q^- = p^- + \lambda^-$ depicts the same relationship in the contraction regime. The purchase price of capital $p^+ = 1.0$, while its resale price $p^- = 0.7$.

3.3 Some Comparative Statics

To gain some intuition into this capital accumulation problem, consider the case of frictionless financial markets, where $\varphi = \phi = \epsilon_z^D = q_k = 0$. In that case, the Euler equation (24) reduces to

$$p^+ - \lambda^+ = \eta \mathbb{E}\Big[m(\mathbf{s}, \mathbf{s}') \left[\pi'_{k^+} + (1-\delta) \min\left\{p^+, \max\left\{p^{-\prime}, (p^+ - \lambda^{+\prime})\right\}\right\}\right] \big| z, \mathbf{s}\Big]. \tag{26}$$

The term on the left side (26) can be interpreted as the marginal Q in the expansionary regime and is shown in Figure 4 by the line labeled $Q^+ = p^+ - \lambda^+$. When the firm's capital stock is relatively small, the optimal investment policy warrants a positive amount of capital expenditures, because the expansion constraint $k' - (1-\delta)k \geq 0$ does not bind (i.e., $\lambda^+ = 0$), and the value of the marginal unit of capital is greater than its purchase price p^+. However, as the firm continues to expand its productive capacity, the expansion constraint eventually binds (i.e., $\lambda^+ > 0$), and the value of the additional unit of capital eventually falls below the purchase price.

Alternatively, consider the case of the firm with a capital overhang, which is shown by the line labeled $Q^- = p^- + \lambda^-$. The relevant investment-optimality condition in the contractionary regime is given by

$$p^- + \lambda^- = \eta \mathbb{E}\Big[m(\mathbf{s}, \mathbf{s}') \left[\pi'_{k^-} + (1-\delta) \min\left\{p^+, \max\left\{p^{-\prime}, (p^- + \lambda^{-\prime})\right\}\right\}\right] \big| z, \mathbf{s}\Big]. \tag{27}$$

When the firm experiences an overcapacity problem, the optimal policy calls for disinvestment: The contraction constraint $k' - (1-\delta)k \leq 0$ does not bind (i.e., $\lambda^- = 0$), and, at the margin, the value of excess capital is below the resale price p^-, which in this example is held constant at 70 percent of the purchase price. At some point, as the firm's capital stock falls sufficiently, the contraction constraint becomes binding (i.e., $\lambda^- > 0$), and the value of the marginal unit of installed capital exceeds the resale price.

It is important to emphasize that Q^+ and Q^-, the marginal Qs implied by the two auxilliary value functions in their respective scenarios, do not necessarily represent the true value of the marginal unit of capital to the firm. The absence of arbitrage opportunities implies that the true marginal Q cannot fall below p^- or rise above p^+. The true marginal Q, therefore, is truncated by the min-max operator and is given by

$$Q = \min\left\{p^+, \max\left\{p^-, (p^+ - \lambda^+)\right\}\right\} = \min\left\{p^+, \max\left\{p^-, (p^- + \lambda^-)\right\}\right\}, \tag{28}$$

where the second equality follows from the fact that the two auxiliary value functions—and their respective derivatives—are equal when the firm's capital stock lies in the investment inaction region. The relationship between the marginal Q and investment in the presence of partial irreversibility is given by the inverted S-curve, connecting points A, B, C, and D in Figure 4.[17]

By comparing (24) with (26) and (25) with (27), we can see how financial market frictions modify the investment Euler equations in the presence of costly reversibility. First, the firm's stochastic discount factor $\tilde{m}(\mathbf{s}; \mathbf{s}') = m(\mathbf{s}, \mathbf{s}')(1+\phi')/(1+\phi)$ reflects the tradeoff between debt and equity financing. Second, the optimal investment policy internalizes the marginal effect of (dis)investment on the cost of debt finance, an effect captured by the term $q_{k^i}(k^i, b^i, z; \mathbf{s})b^i$, for $i = +, -$. Using (16), it is straightforward to show that this term is always nonnegative, because an increase in the firm's capital stock raises the recovery value of the bond in the case default, which boosts the bond's current price. In contrast, the firm's investment policy does not internalize the cost of default, because limited liability protects equity holders from this form of risk, an effect captured by the last term in equations (24) and (25).

An increase in uncertainty causes these two effects to move in opposite directions: In response to heightened uncertainty, bond investors demand greater protection against increased downside risk, while the convexity of the payoff structure for stock market investors—reflecting the limited liability assumption—boosts the expected return on equity. In our baseline calibration, the first

[17] The fact that $\partial Q / \partial k = 0$ in the two action regions is known as the *super-contact condition* in the continuous time irreversible investment literature; see, for example, Abel and Eberly (1996). Note that by using equation (28), equations (26) and (27) can be simplified as $Q^+ = \eta \mathbb{E}\left[m(\mathbf{s}; \mathbf{s}')\left[\pi'_{k^+} + (1-\delta)Q'\right] | z; \mathbf{s}\right]$ and $Q^- = \eta \mathbb{E}\left[m(\mathbf{s}; \mathbf{s}')\left[\pi'_{k^-} + (1-\delta)Q'\right] | z, \mathbf{s}\right]$.

effect dominates, on balance, which implies that financial market frictions amplify the negative impact of uncertainty shocks on capital spending in the standard irreversible investment framework.

A distinct feature of the irreversibility models with perfect financial markets is the fact that the firm's targets for capital stock—in the case when the expansion and contraction constraints are not binding—are independent of the firm's current productive capacity. In this so-called (S, s)-type adjustment rule, the target capital stocks depend only on the fundamentals, such as the firm-specific level of technology and the aggregate state.[18] In the presence of financial market frictions, however, this property will, in general, not hold, because the firm's current capital stock, along with the firm's debt burden, determines the strength of its balance sheet and hence the cost of external finance. The optimal investment policy reaches the *unconstrained* case—that is, it becomes independent of the firm's financial condition and the current level of capital—only when the firm's balance sheet is sufficiently strong.

Figure 5 illustrates the key interactions between investment irreversibility and financial conditions for two types of firms: (1) a "high" productivity firm $(1.15\bar{z})$; and (2) a "low" productivity firm $(0.85\bar{z})$, where $\bar{z}$ denotes the level of technology in the steady-state. That is, the high productivity firm operates a technology that is 15 percent above the steady state, while the low productivity firm has the level of technology 15 percent below the steady-state level. The two left panels show the relationship between the current level of capital and the level of capital in the next period, while the two right panels show the corresponding investment trajectories as a function of the current capital stock. Each panel shows three investment polices, corresponding to three different financial states: (1) "low" debt $(-0.5\bar{b})$; (2) "medium" debt $(\bar{b})$; and (3) "high" debt $(2\bar{b})$, where $\bar{b}$ equals the steady-state level of debt.[19] As in Figure 4, the resale price of capital is held constant at 70 percent of the purchase price.

For the high productivity firm, the contraction constraint is always binding, so selling capital is never optimal (top left panel). The expansion constraint, in contrast, does not always bind. In particular, when the firm's current productive capacity is relatively small, the firm optimally chooses—depending on the level of debt on its balance sheet—the capital expansion targets labeled by "*." Note that the capital expansion targets, in addition to varying with the firm's financial condition, also depend on the current stock of capital, a feature reflecting the presence of financial market frictions.[20]

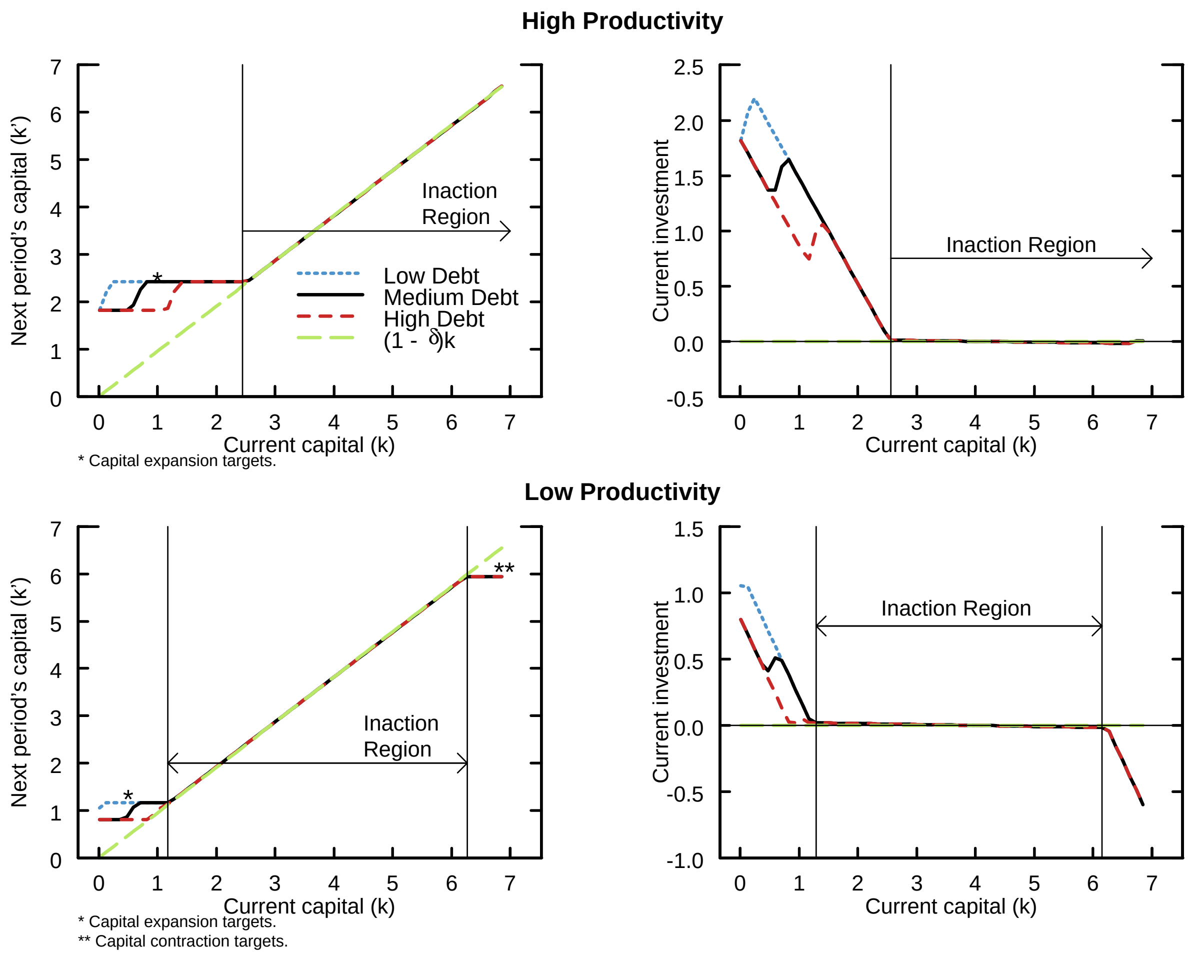
To illustrate these dynamics more fully, consider a high productivity firm with $b = \bar{b}$, the medium debt-burden case, shown by the solid line. At productive capacity below 0.5, the marginal profitability of an additional unit of capital is sufficiently high that the firm is willing to tap equity markets despite the relatively high cost of issuing new shares. When the marginal source of external finance is equity, the linear cost of equity issuence in our model implies a constant marginal cost of capital. As the firm accumulates capital, its debt capacity expands, because of the implicit

[18] This can be seen by setting $\lambda^+ = \lambda^- = 0$ in equations (26) and (27).

[19] In the dynamic programming problem, firms are allowed to accumulate cash—that is, hold negative debt. In equilibrium, however, no firm in our model economy holds positive cash balances.

[20] In the frictionless case, the unique capital expansion target would be represented by a single horizontal line.

Figure 5: Financial Conditions and Irreversible Investment
(*Imperfect Financial Markets*)



NOTE: The high productivity firm has the technology level of $1.15\bar{z}$ and the low productivity firms has the technology level of $0.85\bar{z}$, where $\bar{z}$ denotes the level of technology in the steady-state. The low debt financial state implies a debt burden of $-0.5\bar{b}$, the medium debt state a burden of $\bar{b}$, and the high debt state a burden of $2\bar{b}$, where $\bar{b}$ equals the steady-state level of debt. The solid, dotted, and dashed lines labeled by the * and ** denote the firm's capital expansion and contraction targets, respectively, corresponding to the different financial states. The purchase price of capital $p^{+} = 1.0$, while its resale price $p^{-} = 0.7$.

improvement in the recovery value of its debt. The improvement in the firm's financial condition leads to more attractive borrowing terms, and the firm enters the bond market to finance its capital expansion plans, as evidenced by the increase in investment shown by the solid line in the panel to the right.

Once the firm has accessed the bond market, its optimal investment policy will—for a limited period of time—depend positively on the current level of capital, even though the expansion constraint is not binding. This positive feedback effect occurs because increases in capital financed by bond issuance lower, due to the presence of financial market frictions, the marginal cost of capital, leading to more capital accumulation. Eventually, as productive capacity continues to expand, the firm's investment policy reaches the unconstrained case. In that case, the firm's optimal investment becomes a decreasing function of the current stock of capital—and the level of debt outstanding—a characteristic reflecting the debt overhang problem of Myers (1977).

The bottom two panels examine the interaction between capital and debt capacity in the case of a low productivity firm. Such a firm faces a capital expansion problem that is similar to that of a high productivity firm; in addition, the low productivity firm can also face a capital contraction problem. Note that these two decision problems are separated by the inaction region, in which neither investment nor disinvestment is optimal. In this example, if the firm suffers from the capital capacity overhang, its capital contraction targets are not associated with the debt overhang problem, because the firm's leverage is sufficiently low so that the likelihood of default is essentially zero.

3.4 Aggregation

To fully solve their respective optimization problems, economic agents in our model need to understand how the aggregate state variables evolve over time. One of the aggregate state variables is the joint distribution of capital, debt, and the level of idiosyncratic technology across heterogeneous firms, denoted by $\mu(K, B, Z))$. The exact law of motion of this joint distribution is given by

$$\mu(K_0, B_0, Z_0) = \int_{K_0 \times B_0 \times Z_0} \left[\int_{K \times B \times Z} \mathbf{1}\big((k', b') = g(k, \min\{b, b^R\}, z; \mathbf{s})\big) G(z'|z, \sigma_z) d\mu \right] dk' db' dz', \quad (29)$$

where $K \subseteq \mathbb{R}_{++}$, $B \subseteq \mathbb{R}$, $Z \subseteq \mathbb{R}_{++}$, and μ is a Borel measure defined on the measurable space $(K \times B \times Z, \mathcal{K} \times \mathcal{B} \times \mathcal{Z})$, where $\mathcal{K}$, $\mathcal{B}$, and $\mathcal{Z}$ denote the σ-algebras of Borel sets generated by the subsets of K, B, and Z, respectively.

In equation (29), $G(\cdot|z, \sigma_z)$ denotes the transition function of the idiosyncratic productivity shock z, while the definition of the vector-valued function

$$g(\cdot, \cdot, z; \mathbf{s}) \equiv \big(k'(k, \min\{b, b^R\}, z; \mathbf{s}), b'(k, \min\{b, b^R\}, z; \mathbf{s})\big),$$

expresses the firm's optimal choice of capital and debt next period, conditional on the today's realization of the idiosyncratic productivity shock z and the aggregate state $\mathbf{s}$. The corresponding vector-valued indicator function $\mathbf{1}(\cdot, \cdot)$ implies that $\mu(K_0, B_0, Z_0)$ measures the proportion of firms in the economy whose capital, debt and technology are in $K_0 \times B_0 \times Z_0$ next period, where $K_0 \in \mathcal{K}$, $B_0 \in \mathcal{B}$, and $Z_0 \in \mathcal{Z}$.[21] Note that this law of motion in describes the evolution of the debt distribution in the economy before the process of debt renegotiation takes place—that is, from b to b'.[22]

[21] See Ríos-Rull (1997) for a general discussion of computing equilibria in models with a large number of heterogeneous agents.

[22] Note also that the exogenous exit shock does not affect the law of motion directly, because we have assumed that the exiting firms are replaced by new entrants that inherit all technological and financial characteristics of the

Following the literature on computable general equilibrium models with heterogeneous agents (cf. Krusell and Smith (1998)), we assume that the agents in the model exhibit bounded rationality. Specifically, the agents concern themselves with only a finite number of moments of the distribution μ and use these moments in log-linear functional forms to forecast equilibrium prices. For computational feasibility, the agents in our model are concerned only with the first moments of the joint distribution of capital, debt, and technology. The agents use these state variables to forecast the two prices needed to solve their optimization problems: (1) the marginal utility of consumption for the representative household $u_c(\mathbf{s})$; and (2) the real wage $w(\mathbf{s})$.

In fact, because we have assumed an infinitely elastic labor supply, the equilibrium real wage can be inferred from the first-order condition characterizing the representative household's labor-supply decision: $w(\mathbf{s}) = \zeta/u_c(\mathbf{s})$. Accordingly, it is sufficient for the agents to forecast aggregate debt ($\bar{b}$), aggregate capital ($\bar{k}$), and the marginal utility of consumption (u_c) and then to infer the real wage and the level of consumption ($\bar{c}$) that clear both the labor and goods markets. Formally, we assume that the agents use the following system of forecasting rules:

$$\begin{bmatrix} \log \bar{b}'(\mathbf{s}) \\ \log \bar{k}'(\mathbf{s}) \\ \log u_c(\mathbf{s}) \end{bmatrix} = \mathbf{\Theta}_0 + \mathbf{\Theta}_1 \begin{bmatrix} \log \bar{b} \\ \log \bar{k} \\ \log \bar{c}_{-1} \end{bmatrix} + \mathbf{\Theta}_2 \begin{bmatrix} \log \sigma_z \\ \log a \\ \log p^- \end{bmatrix}, \tag{30}$$

with

$$\bar{c}(\mathbf{s}) = \frac{1}{u_c(\mathbf{s})} - \tau \bar{c}_{-1} \quad \text{and} \quad w(\mathbf{s}) = \frac{\zeta}{u_c(\mathbf{s})}.$$

In equation (30), $\mathbf{\Theta}_0$ is a (3×1)-dimensional vector and $\mathbf{\Theta}_1$ and $\mathbf{\Theta}_2$ are (3×3)-dimensional matrices of coefficients that need to be determined in equlibrium.

To do so, let $c^F(\mathbf{s})$ and $w^F(\mathbf{s})$ denote the consumption and real wage forecasts implied by the posited forecasting rules. The actual consumption and real wage are denoted by $c^E(\mathbf{s})$ and $w^E(\mathbf{s})$, respectively. In equilibrium, consumption and wages must satisfy the following aggregate clearing condition:

$$0 = \int \Big[y_j(\mathbf{s}; w^E(\mathbf{s}), c^E(\mathbf{s})) - k'_j(\mathbf{s}; w^E(\mathbf{s}), c^E(\mathbf{s})) \Big] dj + (1 - \delta - F)\bar{k} - c^E(\mathbf{s}). \tag{31}$$

In deriving the market clearing conditions, we have assumed that all costs associated with the issuance of new equity and the bankruptcy costs associated with default—the total loss of economic efficiency due financial frictions—are rebated to the household in a lump-sum fashion, so that these terms do not enter equation (31).

The general equilibrium consistency requires that the forecasting rules are accurate in the sense that $c^E(\mathbf{s}) = c^F(\mathbf{s}) + \epsilon_c$ and $w^E(\mathbf{s}) = w^F(\mathbf{s}) + \epsilon_w$, where the "forecast errors" ϵ_c and ϵ_w are arbitrarily small. We achieve this consistency by starting with an arbitrary choice of coefficients $\mathbf{\Theta}_0$, $\mathbf{\Theta}_1$ and $\mathbf{\Theta}_2$ in the forecasting rule (30). We then simulate the model economy with a large

exiting firms. The entry/exit process is thus fully frictionless and plays no role in the model, other than creating a wedge between the internal rate of discounting and the risk-free rate, thereby incentivizing firms to take on debt.

number of heterogenous agents using random draws of aggregate and idiosyncratic shocks. The agents learn from their errors and update the forecasting rules, and the process is iterated until full convergence.[23] The accuracy of this approximate aggregation method used to compute the equilibrium of the model is discussed in Appendix C.

4 Calibration

The time period t in the model corresponds to one quarter. To simulate the model, we must calibrate a number of parameters. For the most part, our calibration relies on parameter values that are standard in the literature. There are, however, a number of parameters that are specific to our model, the calibration of which we discuss below.

To calibrate the curvature of the profit function and the parameters governing the stochastic volatility process of the idiosyncratic productivity shock z, we utilize the information from the S&P's quarterly Compustat database. Specifically, from Compustat we selected all U.S. nonfinancial firms with at least 20 quarters of data on sales and capital over the 1976:Q1–2011:Q3 period, a procedure yielding an unbalanced panel of 9,273 firms for a total of 535,587 observations.[24] To calibrate χ, the DRS parameter in equation (9), we use this panel to estimate the following cointegrating relationship between sales (Y_{it}) and capital (K_{it}):

$$\log Y_{it} = \theta \log K_{i,t-1} + \eta_i + \lambda_{st} + u_{it}, \tag{32}$$

where the disturbance term u_{it} represents the empirical counterpart to the stationary productivity shock $\log z_{it}$ in our model.[25] The regression also includes a firm fixed effect η_i that captures any unobservable (time-invariant) differences in the long-run relationship between sales and capital across firms, while the industry-specific (2/3-digit NAICS) time fixed effect λ_{st} controls for shocks that are common to all firms in a specific industry. We estimate equation (32) by OLS, which yields an estimate of $\theta = 0.63$, with a robust 95-percent confidence interval of $[0.62, 0.64]$.

We set α, the share of capital in the Cobb-Douglas production function, to one-third, which together with our estimate of θ implies that $\chi = 0.84$, an estimate of decreasing returns that is within the range of values reported in the literature.[26] The quarterly depreciation rate $\delta = 0.045$,

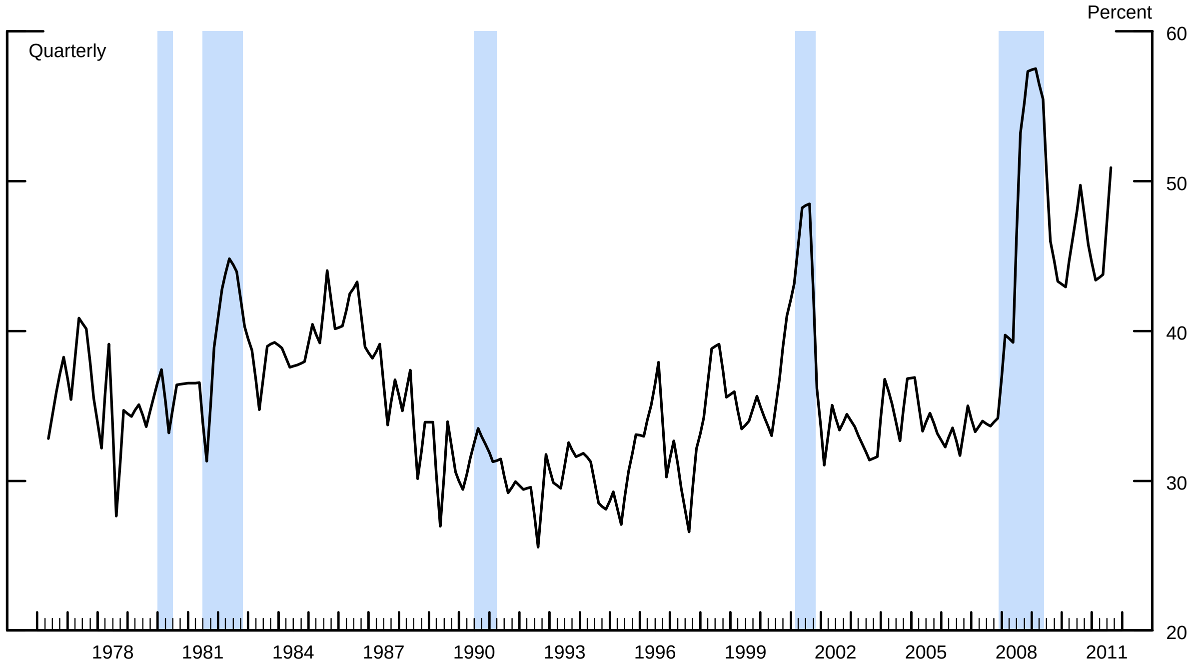
[23]In the simulation, we assume that there are 10,000 heterogenous firms at any point in time, and we simulate the economy for 2,500 periods. In updating the agents' perceived aggregate laws of motion, we discard the first 500 observations and use the remaining 2,000 observations to estimate the coefficients of equation (30). These laws of motion are then used to update the individual agent's policy rules implied by the numerical dynamic programming problem. We use value function iteration with linear interpolation for the points that are off the grid; see Miao (2006) for the exact conditions under which a unique equilibrium exists for this type of economy.

[24]Prior to 1976, most firms in Compustat did not report their capital stock data (i.e., net property, plant, and equipment) on the quarterly basis. To ensure that our results were not driven by a small number of extreme observations, we dropped from the sample all observations with the sales-to-capital ratio below 0.01 and above 20.0 and observations with quarterly growth rates of sales and capital above and below 100 percent.

[25]Under our assumption of a Cobb-Douglas production function, gross profit (i.e., profit before fixed operation costs) differs from sales only up to a constant. Hence, one can estimate the parameter θ using data on either sales or gross profits. We chose sales in order to avoid the occassionally negative gross profit observations.

[26]See, for example, Burnside (1996), Cooper and Ejarque (2003), and Cooper and Haltiwanger (2006).

Figure 6: Uncertainty Based on Productivity Shocks



NOTE: Sample period: 1976:Q2–2011:Q3. The solid line depicts the estimate of idiosyncratic uncertainty (in annualized percent) based on shocks to the long-run cointegrating relationship between sales and capital (see text for details). The shaded vertical bars denote NBER-dated recessions.

a value consistent with the annual depreciation rate for Compustat firms. We set the quasi-fixed cost parameter $F = 0.05$, which implies that fixed costs equal about 10 percent of sales.[27] These parameter choices imply that the dividend-payout ratio (dividends as a proportion of net income) in the model is about 50 percent, roughly the same as the average dividend-payout ratio for the S&P 500 index since 1940.

We use the residuals from the estimation of (32) to calibrate the process for the idiosyncratic productivity shock. First, the persistence of the process is obtained by estimating the following pooled OLS regression:

$$\hat{u}_{it} = \rho_z \hat{u}_{i,t-1} + \epsilon_{it},$$

which yields the estimate of the persistance parameter $\rho_z = 0.87$. Second, if the error term ϵ_{it} is distributed normally, as assumed in the model, then $\sqrt{\pi/2}|\hat{\epsilon}_{it}|$ represents an unbiased estimator of the true standard deviation $\sigma_{\epsilon,it}$.[28]

[27]In our panel of Compustat firms, the median ratio of selling, general, and administrative (SG&A) expenses to sales is about 24 percent. Some part of SG&A expenses could be considered as investment in intangible capital, which is counted as investment by the Bureau of Economic Analysis but as "expenses" by Compustat. We assume that about one-half of this ratio represents the fixed costs of production in the model.

[28]Note that we estimate the volatility of the idiosyncratic productivity shock for each firm i in every quarter t. This approach is similar to that of Kim and Nelson (1999) and McConnell and Perez Quiros (2000), who estimate the volatility of aggregate output growth in each quarter from a single observation.

To obtain a corresponding measure of time-varying uncertainty, we employ an approach similar to that used in Section 2 and estimate a fixed effects regression of the form:

$$\log\left[\sqrt{\frac{\pi}{2}}|\hat{\epsilon}_{it}|\right] = \gamma_i + \delta_i t + \sigma_t + \zeta_{it}; \quad \zeta_{it} \sim N(0, \omega_\zeta^2).$$

In keeping with our earlier approach, a measure of uncertainty based on productivity shocks corresponds to the sequence of estimated time fixed effects $\hat{\sigma}_t$, $t = 1, ..., T$, which captures common movements in the idiosyncratic uncertainty regarding the profitability prospects in the nonfinancial corporate sector.[29] As shown in Figure 6, the estimate of uncertainty based on revenue shocks—like its counterpart based on equity valuations—is clearly countercyclical, increasing significantly before an onset of an economic downturn.

In the model, the volatility of the idiosyncratic productivity shock is assumed to follow a continuous autoregressive process. To calibrate this process, we estimate a discrete analogue of the AR(1) process in equation (11), using $\hat{\sigma}_t$, our estimate of the time-varying idiosyncratic uncertainty based on productivity shocks. This yields the persistence of the process $\rho_\sigma = 0.82$, with a robust 95-percent confidence interval of $[0.71, 0.94]$, a range of values consistent with the previous research; see, for example, Bloom (2009). In our simulations, we let $\rho_\sigma = 0.8$.

We calibrate the steady-state level of uncertainty $\bar{\sigma}_z = 0.15$, a value consistent with that estimated by Hennessy and Whited (2007). The standard deviation of the shocks to the uncertainty process σ_σ is set equal to 0.015. With these parameter values, the uncertainty process in the simulation fluctuates in the $[0.135, 0.165]$ range, and an uncertainty shock of one standard deviation increases the level of uncertainty by about 10 percentage points, a magnitude consistent with the VAR results reported in Section 2.

A far more daunting task is the calibration of the process for the resale value of capital—we are not aware of any data that track the resale value of fixed capital stock over time. As a proxy, we employ the ratio of the CPI for used car sales relative to the CPI index for new car sales. Estimating an AR(1) process for this ratio over the 1998:M2–2011:M11 period yields (at a quarterly frequency) an estimate of persistence $\rho_p = 0.97$ and an estimate of volatility $\sigma_p = 0.015$; we use these two values to calibrate the process for the resale value of capital in equation (13).

The steady-state resale value of capital $\bar{p}^- = 0.5$, which yields an average leverage ratio—that is, $b/(\pi + p^+ k)$—of 0.5, a value that is very close to its empirical counterpart for our matched sample of Compustat firms. These parameter values imply that a one standard deviation shock corresponds to an increase (or decrease) in the resale value of capital of about 3.0 percentage points. The purchase price of capital p^+ is normalized to 1.0. Following Prescott (1986) and a number of subsequent studies, we set the persistence of the aggregate productivity shock $\rho_a = 0.95$ and its volatility $\sigma_a = 0.0075$. The integration of all exogenous AR(1) processes in the model is approximated by a Gaussian quadrature rule.

[29] To ease the interpretation, the sequence $\hat{\sigma}_t$, $t = 1, \ldots, T$, has been re-scaled, seasonally adjusted using the X11 filter, and expressed in annualized percent.

Given our parameterization of the uncertainty process and the process for the resale value of capital, we calibrate the degree of frictions in the bond market—the bankruptcy cost parameter ξ—to match the median credit spread of 170 basis points for the 10-year BBB-Treasury spread over the 1976:Q2–2011:Q3 period. Accordingly, we set $\xi = 0.10$, a value consistent with that used by Bernanke et al. (1999) and the micro-level evidence of Levin et al. (2004), and one that implies a relatively modest degree of additional losses for the bondholders in the case of bankruptcy.

The estimates of the cost of equity issuance vary substantially in the literature, from 0.08 in Gomes (2001) to 0.30 in Cooley and Quadrini (2001). We make a relatively conservative choice by letting $\varphi = 0.12$. With this calibration, we choose the lower bound on dividends $\bar{d}$, such that, on average, 15 percent of firms in the model issue new shares in each period, a proportion that is a bit smaller than the 18 percent seen in the Compustat data. We make a natural choice for the lower bound of networth—that is, $\bar{n} = 0$. And lastly, in calibrating the exogenous exit rate of firms in the model, we set $\eta = 0.946$, a survival probability that is close to that observed in the micro-level data.[30]

With regard to the preferences of the representative household, we set $\tau = 0$ in the baseline specification of the model. We treat ζ, the weight of the disutility of labor hours, as a free parameter that is chosen so that the real wage can be normalized to 1.0 in the steady state. Finally, the time discount factor $\beta = 0.989$, which implies an annualized risk-free rate of 4 percent.

5 Model Simulations

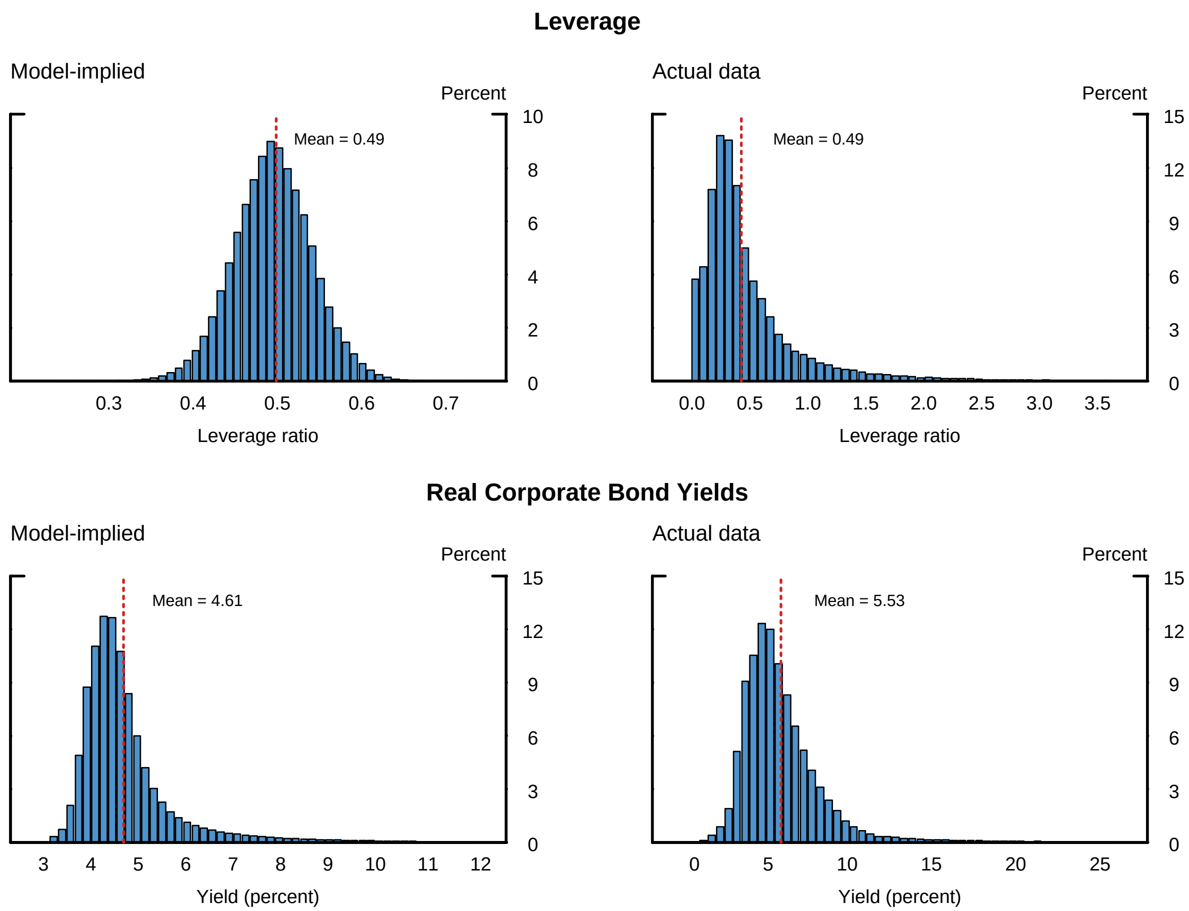
This section presents our main results. In subsection 5.1, we show that the model-implied distributions of leverage and corporate bond yields across heterogeneous firms correspond reasonably well to their empirical counterparts, a result that attests to the soundness of our calibration strategy. The key business cycle properties of the model are examined in subsection 5.2, while in subsection 5.3, we quantify the relative importance of costly irreversibility and financial frictions in the propagation of various economic shocks. We close the results section with the discussion of the magnitude of potential efficiency losses, stemming from the dynamic inefficiency of capital reallocation across heterogeneous productive units in the economy with imperfect capital markets.

5.1 Heterogeneity of Leverage and Corporate Bond Yields

The top left panel of Figure 7 shows the ergodic distribution of the leverage ratio implied by the model, while its empirical counterpart over the 1973:M1–2011:M9 period, based on our matched sample of 1,099 Compustat firms, is shown in the panel to the right. In the model, leverage is defined as the face value of debt (b), relative to the sum of net profits and the book value of capital ($\pi + p^+k$), while its empirical analogue is given by the ratio of the book value of total debt to the

[30] According to the survey of the Business Employment Dynamics conducted by the Bureau of Labor Statistics, the average yearly survival rate of new establishments between 1994 and 2009 was 0.784, which implies a quarterly rate of 0.941.

Figure 7: Distribution of Leverage and Corporate Bond Yields



NOTE: The panels in the left column depict the ergodic distributions of leverage and real corporate bond yields implied by the model. The model-implied distributions are based on the simulated data of 10,000 firms over 2,000 periods. The panels in the right column depict the corresponding distributions of (quarterly) leverage and (monthly) real bond yields for our sample of 1,099 nonfinancial firms that had senior unsecured debt trading in the secondary market ober 1973:M1–2011:M9 period; see text for details.

sum of of gross profits and the book value of gross capital stock.[31]

The ergodic distribution of leverage in the model shows a significant degree of dispersion, an indication that balance sheet conditions vary substantially across firms and time. Moreover, at 0.49, the average leverage ratio in the model exactly matches the average leverage in our sample of bond issuers.[32] However, the model is considerably less successful in replicating the skewness and

[31]We focus on the subset of firms that have senior unsecured debt trading in the secondary market, because a significant fraction of nonfinancial firms covered by Compustat has no debt on their balance sheets. While our theoretical framework allows firms to accumulate negative debts—that is, hold positive cash balances—as part of their optimization problem, in practice, all firms in our model economy choose a positive leverage in equilibrium. Accordingly, comparing the model-implied distribution of leverage with that of our matched sample of bond issuers provides a more meaningful metric by which to judge the empirical relevance of the model.

[32]The model-implied average leverage ratio is also consistent with the time-series average of the aggregate leverage ratio for the U.S. business sector calculated using the Flow of Funds Accounts data published by the Federal Reserve

the heavy right tail of the leverage distribution that is evident in the data.

The bottom two panels of Figure 7 compare the model-implied distribution of corporate bond yields with that seen in the actual data.[33] In this case, the model does a noticeably better job in replicating the general contours of the empirical distribution of real interest rates. Nevertheless, there are two important differences between the two distributions. First, the average firm-specific real interest rate in the model is 4.6 percent, almost a full percentage point less than the average real bond yield for nonfinancial bond issuers over the 1973:M1–2011:M9 period. Relatedly, given our calibration of the risk-free rate at 4 percent, the model generates an average credit spread of only about 60 basis points, a level that is substantially below the average spread of 230 basis points seen in the bond-level data. Second, despite the relatively standard and realistic calibration of the level of idiosyncratic uncertainty, the model generates an insufficient amount of dispersion in real interest rates compared with the actual data.

5.2 Business Cycle Dynamics

We now turn to the business cycle properties of the model. First, we present the impulse response of key macroeconomic aggregates to the three exogenous driving forces in our model economy: aggregate TFP shocks, idiosyncratic uncertainty shocks, and shocks to the resale value of capital (i.e., liquidity shocks). We then consider the model-implied business cycle moments and compare them to the corresponding moments based on the actual data.

Figure 8 displays the impulse response of model variables to an aggregate technology shock. These responses are very similar to those implied by a more standard real business cycle model: A positive innovation in aggregate TFP implies a strong and prolonged expansion in output, along with strong comovement in consumption, hours worked, and labor productivity; and the initial response of investment is about five times as large as that of output, but it dies out rather quickly. These results suggest that the richness of our model structure, which includes the firm-level costly reversible investment and financial market frictions, does not substantially alter dynamics of a technology-driven real business cycle.

As shown by Veracierto (2002) and Sim (2006), the irreversibility constraint alone does not fundamentally modify the dynamics of equilibrium business cycle when the driving force of the business fluctuation is the technology shock.[34] The addition of financial market frictions also does not appear to significantly alter the the dynamic response of the economy. In response to a persistent

Board.

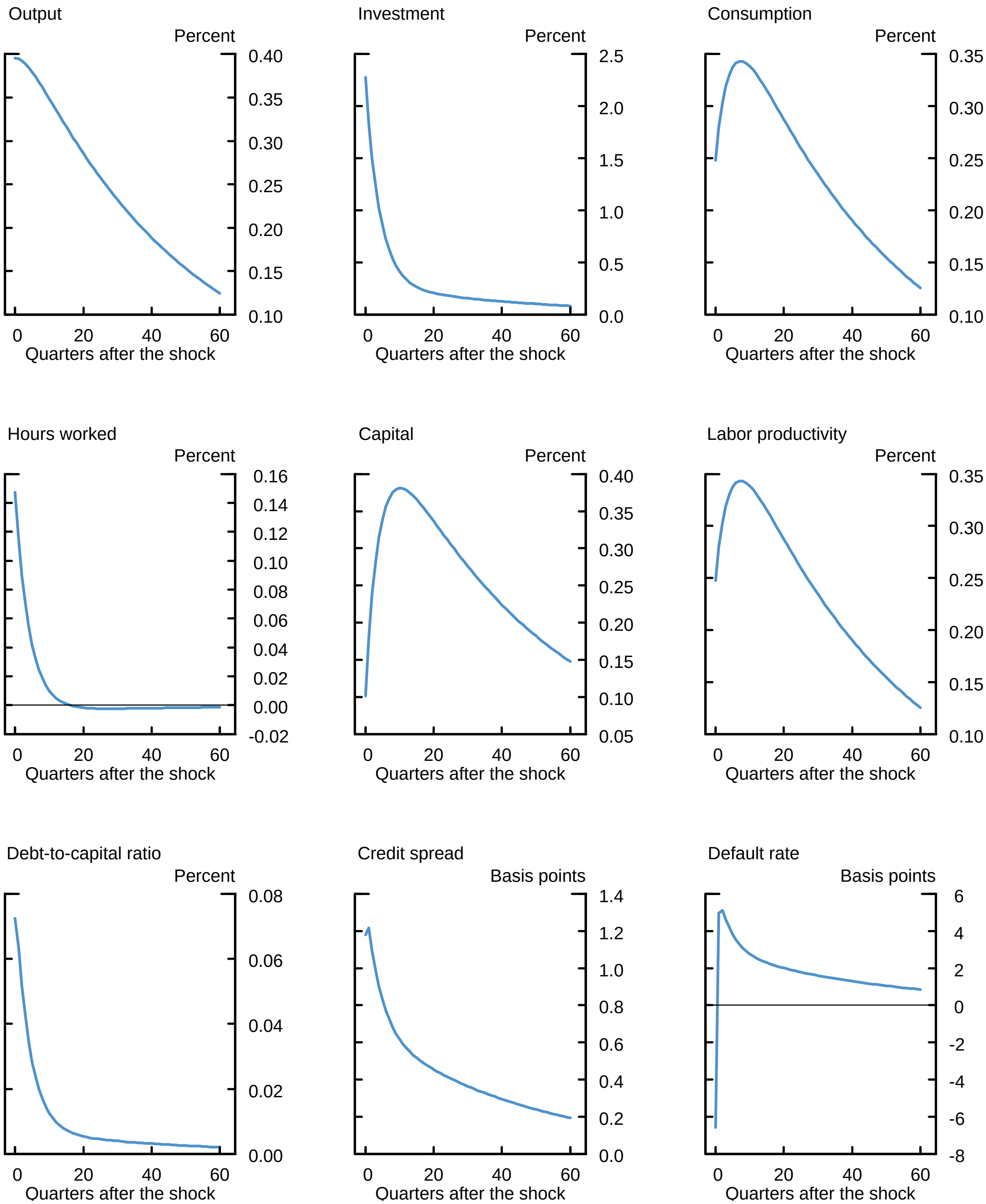
[33]We calculate real yields for our sample of corporate bonds according to

$$r_{it}[k] = r^f_t + s_{it}[k],$$

where $r^f[k]$ denotes the real 10-year Treasury yield in month t and $s_{it}[k]$ is the credit spread on bond k (issued by firm i) in month t. The real 10-year Treasury yield is calculated as the difference between the 10-year nominal Treasury yield and the average expected CPI inflation over the next 10 years, as measured by the Philadelphia Fed's Survey of Professional Forecasters.

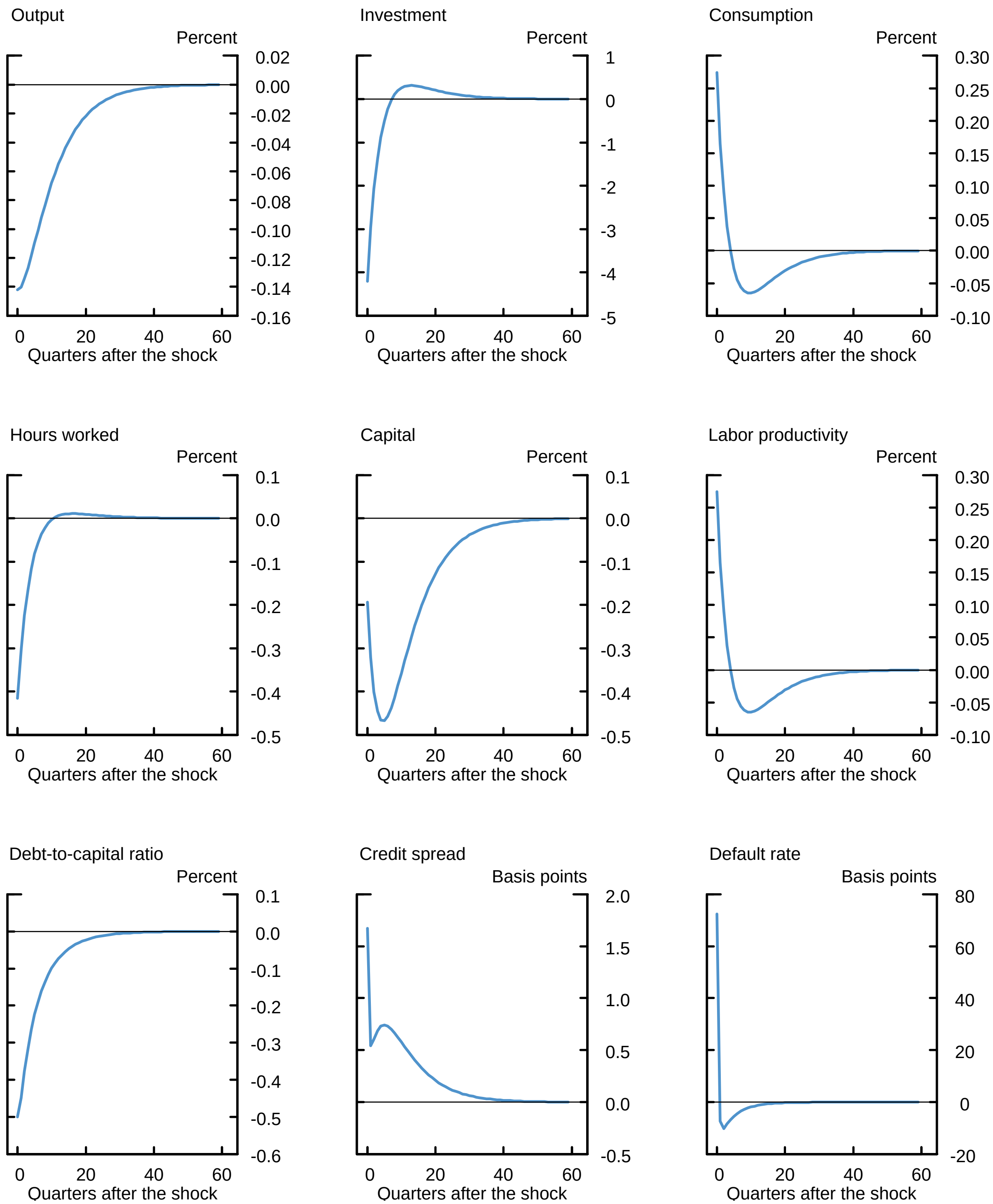
[34]Sim (2006), for example, shows that the excess smoothness—excess relative to a frictionless model—created by the inactive firms with overcapacity is offset by the excess volatility of the target capital stock of the firms with nonbinding constraint.

Figure 8: Model-Based Impulse Responses to an Aggregate TFP Shock



NOTE: The solid line in each panel depicts the model-based impulse response function of a selected variable to a 1 standard deviation positive shock in aggregate TFP (a). All variables are expressed in deviations from their respective steady-state values.

Figure 9: Model-Based Impulse Responses to an Uncertainty Shock



NOTE: The solid line in each panel depicts the model-based impulse response function of a selected variable to a 1 standard deviation positive shock in idiosyncratic uncertainty (σ_z). All variables are expressed in deviations from their respective steady-state values.

technology shock, the debt capacity of the economy expands, implying an improvement in borrowing terms. However, as firms invest more, leverage increases, and borrowing terms tighten as firms move up their loanable funds supply schedule. These two forces roughly offset each other, leaving the response of financial variables such as the leverage ratio, credit spreads, and default probability largely unaffected by the innovation in aggregate TFP.

Figure 9 considers the macroeconomic consequences of an uncertainty shock. As in our VAR analysis, we consider a one standard deviation shock to the volatility of an idiosycratic productivity shock. This rise in uncertainty causes an immediate contraction in investment that is on the order of 4 percent of its steady-state level. The magnitude of the investment response is about twice as large as that obtained when the economy is hit by by a one standard deviation shock to aggregate TFP.

In our environment, the uncertainty shock is a pure second moment shock that is not correlated with the first moment of the shocks to the production technology—formally, the uncertainty shock is a *mean preserving spread*. As shown above, the profit function at the firm level is given by $\pi = az\psi(w)k^{\gamma} - Fk$, which linear in the level of idiosyncratic technology z. In our framework, therefore, there is no direct link between the first and second moments of the shocks affecting the production frontier.[35] In particular, this mean preserving spread should not affect investment policy in the absence of frictions associated with irreversibility and financial market frictions.

Despite the absence of a direct link between uncertainty and aggregate technology, the uncertainty shock nevertheless leads to a reduction in aggregate output. Although investment falls sharply in response to an increase in uncertainty, the effect on output is relatively modest, reflecting a temporary increase in consumption. The positive correlation between uncertainty and consumption during the first year or so after the impact of the shock occurs because heightened uncertainty does not directly affect the resource constraint, but it does raise the cost of investment relative to consumption.[36]

The increase in consumption lowers the marginal utility of consumption, which in turn puts upward pressure on the equilibrium wage to satisfy the labor market clearing condition $w(\mathbf{s})u_c(\mathbf{s}) = \zeta$. As real wages rise, hours worked fall as the economy move up the downward sloping labor demand curve. The large contraction in hours worked relative to output implies an increase in labor productivity. While the exact mechanism is rather stylized, this result is consistent with the surge in labor productivity observed in the past several recessions.

Credit spreads respond positively to the increase in uncertainty. However the magnitude of the

[35] In this sense, our approach differs from others in the literature (e.g., Storesletten et al. (2004) and Bachmann and Bayer (2009)), who assume a negative correlation between the first and second moments of technology shocks.

[36] This result, however, is dependent on the assumption of a closed economy; in an open economy setting, the representative household can borrow and lend to smooth their consumption expenditures, which substantially weakens the tight link between consumption and investment imposed by the resource constraint. In fact, our VAR results indicate that the response of consumption of nondurable goods and services to an increase in uncertainty is statistically undistinguishable from zero, a point also made by Knotek and Khan (2011). However, as shown by Carroll and Dunn (1997), different uncertainty measures—especially those based on the prospect of job security—can be negatively correlated with personal consumption expenditures.

response is surprisingly small—on the order of only a couple of basis points. In comparison, the effect of uncertainty on the realized default rate is large: On impact, the default rate jumps 70 basis points. Because of increased default risk, firms deleverage rapidly, a process that limits the effect of an increase in uncertainty on credit spreads. Hence, a substantial portion of the adjustment in credit markets occurs through quantities rather than prices. Because the stock of debt outstanding drops sharply, the default rate returns to its steady state rather quickly.

Nevertheless, as shown by our empirical analysis, increases in uncertainty are typically associated with much larger movements in credit spreads. This suggests two potential factors that may be missing from the analysis: First, to explain the observed surge in credit spreads during downturns associated with large uncertainty shocks may require a mechanism that significantly damps and prolongs the firms' deleveraging process; and second, our experiment focuses largely on the default risk aspect of uncertainty shocks and does not allow for a significant widening of credit spreads through the price of default risk. In reality, a substantial portion of fluctuations in credit spreads over time appears to be due to the time variation in the price of default risk (i.e., the default risk premiums) rather than the risk of default. Our baseline version of the model with log utility, by construction, does not allow for a meaningful degree of variation in the default risk premiums.

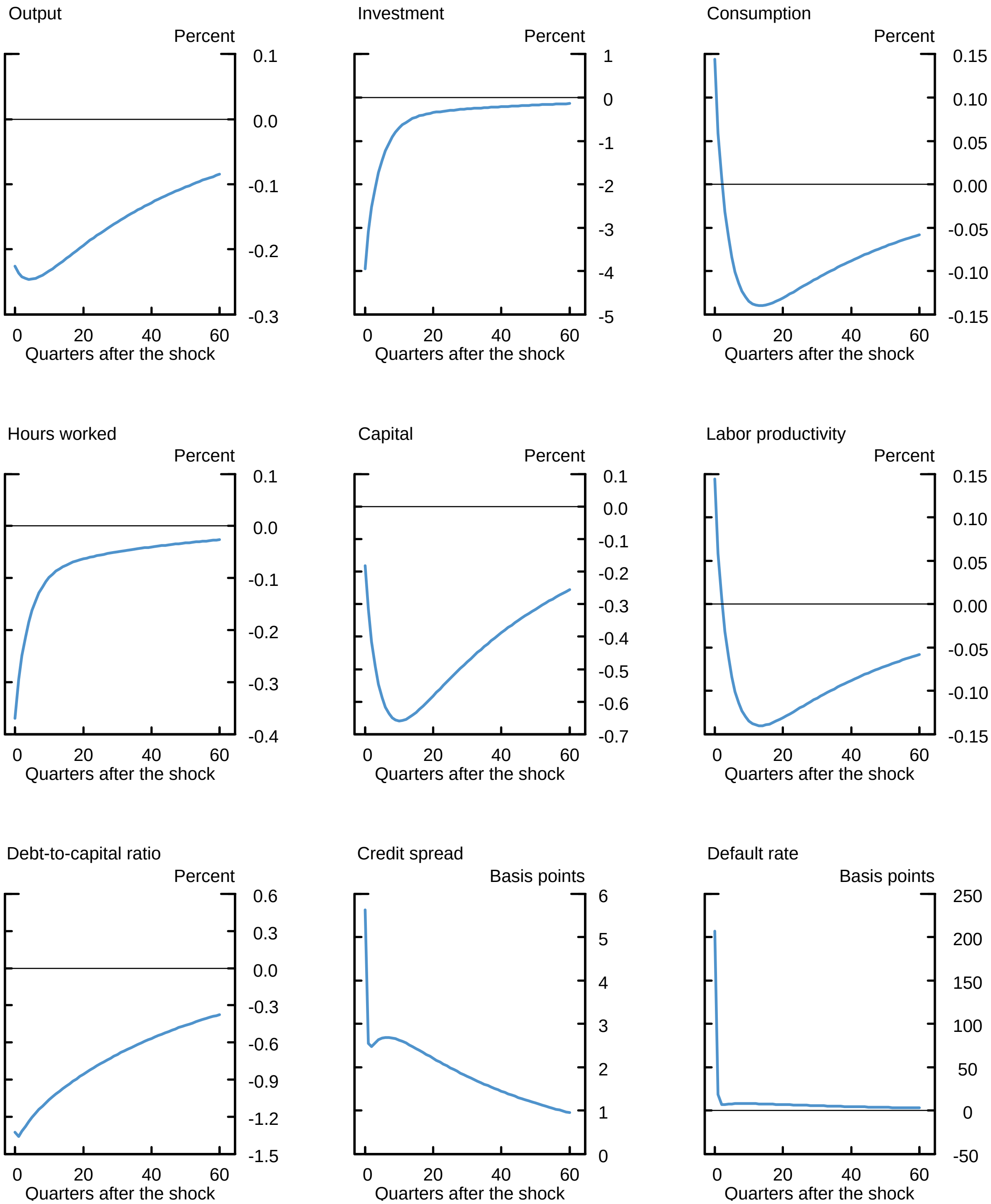
We now turn to the effect of a shock to the liquidity of capital. Figure 10 displays the effects of a negative one standard deviation shock to the resale price of capital. The decline in the liquidity of capital leads to a plunge in aggregate investment, the magnitude of which is comparable to the drop in investment following an uncertainty shock. Again, consumption increases on impact due to the general equilibrium effects discussed above. However, in the case of liquidity shocks, the initial increase in consumption is appreciably smaller than that observed in response to the uncertainty shock, implying a more pronounced decline in aggregate output. Again, the decline in hours worked exceed that of the output, which leads to a temporary increase in labor productivity.

The liquidity shock affects investment in two ways. First, a lower resale value of capital reduces the firms' capital expansion targets. Because the liquidation of capital is now more costly, the option value to reduce unwanted productive capacity falls. Firms reduce their desired capital stock to avoid production overcapacity and hence potentially costly liquidation in the future. On the other hand, some firms that were previously planning to disinvest are now no longer willing to do so given the lower resale value of capital. On balance, these two effects tend to offset each other to a large extent.

Financial market frictions provide an additional channel through which the liquidation shock affects aggregate investment. With the sudden drop in the resale value of capital, the debt capacity of firms shrinks immediately because the recovery value of their debt in the case of default drops sharply, which triggers an immediate increase in the rate of default of about 200 basis points, but only a small increase in credit spreads. As the liquidation value of capital declines, the resulting increase in the cost of capital sets off a large and prolonged period of deleveraging—the decline in aggregate debt is almost five times greater than that of aggregate capital.

In economic terms , a negative one standard deviation liquidity shock corresponds to a decline

Figure 10: Model-Based Impulse Responses to a Liquidity Shock



NOTE: The solid line in each panel depicts the model-based impulse response function of a selected variable to a 1 standard deviation negative shock to the resale price of capital (p^{-}). All variables are expressed in deviations from their respective steady-state values.

in the resale price of capital of about 3 percent. For comparison, consider the recent financial crisis, during which the nation-wide average of house prices declined almost 30 percent. In our model, a decline in the resale price of capital of that magnitude would imply a drop in aggregate investment of about 40 percent, a doubling of credit spreads from their steady-state level, a drop in the leverage ratio of about 15 percentage points, and a massive increase in the default rate. This suggests that the financial shock in our model can capture the dynamics of a severe financial crisis relatively well, conditional on the size of the initial shock.

We close this section by summarizing the main business cycle moments of the model. Table 4 reports the business cycle statistics of the model and the actual data. The data moments are computed over the 1947:Q2–2011:Q4 period, using the quarterly growth rates of the chain-weighted nonfarm business output, consumer expenditures on nondurable goods and nonhousing services, investment in producers' durable equipment, and nonfarm business sector aggregate labor hours. The model-based moments are averages of the simulations of over 2,000 periods, when the model is simulated using all three shocks simultaneously.

Table 4: Unconditional Moments of the Model

	Vol. Relative to Output Vol.		Correlation with Output	
Variable(x)	Data	Model	Data	Model
Consumption	0.41	0.93	0.54	0.97
Investment	2.54	2.63	0.65	0.66
Hours	0.68	0.22	0.71	0.44
Labor Productivity.	0.22	0.93	0.71	0.98
Memo: std(Y), corr(C,I)	1.35	2.16	0.42	0.49

Subject to the caveat that the shock processes are calibrated rather than estimated, the model succeeds at matching the salient features of the data. In particular, the model matches the relative volatility of investment and its comovement with output and consumption: The relative volatility of investment is 2.63 in the model, while it is 2.54 in the data; the correlation between investment and output is 0.66 while it is 0.65 in the data; the correlation between investment and consumption is 0.49 in the model while it is 0.42 in the data.

The uncertainty shock and the liquidity shock play important roles in replicating the comovement of the endogenous variables in the data. Moments conditional only on the aggregate TFP shock, as is typical in the real business cycle literature, suffer from too much comovement. The conditional correlations between investment and output and between investment and consumption are 0.74 and 0.66, respectively. As shown by our model-based impulse response analysis, the uncertainty and liquidity shocks generate negative correlation between consumption and investment for a few initial periods, which dilute the strong comovement of these variables that is typical of the technology-driven business cycles.

The model, however, is less successful in matching the volatility of consumption and hours worked. The relative volatility of hours worked in the data exceed that implied by the model by a factor of three (0.68 in the data vs. 0.22 in the model). In contrast, the relative volatility of consumption in the data is substantially lower than that generated by the model (0.51 vs. 0.93). The excess volatility of consumption is systematically linked to the low volatility of hours worked. When a positive technology shock hits the economy, a large increase in consumption implies a large drop in the marginal utility of consumption, which then requires a sharp increase in the market clearing wage, thereby limiting further expansion in hours worked.

5.3 The Relative Importance of Irreversibility and Financial Frictions

Both the empirical analysis and the simulation results of the theoretical model show that the two major frictions, the irreversible investment and the financial market frictions in this paper play independent roles in the propagation of shocks. However, it is not obvious how much is contributed by each channel depending on different sources of fluctuations. To address this issue, we consider a pure irreversibility model in this section. We modify the model such that it is devoid of any financial frictions but leave the costly reversible investment friction intact. With this change, the firm problem is simplified as

$$\begin{aligned} v(z,k;\mathbf{s}) &= \max_{d,k'} \left\{ d + \eta \mathbb{E}\left[m(\mathbf{s},\mathbf{s}')v(z',k';\mathbf{s}')|z,\mathbf{s}\right] \right\} \qquad (33) \\ &\quad \text{s.t} \\ d &= az\psi(w(\mathbf{s}))k^{\gamma} - Fk - p(k',k)[k'-(1-\delta)k] \\ \mathbf{s}' &= \tilde{\Gamma}(\mathbf{s}) \end{aligned}$$

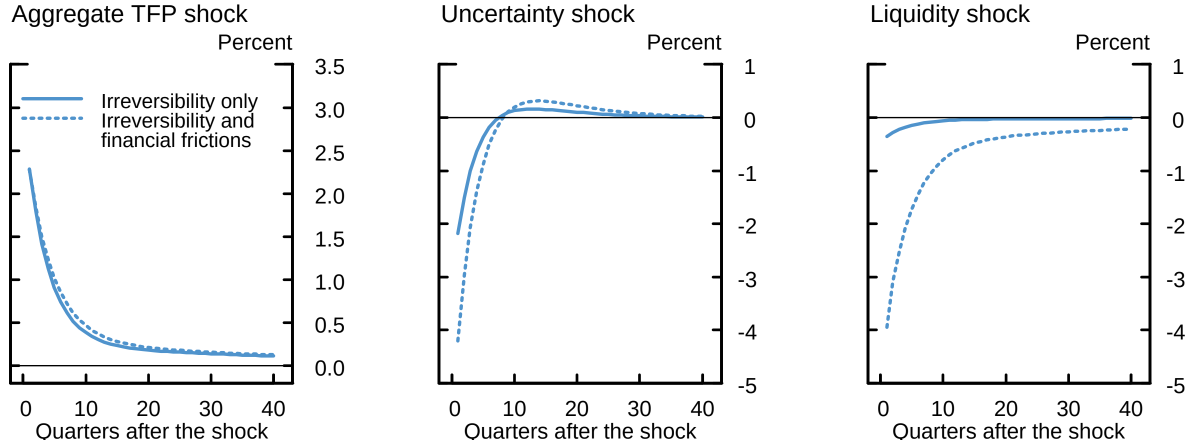
where the laws of motion of the distribution, $\mathbf{s}' = \tilde{\Gamma}(\mathbf{s})$, no longer depend on debt dynamics.

Figure 11 compares the responses of aggregate investment for the baseline model to those obtained from the pure costly reversibility model. The first panel shows the response of investment to the aggregate technology shock. These results confirm that introducing financial market frictions into an irreversible investment model does not fundamentally alter the course of aggregate investment: In both model, aggregate investment increases about 2.3 percent on the impact and the dynamic paths of the impulse responses thereafter are almost indistinguishable from each other.

The second panel depicts the investment response to the uncertainty shock. The investment contraction is twice as large in the model with financial frictions relative to the model with only costly reversibility. This result is consistent with our empirical findings—both at the macro and micro level—that the financial market frictions almost double the impact of uncertainty shock on investment. The increase in uncertainty strengthens the “wait and see” motive under the costly reversible investment friction, leading to a decline in investment on the order of 2 percent.

In the presence of financial market frictions, an additional 2 percent decline in investment is obtained through the credit-supply channel. Reflecting the concave return structure of the standard debt contract—in which the payoff of the creditor resembles that of writing a put option—the increased uncertainty lowers the expected return from holding corporate bonds, which raises the

Figure 11: Model-Based Impulse Responses of Investment to Various Types of Shocks



NOTE: The solid line in each panel depicts the impulse response function of investment to the specified 1 standard deviations shock based on a partial irreversibility model without financial market frictions; the dotted line in each panel depicts the corresponding impulse response function based on a partial irreversibility model with financial market frictions (see text for details).

cost of capital for firms and amplifies the impact of uncertainty on investment. In particular, while the option value of waiting expands the inaction boundaries, the financial mechanism counteracts the wave of "wait and see" by prompting firms with debt overhang and excess captial to liquidate their overcapacity to improve cashflows.

The third panel displays the case of the liquidity shock. In this case, financial frictions aplify the response of investment by a factor of eight—while the model without financial frictions generates about 0.5 percent response in aggregate investment upon impact, the model with financial frictions implies a 4 percent immediate drop in aggregate investment. This reconfirms our conclusion that the expansion of the inaction region brought about by the liquidity shock alone is unlikely to generate a meaningful business cycle. Because financial market imperfections provide a direct link between the debt capacity and the liquidity of capital assets, the model with financial frictions, in contrast, can generate a large and persistent investment decline.[37]

[37]The fact that irreversible investment in general equilibrium can be insensitive to the changes in the resale value of capital was pointed out by other studies, for instance, by Veracierto (2002) using a linear-quadratic solution method and Sim (2006) using an approximate aggregation method such as the one used in the current analysis. In fact, the presence of fixed operation cost in our model substantially strengthens the incentive of inefficient firms to liquidate some portion of their capital because active disinvestment can reduce the operating loss at a time of when the economy is experiencing a persistent decline in productivity. Our results show that even with the specification of fixed operating costs, the direct impact of the liquidity shock on the aggregate investment can be quite moderate in the absence of financial market frictions; Khan and Thomas (2008) tackle a similar issue by combining a fixed investment cost model with a collateral constraint.

5.4 Financial Frictions and Capital Misallocation

The combination of costly investment reversiblity and financial frictions creates a dispersion in the marginal product of capital and labor across firms. In a frictionless environment, these marginal products are equalized. To the extent that there exists dispersion in the marginal productivities of factor inputs, there is a potential to improve upon resource allocation and increase the overall efficiency of the economy. In our theoretical framework, the allocation of labor is frictionless, so we focus on the misallocation of capital.

Because we view time-to-build and costly reversibility as technological constraints, we use the model with pure irreversibility as the benchmark. We then analyze how much inefficiency is created by the capital market imperfection in our model economy and how such inefficiency is related to the forcing variables of the model economy. This can be directly analyzed by comparing the efficiency levels of the two economies, the one with the pure irreversibility and the one with the financial market frictions as well. To measure the efficiency level of a given economy, we construct a measured TFP series for each economy defined as

$$TFP_t^m = \exp[(1-\alpha)\chi(\log \bar{y}_t - \alpha\chi \log \bar{k}_t - (1-\alpha)\chi \log \bar{h}_t)], \tag{34}$$

which can be directly compared with the actual $TFP_t = a_t$. The two measures of TFP do not coincide with each other because there is a gain from reallocating capital, and as a result the former is greater than the latter in general.

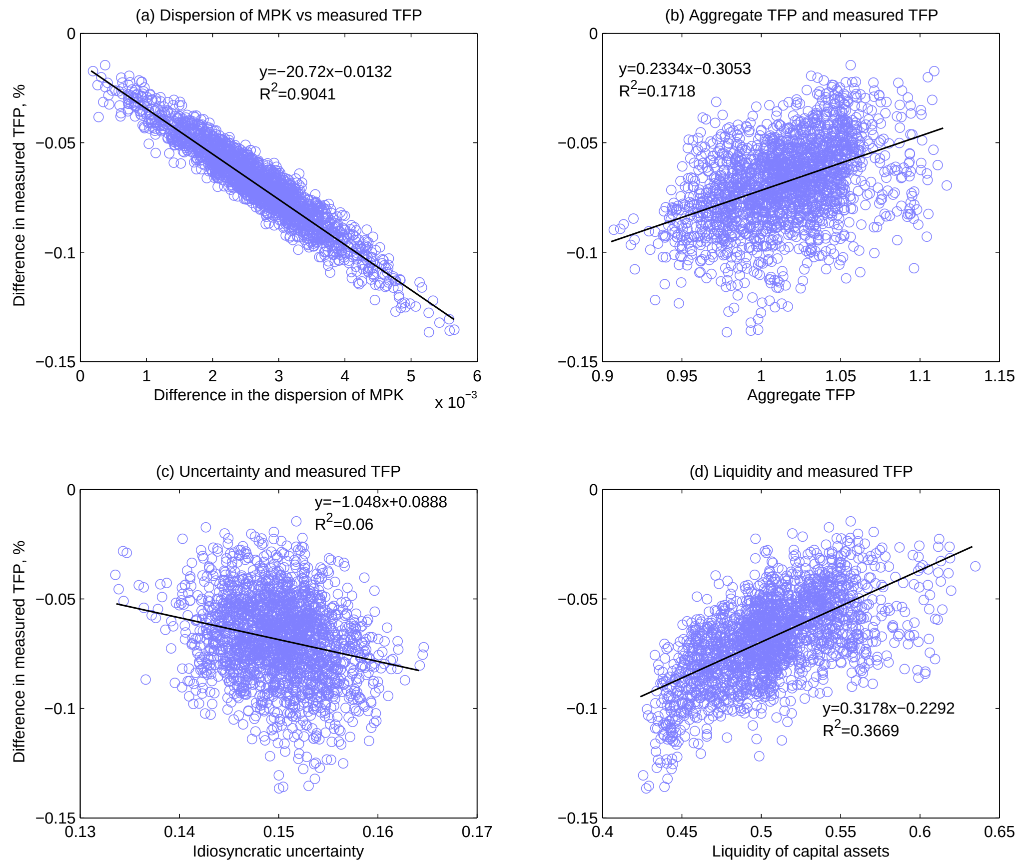
Figure 12 shows the relationship between the differential of the measured TFPs of the two economies and its relationships with: (i) the difference in the dispersion of the marginal productivity of capital (MPK) of the two economies (first panel); (ii) the aggregate TFP shock (a_t) (second panel); (iii) the uncertainty level (third panel); and (iv) the liquidity of capital assets (fourth panel).

The first panel shows the difference of the dispersion of the MPK of the baseline economy from that of the pure irreversibility economy on the x-axis, and the difference of the measured TFP of the baseline economy from that of the pure irreversible economy on the y-axis. Note that the scatter plot belongs to the fourth quadrant ($x > 0$, $y < 0$). Measured TFP of the baseline economy with the financial market frictions is always strictly less than that of the pure irreversibility economy. In addition, dispersion in the MPK is always greater for the economy with the financial market frictions.

A qualitative conclusion that comes out of this exercise is that the financial market frictions considered in this paper creates inefficiency in reallocating capital across heterogenous production units. This results in a greater dispersion in the marginal productivity of capital, which then causes the overall level of the efficiency of the economy to fall below the benchmark level, i.e., that of the pure irreversibility economy. As shown by the simple regression line, the strength of this linear relationship is near perfect.

A quantitative conclusion from the investigation is that the magnitude of the inefficiency created

Figure 12: Aggregate Shocks, Dispersion of MPK, and Measured TFP



by the financial market friction, when the model is calibrated to the U.S. economy, is small: the maximum level of efficiency loss never exceeds 0.15 percent of the benchmark level. It is possible that this is a conservative estimate given that the model does not fully account for dispersion in borrowing costs. In related work, Gilchrist et al (2012), use the dispersion of the firm-level marginal costs of borrowing to infer that the potential efficiency loss due to the capital market imperfection in the U.S. is about 1 percent relative to the efficient level in a model with no real-side frictions to capital allocation. Because this calculation assumes no real-side frictions to capital reallocation, it is likely an overestimate of the true loss in productivity owing to dispersion in borrowing costs however.

The second and the fourth panel show how the inefficiency level changes during business cycles depending on the source of shocks. As expected, aggregate TFP varies positively over the business cycle, expanding in response to positive technology shocks and contracting in response to increases in uncertainty or reductions in liquidity. The variation that occurs is small however, on the order of five to ten basis points depening on the shock.

6 Conclusion

[To be completed.]

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