Monopoly Distortions and the Welfare Gains from Trade

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Abstract

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1 Introduction

When some goods are monopolized and others are not, monopoly mark-ups distort the allocation of resources across goods. If trade barriers fall and firms that would otherwise have monopoly power are forced to compete with foreign firms, the decline in monopoly mark-up raises welfare by more efficiently allocating resources across goods. This paper analyzes this channel of welfare gain from trade, isolating its impact as distinct from standard Ricardian gains from trade.

In an influential recent paper, Arkolakis, Costinot, and Rodriguez-Clare (2009), hereafter ACR, derive a condition summarizing the welfare gains from trade that depends upon the aggregate share of expenditure on imported goods, as well as an elasticity parameter. The condition is valid across a variety of different models of the underlying sources of gains from trade, including the Ricardian framework of Eaton and Kortum (2002), as well as other frameworks like Armington, Krugman (1980), and Melitz (2003). While these models differ in underlying microeconomic structure, in terms of aggregate welfare they look alike, given the same observed aggregate trade flows. Simply put, in the general ACR framework, to get welfare benefits from trade, we need to have trade, and observations on trade flows tell us all we need to know about the benefits.

The starting point of this paper is the observation that a reduction in trade frictions *can* have impact, even if no trade is observed, if domestic firms are forced to change mark-ups to meet increased competition from foreign competitors. The threat of imports can impact resource allocation across domestic firms and these impacts will not get picked up in trade statistics. It is an old idea that trade can matter even when unseen, e.g., Markusen (1981), and the idea has been discussed in a variety of contexts.[1] A new idea here is to put this mechanism into the ACR framework, contrasting the gains from trade when this mechanism is taken into account, to the predicted gains according to the ACR condition where the mechanism is shut down (because mark-up distributions in the ACR analysis are invariant to trade regime).

In the model, the preference structure is CES, the workhorse of trade analysis. To allow mark-up distributions to vary with trade regime, we follow the approach of de Blas and Russ (2010) which generalizes Bernard, Eaton, Jenson, and Kortum (2003). BEJK is a Ricardian model of trade with productivity heterogeneity across firms. As trade frictions are lowered, welfare gains are achieved through increased selection, as production is reallocated from inefficient domestic firms to more efficient foreign firms. This mechanism of BEJK fits into the general framework of ACR.[2] Oligopolistic firms compete in a Bertrand fashion,

[1] Recent examples include Schmitz (2005) and Salvo (2010).

[2] In the presentation, ACR focus on Eaton and Korum (2002) which is a model of perfect competition.

and, with heterogeneity in productivity, mark-ups are positive. However, the distribution of the mark-up is fixed and does not change when trade barriers are reduced. In such an event, prices indeed fall, but with increased selection, costs also fall and in an elegant result, the two forces exactly counterbalance. de Blas and Russ (2010) differ from BEJK in assuming a finite number of firms for each product market; BEJK can be thought of as the limit where this number is large. Away from this limit, mark-up distributions do depend on trade regime. de Blas and Russ (2010) put the structure to work to look at the impact of reducing trade frictions on pricing issues, e.g. pricing-to-market and imperfect pass-through. Our paper is different in that we focus on allocative efficiency, and in particular, derive the connection to the welfare analysis of ACR.

Exploiting a convenient functional form assumption (the Fréchet), we derive an expression for the welfare gain from trade that separates out the impact of Ricardian gains from trade, from the impact of changes in allocative efficiency. The Ricardian gains from trade are as formulated in BEJK and as captured in the ACR statistic. We consider that before a change in trade regime, products may differ as to whether there is one domestic competitor (i.e., monopoly in autarky), two domestic competitors (i.e., duopoly in autarky), and so on. The impact on allocative efficiency can be further decomposed into the impact on allocative efficiency *within* a group of firms with the same initial market structure (i.e. monopoly good or duopoly goods), and *across* goods that vary by initial market structure.

Decreasing trade barriers can potentially *reduce* within group allocative efficiency. Consider for example that if we begin in autarky with all firms being monopolists, setting a constant monopoly mark-up, then first best allocative efficiency across products is achieved. (The equal monopoly taxes on each good exactly cancel out in the marginal conditions for consumer choice, so there are no distortions.) Opening up to trade from this point can only increase the dispersion in mark-ups and thereby reduce allocative efficiency amongst the monopoly goods. The point that a constant monopoly mark-up across all goods eliminates distortions has long been understood.(See Robinson (Ch 27, 1934)).

Decreasing trade barriers raises *across-group* allocative efficiency. Mark-ups in markets that are monopoly without trade fall relatively more than they do if they were say, initially duopoly without trade. Compressing mark-ups across groups reduces the monopoly distortion. In a simple Bertrand model where all firms have the same cost, it is easy to see that the impact of the second firm in a market is bigger than the third, as adding the second lowers price from the monopoly level to marginal cost, while adding more firms beyond that makes no difference. Here with cost heterogeneity, mark-ups are positive, even with many

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BEJK is a Bertrand oligopoly version of Eaton and Kortum and the welfare analysis of trade friction reductions is equivalent.

firms. Nevertheless, the property that the second firm is key carries over into the more general case. Depending on initial conditions, we find that taking into account across-group allocative efficiency can lead to potentially large welfare gains beyond that measured by the ACR statistic.

There has recently been significant interest in the literature in models where changes in trade regime impact the distribution of mark-ups, in addition to de Blas and Russ (2010) mentioned above. Melitz and Ottaviano (2008) develop a framework that captures the selection and reallocation effects of trade, highlighted in Melitz (2003), while at the same time allows for mark-ups to be endogenous. The market structure in Melitz and Ottaviano is monopolistic competition, so firms do not compete head-to-head as they do in oligopolistic competition. Our work is different, not only in having head-to-head competition, but also in particular in the important role that differences in head-to-head competition (some markets being monopoly and others duopoly) plays in the analysis. Atkeson and Burstein (2008) is a recent model in the CES tradition with head-to-head competitors competing in a Cournot fashion. Their focus on pricing issues like imperfect pass-through is very different from the welfare issues about allocative efficiency considered here.

Most closely to our paper is the recent working paper Edmond, Midrigan, and Xu (2011). The papers are complements, written independently of each other, both highlighting allocative efficiency gains from trade through changes in mark-ups. Their focus is quantitative, applying their model to data on Taiwanese manufacturing establishments, while ours is theoretical, in particular in deriving an explicit link to ACR, and exploring how different parameters impact different channels. The modeling structures differ; their firms are Cournot quantity competitors (building on Atkeson and Burstein (2008)), while our firms are Bertrand price competitors. It is an open question as to which is preferable for empirical purposes, but for the theoretical exercise conducted here, the Bertrand approach has proved tractable. Another difference is the distinction we highlight between allocative efficiency *across* markets varying in concentration, and *within* markets with the same concentration, a distinction that plays a central role in our paper.

Our paper is connected to the recent literature relating distortions to aggregate productivity. (See Rustucia and Rogerson (2008), and Hsieh and Klenow (2009).) This literature has focused on policy distortions, such as taxes on some firms and subsidies on others, that distort the allocation of resources across firms. Differential mark-ups across goods, arising through differences in market power, operate like tax differentials in terms of the distortions they create. More generally, there has been great interest in recent years in incorporating heterogeneity into economic models of industry, such as heterogeneity in productivity, as in Melitz (2003), or heterogeneity in distortions faced, as in the distortions literature. This paper highlights a different kind of heterogeneity—that pre-trade some markets will have monopolies and others will have competition—and with such heterogeneity, opening up to trade delivers welfare gains not captured in standard models, and not picked up in trade flows.

Turning to an overview of the paper, Section 2 develops the model and highlights specific results in the literature that we use. Section 3 derives welfare conditions, holding fixed market structure, i.e., the number of firms for each product in each country. Section 4 endogenizes market structure by introducing a fixed cost and allowing free entry. An interesting feature of the model is that it nests two classic trade models as limiting cases. When fixed costs are large, it reduces to the Krugman (1980) model of Dixit-Stiglitz monopolistic competition, because entry is less than the total number of varieties, and firms spread out as monopolists. When fixed costs are very low, there are many firms per variety and the model goes to BEJK. We focus on an intermediate case where in autarky there is a mix of some goods with monopoly and others with duopoly.

2 Model

2.1 Description of the Model

The consumption composite is an aggregation of differentiated goods indexed by ω on the unit interval. It follows the standard CES form,

$$Q = \left(\int_0^1 (q(\omega))^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution. We also consider the Cobb-Douglas case, $\sigma = 1$, in which

$$Q = \exp \left(\int_0^1 \ln q(\omega) d\omega \right).$$

There are I countries indexed by $i \in 1, 2, ..., I$. Each country has a measure L_i workers. Labor is the only factor of production.

In the initial formulation, we take as given the number of firms $n_i(\omega) \in \{0, 1, 2, ..\}$ that are capable of producing product ω in country i. Later, in Section 4, we introduce a fixed cost of entry and make the number of firms endogenous.

There is an iceberg trade friction to ship goods from one country to another. Formally, to deliver one unit of product ω, from source country i to destination j, $\tau_{ij} \geq 1$ units of product need to be shipped. Assume $\tau_{ii} = 1$, so there is no transportation cost for local shipments.

Each firm draws a random level of productivity. A firm capable of producing product j, drawing productivity Z, can produce Z units of product j per unit of labor input. The process of drawing Z is independent and identically distributed across all firms in the same country. We follow de Blas and Russ (2010) and assume firms draw from the Fréchet distribution, with c.d.f.

$$F_i^{draw}(z) \equiv \Pr[Z \leq z] = e^{-\zeta_i z^{-\theta}}, \tag{1}$$

where $\zeta_i > 0$ is a country-level scaling parameter and θ determines curvature. The parameter ζ_i governs a country's efficiency. The parameter θ is a measure of similarity; the bigger θ, the lower the variance in productivity draws. As in Eaton and Kortum (2002), assume $\theta > \sigma - 1$. The role of this assumption is analogous to that of $\sigma > 1$ in a standard Dixit-Stigliz model. If $\theta < \sigma - 1$, heterogeneity in costs is so large relative to curvature of preferences that expressions involving welfare and price indices become undefined.

Suppose we take n independent draws at location i and let $Z_{1,i}$ denote the random realization of the highest value. The distribution is

$$F_{1,i}(z_1; n) \equiv \Pr(Z_{1,i} \leq z_1 | n) = \left(F_{1,i}^{draw}(z_1)\right)^n = e^{-n\zeta_i z_1^{-\theta}} \tag{2}$$

Thus the distribution of the highest value is also Fréchet with scaling parameter $n\zeta_i$ and the original curvature parameter θ. The Fréchet is an extreme value distribution which means that the highest from a set of independent draws remains within the same distribution family. There are three different extreme value distributions and the Fréchet distribution is the applicable one when the underlying draws come from a distribution with a fat right rail.

The assumption of the Fréchet for the highest value by Eaton and Kortum (2002) and BEJK is a core assumption delivering their tractable formulations that have had wide impact. This assumption can be derived from various other, more primitive, assumptions, such as assuming the underlying distribution of draws is some general fat-tailed distribution, which includes the Pareto distribution and the log-normal distribution, and then taking the maximum over a large number of draws. We note that assumption (2) is stronger in that Fréchet is imposed for each individual draw, not just at the extremes.

We impose the Fréchet (2) at the firm level because it appears to be what is needed to draw the analytic connection to ACR, itself a functional form based exercise. We note the Fréchet it is very similar in shape to the Pareto distribution, the distribution which is typically assumed to hold in models of monopolistic competition. In particular, on the right side of the distribution the Fréchet satisfies the property of "tail equivalence" with the Pareto.[3] While the distributions look different on the left (log productivity is bell-shaped under Fréchet, but not under the Pareto), as a model of selection, the right side of the distribution is more important than the left. In any case, the bell-shaped feature of the Fréchet is a desirable property since empirical distributions of productivity are typically reported as being bell-shaped (e.g. Syverson (2004)). While the particular functional form assumption we make here is of course special, the analysis sheds light on broader economic considerations that are likely applicable in general environments.

2.2 Results in the Literature

In this subsection we briefly review the results from BEJK, de Blas and Russ (2010), and ACR, that we need for our analysis. Recall a particular market j has $n_i(\omega)$ firms located in country i that are capable of producing the good. In this subsection we assume that $n_i(\omega)$ is constant across all ω and drop the index ω.

Consider a destination country j. Take a source country i and let the wage at i be w_i. A firm at source country i drawing productivity Z, has a marginal cost of $w_i\tau_{ij}/Z$ to deliver one unit to destination j. We invert marginal cost and call it the firm's *cost-adjusted productivity* of selling in market j,

$$\tilde{Z}_{ij} = \frac{Z}{w_i\tau_{ij}}.$$

Cost-adjusted productivity is the amount of good the firm can deliver to location i, per unit numeraire expenditure on input. Define $\tilde{Z}_{1,ij}$ to be the maximum cost-adjusted productivity for selling to destination j across all n_i firms at source i capable of producing the product, unless $n_i = 0$, in which case set $\tilde{Z}_{1,ij} = 0$. Analogously, let $\tilde{Z}_{2,ij}$ be the second highest value, unless $n_i \leq 1$, in which case $\tilde{Z}_{2,ij} = 0$.

In each destination market, firms from all countries compete in a Bertrand fashion. Let $\tilde{Z}_{1,j}$ be the highest cost-adjusted productivity of delivering to destination j across all sources,

$$\tilde{Z}_{1,j} = \max_u \left\{\tilde{Z}_{1,uj}\right\}.$$

The firm with highest cost-adjusted productivity wins the Bertrand price game, becoming the supplier to destination j. To determine the resulting price, we also need to know the second highest cost-adjusted productivity $\tilde{Z}_{2,j}$ across all sources. Note this might be the second highest in the same originating country as the winning firm, or the first highest in some other country. If the winning firm is a monopolist, meaning there are no other

[3] They are *tail equivalent* in the sense that the tail probabilities on the right are proportional to each other in the limit.

producers from the same origination or any other origination, then $\tilde{Z}_{2,j} = 0$. The resulting price equals

$$p(\tilde{Z}_{1,j}, \tilde{Z}_{2,j}) = \min\left(\frac{1}{\tilde{Z}_{2,j}}, \frac{\mu}{\tilde{Z}_{1,j}}\right). \tag{3}$$

for

$$\mu \equiv \frac{\sigma}{\sigma - 1}.$$

The winning firm matches the delivered cost $1/\tilde{Z}_{2,j}$ of the second-most efficient firm, unless the winning firm's cost advantage is so strong that it can set the monopoly price and still undercut the second-most efficient firm. The monopoly mark-up μ is the standard form when the elasticity of demand is constant and equal to σ.

Country i is the source to destination j, if the most efficient firm at i has higher cost-adjusted productivity of serving j, compared to the most efficient at all other locations. BEJK show the probability of this event equals

$$\pi_{ij} \equiv \Pr\left[\tilde{Z}_{1,ij} > \max_{u \neq i}\left\{\tilde{Z}_{1,uj}\right\}\right] = \frac{n_i \zeta_i \left(w_i \tau_{ij}\right)^{-\theta}}{\sum_{u=1}^{I} n_u \zeta_u \left(w_u \tau_{ui}\right)^{-\theta}}, \tag{4}$$

where $n_i \zeta_i$ corresponds to the country level technology parameter in BEJK notation. For ease of exposition, define

$$\phi_{ij} = \zeta_i \left(w_i \tau_{ij}\right)^{-\theta}. \tag{5}$$

as a summary measure of the "quality" of a productivity draw at source country i for selling to destination j. The quality adjustment is higher with a better technology ζ_i, and is discounted with a higher wage or transportation cost. Define

$$\Phi_j \equiv \sum_{u=1}^{I} n_u \phi_{uj} \tag{6}$$

as the sum across locations of all the quality-adjusted draws for serving j. The formula for the probability i sells to j can then be expressed as $\pi_{ij} = n_i \phi_{ij} / \Phi_j$ which can be interpreted as origination i's share of the (quality adjusted) draws for selling in j.

We turn now to mark-ups. Following (3), the mark-up at country j equals

$$M(\tilde{Z}_{1,j}, \tilde{Z}_{2,j}) = \min\left(\frac{\tilde{Z}_{2,j}}{\tilde{Z}_{1,j}}, \mu\right). \tag{7}$$

Assuming the underlying draws are Fréchet as in (1), and with mark-ups determined through (7), de Blas and Russ (2010) show that the distribution of the mark-up at destination j, conditioned upon the sales originating at source i, is given by

$$F_{ij}^{m}(m) = \begin{array}{c} 1 - \frac{1}{m^{\theta} - \frac{\phi_{ij}}{\Phi_j}\left(m^{\theta}-1\right)}, \text{ for } m < \mu, \\ 1, \text{ for } m \geq \mu. \end{array}$$

To interpret this formula, note first that if there is only a single monopoly firm at location i for each good, then $\phi_{ij} = \Phi_j$, and the formula reduces to $F_{ij}^{m}(m) = 0$, for $m < \mu$, i.e. the distribution is degenerate at the monopoly mark-up. It is worth emphasizing that for *any* assumption about the distribution of draws, the monopoly mark-up is degenerate at μ, so in particular this has nothing to do with our assumption of Fréchet. Next consider what happens if we make all the n_i large, increasing them proportionately. In this case ϕ_{ij} stays the same while Φ_j blows up, and the ratio ϕ_{ij}/Φ_j goes to zero. In particular, if $\phi_{ij} = 1$ all i and j then the ratio equals $1/\sum n_i$. In the limit with large n_i, the mark-up distribution reduces to $F_{ij}^{m}(m) = 1 - \frac{1}{m^{\theta}}$, for $m < \mu$, which is the mark-up distribution in BEJK. It is worth emphasizing that if the underlying distribution of productivity is a general fat-tailed distribution, such as the Pareto or the log normal, then the distribution of the maximum goes to Fréchet, when the number of draws gets large. Thus formula (**??**) holds in the large n limit for this more general distribution class. As the formula also holds at the other extreme of monopoly, we can think of our assumption of the Fréchet as a convenient way to connect the dots in between.

We conclude by deriving the ACR result for the BEJK limiting case of the model. To take limits, suppose that $n_i = n_i^{\circ}\eta$ and $\zeta_i = \zeta_i^{\circ}/\eta$, for scaling parameter η and constants n_i° and ζ_i°. With this scaling, as η gets large, the number of draws goes to infinity while the distribution of the maximum value remains a constant. It is straightforward to show in the limit, as η goes to infinity, that the distribution of the first and second best draw at each location takes the form assumed by BEJK. Hence the BEJK results hold for this limiting case. BEJK show the price index at j is $P_j = \gamma\Phi_j^{-1/\theta}$, for a constant γ that depends upon θ and σ (held fixed here) but is independent of the trade friction. BEJK show the share of variable costs in revenue is $\theta/(1+\theta)$. Welfare at location j can be written

$$Welfare_j = \frac{Y_j}{P_j} = \frac{w_j L_j}{P_j \frac{w_j L_j}{R_j}} = \frac{w_j L_j}{P_j\left(\frac{\theta}{1+\theta}\right)}, \tag{8}$$

where Y_j denotes income at j and R_j denotes total spending at j. The second equality is a straightforward manipulation using income equal to revenue, $Y_j = R_j$. The final equality plugs in the BEJK result on the variable cost share.

We examine the impact of a change in trade friction. Let τ_{ij}° denote the initial trade friction between i and j and τ'_{ij} be the new level. Without loss of generality we examine the welfare impact on country 1 and let the wage in county 1 be the numeraire. The ratio of welfare in the new case relative to the initial is

$$W_{BEJK} = \frac{\frac{L}{P'\left(\frac{\theta}{1+\theta}\right)}}{\frac{L_1}{P^\circ\left(\frac{\theta}{1+\theta}\right)}} = \frac{\frac{1}{P'}}{\frac{1}{P^\circ}} = \left(\frac{\Phi'}{\Phi^\circ}\right)^{\frac{1}{\theta}}, \tag{9}$$

where we leave the country index implicit. Note that since the variable cost share is invariant, incomes are not impacted by the change and the welfare ratio is simply the ratio of the inverse price indices.

Following the notation in ACR, let λ be the fraction of expenditure on domestic goods. BEJK show this fraction equals

$$\lambda = \pi_{11} = \frac{n_1\zeta_1}{\Phi_1}, \tag{10}$$

where we use (5) to substitute in for ϕ_{11} in the definition of π_{11}. Let R_{ij} be the total spending of j's consumers on i's goods. Following ACR, let the trade elasticity be

$$\varepsilon \equiv \frac{\partial \ln\left(R_{21}/R_{11}\right)}{\partial \ln \tau}, \tag{11}$$

Note that

$$\frac{R_{21}}{R_{11}} = \frac{\pi_{21}R_1}{\pi_{11}R_1} = \frac{\pi_{21}}{\pi_{11}} = \frac{n_2\phi_{21}}{n_1\phi_{11}} = \frac{n_2}{n_1}\left(\frac{w_2}{w_1}\right)^{-\theta}\tau^{-\theta}.$$

With symmetric countries $n_2/n_1 = 1$ and $w_2/w_1 = 1$, so the trade elasticity ε equals $-\theta$. Or if the ratios n_2/n_1 and w_2/w_1 are held fixed, $\varepsilon = -\theta$. Hence,

$$W_{BEJK} = \left(\frac{\Phi'_1}{\Phi^\circ_1}\right)^{\frac{1}{\theta}} = \left(\frac{\lambda^\circ}{\lambda'}\right)^{\frac{1}{-\varepsilon}} \equiv W_{ACR}, \tag{12}$$

which is the summary statistic reported in ACR. The second equality follows from (10) and the fact that n_1 and ζ_1 are held fixed, and $\varepsilon = -\theta$.. We can see that the statistic (12) reported in ACR is in fact equal to the welfare ratio calculated in (9).

3 Results when Market Structure is Fixed

This section determines the welfare impact of trade liberalization taking the market structure, i.e., the firm counts for each product, as given. Suppose the interval of products can be broken up into K groups, with shares α_k in each group, $\sum_{k=1}^{K}\alpha_k = 1$. Group 1 is the subinterval $\omega \in [0, \alpha_1)$, group 2 is $\omega \in [\alpha_1, \alpha_2)$, group 3 is $\omega \in [\alpha_1 + \alpha_2, \alpha_3)$, and so on.

Assume the higher the k, the proportionately higher the firm counts. Formally, assume the products in group k in country i have $n_{k,i} = kn_{1,i}$ firms. In the leading case we will consider there are two groups; group 1 has one firm for each product in each country, and group 2 has two firms.

With this proportional structure across groups, the variable Φ_j^k for group k at country j will be proportional to k,

$$\Phi_j^k = k\Phi_j^1.$$

Recall this variable can be interpreted as the quality adjusted number of draws summed across all suppliers selling at j. With this proportional structure, the probability that country i is the lowest cost supplier to destination j is the same across groups k, and so we can denote it π_{ij}, the same as before, with no index for k.

The first step of the analysis is to derive an expression for the price index of each product group.

Lemma 1. For $\sigma > 1$, the price index of group k products in country j can be written as

$$P_{k,j} = \left(k\Phi_j^1\right)^{-\frac{1}{\theta}} \left(E_j^k[m^{-(\sigma-1)}]\right)^{\frac{-1}{\sigma-1}}$$

for

$$E_j^k[m^{-(\sigma-1)}] = \sum_{i=1}^{I} \pi_{ij} \int_0^\infty \left[\int_m m^{-(\sigma-1)} dH_{ij}^k(z)\right] z^{(\sigma-1)} f(z)dz.$$

The distribution $H_{ij}^k(m,z)$ is the c.d.f. of mark-ups for group k, given a sale at j originates at i, and given the winner has cost adjusted productivity $\tilde{Z}_{1,ij}^k = \Phi_j^k z$. The density $f(z)$ is Fréchet, with curvature θ and a unit scaling parameter. For the Cobb-Douglas case, the price index for group k at j is

$$\begin{aligned}
P_{k,j} &= \xi \left(k\Phi_j^1\right)^{-\frac{1}{\theta}} \exp\left(E_j^k[\ln m]\right) \\
E_j^k[\ln m] &= \sum_{i=1}^{I} \pi_{ij} \int_0^\infty \left[\int_m \ln m dH_{ij}^k(z)\right] f(z)dz \\
\xi &\equiv \exp\left(-E \ln z\right)
\end{aligned}$$

Proof. See appendix.

Two points are worth noting. First, we see that the price index for each product group can be written in a form that is multiplicatively separable in a first term $\Phi_j^k = k\Phi_j^1$ that subsumes the trade frictions, firm counts, the technology scaling parameter ζ_i for each country i, and a second term that is a function of the distribution of mark-ups. In BEJK, the mark-up distribution is independent of the trade frictions, and independent of the source and destination country. Thus in BEJK, the expectation within the parenthesis is the same for each i and j and invariant to trade frictions, and so it is a constant term in the price index, that factors out in welfare analysis. In contrast, here a change in the trade friction changes the distribution of mark-ups, so we need to take this term into account.

Second, the price index has a particularly convenient form in the $\sigma = 1$ case. The multiplicative term with the mark-up equals the exponential of the expected value of the log mark-up. Each product is equally weighted in the expected value, since with $\sigma = 1$, the expenditure shares are the same. The case of $\sigma > 1$ is more complicated since expenditure shares depend upon the relative productivity draw of the supplying firm. The integral over a standardized value of the productivity draw takes this into account.

To present our welfare result, we need to introduce some additional notation. For a particular product with mark-up m, let $c = 1/m$ be the inverse of the mark-up. This equals the share of variable cost in the price for the particular product. Let $\bar{c}^k_{buy}$ denote the revenue-weighted value of c for all products in group k purchased at county 1. (For notational simplicity, we leave the index for country 1 implicit.) Analogously, let $\bar{c}^k_{sell}$ be the revenue-weighted average of the cost share c across all products in group k originating at location 1. As noted earlier, in the BEJK limit, these shares are a constant, $\bar{c}^k_{sell} = \bar{c}^k_{buy} = \theta / (1 + \theta)$.

Let W_{total} be the ratio of welfare in the prime (new) case to the naught (initial) case. Our result is

Proposition 1. (i) For $\sigma = 1$, we can write the welfare ratio in the following form

$$W_{total} = W_{BEJK} \times \prod_{k=1}^{K} \left(\frac{\bar{c}^{k\prime}_{buy} / \bar{c}^{k\prime}_{sell}}{\bar{c}^{k\circ}_{buy} / \bar{c}^{k\circ}_{sell}} \right)^{\alpha_k} \times \prod_{k=1}^{K} \left(\frac{A^{k\prime}_{Within}}{A^{k\circ}_{Within}} \right)^{\alpha_k} \times A_{across} \tag{13}$$

for

$$\begin{aligned} W_{BEJK} &= \left(\frac{\Phi^{1\prime}}{\Phi^{1\circ}} \right)^{1/\theta} \\ A^k_{Within} &\equiv \frac{\exp\left(E^k[\ln c]\right)}{\bar{c}^k_{buy}} \\ A_{Across} &\equiv \prod_{k=1}^{K} \left(\frac{L_k / L}{\alpha_k} \right)^{\alpha_k}. \end{aligned}$$

(ii) For the $\sigma > 1$ case, the welfare ratio can be written,

$$W_{total} = W_{BEJK} \times \frac{\bar{c}^{\prime}_{buy} / \bar{c}^{\prime}_{sell}}{\bar{c}^{\circ}_{buy} / \bar{c}^{\circ}_{sell}} \times \frac{A^{\prime}}{A^{\circ}},$$

$$A \equiv \left[\sum_{k=1}^{K} \left(\frac{\alpha_k k^{\frac{(\sigma-1)}{\theta}}}{\sum_{u=1}^{K} \alpha_u u^{\frac{(\sigma-1)}{\theta}}} \right) \left(\frac{\bar{c}_{buy}^k}{\bar{c}_{buy}} \right)^{(\sigma-1)} \left(A_{Within}^k \right)^{(\sigma-1)} \right]^{\frac{1}{(\sigma-1)}},$$

for

$$A_{Within}^k \equiv \frac{\left(E^k[c^{(\sigma-1)}] \right)^{1/(\sigma-1)}}{\bar{c}_{buy}^k}.$$

Proof. For $\sigma = 1$, the aggregate price index can be expressed using the price index of each group k through $P = \prod_{k=1}^{K} \left(P^k \right)^{\alpha_k}$. From Lemma 1, we can substitute in for P^k to write welfare as

$$Welfare = \frac{Y}{\prod_{k=1}^{K} \left(P^k \right)^{\alpha_k}} = \frac{Y \bar{k}}{\xi} \left(\Phi_j^1 \right)^{\frac{1}{\theta}} \prod_{k=1}^{K} \left[\exp \left(E^k [\ln c] \right) \right]^{\alpha_k}$$

where $\bar{k} \equiv \prod_{k=1}^{K} k^{\alpha_k}$, and noting $-\ln m = \ln c$.

Let R_j denote spending at j. On account of $\sigma = 1$, spending on each product is the same. Since a fraction π_{1j} of all products purchased at j are produced at 1, sales originating at 1 with destination j equal $\pi_{1j} R_j$. Total income at 1 is the aggregate of sales across all destinations,

$$Y = \sum_{j=1}^{I} \pi_{1j} R_j.$$

The quantity of labor at location 1 allocated to group k must equal

$$L^k = \bar{c}_{sell}^k \sum_{j=1}^{I} \pi_{1j} \alpha_k R_j = \bar{c}_{sell}^k \alpha_k Y.$$

The first equality follows from the definition $\bar{c}_{sell}^k$ as variable cost at country 1 to produce group k as a fraction of total sales revenue. (Note also the numeraire assumption, $w_1 = 1$). The second equation substitutes in the previous equation for Y. Multiplying welfare by $1 = \left(L^k / \bar{c}_{sell}^k \alpha_k Y \right)^{\alpha_k}$ for each k, and cancelling Y, yields (13), completing the proof of part (i).

To prove (ii), start by writing welfare equal to $L / (\bar{c}_{sell} P)$. Using the standard formula for the overall price index P as a function of the group-level prices P^k, substituting in the formula for P^k in Lemma 1, and straightforward manipulation yields the result. Q.E.D.

To discuss the result, we begin with the $\sigma = 1$ case which is the cleanest. The welfare impact is decomposed into four parts. The first component is the Ricardian welfare gain through the BEJK channel, where the distribution of mark-ups is fixed. The remaining three components are all related to changes in the distribution of mark-ups. The first mark-up component can be thought of as a terms of trade effect through mark-ups. A country benefits if the variable cost share of revenue of what it buys is higher than the share of what it sells. The remaining two mark-up-related welfare components are both measures of allocative efficiency, one a *within* group measure, the other an *across* group measure.

Consider the formula for A^k_{within}, the index of within-group allocative efficiency for group k. The numerator is the exponential of the expected log cost share. The denominator is the expected cost share, without logs or exponentials. If c were constant across all products in the group, i.e., a constant markup, then $A^k_{within} = 1$. In this case, we get first-best allocative efficiency within the group, as a consumer's marginal rate of substitution between any pair of goods in the group equals the ratio of marginal costs. Increasing the mark-up, leaving it constant across products, does not lead to distortions, because the constant mark-up cancels out in the marginal conditions between any pair of products within the group. Next note that if there is any variation in c across products in group k, then $A^k_{within} < 1$. (To see this, take $\ln\left(A^k_{within}\right)$ and observe that $E^k[\ln c] < \ln\left(E^k[c]\right)$ because of Jensen's inequality.) Thus changes that reduce the dispersion of c within a group increase within-group allocative efficiency and thereby increase welfare, everything else fixed.

Next consider A_{across}, the index of allocative efficiency across groups. It is a function of the employment share of each group. A social planner maximizing total surplus will allocate employment across groups in proportion to the shares α_k, in which case the index $A_{across} = 1$. For any other allocation, the index is strictly less than one. Changes that reallocate employer shares closer to the social planner's level thereby raise welfare, everything else fixed.

Turning to the $\sigma > 1$ case, like the $\sigma = 1$ case we can separate out the effect on allocative efficiency from the Ricardian welfare component and a mark-up terms of trade component. Overall allocative efficiency, in turn, can be expressed as a function of with-in group allocative efficiency and terms related to across-group efficiency.

Our next task is to connect the analysis to the statistic derived in ACR, which again is

$$W_{ACR} = \left(\frac{\lambda^{\circ}}{\lambda'}\right)^{\frac{1}{-\varepsilon}},$$

for the trade elasticity

$$\varepsilon \equiv \frac{\partial \ln\left(R_{21}/R_{11}\right)}{\partial \ln \tau},$$

where R_{ij} is expenditure at j on goods from i. We have an analytic result for σ at its upper and lower bound.

Proposition 2. At $\sigma = 1$ or the limit as σ goes to $\theta + 1$, $W_{BEJK} = W_{ACR}$.

Proof. See the Appendix.

At the limits where $\sigma = 1$ and $\sigma = \theta + 1$, the trade elasticity ε equals θ and the ratio Φ'/Φ° equals the ratio of the domestic shares λ'/λ°. In between these limits, the ratios differ and ε does not equal θ. We don't have an analytic result for the range in between, but our numerical analysis indicates that W_{BEJK} is less in absolute value than W_{ACR}.

To illustrate these numerical results, we consider the benchmark cases considered in ACR. ACR consider $\varepsilon = -5$ and $\varepsilon = -10$ as representative values of the trade elasticity from the literature. The paper considers $\lambda = .93$ as a representative value for the spending share on domestic goods (in the U.S.). With this benchmark, the gain from trade relative to autarky for $\varepsilon = -5$ is $W_{ACR} = (1/.93)^{1/5} = 1.014$. For the higher value of the trade elasticity, the gain is $W_{ACR} = (1/.93)^{1/100} = 1.007$.

For this discussion, we compare W_{ACR} to the value of W_{BEJK} calculated in the symmetric two-country case with a single firm for each product at each location, $n = 1$. For a given value of σ, we back out the value of τ and θ consistent with $\lambda = .93$ and a trade elasticity of $\varepsilon = -5$ (or $\varepsilon = -10$). Let the τ so calculated be τ' and let $\tau^{\circ} = \infty$, and we calculate W_{BEJK} with these. Figure 1 plots W_{BEJK} and W_{ACR}. Consistent with the proposition at the limits of the range of σ, $W_{BEJK} = W_{ACR}$. In between W_{BEJK} is less than W_{ACR}, with the maximum discrepancy a little less than two thirds the percentage gain.

Having examing the BEJK welfare term and connected it to the ACR statistic, our next step is to consider the remaining terms of the overall welfare gain. We start by looking where σ is close to its upper limit.

Proposition 3 In the limit as σ goes to its upper bound all the components of W_{total} go to one except W_{BEJK}, and thus

$$\lim_{\sigma \to \theta+1} W_{total} = \lim_{\sigma \to \theta+1} W_{BEJK} = \lim_{\sigma \to \theta+1} W_{ACR}.$$

Proof. See Appendix.

According to the result, if σ is at its upper limit, the total impact on welfare is entirely accounted for by the Ricardian term. And given the equivalence between the ACR statistic and the Ricardian term from Proposition 2, the ACR statistic matches the exact welfare gain. We can get intuition for this result by bounding cost as a share of revenue. The monopoly mark-up is $\sigma/(\sigma - 1)$, so cost as a share of revenue is bounded below by $(\sigma - 1)/\sigma$. It is also bounded above the share $\theta/(1 + \theta)$ in BEJK. As σ is raised to $\theta + 1$, the lower bound from monopoly converges to the upper bound from BEJK, and the cost share is pinned down in the limit at $\theta/(1 + \theta)$, regardless of the trade friction or firm counts. As the cost share distribution is invariant, the within and across allocation efficiency is invariant, and cancel out in the welfare ratio, leaving only the BEJK Ricardian component.

Because the limit where σ goes to $1+\theta$ eliminates interesting issues about mark-up distributions, we place our focus away from the limit so that mark-up distributions are in play. We start with the other extreme of $\sigma = 1$. We look at symmetric two-country examples, which eliminates terms of trade issues. Let $\tau \geq 1$ be the trade friction between the two countries. Assume two product groups. In product group 1, there is one firm in each country and in product group 2 there are two firms in each country.

Figure 2 plots the within allocative efficiency index A^1_{within} for group 1 as a function of τ, for several different values of θ and for $\sigma = 1$. The index monotonically declines with the friction τ. For intuition, consider the case of frictionless trade, $\tau = 1$. With $\sigma = 1$, the pure monopoly mark-up is infinite, so all prices equal the limit price of the second highest cost. With symmetric countries and frictionless trade, a seller of a particular product is equally likely to be the foreign firm as the domestic firm (there is one of each) and the mark-up distributions are the same. If we add a trade friction, when the seller is domestic, the mark-up rises and when the seller is foreign the mark-up falls. This force spreads mark-ups apart, lowering allocative efficiency. A plot of the analogous within index for group 2 (not shown) is also downward sloping, but much flatter.

Suppose half of all products have one firm in each country and the other half have two in each country, $\alpha_1 = \alpha_2 = \frac{1}{2}$. Figure 3 plots the across allocative efficiency index A_{across}, showing that it is a decreasing function of τ. A social planner allocates half of the labor force to each group. In contrast, in the market allocation, with higher mark-ups in group 1 than group 2, less than half of the labor force ends up in group 1. Raising τ spreads mark-ups between the two groups further apart, lowering A_{across}.

We conclude that for $\sigma = 1$, increases in τ impact within and across allocative efficiency in the same direction, decreasing both. Recall also from Proposition 2 that the ACR statistic coincides with the BEJK Ricardian term. Thus the ACR statistic, which captures only the Ricardian component, understates the welfare gains from trade in this case.

Next we consider larger values of σ. Figure 4 plots the within index for group 1 as a function of τ for various σ. While the index decreases for $\sigma = 1$, it actually increases when σ and τ are both high. Recall when we discussed the intuition for the $\sigma = 1$ case, we noted that price always equals the limit price of the second highest cost. In contrast, when σ is high, price is relatively likely to hit the cap of the monopoly mark-up. In fact, in the pure autarky case of $\tau = \infty$, every price in group 1 has the monopoly mark-up and perfect within-group allocation efficiency is achieved, $A^1_{within} = 1$. Variations in mark-up induced by opening to trade can only negatively impact the within index in this extreme case.

Our final plot looks at the impact of trade on total allocative efficiency for various σ.

Again, half of the products are in each group. At $\sigma = 1$, the within and across effects work together and the index sharply decreases throughout the range of τ. For higher σ and large τ, the across effect works opposite to the within effect. For $\sigma = 2$, the net effect of higher τ is negative throughout. For high values of σ, total allocative efficiency actually increases in τ. But note that when this happens the relationship is fairly flat. In the limit, when σ goes to its upper bound of $\theta + 1$ (here $\sigma = 6$), the relationship is perfectly flat, consistent with Part (i) of Proposition 2. The main takeway from Figure 5 is that when the impact of an increase in τ on allocative efficiency is substantial, the impact is negative.

Our last step is to compare the impact on allocative efficiency with the Ricardian welfare impact. As above, for concreteness, we focus on the illustrative numbers reported in ACR. $\varepsilon = -5$ and $\varepsilon = -10$ for alternative trade elasticities and $\lambda = .93$ as a representative value for the spending share on domestic goods. Again, the ACR statistic is $W_{ACR} = .93^{-1/\varepsilon}$ equals 1.014 and 1.007 for the two different values of ε. For each value of σ and each value of ε, we solve for the θ and the τ consistent with this ε and $\lambda = .93$. As discussed above, at the extreme where $\sigma = 1$ or $\sigma = 1 + \theta$, $\varepsilon = -\theta$. Otherwise, in between these extremes, the absolute value of ε is slightly less then θ.[4]

Table 1 reports the components of the welfare gains from trade, as well as the overall gains. The benefit reported is the gain delivering a domestic share of $\lambda = .93$ relative to autarky. For example, if $\sigma = 2$ and $\varepsilon = -5$ the allocative efficiency component contributes 1.5 percent to welfare, approximately the same as the 1.2 percent contribution of the Ricardian component. Thus, taking into account this additional effect doubles the welfare gain. Note also that while the allocative efficiency impact is actually negative at the high value of σ ($\sigma = 5$), equalling $-.1$ percent, this is negligible in comparison to the 1.4 percent positive Ricardian component.

Things get even more interesting at $\theta = 10$.[5] The W_{BEJK} term gets smaller, but the allocative efficiency term becomes larger for low σ. For example, at $\sigma = 2$, higher allocative efficiency contributes 5.9 percent to welfare, while the Ricardian gain is only .9 percent, a factor eight difference. The ACR procedure applied in this case will only get (approximately) the small Ricardian gain. The welfare estimate comes in small given the high trade elasticity and the relatively small increase in trade flows. The welfare gain of more efficient resource allocation within the country is not picked up by the trade flow data.

The welfare gains missed by ACR can potentially be measured with data on mark-ups or cost shares by product group. Looking at the last four columns of Table 1, we can see that

[4] For fixed θ and $\sigma \in (1, \theta + 1)$, ε will vary with the choice of τ. We select the θ so ε equals its given value at $\lambda = .93$.

[5] Here τ is reduced from $\tau^{\circ} = 3$ to $\tau' = 1.30$ to get $\lambda' = .93$ at $\sigma = 1$ and approximately the same λ' for other σ.

while trade raises the variable cost share of revenue for both product groups, the impact on group 1 is disproportionate, meaning trade tends to equalize margins across groups.

We comment about the use of low values of σ. In models of monopolistic competition, low values of σ imply high mark-ups, or equivalently low variable cost shares of revenue. Here, with Bertrand competition between multiple firms producing the same product, it is possible to have low values of σ and still have cost shares similar in magnitude to those found in the literature. For example, BEJK use $\varepsilon = -3.5 = -\theta$, implying a variable cost share of $\theta/(\theta+1)$ =.78. In Table 1, when $\sigma = 2$ the average cost share across the two product groups under trade is .72 if $\varepsilon = -5$ and .83 if $\varepsilon = -10$, bracketing the level in BEJK. This illustrates that it is possible in this model to have plausible cost shares and high allocative efficiency welfare gains from trade.

4 Endogenous Market Structure

This section introduces a fix cost of entry and makes the level of entry endogenous. Whereas above we took as given, for example, that half of the products had one firm in a country and half had two, here difference in market structure across products will arise in free entry equilibrium. Trade liberalization will impact the equilibrium number of firms. Despite this modeling difference, we show a basic force isolated in the previous section is retained. Opening to trade when initially some products are monopoly and others are duopoly leads to potentially significant impacts on welfare through impacts on allocative efficiency that are not picked up in the ACR analysis of trade flows.

Let β be the fix cost in labor units to start a firm and obtain a productivity draw. We model the entry process as being a simultaneous move game taking place prior to the sequence of events in the earlier description of the model. In particular, for each good in each country, entrepreneurs make an simultaneous entry decision taking as given the entry decisions of the other entrepreneurs. Suppose as a result of this process $n_i(\omega)$ firms enter good ω at country i. Each of these firms pays a fixed cost β and obtains a productivity draw. The firms then compete in a Bertrand fashion, as described earlier.

Consider a closed economy with total labor force L. Define a cut-off value of β by

$$\hat{\beta}_{DS} = \frac{L}{\sigma}.$$

If $\beta > \hat{\beta}_{DS}$, the equilibrium outcome is Dixit-Stitliz monopolistic competition, with every firm a monopolist with zero ex ante expected profits. (With trade and high β, the model corresponds to Krugman (1980)). To see this, observe that with Dixit-Stigliz, the share of labor allocated to fixed cost is $1/\sigma$, so the volume of entry is $(L/\sigma)/\beta$, which is less than one if $\beta > \hat{\beta}_{DS}$. That is, the volume of entry is less than the unit set of possible products, so the entrants spread out and don't overlap.

Next consider the opposite extreme in which the fixed cost is very low. There will be many firms per each good and with large n the equilibrium outcome is approximately the BEJK outcome. Specifically, in the BEJK limit the fraction of revenue going to variable cost is $\theta/(1+\theta)$. With free entry, the residual fraction $1/(1+\theta)$ of revenue will (approximately) go to fixed cost. Thus the number of firms for each good will (approximately) be $n \approx \frac{L}{1+\theta}/\beta$, when β is small. We see that n will indeed be large when β is small, so pricing will indeed approximate the BEJK outcome.

Thus this model nests two classic models, Dixit Stiglitz and BEJK, as limiting cases of high and low β. We are interested in what happens in between. In particular, suppose that, in autarky, a fraction $\alpha_1 \in (0,1)$ of products have monopoly and the remaining fraction $\alpha_2 = 1 - \alpha_1$ have duopoly. We first show how to solve for per capita expected welfare and then examine welfare changes from trade liberalization.

The expected profits of duopoly markets must be zero, otherwise it would be profitable for a second entrant to come into one of the monopoly markets. Let R_n be expected sales revenue for goods with n firms and let $\bar{c}_n$ be the (sales weighted) share of variable cost in revenue, given n firms. The zero profit condition implies expected revenue for a duopoly firm net of variable costs equals the fixed entry cost,

$$\frac{R_2}{2}(1-\bar{c}_2) = \beta.$$

The 2 in the denominator arises because a given duopolist is the Bertrand winner half the time. Revenue across the monopoly and duopoly products must equal total income,

$$\alpha_1 R_1 + (1-\alpha_1) R_2 = L + \alpha_1 \left(\frac{R_1}{\sigma} - \beta\right),$$

where income equals total wages plus the number α_1 of monopolists times expected monopoly profit. The later is monopoly revenue net of variable costs R_1/σ, less the fixed entry cost β. Solving for L and substituting for β, we can write per capita income as

$$\frac{R}{L} = \frac{\alpha_1 + (1-\alpha_1) r}{\alpha_1 + (1-\alpha_1) r - \alpha_1 \frac{1}{\sigma} + \alpha_1 \frac{(1-\bar{c}_2)}{2} r}$$

where $r \equiv R_2/R_1$ is a function only of σ and θ. The price index is

$$P = \left(\alpha_1 P_1^{-(\sigma-1)} + \alpha_2 P_2^{-(\sigma-1)}\right)^{-1/(\sigma-1)}$$

where P_n is the price index conditioned on firm count n, also a function of σ and θ. Per capital welfare in autarky is

$$Welfare^{\circ}(\alpha_1, \sigma, \theta) = \frac{R}{L} \times \frac{1}{P}.$$

where R/L and P are implicitly functions of α_1, σ, and θ.

We consider a liberalization that integrates the particular country under consideration into a larger economy. For simplicity, we assume that the trade friction is zero for shipments across the various countries in the new larger economy. Let L' denote the aggregate labor endowment of the larger economy. Let L' be large, so that the equilibrium is approximately the BEJK outcome. The price index in the BEJK outcome is

$$P'_{BEJK} = \gamma(\sigma, \theta)\Phi'^{-1/\theta}$$

where the formula for $\gamma(\sigma, \theta)$ is from BEJK and where $\Phi' = \frac{1}{1+\theta} L'/\beta$ is the equilibrium entry per product. (Note that using (6) with no trade friction yields $\Phi' = n'$) Per capita welfare in BEJK is

$$Welfare'(\Phi', \sigma, \theta) = \frac{1}{\gamma(\sigma, \theta)} (\Phi')^{1/\theta}$$

We can write the ratio of the BEJK welfare from integration to the initial welfare as

$$W_{total} = f^{total}(\alpha_1, \sigma, \theta) (\Phi')^{1/\theta}.$$

Next note that in autarky the domestic share equals one, $\lambda^{\circ} = 1$. With integration, the domestic share equals the country under consideration's share of the aggregate labor force, $\lambda' = L^{\circ}/L'$. In the BEJK limit, the trade elasticity equals θ. The ACR formula is

$$\begin{aligned} W^{ACR} &= \left(\frac{\lambda'}{\lambda^{\circ}}\right)^{-\frac{1}{\theta}} = \left(\frac{L'}{L^{\circ}}\right)^{1/\theta} \\ &= f^{ACR}(\alpha_1, \sigma, \theta) (\Phi')^{1/\theta}, \end{aligned}$$

where we show in the appendix that we can pull out the $(\Phi')^{1/\theta}$ in a multiplicatively separable fashion.

Therefore, the ratio between the actual gain in welfare and the ACR predicted level can be written as

$$relative\_descrepancy(\alpha_1, \sigma, \theta) = \frac{W_{total}}{W^{ACR}} = \frac{f^{total}(\alpha_1, \sigma, \theta)}{f^{ACR}(\alpha_1, \sigma, \theta)}. \tag{14}$$

Notably, the Φ' term cancels out in this ratio. The Φ' term scales up proportionately with the size L' of the integrated economy. This says that conditioned on the trade opening being large enough that the outcome approximates BEJK, the relative discrepancy between the actual welfare change and the ACR predicted level does not depend upon L'. Of course, the overall welfare gain is larger the larger L', but this is fully picked up in the ACR term. In summary, the relative difference between the actual and predicted can be written as a function of the monopoly market share under autarky, the demand elasticity σ and the trade elasticity θ.

Figure 6 illustrates numerical values of the relative discrepancy. It plots it as a function of the initial monopoly share α_1 for different values of θ, for low demand elasticity in part A and high demand elasticity in part B. The first thing to note is that the values are close to one when $\alpha_1 = 0$. At this endpoint, there are two firms for every product in autarky. With two firms the initial mark-up distribution is relatively close to the BEJK level, so at $\alpha_1 = 0$, things are relatively close to BEJK to begin with, and the ACR formula works well. The second thing to note is that for high α_1 (close to 1) and low θ (close to its lower bound $\sigma - 1$), the ratio is less then one, i.e. the actual gain is below the ACR prediction. At the end point $\alpha_1 = 1$ where every firm is a monopolist, the mark-up is a constant μ across all products. As noted earlier, in this case allocative efficiency across products is maximized. As the economy become integrated, variance is introduced into the mark-up, and this introduces a distortion not picked up in the ACR formula.

The most striking feature of Figure 6 is the inverted U-shape of the relationship.[6] It is at intermediate levels of the monopoly market share where the actual gain from integration is the highest relative to the predicted gain from ACR. In these intermediate cases where in autarky we have some goods that are monopoly with high mark-ups and other goods that are duopoly with low mark-ups, we obtain allocative inefficiency across these sets of goods. By opening up to trade, we even out the mark-up distributions across goods. For relatively high trade elasticity θ, and relatively low values of σ, large positive discrepancies are possible. For example, suppose $\sigma = 2$. If $\theta = 10$, the maximum discrepancy is a factor 1.098, attained at a monopoly share of .67, and for $\theta = 20$, the maximum is a factor 1.113, attained at a monopoly share of .69. Suppose we are willing to take a stand on the following:

[6] We comment about the perfectly flat line in Figure 6B at $\theta = 3$ and $\sigma = 4$. This is the boundary case where σ hits its lower bound of $1 + \theta$. Following Proposition 3, the BEJK formula is exact in this limit.

that in autarky there is a mix of duopoly and monopoly, that the demand elasticity $\sigma \geq 2$ and that the trade elasticity $\theta \leq 20$. Then an upper bound on the understatement of the ACR condition is a factor 1.113. As the conditions on σ and θ would generally be regarded as conservative, this is a useful upper bound on the nature of the magnitudes involved.

If one is willing to make stronger assumptions, the upper bound is of course tighter. If $\sigma \geq 4$ and $\theta \leq 10$, then the maximum understatement is a factor 1.016 (attained at a monopoly share of .50). Given the small magnitudes of gains generally calculated, an understatement of the relative gain by a factor 1.016 would still be significant.

In summary, this section shows that opening to trade can impact welfare through changes in allocative efficiency that are not captured by changes in trade flows. This discrepancy is more likely to be positive and large when the pre-trade market structure has a mix of monopoly and duopoly products, as this leads to allocative distortions across these groups of products.

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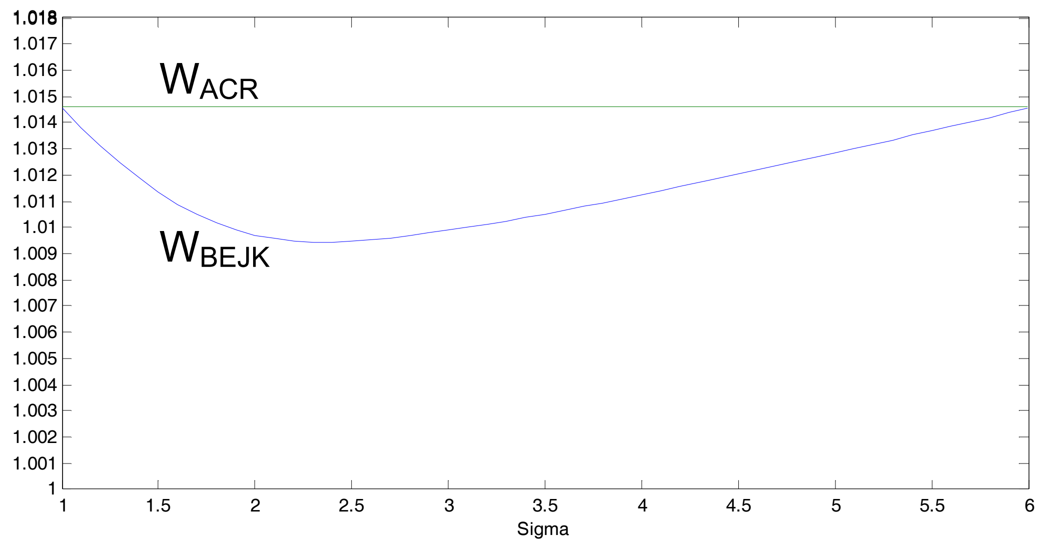
Figure 1
W_{BEJK} and W_{ACR} for Various Values of σ

Holding Fixed ε=5 and λ° = .93



Holding Fixed ε=10 and λ° = .93

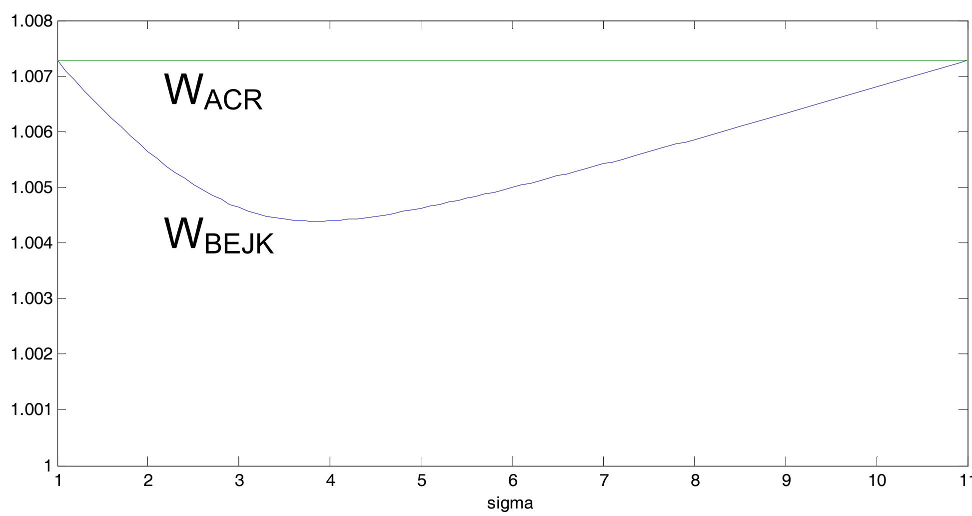


Figure 2

Index A^{1}_{Within} of Within Group 1 Allocative Efficiency
Plotted as Function of Trade Friction τ for Various θ and σ =1

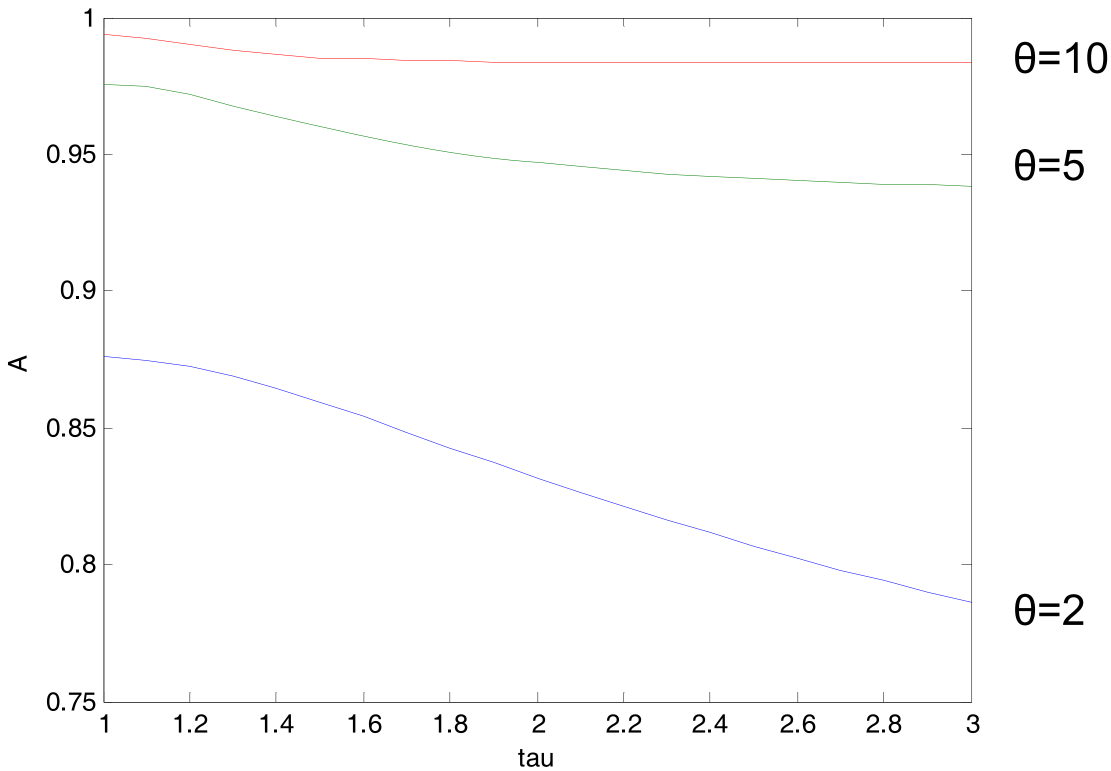


Figure 3
Index A_{across} of Across Group Allocative Efficiency
Plotted as a function of Trade Friction τ
For various θ and $\sigma = 1$

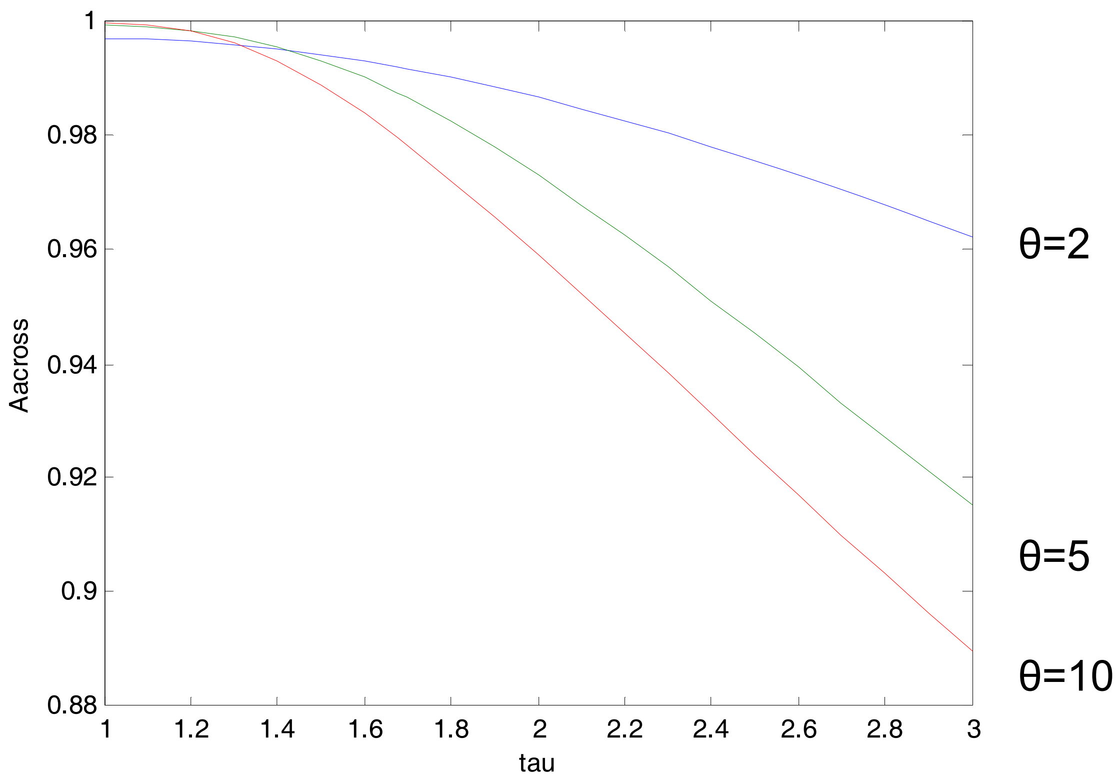


Figure 4
Index A^1_{Within} of Within Group 1 Allocative Efficiency
Plotted as Function of Trade Friction τ for Various σ and $\theta = 5$

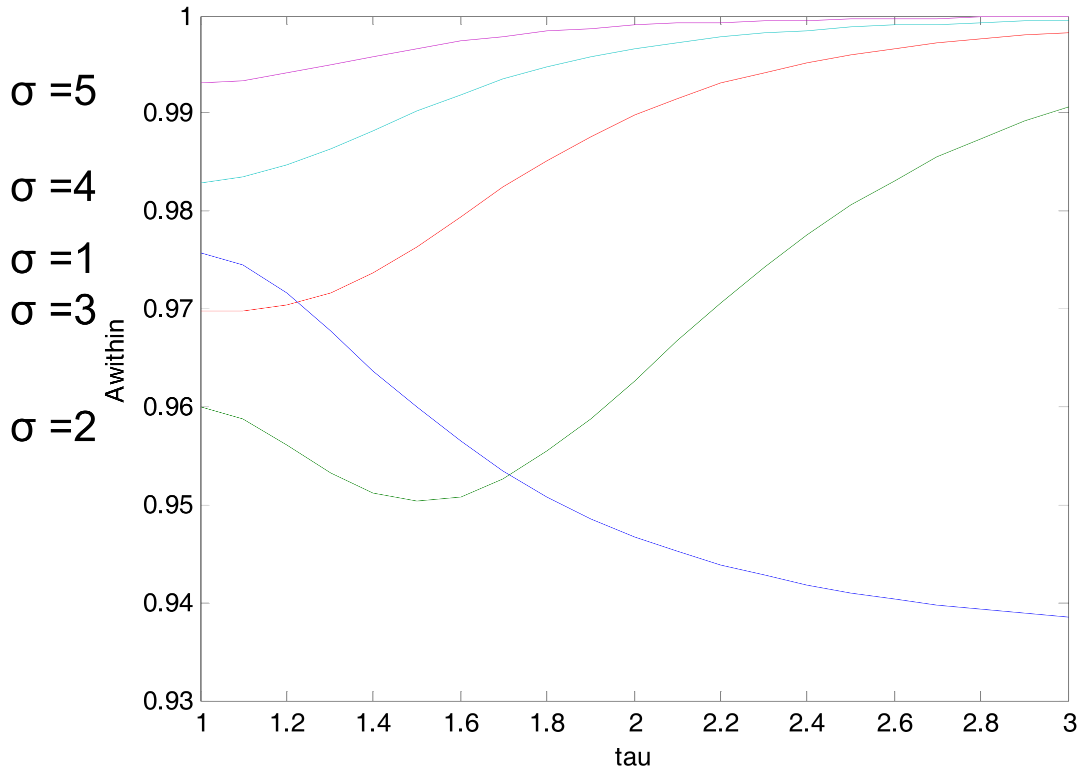


Figure 5
Index of Total Allocative Efficiency as Function of the Trade Friction τ
For Different Values of σ and θ= 5
Half of Products in Group 1, Half of Products in Group 2

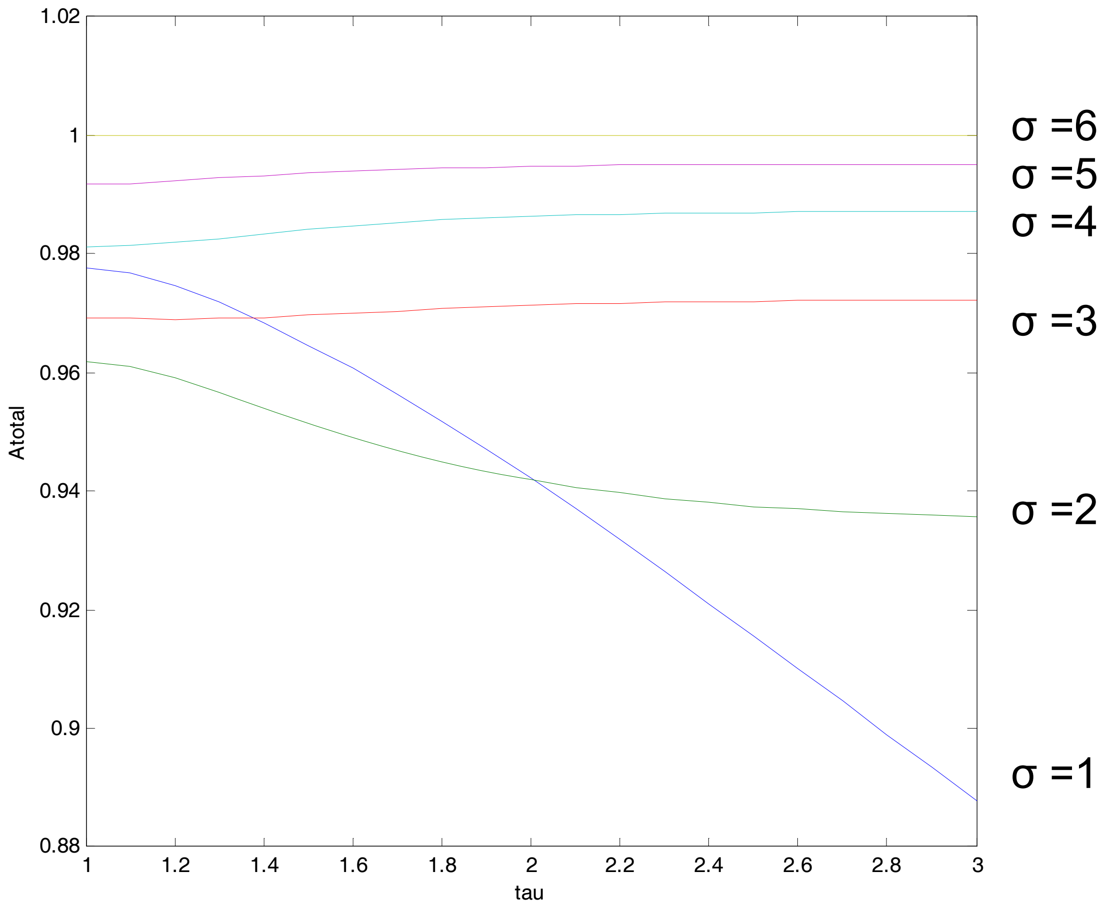


Figure 6A
Relalative Discrepancy with Free Entry
Ratio of Actual Welfare Gain to ACR Predicted Gain
How it Depends upon Market Structure Under Autarky, the Trade Elasticity θ

Low Demand Elasticity (σ = 2)

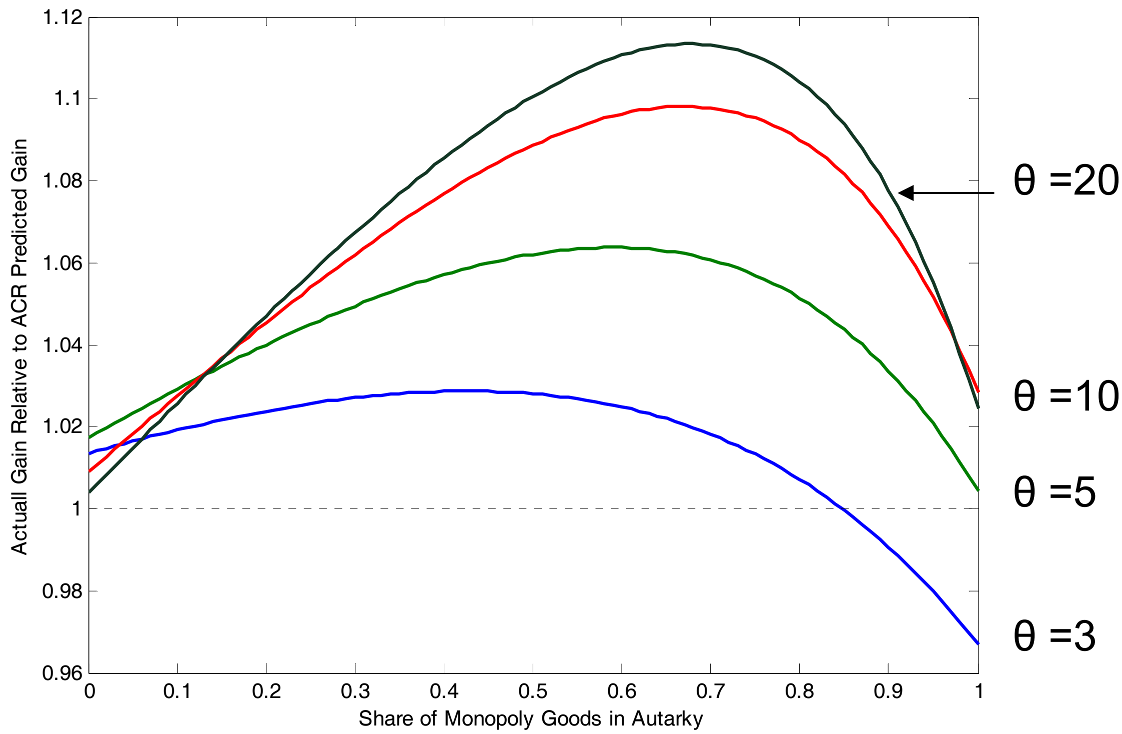


Figure 6B
Relalative Discrepancy with Free Entry
Ratio of Actual Welfare Gain to ACR Predicted Gain
How it Depends upon Market Structure Under Autarky and the Trade Elasticity θ

High Demand Elasticity (σ = 4)

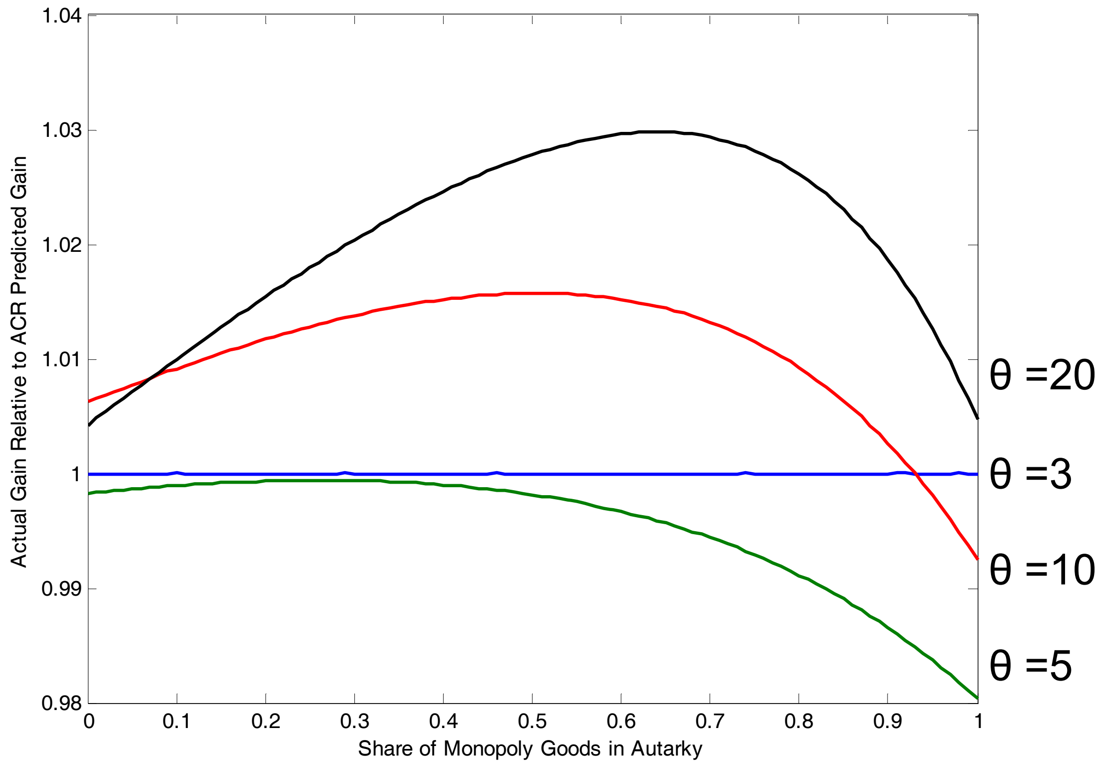


Table 1
Welfare Gain of Trade
Symmetric Countries
Half of Goods Have One Firm in Each Country (Group 1)
Half Have Two in Each Country (Group 2)

	W_{ACR}	Ricardian Term W_{BEJK}	Allocative Efficiency A'/A°	Total Effect W_{Total}	Variable Cost Shares by Product Group No Trade Case		Variable Cost Shares by Product Group Trade Case	
					$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_1$	$\bar{c}_2$
$\varepsilon = 5$								
$\sigma = 1$	1.014	1.014	1.078	1.093	.355	.777	.607	.786
$\sigma = 2$	1.014	1.012	1.015	1.026	.503	.790	.641	.797
$\sigma = 5$	1.014	1.014	.999	1.013	.800	.821	.806	.823
$\lim \sigma = \theta+1$	1.014	1.014	1.000	1.014	.833	.833	.833	.833
$\varepsilon = 10$								
$\sigma = 1$	1.007	1.007	1.115	1.123	.339	.876	.770	.882
$\sigma = 2$	1.007	1.006	1.059	1.066	.501	.882	.780	.887
$\sigma = 5$	1.007	1.006	.999	1.005	.800	.889	.832	.892
$\lim \sigma = \theta+1$	1.007	1.007	1.000	1.007	.909	.909	.909	.909

For each value of ε and σ, we solve for θ and τ such that λ=.93 and ε equals the given value.