A Theory of Power Wars*

Helios Herrera[1], Massimo Morelli[2], and Salvatore Nunnari[3]

[1]HEC Montréal
[2]Columbia University
[3]Columbia University

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PRELIMINARY AND INCOMPLETE

Abstract

This paper provides a theory of how the frequency of war onset and war duration depend on technology and endowments when winning a war induces power shifts but the loser is not eliminated. In the model, agents take into account not only the expected consequences of war on the current distribution of resources, but also its expected consequences on the future distribution of military and political power. The key driver of war turns out to be the *mismatch* between military and political power, and how such a mismatch is affected by a war. Additionally, we show that wars characterized by fixed military power shifts but endogenous political power shifts are more frequent and shorter than wars characterized by fixed political power shifts but endogenous military power shifts.

1 Introduction

Countries (in interstate disputes) and groups (in domestic disputes) decide whether to wage a war or not based not only on the effects on the distribution of revenues, but also thinking about the effects on future relative power, which in turn will affect the relative

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incentives to go to war again in the future. To make progress in the direction of endogenous power *after* wars, we need first of all to distinguish between two types of power: military power and political power. After a war, the winner typically obtains control on more weapons, as well as on more resources or institutions that determine indirectly the distribution of resources. In other words, a victory determines both a shift in military power and a shift in political power, a catch-all term we use to encompass all the means to obtain a higher share of resources (for example, through a greater control of the democratic institutions). Beside the fact that a victory has effects on both types of power, and in different ways when comparing civil wars and interstate wars, the two types of power also play different roles *before* wars. While only military power matters for the odds of winning a given war, the relative political power before the war acts as a status quo outside option to war. The value of winning a war is determined by the degree to which victory affects political power (which affects the future outside option), as well as military power (which affects the likelihood of winning future wars).

In some civil wars, namely in those where ethnic groups' relative strength is tightly connected to their relative group sizes, especially in underdeveloped countries and going back in history, the effect of a victory does not alter that much the relative military power of groups.[1] On the other hand, the swings of political power induced by decisive war outcomes can be enormous, with previously exploited groups gaining access to the key determinants of distribution of surplus. Some other wars, especially interstate wars, can instead be thought as having the opposite characteristics: the relative political power of two countries cannot be affected much in terms of effects on relative revenue sources (especially if there is no border change), whereas the relative military power can change dramatically (from no fly zones to full destruction of arsenals and destruction of plants to build bombs). The "production function" of victory in an interstate war is typically much more capital intensive, especially in modern warfare.[2] One of the benefits of introducing multiple types of endogenous power and a connection between them and the different types of war is that we can obtain predictions on (A) duration of wars, as a function of different types of power shifts; (B) differences in the expected frequency of different types of wars. More generally, the initial conditions in terms of inequality and unequal distribution of powers matter for war onset in one direction or the other depending on the technology of war and power shifts that are feasible after a victory.

[1]See Buhaug, Gates and Lujala (2009) for evidence on the connection between probability of winning and group sizes in civil wars.

[2]See Tilly (2009) and Harrison (2005), among others, for historical evidence of the growing importance of capital in interstate wars.

The model that we introduce is the simplest possible model to address all such issues. We consider a two period model that can be described informally as follows: given any initial distribution of military and political power between two players, and given a random draw of a cost of conflict, the players choose (sequentially, in order to avoid wars due to coordination problems) whether to go to war or not; if peace prevails in the period, then the players consume their relative political power; while if war takes place the winner consumes all surplus of the period minus the conflict costs, and both types of powers have a shift going into the next period. In the second period, starting with the relative power endowments determined in the previous period, there is a new random draw of conflict costs, and then again players choose whether to accept the current political power (and hence a peaceful distribution of surplus) or else go to war in order to consume all the surplus of that period, minus the new cost of war.[3]

No bargaining to reallocate political power after a realization of a cost of war is allowed. In other words, we implicitly assume frictions which make it impossible to reallocate political power at all times: only *after* a war there is an opportunity to modify political power.[4]

We will study first the case in which the victory of a war induces an exogenous shift in both powers. Then we will consider the case in which military power shifts are exogenous but political power shifts are chosen by the winner and the case in which the political power shifts induced by a war outcome are exogenous but the military power shifts are decided by the winner.

Our main result, in the static as well as in the dynamic case, is that the key cause of wars of all kinds is what we call the *mismatch* between military and political power. In other words, what matters for the probability of a war onset is not really whether any type of power is balanced, but rather whether the two types of powers are similarly distributed. This main result has implications for the debate on the role of balance of power in international relations, and provides a rational framework for the early insights of realists like Gilpin.[5] The debate on whether peace is more likely with balance of power or preponderance of power is huge,[6] but we point out that neither of such descriptions is sufficient to rationalize the likelihood of wars. Two super powers militarily constitute a likely peaceful

[3]The fluctuations of power and uncertain costs have also been emphasized as relevant for civil war in Van Evera (2013).

[4]See e.g. Chadefaux (2011) for a simple complete information model with bargaining over power distributions where there are multiple reasons to expect bargaining to break down.

[5]Gilpin (1983, 1988) suggested a historical perspective compatible with our main finding.

[6]See e.g. Powell (1996) and the references therein, and the prior seminal contributions by Waltz (1979) on one side and Blainey (1988) on the other.

duo if also the political power is balanced, and two countries in a preponderance imbalance of military power may be equally peaceful if the political power imbalance is proportional to the military one.

The fact that the likelihood of war onset depends on the mismatch between political and military power, also has implications for the debate on the role of inequality for civil war onset: grievances about inequality are basically captured by a preponderance of political power for one group over another, but our prediction is that such an inequality can lead to war only if it is not paralleled by a similar imbalance in military power.[7]

Second, the main results on the expected frequency of wars are that wars with endogenous political shifts are more frequent and more sensitive to the *mismatch* between military and political power initial conditions. Third, we obtain a number of interesting results on war duration. We capture "duration" of wars in the model by looking at the equilibrium probability of a second war conditional on a war taking place in the first period. We show that, in all cases, the duration of wars is lower the greater are the exogenous power shifts induced by a war outcome. In particular, for the case of endogenous political power shifts, the prediction is that the lower is the change in military power after a war, the higher the duration of conflicts.[8] Moreover, we show that the duration of wars with endogenous military power shifts is very sensitive to the size of political power shifts, while duration of wars with political power shifts is not sensitive to the size of military power shifts when starting from a balanced strength initial condition.

In terms of the relationship with traditional schools of thought in International Relations theory, the perspective endorsed by our model is clearly a realist one, as winning a war matters both in terms of acquired resources and in terms of enhancing security going forward. As Liberman (1993) puts it, spoils of conquest can be significant and cumulative sequences of wars should be expected.[9] However, as mentioned above, we move away from the debate on balance of power by breaking power into two separate variables, formalizing the insights of Gilpin.

[7]See for example Cederman, Weidmann and Gleditsch (2011) for the empirical findings on the importance of between groups inequality for civil conflict. Huber and Mayoral (2013) find instead that between group inequality is not an important driver of war, and it would be interesting to check whether on their data the role of between group inequality becomes relevant again when interacted with some measure of mismatch.

[8]Remarkably, this finding provides a theoretical explanation of an otherwise unexplained empirical result by Collier, Hoeffler and Soderbom (2004), that civil wars last longer if per capita income is low and inequality is high. In fact, if economic and technological developments are low, the production function of war depends more heavily on the relative group sizes, and hence the military power shifts after a victory are lower than in a more capital intensive context. This, as we prove, induces longer wars.

[9]In the realist tradition, Morgenthau (1948) and Waltz (1979) are seminal contributions alternative to the Gilpin view we endorse.

Standard theories of war onset take the relative power of players as a fixed parameter.[10] There are only two strands of literature where power is in some sense endogenous: one is the literature on strategic militarization, where the endogenous power we are talking about is the relative militarization levels chosen by the players in anticipation of war (Powell 1993, Jackson and Morelli 2009, Meirowitz and Sartori 2009). The other strand, closer to us, focuses more explicitly on power shifts. The seminal paper in the rationalist theory of endogenous power and wars is Fearon (1996). There are two important differences between this paper and our approach: first, in Fearon's model, a decisive war is an end state; and, second, in his model players bargain every period on territory (which plays the same role as our political power) but today's political power maps one to one into tomorrow's military power, hence eliminating by assumption the possibility of a mismatch, which is what determines perpetual peace in his equilibrium path. In Powell (2013) and Debs and Monteiro (Forthcoming) wars can occur to avoid an anticipated power shift that could be caused by a militarization strategy.[11] All these papers that deal with endogenous power, in one sense or another, have in common one thing: a decisive war is an end state, hence power related strategies always relate to preparation for war or deterrence of war, but once war happens and a player wins, the game is over.[12] We believe that our modeling innovation captures an important characteristic of the real world, where wars are rarely an end state, almost never in history. States rarely fully disappear after a defeat, and ethnic groups within a country are (fortunately) only rarely eliminated from the contest of power altogether.[13]

The paper is organized as follows: in section 2 we analyze the benchmark case in which a war outcome induces exogenous power shifts; section 3 characterizes the equilibrium when one of the two power shifts can be freely chosen by the winner; in section 4 we compare the two endogenous cases; and section 5 concludes.

[10] See Jackson and Morelli (2011) for a recent survey of rationalist theories of war.

[11] For a summary on the related literature on exogenous power shifts and the consequent incentives to start preventive wars, see Powell (2012) and references therein.

[12] In Powell (2013) wars can be followed by a continuation of the game, but only if the outcome of a war is a stalemate. Moreover, power shifts are basically a "substitute" of war, rather than being a consequence. A related contribution is Yared (2010), which displays repeated wars in equilibrium, but wars are always triggered by an aggressive country in the absence of public information, hence the setting is completely different.

[13] See Esteban, Morelli and Rohner (2013) for an analysis of strategic mass killings.

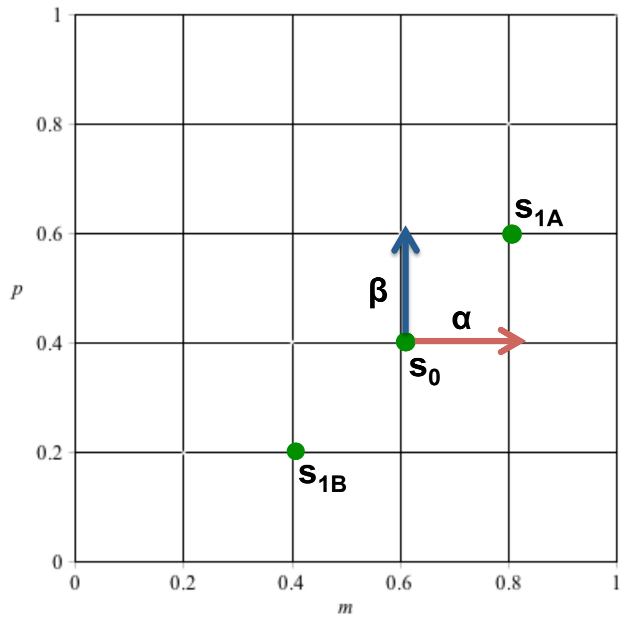

Figure 1: Power states when $\alpha = \beta = 0.2$. Relative military power (m) is on the horizontal axis; relative political power (p) on the vertical axis. The point $s_0 = (0.6, 0.4)$ depicts an arbitrary initial state. In case of war, the new state will be s_{2A} if A wins and s_{2B} if B wins.

2 Exogenous Power Shifts

2.1 Model

Consider the standard problem of two players (groups or countries) having to share a per period surplus, which may come from exploitation of the local resources or control of the state. At the start of any period t, player A has a share of military and political power equal to, respectively, $m_t \in [0, 1]$ and $p_t \in [0, 1]$. Faction B therefore has powers $(1 - m_t), (1 - p_t)$.

We consider a two period model, with the expected utility from the second period discounted by a factor $\delta \in [0, 1]$. In each period, the two factions sequentially choose whether to go to war or not, in a random order. If neither of them attacks, then we have a period of peace and the two factions consume, respectively, p_t and $(1 - p_t)$, the distribution of resources dictated by the peace-time institutions.[14] If instead a war occurs, the outcome is determined by the distribution of military power in the current period: A wins with probability m_t and B wins with probability $(1 - m_t)$. A stalemate is not possible. If A

[14] In future work, it would be interesting to relax this linearity, for example, distinguishing between winner take all and proportional democratic institutions, or exploring the distributional consequences of different bargaining protocols with three or more groups.

wins, the immediate post war consumptions are $1 - c$ for A, and $-c$ for B.

In addition to the short run effect on consumption, war affects the future distribution of powers. For example, consider first a setting in which power shifts depend on war outcomes (hence they are endogenous in that sense) but are fixed in size. If A wins (respectively loses) a war at time t, and there is still a period $t+1$ in the game, we call $\alpha \geq 0$ and $\beta \geq 0$ the fixed power effects of a war, so that the future military power is $m_{t+1} = m_t + (-)\alpha$ and the future political power is $p_{t+1} = p_t + (-)\beta$.[15]

The cost of war $c \in [0, 1]$ is known when the factions decide whether to attack or not in period t, but is unknown at the end of period $t - 1$. In other words, each period starts with a realization of a new cost of war, c_t, drawn from a uniform distribution between 0 and 1. This means that the expected utility from the new state variable determined at the end of period t will be a continuous function of α and β. The second period is identical to the first, but this time the new realization of power in case of a war does not matter because the game ends there.

2.2 The Static Game

We start from the basic analysis of the static game, or the last period in the two period model. Since in the remainder of the analysis, we will call the two periods $t = 0$ and $t = 1$, it is convenient to use the subscript 1 in the analysis of the static game.

When factions only care about their current consumption, they decide whether to attack or not by comparing the share of resources assigned to them by peace-time institutions and the expected utility from a conflict, which is a function of their relative military power and of the cost of war in the current period. In particular, A prefers a peaceful outcome if and only if:

$$p_1 > m_1 (1 - c) + (1 - m_1) (-c) = m_1 - c$$

Similarly, B prefers peace if and only if:

$$1 - p_1 > (1 - m_1)(1 - c) + m_1(-c) = 1 - m_1 - c$$

Therefore, we have peace when, combining the two conditions above:

$$M := |p_1 - m_1| < c$$

[15] If the initial conditions are interior and α and β are small, then we do not have to worry about corners for the basic description of the game and dynamics.

M represents the *mismatch* between political and military power and it coincides with the probability of conflict:

$$\Pr(M > c) = |p_1 - m_1|$$

The chance of peace is given by:

$$\Pr(M < c) = 1 - M$$

Proposition 1 *The probability of war in the absence of concerns for the future is equal to the mismatch between political and military power, M.*

2.3 The Two-Period Game

Using the equilibrium strategies and the associated probabilities of war and peace in the second period derived in the previous section, we can derive the value of a state (m_1, p_1) for player A, evaluated at the end of the first period:

$$V^A(m_1, p_1) = (1 - M_1)\,p_1 + M_1 \int_0^{M_1} \frac{m_1 - c}{M_1} dc = (1 - M_1)\,p_1 + M_1 \left(m_1 - \frac{M_1}{2}\right)$$

The first term is the value of the state (m_1, p_1) in peace, weighted by the probability of peace. The second term is the value of the state in war, weighted by the probability of war. The value of the state in war consists of the chance of winning the war m_1 minus the average cost of war, $M_1/2$.

We can also write this value function as a step function, depending on whether player A is relatively advantaged or disadvantaged in terms of political vs. military power:

$$V^A(m_1, p_1) = \begin{cases} p_1 + \frac{M_1^2}{2} & \text{if } p_1 < m_1 \\ p_1 - 3\frac{M_1^2}{2} & \text{if } p_1 > m_1 \end{cases}$$

Similarly, for player B we have

$$V^B(m_1, p_1) = (1 - M_1)(1 - p_1) + M_1 \left(1 - m_1 - \frac{M_1}{2}\right)$$

$$V^B(m_1, p_1) = \begin{cases} (1 - p_1) + \frac{M_1^2}{2} & \text{if } p_1 > m_1 \\ (1 - p_1) - 3\frac{M_1^2}{2} & \text{if } p_1 < m_1 \end{cases}$$

The total welfare is equal to:

$$V^A(m_1, p_1) + V^B(m_1, p_1) = 1 - M_1^2$$

This sums to 1 only when the chance of war is zero, namely on the diagonal where $p_1 = m_1$ and there is no mismatch, and then it decreases as the chance of war increases.

Let us consider the case in which a conflict in period 0 yields a fixed size shift of power for the winner, assuming $\alpha = \beta = g$. In this case, we have war initiated by agent A when

$$p_0 < m_0 + \delta g(2m_0 - 1) - c = m_0(1 + 2g\delta) - g\delta - c$$

The indifference region (that is, the set of all states where A is indifferent between peace and war) is a line with slope $(1+2g\delta) > 1$. Therefore, given a realized value of c, the war-peace boundary is a strip steeper than the diagonal (see Figure 2). This is because the static incentives to launch a war, given by the mismatch, is complemented by a dynamic incentive to attack. When $g\delta > c$, A may start a war even if he is relatively advantaged by the peace-time institutions, that is, when $p_0 > m_0$. This happens when m_0 is large enough to make war appealing.

If c is uniformly distributed between 0 and 1, the ex-ante probability of war initiated by A is equal to $Pr\left[c < m_0 - p_0 + \delta g\left(2m_0 - 1\right)\right]$, or:

$$F\left(m_0 - p_0 + \delta g\left(2m_0 - 1\right)\right) = \begin{cases} 0, & \text{if } m_0 - p_0 + \delta g\left(2m_0 - 1\right) < 0 \\ \left(m_0 - p_0\right) + \delta g\left(2m_0 - 1\right), & \text{if } m_0 - p_0 + \delta g\left(2m_0 - 1\right) \in [0, 1] \\ 1, & \text{if } m_0 - p_0 + \delta g\left(2m_0 - 1\right) > 1 \end{cases}$$

Similarly, the incentive compatibility constraint of peace for player B is:

$$c > p_0 - m_0 - \delta g\left(2m_0 - 1\right)$$

If c is uniformly distributed between 0 and 1, the ex-ante probability of war initiated by B is equal to $Pr\left[c < p_0 - m_0 - \delta g\left(2m_0 - 1\right)\right]$, or:

$$F\left(p_0 - m_0 - \delta g\left(2m_0 - 1\right)\right) = \begin{cases} 0, & \text{if } p_0 - m_0 - \delta g\left(2m_0 - 1\right) < 0 \\ p_0 - m_0 - \delta g\left(2m_0 - 1\right), & \text{if } p_0 - m_0 - \delta g\left(2m_0 - 1\right) \in [0, 1] \\ 1, & \text{if } p_0 - m_0 - \delta g\left(2m_0 - 1\right) > 1 \end{cases}$$

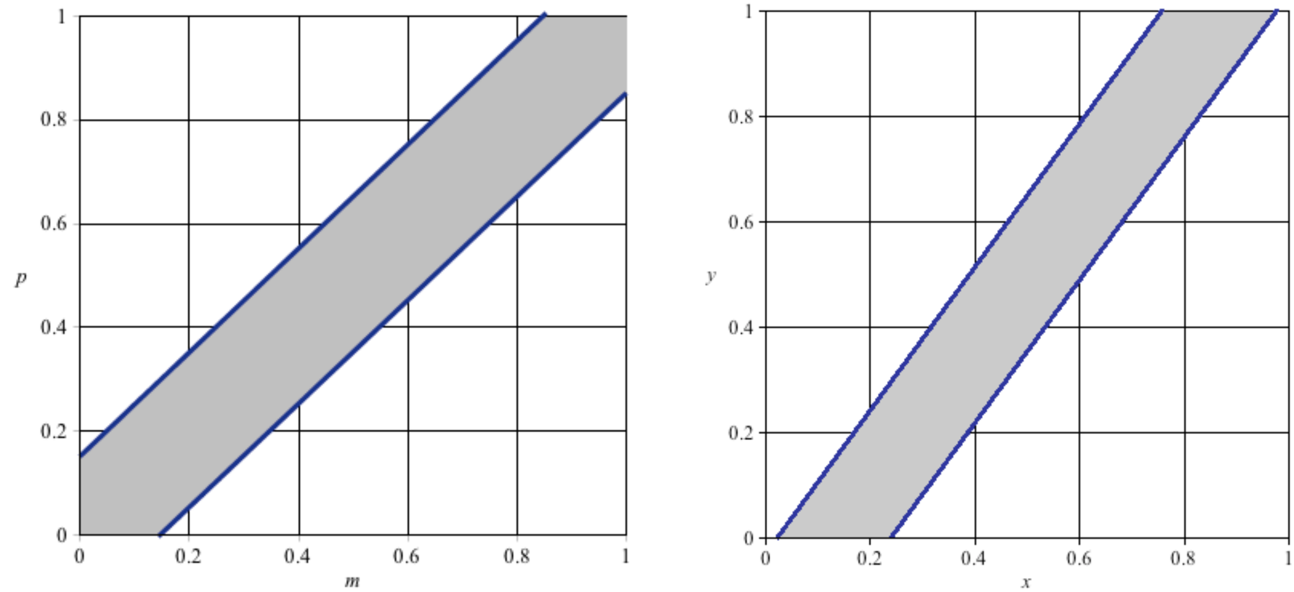

Figure 2: War and peace regions in the static game (right-hand panel) and in the dynamic game (left-hand panel). The shaded area is the set of all power states where there is peace, given a realized cost of war of 0.15 (and, for the dynamic game, $\delta = 0.9$).

Proposition 2 *In the two-period game with fixed exogenous power shifts we have peace (that is, neither player provokes the war) in the first period if:*

$$c > |\ m_0 - p_0 + \delta g\,(2m_0 - 1)|$$

Thus, the chance of peace is:

$$\max\,(0, 1 - |\ m_0 - p_0 + \delta g\,(2m_0 - 1)|)$$

Correspondingly, the chance of war is:

$$\min\,(1, |\ m_0 - p_0 + \delta g\,(2m_0 - 1)|)$$

Corollary 1 *When players are forward-looking and consider the impact of winning a war on power shifts, the probability of a war is weakly increasing in the initial military power imbalance ($|m_0 - 1/2$). The impact of the initial mismatch (M_0), the magnitude of the power shifts (g), and the discount factor (δ) depends on the initial condition. The impact of the initial military power imbalance on the frequency of war onset increases in the importance of the future, captured by an increase in g or δ.*

3 Endogenous Power Shifts

In some wars, the power shifts after a victory can be a choice variable for the winner, rather than being determined in an exogenous manner like in our benchmark analysis. In this section we analyze first the wars where the winner can choose the endogenous shift in political power (which we will refer to as *p-wars*) and then those wars where instead the winner can choose the military power shift (which we will refer to as *m-wars* for brevity). In the next section we will then compare the benchmark power wars with the p-wars and m-wars, both in terms of frequency ex ante and in terms of duration.

3.1 Endogenous Political Power Choice (P-Wars)

We call *p-wars* those wars where the winner is able to conquer the possibility to adjust political power with more flexibility than in the fixed size power shift benchmark. This is particularly true of some civil wars: within a country, the relative power in terms of probability of winning a conflict is mostly related to population sizes of ethnic or religious groups, which, excluding mass killings, do not vary much after a conflict. On the other hand, the potential exists for a winner to secure all political power next period, if she wants to.

The modified timing of the game for this case is as follows:

1. Agents see c and choose war or peace;
2. The winner chooses p^* given the realized $m_0 + \alpha$ (if A wins) or $m_0 - \alpha$ (if B wins);
3. Agents see a new c and choose war or peace.

3.2 Equilibrium Analysis

3.2.1 Subgames Following a First-Period War

A victory by A implies $m_1 = m_0 + \alpha$, and, at the beginning of the second period, A chooses a value $p_1^A \geq m_1$, maximizing the objective $V^A(m_1, p_1) = p_1 - 3\frac{M^2}{2}$ hence:

$$p_1^A = \begin{cases} m_0 + \alpha + 1/3 & \text{if } m_0 < 2/3 - \alpha \\ 1 & \text{if } m_0 \geq 2/3 - \alpha \end{cases}$$

A victory by B implies $m_1 = m_0 - \alpha$, and, at the beginning of the second period, B chooses a value $p_1^B \leq m_1$, maximizing the objective $V^B(m_1, p_1) = 1 - p_1 - 3\frac{M^2}{2}$ hence:

$$p_1^B = \begin{cases} m_0 - \alpha - 1/3 & \text{if } m_0 > 1/3 + \alpha \\ 0 & \text{if } m_0 \leq 1/3 + \alpha \end{cases}$$

Remark 1 *In the case of adjustable political power, the winner of a conflict does not always exclude the loser completely from the distribution of resources. In particular, this is not the case, when the initial military power is balanced and the exogenous military shift after a war is small. In this case, the winner trades off an increase in his current resources and a decrease in tomorrow's chances of a conflict.*

The indirect utilities after a victory by A are:

$$V^A\left(m_0 + \alpha, p_1^A\right) = \begin{cases} m_0 + \alpha + 1/6 & \text{if } m_0 < 2/3 - \alpha \\ 1 - 3\frac{(1-m_0-\alpha)^2}{2} & \text{if } m_0 > 2/3 - \alpha \end{cases}$$

$$V^B\left(m_0 + \alpha, p_1^A\right) = (1 - p_1) + \frac{M^2}{2} = \begin{cases} 13/18 - m_0 - \alpha & \text{if } m_0 < 2/3 - \alpha \\ \frac{(1-m_0-\alpha)^2}{2} & \text{if } m_0 > 2/3 - \alpha \end{cases}$$

The indirect utilities after a victory by B are:

$$V^A\left(m_0 - \alpha, p_1^B\right) = \begin{cases} m_0 - \alpha - 5/18 & \text{if } m_0 > 1/3 + \alpha \\ \frac{(m_0-\alpha)^2}{2} & \text{if } m_0 \leq 1/3 + \alpha \end{cases}$$

$$V^B\left(m_0 - \alpha, p_1^B\right) = \begin{cases} 7/6 + \alpha - m_0 & \text{if } m_0 > 1/3 + \alpha \\ 1 - 3/2(m_0 - \alpha)^2 & \text{if } m_0 \leq 1/3 + \alpha \end{cases}$$

The total welfare in case of a victory by A is:

$$V^A(m_0 + \alpha, p_1^A) + V^B(m_0 + \alpha, p_1^A) = \begin{cases} 1 - (1/3)^2 = 8/9 & \text{if } m_0 < 2/3 - \alpha \\ 1 - (1 - m_0 - \alpha)^2 & \text{if } m_0 > 2/3 - \alpha \end{cases}$$

The total welfare in case of a victory by B is:

$$V^A(m_0 - \alpha, p_1^B) + V^B(m_0 - \alpha, p_1^B) = \begin{cases} 8/9 & \text{if } m_0 > 1/3 + \alpha \\ 1 - (m_0 - \alpha)^2 & \text{if } m_0 \leq 1/3 + \alpha \end{cases}$$

Remark 2 *If the initial mismatch is sufficiently large, the total welfare conditional on a first period war is higher in the flexible political power case, whereas for sufficiently small mismatch the fixed power shifts are better.*

3.2.2 War Duration

To study the duration of a war, we can look at the probability of a conflict in the second period, conditional on a conflict in the first period. In the remainder of this section we make the following assumption on the size of the military power shift, the dimension not controlled endogenously by the conflict winner:

Assumption 1: $\alpha \leq 1/6$.

Proposition 3 *In a p-war, the expected mismatch in the second period, conditional on war in the first period is:*

$$E[M_1^w] = \begin{cases} 0 & \text{if } m_0 \leq \alpha \\ m_0(4/3+\alpha) - \alpha - m_0^2 & \text{if } \alpha < m_0 < 1/3+\alpha \\ 1/3 & \text{if } 1/3+\alpha \leq m_0 \leq 2/3-\alpha \\ m_0(2/3-\alpha) + 1/3 - m_0^2 & \text{if } 2/3-\alpha < m_0 < 1-\alpha \\ 0 & \text{if } 1-\alpha \leq m_0 \end{cases}$$

Corollary 2 *Conditional on war in the first period, the probability of a second p-war is strictly increasing in m_0 for $m_0 \leq 1/3+\alpha$, constant in m_0 for $m_0 \in [1/3+\alpha, 2/3-\alpha]$, strictly decreasing in m_0 for $m_0 \geq 2/3-\alpha$, and weakly decreasing in α for every initial condition m_0.*

This result tells us that ethnic conflicts where the relative military power of the fighting factions is fully determined by their relative population size (that is, $\alpha = 0$), tend to have a higher duration than wars where conflict is accompanied by a shift in relative military power.[16] It also establishes that the duration of p-wars is higher when the groups fighting are of close to equal size, which reflects polarization.[17]

[16]This statement concerns a comparative static within the assumed range $\alpha \in [0, 1/6]$. For $\alpha > 1/6$, we have more cases to take into account, but a similar comparative static result can be proven.

[17]See Esteban and Ray (2004) and Montalvo and Reynall-Querol (2005, 2010) for the role of polarization in civil wars. Formally, we could decompose the relative military power m_0 as a convex combination $kn_0 + (1-k)a_0$, where n_0 is the relative population size of the two groups and a_0 their relative strength in terms of capital or weapons, and k a weight between 0 and 1. In the case of civil wars, k is likely to be close to 1, so that m_0 in the text is indeed interpretable as relative group sizes and an intermediate m_0 reflects polarization. When fighting effort is endogenous, like in the traditional Hirshleifer appropriation technology models and in the continuous time extensions of it [ADD CITES], the fact that duration may be higher in more polarized contexts is a natural result, because any war of attrition lasts longer if the two parties in the tug of war are equal strength. However, the novelty of our result is that polarization and duration are correlated in p-wars even when not taking into account efforts, only ex ante incentives.

3.2.3 First Period and Ex-Ante Probability of War

At the beginning of the first period after the cost of war for the period has been realized, player A faces the following expected utilities from peace and from war, when we assume that the winner of the first-period war chooses p_1:

$$\begin{aligned} EU^A(P|m_0,p_0) &= p_0 + \delta V^A(m_0,p_0) \\ EU^A(W|m_0,p_0) &= (m_0-c) + \delta\left[m_0 V^A\left(m_0+\alpha,p_1^A\right) + (1-m_0)V^A\left(m_0-\alpha,p_1^B\right)\right] \end{aligned}$$

where p_1^B is the political power optimally chosen by B, p_1^A is the political power optimally chosen by A, and the corresponding value functions are the one characterized above.

A prefers peace in the first period if and only if:

$$p_0 + \delta V^A(m_0,p_0) > (m_0-c) + \delta\left[m_0 V^A\left(m_0+\alpha,p_1^A\right) + (1-m_0)V^A\left(m_0-\alpha,p_1^B\right)\right]$$

Plugging in the value functions, we have:

$$\begin{gathered} p_0 + \delta\left[(1-M_0)p_0 + M_0\left(m_0 - \frac{M_0}{2}\right)\right] > \\ (m_0-c) + \delta\left[m_0\begin{cases} m_0+\alpha+1/6 & \text{if } m_0+\alpha<2/3 \\ 1-3\frac{(1-m_0-\alpha)^2}{2} & \text{if } m_0+\alpha>2/3 \end{cases}\right] \\ +\delta\left[(1-m_0)\begin{cases} m_0-\alpha-5/18 & \text{if } m_0-\alpha>1/3 \\ \frac{(m_0-\alpha)^2}{2} & \text{if } m_0-\alpha<1/3 \end{cases}\right] \end{gathered}$$

We have 3 cases: 1) $m_1 < 1/3$, 2) $m_1 \in [1/3, 2/3]$, and 3) $m_1 > 2/3$. In the remainder of this section, we assume that, initially, there is a perfect balance of military power. This allows us to focus the analysis on the second case.

Assumption 2: $m_0 = 1/2$.

The condition above becomes:

$$c > \left(\frac{1}{2} - p_0\right)(1+\delta) + \delta\left[\left(p_0 - \frac{1}{2}\right)M_0 + \frac{M_0^2}{2} - \frac{1}{18}\right] = c_A^*$$

where $M_0 = \left|\frac{1}{2} - p_0\right|$.

The same condition for agent B is given by:

$$c > \left(p_0 - \frac{1}{2}\right)(1+\delta) + \delta\left[\left(\frac{1}{2} - p_0\right) M_0 + \frac{M_0^2}{2} - \frac{1}{18}\right] = c_B^*$$

Combining the two conditions above, the ex-ante probability of p-war is equal to:

$$Pr(\text{Civil War}) = F(\max\{c_A^*, c_B^*\})$$

where $F(\cdot)$ is the cumulative distribution function of c. It is straightforward to prove that $c_A^* \geq c_B^*$ if and only if $p_0 \leq 1/2$. Using this and the fact that c is uniformly distributed between 0 and 1, the ex-ante probability of war initiated by A is equal to:

$$Pr(\text{Civil War}) = \begin{cases} 0, & \text{if } (p_0 > 1/2 \text{ and } c_B^* \leq 0) \text{ or } (p_0 \leq 1/2 \text{ and } c_A^* \leq 0) \\ c_A^*, & \text{if } p_0 \leq 1/2 \text{ and } c_A^* \in [0,1] \\ c_B^*, & \text{if } p_0 > 1/2 \text{ and } c_B^* \in [0,1] \\ 1, & \text{if } (p_0 > 1/2 \text{ and } c_B^* > 1) \text{ or } (p_0 \leq 1/2 \text{ and } c_A^* > 1) \end{cases}$$

Proposition 4 *The ex-ante probability of p-war in a militarily polarized context is weakly increasing in the initial mismatch between political and military power (that is, in the distance between p_0 and $1/2$), weakly increasing in the discount factor for $p_0 < 0.44$ and $p_0 > 0.56$, weakly decreasing in the discount factor for $p_0 \in [0.44, 0.56]$.*

Note that in the case of a civil war $|p_0 - 1/2|$ is not only a measure of mismatch but also a proxy of inequality: in a polarized country where the two major groups fighting for power are equally large or strong, $p_0 = 1/2$ would be the equal distribution of political power between groups. Simply looking at agent A, the intuition behind Proposition 4 lies in the fact that, while the EU of peace is increasing and convex in p_0, the EU of conflict is independent of p_0 and α. As the mismatch decreases (that is, as p_0 gets closer to $1/2$), the distance between the two EUs decreases and peace is eventually preferable.

3.3 Endogenous Shift of Military Power (M-Wars)

The modified timing of the game for this case is as follows:

1. Agents see c and choose war or peace;
2. The winner chooses m given the realized $p + \beta$ (if A wins) or $p - \beta$ (if B wins);

3. Agents see a new c and choose war or peace.

3.4 Equilibrium Analysis

3.4.1 Subgames Following a First-Period War

If the faction which can endogenously choose m_1 is A, A chooses given p_1 a value $m_1 > p_1$, namely maximizing the objective $V^A(m_1, p_1) = p_1 + \frac{M^2}{2}$. Since $V^A(m_1, p_1)$ is strictly increasing in m_1, A will pick $m_1^* = 1$. The value function for A in this case is:

$$V^A(m_1^*, p_1) = p_1 + \frac{(1-p_1)^2}{2} = \frac{1+p_1^2}{2}$$

Similarly, if the faction which can endogenously choose m_1 is B, B chooses $m_1 < p_1$, namely maximizing the objective $V^B(m_1, p_1) = (1-p_1) - 3\frac{M^2}{2}$. Since $V^B(m_1, p_1)$ is strictly decreasing in m_1, B will pick $m_1^* = 0$. The value function for A in this case is:

$$V^A(m_1^*, p_1) = p_1 - 3\frac{(0-p_1)^2}{2} = p_1 - \frac{3}{2}(p_1)^2$$

Remark 3 *In the case of adjustable military power, the winner of a conflict disarms completely the loser, regardless of the initial military or political power. Contrary to the case with adjustable political power, the winner is not concerned with exacerbating the mismatch since a disarmed opponent poses no threat.*

3.4.2 War Duration

The probability of war in the second period, conditional on war in the first period, is equal to the mismatch M_1. If A wins the first-period war, then $m_1 = 1$ and M_1 is equal to $1 - p_1$. On the other hand, if B wins the first-period war, then $m_1 = 0$ and M_1 is equal to p_1. The expected mismatch at the beginning of the second period (conditional on war in the first period) is therefore given by:

$$E[M_1] = m_0(1-p_0-\beta) + (1-m_0)(p_0-\beta) = m_0 + p_0 - 2p_0m_0 - \beta$$

This is increasing (decreasing) in m_0 if p_0 is smaller (greater) than $1/2$; increasing (decreasing) in p_0 if m_0 is smaller (greater) than $1/2$; always decreasing in β.

Proposition 5 *In the case of adjustable military power, the probability a war continues in the second period is larger the smaller the initial political power of the first-period winner, and the smaller the sensitivity of political power to military conflict.*

So, while in p-wars the duration of conflict is expected to be highest in the polarized world (intermediate m_0) where it does not depend on the identity of the winner, in m-wars duration depends on the imbalance of power in a very specific and intuitive way: if there is an initial imbalance but the politically weaker wins, then the war is likely to continue.

3.4.3 First Period and Ex-Ante Probability of War

At the beginning of the first period, after the cost of war for the period has been realized, player A faces the following expected utilities from peace and from war:

$$\begin{aligned} EU^A(P|m_0,p_0) &= p_0 + \delta V^A(m_0,p_0) \\ EU^A(W|m_0,p_0) &= (m_0-c) + \delta\left[m_0 V^A(1,p_0+\beta) + (1-m_0)V^A(0,p_0-\beta)\right] \end{aligned}$$

A prefers peace in the first period if and only if:

$$p_0 + \delta V^A(m_0,p_0) > (m_0-c) + \delta\left[m_0 V^A(1,p_0+\beta) + (1-m_0)V^A(0,p_0-\beta)\right]$$

After plugging in the value functions and re-arranging this condition becomes:

$$\begin{aligned} c > (m_0-p_0) + \delta\left[(p_0-m_0)M_0 + \frac{M_0^2}{2} + m_0\left(\frac{1}{2}-p_0\right) + \left(2m_0-\frac{3}{2}\right)p_0^2\right] + \\ + \delta\left[\beta\left(m_0(1-2p_0) + \beta\left(2m_0-\frac{3}{2}\right) + 3p_0 - 1\right)\right] \end{aligned}$$

When we consider the case with initially balanced military power ($m_0 = 1/2$), we have:

$$c > \left(\frac{1}{2}-p_0\right) + \delta\left[\left(p_0-\frac{1}{2}\right)M_0 + \frac{M_0^2}{2} + \frac{1}{4} - \frac{p_0}{2} - \frac{p_0^2}{2} + \beta\left(2p_0 - \frac{1}{2} - \frac{\beta}{2}\right)\right] = c_A^{**}$$

where $M_0 = |p_0 - 1/2|$. Similarly, B prefers peace in the first period if and only if:

$$c > \left(p_0-\frac{1}{2}\right) + \delta\left[\left(\frac{1}{2}-p_0\right)M_0 + \frac{M_0^2}{2} - \frac{3}{4} + \frac{3p_0}{2} - \frac{p_0^2}{2} + \beta\left(-2p_0 + \frac{3}{2} - \frac{\beta}{2}\right)\right] = c_B^{**}$$

Combining these two conditions, the ex-ante probability of m-war is equal to:

$$Pr(\text{Inter-State War}) = F(\max\{c_A^{**}, c_B^{**}\})$$

where $F(\cdot)$ is the cumulative distribution function of c. It is straightforward to prove that $c_A^{**} \geq c_B^{**}$ if and only if $p_0 \leq 1/2$. Using this and the fact that c is uniformly distributed between 0 and 1, the ex-ante probability of war initiated by A is equal to:

$$Pr(\text{Inter-State War}) = \begin{cases} 0, & \text{if } (p_0 > 1/2 \text{ and } c_B^{**} \leq 0) \text{ or } (p_0 \leq 1/2 \text{ and } c_A^{**} \leq 0) \\ c_A^{**}, & \text{if } p_0 \leq 1/2 \text{ and } c_A^{**} \in [0,1] \\ c_B^{**}, & \text{if } p_0 > 1/2 \text{ and } c_B^{**} \in [0,1] \\ 1, & \text{if } (p_0 > 1/2 \text{ and } c_B^{**} > 1) \text{ or } (p_0 \leq 1/2 \text{ and } c_A^{**} > 1) \end{cases}$$

Proposition 6 *The ex-ante probability of m-war in a context of military polarization is weakly increasing in the initial mismatch between political and military power (that is, in the distance between p_0 and $1/2$) as long as β is sufficiently small ($\beta \leq \max(p_0, 1-p_0) + \frac{1}{2\delta}$), increasing in β if $p_0 \in [1/4, 3/4]$, decreasing otherwise, weakly increasing in the discount factor for extreme values of p_0, and weakly decreasing in the discount factor for intermediate values of p_0.*

In words, when the fixed jump in political power is not too large, the probability of m-wars is (weakly) increasing in the mismatch. When the mismatch itself is large, the probability of conflict is increasing in the value of the future, whereas in a more balanced world with less mismatch, the shadow of the future reduces the probability of war.

4 Comparison between P-Wars and M-Wars

We are now ready to compare p-wars and m-wars, in terms of their duration and the likelihood of their occurrence. For the results presented in this section, we assume that the initial military power is balanced, that is, $m_0 = 1/2$ and that the exogenous shift in political power after an m-war is small:

Assumption 3: $\beta \leq 1/6$.

Proposition 7 *An m-war lasts longer than a p-war.*

The intuition for this result is quite simple: the main future related goal of going to war is the readjustment of political power. Hence, conditional on a p-war occurring, the likelihood of having another one after the decisive outcome of the first is low, because the political power adjustment is endogenous. On the other hand, if an m-war occurs and the jump in political power from a victory is small, the winner may desire to continue the re-adjustment process.

Proposition 8 *P-wars are more likely than m-wars for any p_0 and δ.*

The intuition for this result is a bit more involved. Ex ante, when evaluating a status quo, the mismatch is particularly conducive to p-wars, as we show below, because the possibility to grab power and fix the mismatch is very tempting, and in relative terms particularly so when β is low. Thus, the two propositions are consistent: conditional on a war the m-war is more likely to continue if the winner has not completed the desired readjustment, but ex ante it is more likely that a p-war will start for a given mismatch because the chance of achieving such a readjustment immediately is higher. Future political power is more valuable then future military power: an m-war does not guarantee a permanent shift in political power, only a sure success in a future costly war (which will not always occur).

Proposition 9 *Both p-wars and m-wars are more likely when the initial mismatch between political and military power is larger. The likelihood of p-war onset is more sensitive to a change in the power mismatch than the likelihood of m-war onset.*

5 Conclusions

This paper provides the first comparative analysis of the frequency and duration of wars in different scenarios concerning the effects of a war on the changes in military and political power. In wars where groups' relative military power is sticky but winning the war may give full access to political power, the frequency of wars is higher and the duration is lower than in contexts in which a war induces small shifts in political power while the winner can determine the new military power balance. The effects that wars of these different types have on power shifts is an important and neglected key variable. The contexts where a war has the characteristics of the first type of wars (p-wars) tend to be more sensitive to inequality and development; on the other hand, m-wars, when they occur, last longer. All these theoretical predictions provide potentially useful nuances for future empirical work.

Appendix

Proof of Proposition 4

We have:

$$c_A^* = \left(\frac{1}{2} - p_0\right)(1+\delta) + \delta\left[\left(p_0 - \frac{1}{2}\right)\left|\frac{1}{2} - p_0\right| + \frac{\left(\left|\frac{1}{2} - p_0\right|\right)^2}{2} - \frac{1}{18}\right]$$
$$c_B^* = \left(p_0 - \frac{1}{2}\right)(1+\delta) + \delta\left[\left(\frac{1}{2} - p_0\right)\left|\frac{1}{2} - p_0\right| + \frac{\left(\left|\frac{1}{2} - p_0\right|\right)^2}{2} - \frac{1}{18}\right]$$

As explained in the text, the relevant cost cutoff for the ex-ante probability of war is c_A^* for $p_0 \leq 1/2$ and c_B^* for $p_0 > 1/2$. To determine the comparative static on the ex-ante probability of war with respect to p_0 and δ, we can thus look at the partial derivatives of the relevant cutoff in each interval.

For $p_0 \leq 1/2$, we have:

$$\frac{\partial c_A^*}{\partial p_0} = -1 - \delta\left(p_0 + \frac{1}{2}\right)$$
$$\frac{\partial c_A^*}{\partial \delta} = \frac{23}{72} - \frac{p_0}{2} - \frac{p_0^2}{2}$$

The first derivative is always negative. The second derivative is negative if $p_0 \in [0.44, 0.5]$ and positive if $p_0 \in [0, 0.44]$.

For $p_0 > 1/2$, we have:

$$\frac{\partial c_B^*}{\partial p_0} = 1 + \delta\left(\frac{3}{2} - p_0\right)$$
$$\frac{\partial c_B^*}{\partial \delta} = -\frac{49}{72} + \frac{3p_0}{2} - \frac{p_0^2}{2}$$

The first derivative is always positive. The second derivative is negative if $p_0 \in [0.5, 0.56]$ and positive if $p_0 \in [0.56, 1]$.

Proof of Proposition 6

We have:

$$c_A^{**} = \left(\frac{1}{2} - p_0\right)(1+\delta) + \delta\left[\left(p_0 - \frac{1}{2}\right)\left|\frac{1}{2} - p_0\right| + \frac{\left(\left|\frac{1}{2} - p_0\right|\right)^2}{2} + \frac{1}{4} - \frac{p_0}{2} - \frac{p_0^2}{2} + \beta\left(2p_0 - \frac{1}{2} - \frac{\beta}{2}\right)\right]$$
$$c_B^{**} = \left(p_0 - \frac{1}{2}\right)(1+\delta) + \delta\left[\left(\frac{1}{2} - p_0\right)\left|\frac{1}{2} - p_0\right| + \frac{\left(\left|\frac{1}{2} - p_0\right|\right)^2}{2} - \frac{3}{4} + \frac{3p_0}{2} - \frac{p_0^2}{2} + \beta\left(-2p_0 + \frac{3}{2} - \frac{\beta}{2}\right)\right]$$

As explained in the text, the relevant cost cutoff for the ex-ante probability of war is c_A^{**} for $p_0 \leq 1/2$ and c_B^{**} for $p_0 > 1/2$. To determine the comparative static on the ex-ante probability of war with respect to p_0, δ and β, we can thus look at the partial derivatives of the relevant cutoff in each interval.

For $p_0 \leq 1/2$, we have:

$$\frac{\partial c_A^{**}}{\partial p_0} = -1 - 2\delta p_0 - 2\delta\beta$$
$$\frac{\partial c_A^{**}}{\partial \delta} = \frac{1}{8} - p_0^2 + \beta\left(2p_0 - \frac{1}{2} - \frac{\beta}{2}\right)$$
$$\frac{\partial c_A^{**}}{\partial \beta} = \delta\left(2p_0 - \frac{1}{2} - \beta\right)$$

The first derivative is negative if and only if $\beta < \frac{1+2\delta p_0}{2\delta}$. The second derivative is positive if and only if $p_0 \in \left[\beta - \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2}\beta, \beta + \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2}\beta\right]$. The third derivative is negative if $p_0 < 1/4 + \beta/2$, positive otherwise.

For $p_0 > 1/2$, we have:

$$\frac{\partial c_B^{**}}{\partial p_0} = 1 + 2\delta - 2\delta p_0 - 2\delta\beta$$
$$\frac{\partial c_B^{**}}{\partial \delta} = -\frac{7}{8} + 2p_0 - p_0^2 + \beta\left(\frac{3}{2} - 2p_0 - \frac{\beta}{2}\right)$$
$$\frac{\partial c_B^{**}}{\partial \beta} = \delta\left(\frac{3}{2} - 2p_0 - \beta\right)$$

The first derivative is positive if and only if $p_0 + \beta > 1 + \frac{1}{2\delta}$. The second derivative is positive if and only if $p_0 \in \left[1 - \beta - \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2}\beta, 1 - \beta + \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2}\beta\right]$. The third derivative is negative if $p_0 > 3/4 - \beta/2$, positive otherwise.

Proof of Proposition 7

In order to prove the statement in the proposition, we need to compare the expected mismatches conditional on first period war in the two types of war.

$$E_{ISW}[M_1] = \frac{1}{2} - \beta \geq E_{CW}[M_1] = \frac{1}{3}$$

The inequality above is satisfied as long as $\beta \leq \frac{1}{6}$.

Proof of Proposition 8

In order to prove the statement in the proposition, we need to compare the two relevant cost cutoffs for the two types of war. When $p_0 \leq 1/2$, we want to find the conditions under which $c_A^* \geq c_A^{**}$. After re-arranging, this inequality becomes:

$$\frac{7}{36} \geq \frac{1}{2}(p_0 - p_0^2) - \beta \left(\frac{1}{2} - 2p_0 + \frac{\beta}{2} \right)$$

This inequality is satisfied for any $p_0 \in [0, 1/2]$ if $\beta \leq 1/6$ or $\beta \geq 5/6$. For intermediate values of β the inequality is satisfied for $p_0 \leq \frac{1}{2} + 2\beta - \frac{1}{6}\sqrt{108\beta^2 + 36\beta - 5}$.

When $p_0 \leq 1/2$, we want to find the conditions under which $c_A^* > c_A^{**}$. After re-arranging, this inequality becomes:

$$\frac{7}{36} \geq \frac{1}{2}(p_0 - p_0^2) + \beta \left(\frac{3}{2} - 2p_0 + \frac{\beta}{2} \right)$$

This inequality is satisfied for any $p_0 \in [1/2, 1]$ if $\beta \leq 1/6$ or $\beta \geq 1/6$. For intermediate values of β the inequality is satisfied for $p_0 \geq \frac{1}{2} - 2\beta + \frac{1}{6}\sqrt{108\beta^2 + 36\beta - 5}$.

Proof of Proposition 9

For $p_0 \leq 1/2$, we have:

$$\frac{\partial c_A^*}{\partial p_0} = -1 - \delta \left(p_0 + \frac{1}{2} \right)$$
$$\frac{\partial c_A^{**}}{\partial p_0} = -1 - 2\delta p_0 - 2\delta\beta$$

We want to show that:

$$\left|\frac{\partial c_A^*}{\partial p_0}\right| \geq \left|\frac{\partial c_A^{**}}{\partial p_0}\right|$$

The argument of the LHS absolute value is always negative. If both derivatives are negative, then this inequality is always satisfied since in this interval $p_0 + 1/2 \geq 2p_0$. If $\frac{\partial c_A^{**}}{\partial p_0}$ is positive, then we want to show that:

$$\begin{aligned} 1 + \delta(p_0 + 1/2) &\geq 2\delta\beta - 1 - \delta 2p_0 \\ 2 + \delta(3p_0 + 1/2) &\geq 2\delta\beta \end{aligned}$$

which is always satisfied.

For $p_0 > 1/2$, we have:

$$\begin{aligned} \frac{\partial c_B^*}{\partial p_0} &= 1 + \delta\left(\frac{3}{2} - p_0\right) \\ \frac{\partial c_B^{**}}{\partial p_0} &= 1 + 2\delta - 2\delta p_0 - 2\delta\beta \end{aligned}$$

We want to show that:

$$\left|\frac{\partial c_B^*}{\partial p_0}\right| \geq \left|\frac{\partial c_B^{**}}{\partial p_0}\right|$$

The argument of the LHS absolute value is always positive. If both derivatives are positive, then this inequality is always satisfied since in this interval $p_0 \geq 1/2$. If $\frac{\partial c_B^{**}}{\partial p_0}$ is negative, then we want to show that:

$$\begin{aligned} 1 + \delta(3/2 - p_0) &\geq 2\delta\beta - 1 - \delta(2 - 2p_0) \\ 2 + \delta(3/2 - p_0) + \delta(2 - 2p_0) &\geq 2\delta\beta \end{aligned}$$

which is always satisfied since he LHS is bounded below by 2 and the RHS is bounded above by 1.

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