

December 2006

# Schooling, Inequality and Government Policy

by

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The views expressed in this paper are those of the authors.  
No responsibility for them should be attributed to the Bank of Canada.

## Acknowledgements

We are grateful to Rui Castro, Christopher Ferrall, Paul Gomme, Allen Head, Jonathan Heathcote, Larry Jones, Sharon Kozicki, Tatyana Koreshkova, Burhanettin Kuruscu, Cesaire Meh, Victor Rios-Rull, Francisco Ruge-Murcia, Raphael Solomon, Malik Shukayev, Yasuo Terajima, and seminar participants at the Bank of Canada, Canadian Economic Association Meetings in Montreal, 2006 North American Econometric Society Meetings in Minneapolis, Queen's University, Universite de Montreal, Wilfrid Laurier University for their comments and suggestions. We thank Andrew Au, Amanda Armstrong and Tim Grieder for excellent research assistance. All remaining errors are ours.

## Abstract

This paper addresses the question: What is the effect of government policy on wage and income inequality in an environment with education and labor supply choice? The answer is given in a general equilibrium model, consistent with the main facts on wage inequality and labor supply of males in the U.S. for the post 1960 period. In the model, both schooling and labor supply choices are determined by progressive income taxation, the social security system, and technology driven wage differentials. The paper emphasizes two new channels through which government policy effects wage inequality and output. First, government policy typically has an asymmetric effect on working hours and savings of schooled and unschooled individuals due to the change in redistribution of income or through its effect on the accumulation of skill intensive capital stock. Second, a differential response of private savings and working hours after a policy change implies different incentives of completing college leading to a redistribution of the work force between schooled and unschooled workers. Using a battery of proposed government policies we quantify that these two channels can reverse some of the increase in wage inequality and raise long-run output by an additional 1 to 2 percent.

*JEL classification: H52, J31, J38*

*Bank classification: labour markets, potential output, productivity*

# 1. Introduction

This paper studies the effects of policy change on education choice, labor supply and wage inequality. The paper consists of two parts. First, we build a theoretical framework that is applicable for policy analysis. To this end we develop a general equilibrium model that is consistent with the recent U.S. history of labor hours and earnings. The distinguishing feature of the model is the explicit treatment of individual choices to educate and work. We then use our framework to study the effects of policy change on labor supply and wage premium.

There are two distinct trends in the U.S. post-war aggregate data on labor supply. First, education premium for men, measured as the ratio of median wage per hour of college graduates to that for men with less than 4 years of college, increased from 1.43 in 1961 to 1.80 in 2002, a 26% change. And second, the large increase in the relative wages across education groups was accompanied by the comparable increase in the relative total working hours. The ratio of schooled to unschooled total hours worked more than doubled from 0.16 to 0.47.<sup>1</sup>

A large body of literature has emerged in attempt to explain these facts. A popular modern hypothesis attributes positive secular comovement in hours and compensation across groups with different schooling to the skill-biased technological change (SBTC). According to this hypothesis, technological change favours the demand for skilled hours over unskilled hours, inducing the increase in both relative price and relative quantity of skilled labor.<sup>2</sup> An appealing implementation of the SBTC hypothesis is presented in Krusell et al. (2000), or KORV in short. In KORV's aggregate production function, technological progress is embodied in capital equipment. Capital equipment, in turn, is complementary with skilled hours. Thus, secular increase in the demand for equipment due to the embodied change boosts the demand for skilled labor. Krusell et al. (2000) show that their aggregate technology predicts well the historical change in education premium *given* the observed time series of hours worked by schooled and unschooled individuals, and stocks of capital structures and (quality adjusted) capital equipment. The general equilibrium framework developed in this paper adopts empirically plausible KORV technology to drive up the demand for schooled working hours. Hence the key to the empirical success of this framework is to account for the behaviour of the supply of aggregate hours and savings *given* relative wages and returns on capital.

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<sup>1</sup>See Eckstein and Nagypal (2004) and the survey by Katz and Autor (1999) for a detailed analysis of macroeconomic trends in comensation and labor supply.

<sup>2</sup>See Katz and Murphy (1992) and references therein.

Remarkably, the study of the supply of working hours across skill groups in the aggregate setting has received by far much less attention in the literature than the demand.<sup>3</sup> In the U.S. data, most of the increase in the relative hours comes from the increase in the number of individuals completing college. The fraction of college graduates in employed population more than doubled, increasing from 12.3% to 29.3% from 1961 to 2002. In contrast, the mean workweek length for both skill groups changed only a little in that period, decreasing from 44.5 to 43.2 hours per week for schooled, and from 39.8 to 38.3 hours per week for unschooled workers. Therefore individual choices of attending school and working should be an inherent ingredient of any aggregate model of labor supply and inequality. In our model, each individual chooses the number of hours to work per week as well as whether to obtain a college degree given his realized utility cost of attending college. This cost is a combination of random ("psychic") cost and "studying" cost, which decline in individual's cognitive ability. Given the realization of the schooling cost, an individual in the model decides to go to college if the realized cost is smaller than the gain in the lifetime stream of utility. *Ceteris paribus*, an individual with higher cognitive ability faces higher probability of finishing college. Based on this feature, we calibrate parametric specification of the schooling cost to match the incidence of college completion among young men in the National Longitudinal Survey of Youth 1979.

In the model, individuals' supply of hours and savings are affected by the government policy in the form of progressive income taxation and retirement system. We employ the census data on individual tax returns and social security payments to obtain empirically plausible progressive income tax function as well as retirement benefit payments to retirees in each education group. We demonstrate that the calibrated model is able to generate empirically plausible changes in the supply of total hours and savings in each skill group in the U.S. from 1961 to 2002 given the historical changes in productivity (neutral and skill-biased). This step completes the development of our framework applicable for policy analysis.

In the second, and main, part of the paper we proceed by employing the developed framework to analyze the effect of public policy change on the number of individuals finishing college, working hours and education premium. In particular we emphasize two effects of government policy on wage inequality and output. First, a policy change has a differential effect on working hours and savings of schooled and unschooled workers due to (i) redistribution of income under progressive income taxation and the incumbent pension system, and (ii) the change in demand for schooled hours due to the change in skill intensive capital stock.

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<sup>3</sup>A few examples include Heckman et al. (1998a), Guvenen and Kuruscu (2006), He (2006), Arpad (2004).

Second, the resulting difference in lifetime streams of consumption and leisure imply different incentives of attending college. The ensuing change in college enrollement will lead in the long run to the adjustment in the fraction of schooled workers in the employed population. Higher relative supply of schooled worker-hours puts a downward pressure on wage premium and gives an additional boost to the level of output.

In order to quantify the effects of the change in government policy on labor supply and wage inequality, we conduct a set of policy experiments mimicking some of the proposed policy reforms today. Our policy experiments can be categorized into three groups. First, we study the income tax reforms that lower the level and progressivity of the income tax. Second, we look at changes in the retirement system: (i) reforms that improve viability of the pension fund (increase in social security contributions and the increase in mandatory retirement age); and (ii) reforms that affect redistribution features of the pension system (a move to proportional pension benefits or a switch from a pay-as-you-go-system to a fully-funded pension system). Finally, we investigate policies that improve the efficiency of the schooling system.

Two regularities stand out in the policy experiments in our model. First, the long run reallocation of the workforce between unschooled and schooled jobs can be quite substantial, reaching on several occasions a million or more workers. And second, the skill reallocation in most experiments reverses the initial effect on wage inequality and implies an additional 1-2% change in the level of output in the long run.

The rest of the paper proceeds as follows. Section 2 describes the main facts in data. Section 3 presents the life cycle model. In Section 4 the model is calibrated to the U.S. data in 1961 and 2002. We demonstrate the model's success in matching the key macroeconomic facts and evaluate the importance of labor supply and education choice for wage and working time inequality. In Section 5 we subject the developed model to a sequence of policy experiments to study the effect of government policy change on schooling and inequality. Section 6 concludes.

## 2. Data

In this section we present data on working time, earnings per hour and productivity in the U.S. from 1961 to 2002. We focus on long-run trends and mostly consider changes between 1961 and 2002.

## 2.1 Main facts for education premium and hours from 1961 to 2002

The data on working hours and compensation are based on the March supplement of the Current Population Statistics (CPS) as provided by the IPUMS project.<sup>4</sup> Following a convention in the literature, we restrict our sample to males age 16 to 64. Before age 16, 85% of men go to school and after 64 (the normal retirement age) the fraction of males working decreases by about 80%.

The main reason for restricting our attention to males instead of the whole population is that earnings per hour of women are catching up with those of males. This is especially relevant for the less schooled in the 1980s. If we included women, this catching up would add, we believe, a completely separate set of problems that is unrelated to the fundamentals that drive the education premium change.<sup>5</sup>

The population in our sample is divided into two education groups: college graduates (or “schooled”) and less than college graduates (“unschooled”). This is a convenient division and has been used before, for example by Krusell et al. (2000). We define a college graduate as a person who has completed at least 4 years of college. In contrast a less than college graduate is a person in the population with 0 years of schooling up to 3 years of college completed. A finer division of the group of less than college graduates does not alter our main facts. For example, the group of persons with 1 to 3 years of college is much closer in terms of their earnings per hour to the group of high school graduates, than to the group of college graduates.<sup>6</sup>

Given these groups, we are interested in documenting the evolution of the average hours worked per employed person, the number of workers, and earnings per hour for each education group in the last four decades.

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<sup>4</sup>Appendices A and B provide detailed descriptions and transformations of the data.

<sup>5</sup>For a treatment of the gender gap we refer to Jones et al. (2003). Jones et al. point out that most of the increase in market hours is due to married women entering the labor market and based on a decrease of the gender-wage gap.

<sup>6</sup>As pointed out by Eckstein and Nagypal (2004). There appears to be a considerable difference between pure undergraduate degree holders and graduate degree holders in terms of their earnings per hour. Unfortunately, given the data we have, a subdivision of the group of college graduates is somewhat arbitrary and we abstain from it.

Consider the following decomposition of average hours worked:

$$\begin{aligned} \frac{L}{N} &= \frac{L}{E} \times \frac{E}{N} \\ &= \left[ \frac{E^S}{E} \frac{L^S}{E^S} + \frac{E^U}{E} \frac{L^U}{E^U} \right] \times \frac{E}{N}, \end{aligned} \quad (1)$$

where  $L^i$ ,  $E^i$ ,  $N$  stand, respectively, for the total hours worked by group  $i$ , the number of employed<sup>7</sup> in group  $i$ , and the working age population. The following identities hold:  $L^S + L^U = L$  and  $E^S + E^U = E$ . We define as the workweek length of group  $i$ :  $l^i \equiv L^i/E^i$ . We refer to  $e \equiv E^S/E$  as the fraction of schooled workers in the employed population. The earnings per hour for each education group are measured by the (hours weighted) median earnings per hour of employed persons in the respective group. Using the median and not the mean helps us to navigate around the change in the CPS' top coding procedure in the 1990s. For convenience, we refer to the median earnings per hour as wage. Finally, we define the education premium as the wage of a college graduate to that of a less than college graduate.

From 1961 to 2002, the average hours worked by males decreased by about 9% (see Table 1). This is mostly due to a decrease in the fraction of employed in the male population, -6.5 percentage points. Even though the employment to population change for males is interesting, it is dwarfed in size and importance by the changes that happened within the square brackets of equation (1). Taking this observation into account, we focus on the compositional changes of the hours worked by employed persons.

There are three main observation for the period 1961 to 2002 that we wish to highlight:

- (i) wages of schooled increased dramatically by 26.1% relative to those of unschooled,
- (ii) the fraction of employed schooled persons among all employed persons increased considerably by 17 percentage points,
- (iii) the workweek length<sup>8</sup> of both schooled and unschooled employed persons decreased, but only slightly: -1.4 hours per week (-2.9%) for schooled, and 1.5 hours per week (-3.8%) for unschooled.

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<sup>7</sup>We consider a person to be employed, if that person workes more than zero hours per year, independent of whether they work part-time or full-time.

<sup>8</sup>The workweek length is measures as the average annual hours worked per employed person at the weekly rate. We take the actual hours worked per week and the actual weeks worked per year into account in our calculation of annual hours.

Figure 1 contrasts changes in wages of college graduates and less than college graduates from 1961 to 2002. While the wage of schooled increased by three quarters from \$13.3 to \$23.0<sup>9</sup>, the wage of unschooled increased only slightly by a quarter from \$9.3 to \$12.8. Accordingly, the education premium increased from 1.43 in 1961 to 1.80 in 2002 (see Figure 2). The 26.1% increase in relative wages is sizeable, suggesting that a large part of the 47.7% overall wage growth for males over that period was unevenly distributed.

The observed change in wages was accompanied by a large shift in working hours from unschooled to schooled workers. The fraction of total hours supplied by the college graduates has more than doubled over the last 40 years, reaching now one third of all hours worked in the market place (see Figure 3). Nearly the whole reallocation of total hours is due to a change in the fraction of college graduates in the population and especially in the employed population. From 1961 to 2002 the fraction of schooled in the employed population more than doubled, increasing from 12% to 29%. In contrast, the workweek in each education group stayed virtually unchanged. Figure 4 plots the workweek for the two groups from 1961 to 2002. Schooled work more on average with 44.5 hours per week in 1961 compared to 39.8 hours per week for unschooled<sup>10</sup>. Both time series observe a minor fall by 2002 down to 43.2 and 38.3 hours per week, respectively, for schooled and unschooled employed persons.

In Section 3, we present a model which can be used to account for the three facts just presented. In particular, the model has a schooling and a working time choices. To generate the transition from a low to a high fraction of schooled workers, we appeal to an idea developed by Katz and Murphy (1992) and later extended by Krusell et al. (2000). The partial equilibrium approach taken by KORV attributes the education reallocation to skill-biased technological change which in turn is driven by capital-skill complementarity together with capital-embodied technological change. In the next subsection, we describe the measurement of capital-embodied technological progress in the data.

## 2.2 Capital-embodied technological progress

The stock of capital equipment is reported in NIPA tables and for most of our period of interest the Bureau of Economic Analysis does not adjusted for quality changes. Gordon (1990) conducts a hedonic regression analysis to document the quality component of the growth of equipment. He finds that the quality of capital equipment increased by a factor of 4 from 1961 to 1982. KORV use Gordon's quality time series and extrapolate it to cover their

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<sup>9</sup>Unless otherwise noted, all quantities are in constant (year 2000) US dollars.

<sup>10</sup>We assume 100 hours available for work in a week.

period of interest. We use KORV's results and extend Gordon's time series through 2002 (see Appendix B.2). Equations (2) and (3) capture the measurement and quality adjustment of capital stocks:

$$\bar{K}^{st} = \frac{\text{Structures Stock}}{\text{Consumption Deflator}} \quad (2)$$

$$\bar{K}^{eq} = \underbrace{\frac{\text{Equipment Stock}}{\text{Qual. adj. Equipment Deflator}}}_{K^{eq}} \cdot \underbrace{\frac{\text{Qual. adj. Equipment Deflator}}{\text{Consumption Deflator}}}_q \quad (3)$$

Equation (3) captures the quality adjustment of the stock of equipment, where  $\bar{K}^{st}$  and  $\bar{K}^{eq}$  are, respectively, measured real stocks of capital structures and equipment (in consumption units),  $K^{eq}$  denotes the quality adjusted stock and  $q$  is the quality adjustment factor.

Figure 5 presents the two stocks from 1961 to 2002. Even though the stock of capital structures grew by 136%, it is dwarfed by the 814% growth in the quality adjusted stock of capital equipment<sup>11</sup>. The quality adjustment factor  $q$ , which is equal to the relative price of the quality adjusted stock of equipment in terms of the consumption good, decreased by a factor of four from 0.93 in 1961 to 0.26 in 2002. The decrease in the relative price of equipment is used in the model in Section 3 as a stand-in for (the reciprocal of) capital-embodied technological progress. It is a key force behind the increase in the demand for schooled labor in the model.

### 3. Model

In this Section we present a general equilibrium life cycle model with education choice, labor supply, and capital-embodied, skill-biased technological change.

There is a continuum of ex-ante identical individuals. The diagram in Figure 6 summarizes the life cycle of individuals in the economy. An individual is born at age 18.<sup>12</sup> At birth he

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<sup>11</sup>There are mainly two alternative measures of capital embodied technological progress covering about the same period we consider: Cummins and Violante (2002) and Polgreen and Silos (2005). The later paper provides two main series for capital embodied technological progress, one that predicts strong growth during the 1990s (similar to Cummins and Violante) and one with very weak growth in quality adjusted capital equipment. We are closer to the more conservative estimate in capital embodied technological growth for the 1990s but differ only slightly from the other papers' measures for the pre 1990 period.

<sup>12</sup>We choose the age 18 as the beginning in the model since it is the average time for highschool graduation and for most young males to begin college. This is in contrast to 16 years in the sample we considered in the data. The difference in the data facts due to this age choice is minor and compensated by the comparability of our numbers with standard publications, for example by the Bureau of Labor Statistics.

faces a cost of education and decides whether or not to educate. If the individual decides to educate, she spends 4 years at college. The individual starts working at 22, or 18 if he did not go to college. All workers retire after turning 65 and die after 78.<sup>13</sup>

In this paper we focus on the steady state economy. This implies that all aggregate prices and quantities are constant. Besides allowing for a less cluttered notation in the model, the steady state assumption is reasonable given our focus on the secular change in the U.S. economy between two distant points of time - the years 1961 and 2002. Furthermore, we find that the main results are qualitatively robust even on the transition path. We start by considering an individual's choice of working time, saving, and consumption conditional on the education choice. Afterwards, we present the education choice problem.

### 3.1 Working time choice

An individual's age is represented by an index  $a = 1, \dots, 63$ . The problem of an unschooled individual is to choose life time sequences of consumption  $c_a^U$ , working hours  $l_a^U$ , and asset holdings  $k_{a+1}^U$  to solve :

$$V^U(k^U) = \max_{\{c_a^U, l_a^U, k_{a+1}^U\}} \sum_{a=1}^{63} \beta^{a-1} [\ln(c_a^U) + \phi \ln(1 - l_a^U)] \quad (4)$$

subject to

$$(1 + \tau_c) c_a^U + k_{a+1}^U \leq w^U l_a^U + r k_a^U - T(w^U l_a^U + r k_a^U) + TR_a^U + \Pi_a^U \quad (5)$$

$$l_a^U = 0, \text{ for } \forall a \geq 48 \quad (6)$$

$$k_1^U = k^U \quad (7)$$

Here (5) is the budget constraint of the unschooled at age  $a$ . She consumes, saves, receives wage income at wage rate  $w^U$ , asset returns at rate  $r$ , transfers from the government  $TR_a^U$  and dividend payments  $\Pi_a^U$ . She faces tax payments as a (smooth) function  $T(\cdot)$  of her wage and capital income, and pays a proportional consumption tax with the rate  $\tau_c$ .<sup>14</sup> Equation (6) represent a no working constraint for retirees, and (7) is the constraint on the initial asset

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<sup>13</sup>In the model the life length is chosen to equal the life expectancy at the age of 65 reported in Arias (2004). For our steady state simulations it is interpreted as children expect to live as long as their parents: 78 years in 1961 and 82 years in 2002.

<sup>14</sup>We assume that capital income is taxed only as a part of the personal income. In the U.S. the revenue from tax of capital part of the personal income is larger than the corporate income tax revenue. See, for example, Castaneda et al. (1998) or Altig and Carlstrom (1999) for the same approach.

holdings.

The problem of a schooled individual is similar:

$$V^S(k^S) = \max_{\{c_a^S, l_a^S, k_{a+1}^S\}} \sum_{a=1}^{63} \beta^{a-1} [\ln(c_a^S) + \phi \ln(1 - l_a^S)] \quad (8)$$

subject to

$$(1 + \tau_c) c_a^S + k_{a+1}^S \leq w^S l_a^S + r k_a^S - B_a - T(w^S l_a^S + r k_a^S) + TR_a^S + \Pi_a^S \quad (9)$$

$$l_a = 0, \text{ for } \forall a \geq 48 \quad (10)$$

$$l_a = 0, \text{ for } \forall a \leq 4 \quad (11)$$

$$B_a = B > 0, \text{ for } \forall a \leq 4 \quad (12)$$

$$k_1^S = k^S \quad (13)$$

DO WE ASSUME THAT STUDENTS HAVE ALL THEIR TIME AS LEASURE? IF SO CAN WE DO SOMETHING ABOUT THAT? AT LEAST PART OF A STUDENTS TIME IS ACTUALLY COMPARABLE TO WORKING TIME.

In addition to the budget (9), the retirement time (10) and initial asset holdings (13), individuals who go to college cannot work (11), and pay tuition cost  $B$  (12) while studying. Note that all individuals face the same income tax function and consumption tax rate.

We summarize the factors that contribute to the difference in lifetime utility across education groups: wages, income tax rates (if the tax function is progressive), government transfers, tuition cost, and education time. Now, the problem of education choice presented to a newborn individual.

### 3.2 Education choice

Newborn individuals face a cost of education  $d$ . This cost of education is a decreasing function of an individual's cognitive ability  $Q$ :

$$d(Q, z) = d_{\max} - \delta_d (\iota(Q) - z) \quad (14)$$

where  $Q$  and  $z$  are independent random draws from  $N(0, 1)$ . The following restrictions apply:  $d_{\max} \in \mathbb{R}_{++}$ ,  $\delta_d \in \mathbb{R}_{++}$ , and function  $\iota : \mathbb{R} \rightarrow \mathbb{R}$  is continuous and increasing.

As we point out in Section 4, specification (14) with two random draws allows us to capture two aspects from the data on test performance and educational attainment: (1) individuals with higher test performance have (on average) higher educational attainment; and (2) for low (high) test performance, there are individuals who have (have not) completed at least 4 years of college.

The education choice problem of a newborn individual is:

$$V(k) = \max_{\text{study / work}} \left\{ \begin{array}{l} V^S(k) - d(Q, z) \text{ if study at 18} \\ V^U(k) \text{ if work at 18} \end{array} \right\}. \quad (15)$$

Given continuity of  $d(\cdot, \cdot)$  with respect to its second argument, for any realization of cognitive ability  $Q$ , there exists a draw  $z^*$  such that the individual is indifferent between going to college and working. Hence the education choice for an individual with realized ability  $Q$  and cost draw  $z$  is to educate if  $d(Q, z) < d(Q, z^*)$ , and work otherwise. We apply this decision rule to derive the probability of going to college for an individual with cognitive ability  $Q$ :

$$\Pr(d(Q, z) \leq \Delta V(k)) = F\left(\frac{\Delta V(k) - d_{\max}}{\delta_d} + \iota(Q)\right). \quad (16)$$

where  $F$  is the c.d.f. of  $N(0, 1)$ , and  $\Delta V(k) \equiv V^S(k) - V^U(k)$  is the difference in lifetime utility across two education groups. Equation (16) implies that the fraction of households that decide to educate in each period, the enrollment rate, is given by:

$$\varepsilon_a = \int_{-\infty}^{\infty} F\left(\frac{\Delta V(k) - d_{\max}}{\delta_d} + \iota(Q)\right) dF(Q). \quad (17)$$

Since cohorts in the model have equal size,  $1/63$ , the enrollment rate is constant:

$$\varepsilon_a = \frac{\varepsilon}{63} \quad (18)$$

where  $\varepsilon$  denotes the measure of people ever enrolled in college. Given individuals' life cycle (see Figure 6), the fraction of schooled in the employed population,  $e$ , is

$$e = \frac{\varepsilon(I_R - I_e)}{I_R - \varepsilon I_e} \quad (19)$$

### 3.3 Bank's problem

There is a competitive industry of banks that pool individuals' savings in the form of asset  $k$  and invest them in two types of physical capital: structures and equipment. Structures are a generic form of physical capital, whereas equipment undergoes embodied technological progress - a key source of rising demand for schooled labor in the model. The problem of the representative bank is

$$\max_{K^{st}, K^{eq}, \{k_a^S, k_a^U\}} (r^{st} - \delta_{st}) K^{st} + (r^{eq} - \delta_{eq}q) K^{eq} - r \sum_{a=1}^{63} \left( \varepsilon_a k_a^S + \left( \frac{1}{63} - \varepsilon_a \right) k_a^U \right)$$

subject to

$$K^{st} + qK^{eq} \leq \sum_{a=1}^{63} \left( \varepsilon_a k_a^S + \left( \frac{1}{63} - \varepsilon_a \right) k_a^U \right)$$

Here  $K^{st}$ ,  $K^{eq}$  denote aggregate stocks of structures and equipment,  $r^{st}$ ,  $r^{eq}$  are their respective rental rates;  $\delta_{st}$ ,  $\delta_{eq}$  are their respective depreciation rates; and  $q$  is the price of equipment in terms of the consumption good (with the price of structures in terms of the consumption good normalized to one).

### 3.4 Final good producing technology

We adopt the final goods producing technology from Krusell et al. (2000). This technology admits both the secular rise in relative wages and the rise in schooled hours (see the discussion in footnote 15). The production function  $F(\cdot)$  has four factors: aggregate stocks of capital equipment  $K^{eq}$  and structures  $K^{st}$ , total hours of schooled workers  $L^S$  and total hours of unschooled workers  $L^U$  :

$$F(K^{eq}, K^{st}, L^S, L^U) = A (K^{st})^\alpha \times \left[ \theta_u (A_S L^U)^\eta + (1 - \theta_u) (\theta_k (K^{eq})^\kappa + (1 - \theta_k) (A_S L^S)^\kappa)^\eta \right]^{(1-\alpha)/\eta} \quad (20)$$

We constrain the parameters of the technology to guarantee two input relationships. Capital equipment and labor services of schooled are complementary if the elasticity of substitution  $\kappa < 0$ . Total hours of unschooled, on the other hand, are substitutes with the capital-skill aggregate, with elasticity  $\eta > 0$ . Technological progress is embodied in capital equipment and is the source of increasing demand for hours of college graduates in the model. The remaining parameters include: share parameters  $\alpha, \theta_u, \theta_k \in [0, 1]$ , total factor productivity

$A \in \mathbb{R}_{++}$ , and the labor-efficiency parameter  $A_S \in \mathbb{R}_{++}$ .

Under competitive factor markets, rental prices of structures,  $r^{st}$ , equipment,  $r^{eq}$ , and wages of schooled,  $w^S$ , and unschooled,  $w^U$ , hours are equal to their respective marginal products:

$$\frac{r^{st} K^{st}}{Y} = \alpha \quad (21)$$

$$\frac{r^{eq} K^{eq}}{Y} = \frac{(1 - \alpha) \Theta \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\frac{\eta - \kappa}{\kappa}}}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}} \cdot \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa \quad (22)$$

$$\frac{w^S L^S}{Y} = \frac{(1 - \alpha) \Theta \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\frac{\eta - \kappa}{\kappa}}}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}} \quad (23)$$

$$\frac{w^U L^U}{Y} = \frac{(1 - \alpha) \left( \frac{L^S}{L^U} \right)^\eta}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}} \quad (24)$$

where  $\Theta \equiv \frac{(1 - \theta_u)(1 - \theta_K)^{\frac{\eta}{\kappa}}}{\theta_u}$  and  $Y$  denotes aggregate output,

$$Y = F(K^{eq}, K^s, L^S, L^U) \quad (25)$$

Equations (23) and (24) imply the following expression for the education premium:

$$\frac{w^S}{w^U} = \Theta \left( \frac{L^S}{L^U} \right)^{\eta - 1} \left( \frac{\theta_K}{1 - \theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\frac{\eta - \kappa}{\kappa}} \quad (26)$$

According to (26), the education premium is ceteris paribus negatively related to relative total hours (note that  $\eta - 1 < 0$ ). Hence, for our model to be consistent with the positive secular movement of relative wages and hours (Fact 2 in Section 2), capital equipment  $K^{eq}$  must grow fast enough relative to skilled hours (in efficiency units),  $A_S L^S$ . In our data, the ratio of quality-adjusted capital equipment stock to total schooled hours indeed increases by a factor of four from 1961 to 2002. Therefore, the KORV technology, as opposed to other standard aggregate technologies, is consistent with the secular increases in relative wages and

hours.<sup>15</sup>

### 3.5 Government

The government in the model collects tax revenues, pays for government expenditures  $G$  and makes transfers to individuals. The government budget is balanced in each period:

$$\begin{aligned} & \tau_c \sum_{a=1}^{63} \left( \varepsilon_a c_a^S + \left( \frac{1}{63} - \varepsilon_a \right) c_a^U \right) + \sum_{a=1}^{63} \varepsilon_a \tau_a^S y_a^S + \sum_{a=1}^{63} \left( \frac{1}{63} - \varepsilon_a \right) \tau_a^U y_a^U \\ = & G + \sum_{a=1}^{63} \varepsilon_a TR_a^S + \sum_{a=1}^{63} \left( \frac{1}{63} - \varepsilon_a \right) TR_a^U \end{aligned} \quad (27)$$

where  $y_a^S$  ( $y_a^U$ ) is taxable income of age  $a$  schooled (unschooled) individual;  $\tau_a^S$  ( $\tau_a^U$ ) is the corresponding average income tax rate, defined as  $\tau_a^S = T(y_a^S)/y_a^S$  and  $\tau_a^U = T(y_a^U)/y_a^U$ . The part of the income tax revenue due to retirement contributions is distributed among the retirees. In the benchmark version of the model, per capita transfers are equalized, implying a redistribution of retirement payments in favour of unschooled retirees. This captures the redistributive feature of the pay-as-you-go pension system currently in place for example in the United States. We consider other specifications of the retirement system in Section 5.

### 3.6 Market clearing and equilibrium

There are 4 markets in the model: capital rental markets, and markets for labor services of schooled and unschooled individuals, and the final goods market. The first three markets'

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<sup>15</sup>The education premium derived from a Cobb-Douglas technology is

$$\frac{w^S}{w^U} = \text{const} \cdot \left( \frac{L^S}{L^U} \right)^{-1}$$

so that the education premium and relative total hours are negatively related.

If the CES technology is symmetric (see Katz and Murphy (1992)), relative wages and hours are (as in the Cobb-Douglas case) negatively related, unless labor efficiency  $A_S$  grows fast enough:

$$\frac{w^S}{w^U} = \text{const} \cdot A_S^\eta \left( \frac{L^S}{L^U} \right)^{\eta-1}$$

Krusell et al. (2000) argue that the growth in labor efficiency required to account for facts 1-3 is implausibly large: 11% per year, or a factor of 25 over 30 years.

clearing conditions are:

$$\sum_{a=1}^{63} \varepsilon_a k_a^S + \sum_{a=1}^{63} \left( \frac{1}{63} - \varepsilon_a \right) k_a^U = K^{st} + qK^{eq} \quad (28)$$

$$\sum_{a=1}^{63} \varepsilon_a l_a^S = L^S \quad (29)$$

$$\sum_{a=1}^{63} \left( \frac{1}{63} - \varepsilon_a \right) l_a^U = L^U \quad (30)$$

with the final goods market clearing by Walras law.

We define the **steady state competitive equilibrium** as an allocation  $\{c_a^X, l_a^X, k_{a+1}^X\}_{a=1, X=\{S,U\}}^{63}$ ,  $\{Y, K^{st}, K^{eq}, \varepsilon, e, L^S, L^U\}$  and prices  $r, r^{st}, r^{eq}, w^S, w^U$  such that, given prices, as well as the relative price of equipment  $q$ , productivity  $A$ , tuition cost  $N$ , income tax function  $T(\cdot)$ , and government purchases  $G$ , the allocation solves the problems of unschooled individuals (8)-(7) and schooled individuals (8)-(12); education choice equations (17)-(19) are satisfied; input prices are equal to marginal products (21)-(24); and all markets clear (28)-(30).

## 4. Model and the U.S. data

This section consists of two parts. We first calibrate the model to key characteristics in the U.S. data from 1961 to 2002. Then we investigate the capability of the benchmark model to match facts 1 through 3 outlined in Section 2: increase in the education premium, rise in the fraction of college graduates, and slight decrease in the workweek length. We emphasize the role of education choice and labor supply together with productivity (embodied and neutral) for matching these facts.

### 4.1 Calibration

Our calibration proceeds in three steps. We first calibrate the parameters of the aggregate technology following Krusell et al. (2000). Then we calibrate preference parameters and cost of education. And finally, we determine the income tax functions.

### 4.1.1 Technology

As noted in the previous Section, KORV's aggregate technology in combination with competitive factor markets implies that the education premium is a (nonlinear) function of two ratios: total hours of schooled workers per total hours of unschooled workers,  $\frac{L^S}{L^U}$ , and equipment per total schooled hours (in efficiency units),  $\frac{K^{eq}}{A_S L^S}$ . The education premium function contains six parameters: factor demand elasticities  $\eta$ ,  $\kappa$ , share parameters  $\alpha$ ,  $\theta_u$ ,  $\theta_k$ , and the labor-efficiency parameter  $A_S$ . We choose values for these six key characteristics of the U.S. economy: the education premium in 1961 and 2002; the total labor share in 1961 and 2002; the ratio of stocks of capital structures and equipment in 1961; and real GDP in 1961. Table 3 contains the calibrated parameter values. We borrow values for capital depreciation rates  $\delta_{st}$ ,  $\delta_{eq}$  directly from KORV, 0.050 and 0.125, respectively. The initial asset holdings of individuals are zero.

After calibrating the parameter values of the aggregate technology, we use NIPA data for output and capital inputs as well as CPS data for hours worked by education group to infer the change in neutral productivity  $A$  from 1961 to 2002. We find that the neutral productivity level increased by 42%, or about 1% per year.

This calibration procedure identifies the aggregate technology that is consistent with secular movements of relative prices and aggregate quantities of labor and capital in the U.S. from 1961 to 2002. By construction, the ability of our general equilibrium model to match the observed trend in the education premium depends on its ability to match the historic levels of aggregate supplies of total hours and savings given wages and rental rates, changes in exogenous factors (such as productivity, taxes, etc.) and the specification of aggregate technology. At the end of this Section, we show that the complete model, when calibrated, fits the data well. We then use the model in Section 5 to investigate (a) the importance of education choice and labor supply for wage inequality; and (b) the effects of alternative government policies on schooling, supply of hours and welfare.

### 4.1.2 Income tax function and retirement benefits

Let  $y$  denote an individual's taxable income. The effective average income tax rate  $T(y)/y$ , denoted by  $\tau(y)$ , has three components: federal  $\tau_{fed}(y)$ , state  $\tau_{st}(y)$  and social security tax rates  $\tau_{ss}(y)$ . Given data availability, we first estimate each component separately and then

combine them to obtain the estimated income tax rate:

$$\tau(y) = \underbrace{\tau_{fed}(y)}_{\text{federal}} + \underbrace{\tau_{st}(y)}_{\text{state}} + \underbrace{\tau_{ss}(y)}_{\text{social security}} \quad (31)$$

The federal tax function is approximated by a polynomial of degree five. The polynomial form is easy to estimate and it fits fairly well to both macro and micro data on income tax rates <sup>16</sup>:

$$\tau_{fed}(y) = a_0 + a_1y + a_2y^2 + a_3y^3 + a_4y^4 + a_5y^5$$

The parameters are determined using micro data from the Statistics of Income (SOI) as provided by the Internal Revenue Services. To estimate parameters  $a_0$  through  $a_5$  we use Internal Revenue Service data on individual federal income tax returns in 1961 and 2002. We regress the average tax rate for each person on her respective income.<sup>17</sup> Table 4 provides the estimated parameter values. To obtain the state tax rates, we use the State Tax Handbooks for 1964 and 2002 to find the states' statutory tax schedules. From those we take the highest tax rates for each state and determine a population weighted average,  $\overline{tx}_{st}$ . This is then used to determine a state tax function of the following form:

$$\tau_{st}(y) = \begin{cases} \frac{y}{20,000} \overline{tx}_{st} & \text{for } y \begin{cases} \leq \\ > \end{cases} 20,000 \end{cases} .^{18}$$

For the social security tax, we make use of data provided by the U.S. Social Security Agency on social security tax rates and the social security tax base to determine the proportional rates. There are some social security taxes that apply for all levels of income above a certain minimum,  $\underline{y}$ . The respective tax rate is denoted by  $\overline{tx}_{other\ SS}$ . This is not the case for social security taxes that are counted toward the retirement fund. There exists an upper bound on income,  $\bar{y}_{Retirement}$ , up to which the statutory retirement tax rate,  $\overline{tx}_{Retirement}$ , is applicable. All income above that bound is not taxed by the retirement part of social security. This leads to a lower average social security tax rate for persons with income beyond  $\bar{y}_{Retirement}$ . These facts, taken together, lead to the following formula for the social security tax rate:

$$\tau_{ss}(y) = \begin{cases} 0.7 (\overline{tx}_{Retirement} + \overline{tx}_{other\ SS}) & \text{for } \underline{y} \leq y \begin{cases} \leq \\ > \end{cases} \bar{y}_{Retirement} \end{cases} .$$

<sup>16</sup>See, for example, Gouveia and Strauss (1994).

<sup>17</sup>Throughout the determination of the tax function we work with real US\$ 2000.

<sup>18</sup>The number 20,000 refers to year 2000 US\$ and capture the approximate level of income beyond which the state taxes lose their progressive nature.

Note that the factor of 0.7 is required since the social security taxes apply only to wage income and thus to about 70% of the total income a person earns.

Finally, we smooth out the resulting function  $\tau(y)$  by a polynomial of degree five in order to ensure continuous differentiability of the tax function. It turns out that the difference between the approximation and the original is fairly small. Both the  $R^2$  and the adjusted  $R^2$  are 0.999.<sup>19</sup>

Figure 8 plots the estimated effective average income tax rates as a function of disposable income for 1961 and 2002. Two features stand out. First, the level of the income tax function for 2002 is virtually the same as in 1961. To illustrate this we consider college graduates' income tax. The median income of college graduate in 2002 was around 60 thousand US\$ 2000. So, the average tax rate for a median income college graduate was 20.7%, whereas in 1961 it would have been 17.1%. Second, the income tax schedule is fairly progressive. In 2002 the average tax rate of the median income unschooled individual was 17.5%, which is lower than the average income tax rate of a median income schooled person, 20.7%. When we consider tax policies in Section 5, we find that the progressivity of income taxation is an important factor for an individual's decision to go to college, and has a large impact on the education premium.

In the model, tax revenues are used to pay for government expenditures and lump-sum transfers to workers and retirees. In the NIPA, the government expenditures (plus net exports) to GDP ratio decreased from 25% in 1961 to 16% in 2002. In the benchmark model, the revenue from social security tax is distributed equally among the retired population in a lump-sum fashion. This way of redistribution captures the pay-as-you-go pension benefit system in the U.S. well. It favours income-poor unschooled relative to income-rich schooled retired individuals. The residual budget balance is distributed equally among all workers.

#### ***4.1.3 Preferences and education cost***

The discount factor  $\beta$  is 0.96 so that the interest rate is 4%. We determine the share of leisure in the utility function,  $\phi$ , by matching the workweek of college graduates in 1961 at 44.5 hours per week. The parameter value, 1.78, is consistent with values found in the

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<sup>19</sup>The procedure just described is based on an analysis of Current Population Statistics data on taxes for the year 2002. We use the procedure just outlined and compare the results with the one in which we estimate a fifth order polynomial for the combined federal, state, and social security tax rates directly from the CPS. The difference is small. We would have preferred to use the CPS-micro approach both for the year 1961 and the year 2002, but the CPS does not provide tax data for the 1960s. To preserve consistency we decided to use the approach outlined in the text.

empirical literature.<sup>20</sup>

To calibrate the utility cost of education, we employ data on the cognitive ability test performance for individuals in The National Longitudinal Survey of Youth 1979 (NLSY79). It is a nationally representative panel of more than 12,000 men and women who are between the age of 14 and 22 when they are first interviewed. In 1980, 94% of the NLSY79 respondents took the Armed Services Vocational Aptitude Battery (ASVAB) - a series of tests measuring knowledge and skill in 10 areas, ranging from general science and knowledge of mathematics to word knowledge and paragraph comprehension (see Appendix C for details). A composite Armed Forces Qualifications Test score (AFQT) was constructed for each youth based on selected sections of the ASVAB. We use the score distribution of respondents to the NLSY79 to estimate the  $\iota(\cdot)$  function that relates an individual's cognitive ability in the model to their utility cost of education (see equation (14)). HERE SHOULD BE AN ARGUMENT STATING THAT IT IS REASONABLE TO USE THE 1979 TEST AS A MEASURE OF ABILITY IN OUR CONTEXT. SOMETHING ABOUT THE ASSUMPTION THAT THE DISTRIBUTION OF COGNITIVE ABILITY STAYS THE SAME OVER TIME AND THE TEST SERVES AS A GOOD MEASURE OF COGNITIVE ABILITY.

For our purposes, we restrict the sample to white males who took the ASVAB tests and who were not in college in 1979. We then define standardized test scores by taking out age-specific weighted means from the raw AFQT scores and normalize by the (weighted) standard deviation of scores across the entire sample:

$$Q_i = \frac{Q_i^{AFQT}}{std(Q_i^{AFQT})}$$

Here  $Q_i^{AFQT}$  is the raw (demeaned by age) AFQT score and  $Q_i$  is the standardized AFQT score for individual  $i$ .

For each individual in this sample we document her highest grade completed as of May 1, 1988. Let  $HGR_i$  be equal to 1 if individual  $i$ 's highest grade completed was 4 or more years in college, and 0 otherwise. We estimate the parameters of the following weighted probit equation:

$$HGR_i = h_0 + h_1Q_i + h_2Q_i^2 + h_3Q_i^3 \tag{32}$$

Table 5 contains regression results. Given a normality assumptions on cognitive ability in

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<sup>20</sup>We rescale taxable income in the model to correspond to that in the data. The scale parameter is picked to match the average tax rate faced by the median income individual in the data in 1961 at 15.5%.

the model, the right hand side of (32) is a third-order polynomial approximation of function  $\iota(\cdot)$ . For the estimated parameter values  $h_i$ , Figure 7 plots the probability of going to college conditional on the test score. According to our estimates, an individual with the median score (65 AFQT points, or 0.4 standardized AFQT points) completes college with probability 17%. Whereas this probability for an individual in the 75th percentile (86 AFQT points, or 1.4 standardized AFQT points) finishes college almost half the time, 46%. According to equation (17), the area below the probability line gives the enrollment rate.

It only remains to calibrate two parameters of the cost function (14),  $d_{\max}$  and  $\delta_d$ . We use equations (17) and (18) relating education cost and the utility difference between two education groups to the fraction of college graduates. To infer the utility difference consistent with the observed macro data, we calculate the steady state equilibria such that the model matches the fraction of schooled workers and quality adjusted capital equipment stock in the data in 1961 and 2002.

This procedure gives us two pairs of enrollment rates and utility difference values, which together with the function  $\iota(\cdot)$  are plugged into (17) and (18) to infer the values for  $d_{\max}$  and  $\delta_d$  (see Table 3).<sup>21</sup>

Finally, we take pure monetary tuition cost from the data. These cost include tuition and fees of full time undergraduate students and were taken from the College Board’s publication “Trends in College Pricing 2005”. The numbers supplied are split into public and private college cost. To get an average rate, we weighted the average of the private and public universities student tuition and fees series by the fraction of private university compared to all students ( $0.2 \text{ tuition}_{\text{private}} + 0.8 \text{ tuition}_{\text{public}}$ ). We find that the ratio of average annual college tuition to median annual wage income of an unschooled worker is 0.16 for 1961 and increased to 0.37 in 2002.

## 4.2 How well does the model match the data?

We first ask how well does the model capture the observed changes in the education premium, fraction of college graduates and average working time, given the estimated education cost function (14) and given the change in exogenous factors, reported in Table 6. The driving force of wage inequality is skill-biased technological change visible in the dramatic 4-fold fall in the price of capital equipment from 0.96 in 1961 to 0.26 in 2002. Neutral productivity also increased by a solid 50%. A contribution of this paper is to study factors affecting the *supply*

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<sup>21</sup>We assume that the intercept in (32) is included in the calibrated value of parameter  $d_{\max}$ .

of schooled working time. These factors in the model include progressive income taxes (see Figure 8), consumption taxes ( $-1.7$  percentage points change), government purchases (plus net exports) to GDP ( $-37\%$  change), tuition relative to labor income of unschooled ( $131\%$  increase).

Table 7 compares simulation results for the benchmark model and the data in 1961 (rows 1 and 2) and 2002 (rows 3 and 4).

By construction of our calibration exercise, the success of the 1961 model simulation to match the data hinges on the model's ability to account for the average time of unschooled workers. The model is successful in predicting 39.3 hours per workweek for an unschooled individual, which is close to 39.8 hours per week in the data. As a result, the predicted education premium matches 1.43 in the data.

The model for 2002 is quite close to the data, too. It predicts the increase in the education premium up to 1.76 (compared to 1.80 in the data), accounting for 89% of the observed change. It does well in predicting the large increase in the fraction of schooled working hours and almost no change in the average hours worked. The fraction of schooled workers in the 2002 model increases to 27.6% (compared to 29.3% in the data), whereas working time of schooled (unschooled) workers decreases somewhat to 42.8 and 36.2 respectively in the model (43.2 and 38.3 in the data). We summarize that the calibrated model is successful in generating the main labor facts 1 through 3 presented in Section 2.

To understand the importance of education choice and productivity for wage and working time inequality in the model, we conduct a series of counterfactual experiments. In each experiment, we simulate a steady state equilibrium for the year 2002 keeping one of the exogenous factors at its 1961 level. We compare outcomes for this counterfactual model to those of the 2002 benchmark model. If the difference between the outcomes of the two experiments is large, then the factor that was "shut down" in the counterfactual simulation is interpreted to be important for wage inequality and education choice. We present only the main factors and their impact on labor supply, fraction of college graduates, capital equipment, and the education premium. We can see in Table 6 that various of the exogenous factors changed considerably. There is the price of capital equipment that fell by 73 percentage points from 1961 to 2002, and there is neutral TFP. The remaining forces: income taxes, consumption taxes, government spending, and tuition cost also changed, but not nearly as much as the previous three forces. As a first cut we focus on the main forces and later consider other aspects. Table 8 provides the results.

When productivity (embodied and neutral) are at the 1961 level, any change in hours and in the number of workers is driven by the remaining factors from Table 6: income and consumption taxes, government expenditures and tuition costs. Tuition affects only a small fraction of population, and the rest of the changing factors experienced only small changes from 1961 to 2002. Hence in this experiment all the moments are close to the 1961 level: the fraction of schooled workers is 11.5%, average hours are 44.3 and 38.1 for schooled and unschooled workers, capital-output ratio is 2.71, output level (relative to output level in the benchmark 1961 model) is 0.97 and, finally, the education premium is 1.46. Hence, in the model without productivity changes, there is little to no changes relative to the 1961 benchmark model. One interesting finding is in regard to factor neutral productivity gains. Here we can see that not only capital embodied technological progress has a large impact on the schooling rate and the stock of equipment, but neutral progress has a sizable impact too. Subsequently it also accounts for some of the education premium.

As an extra experiment, we consider a world where the supply of schooled individuals is completely inelastic and thus the schooling rate cannot change from its 1961 level. This might be seen as a country that has problems to increase its university capacities, has restrictions on university access, or the schooling system is incapable of providing more than a bare minimum of the population with the ability to go to college. In this case, the increase in demand for capital equipment and schooled labor is not accompanied by the increase in the supply of schooled workers. As a result, the education premium skyrockets to 2.53. This increase is 41% higher than the 2002 benchmark model and 77% higher than the 1961 level. The workweek length of schooled sees almost no increase with 43.0 hours per week (only a quarter of hour increase compared to the 2002 benchmark model). Regarding capital equipment, due to capital-skill complementarity in the production technology, a scarce supply of schooled hours decreases the demand for capital equipment. The stock of equipment is half of that in the benchmark 2002 model. To summarize: education choice, in addition to technological change, is critical for accounting for the change in the education premium and schooled hours.

## **5. The effects of government policy on schooling and inequality**

In this Section, we study the importance of public policy (income taxes, social security taxes, retirement benefits, tuition subsidies, etc.) for shaping secular changes in the education premium and hours, labor productivity and the accumulation of physical capital.

After having documented the overall success of the model in generating the main macro facts, we turn to the analysis of the effects of alternative social policies on education choice, education premium and output level. We group our policy experiments in three categories. First, we study income tax reforms that lower the level and progressivity of the income tax. Second, we look at changes in the retirement system: (i) reforms that improve viability of the pension fund (increase in social security contributions and the increase in mandatory retirement age); and (ii) reforms that affect redistribution features of the pension system (a move to proportional pension benefits or a switch from a pay-as-you-go-system to a fully-funded pension system). Finally, we derive consequences of policies that improve the efficiency of the schooling system.

Our focus will be on two effects of policy change that distinguish our analysis from the previous studies. Both of these effects turn out to be important for shaping responses in output and inequality to the government policy reforms. We distinguish two effects by the timing of their impact on the fraction of schooled workers on the economy. The fraction of schooled workers is a moving average of school enrollment rates over four or five decades. So we will think of the medium run effect as the one occurring within one or two decades after a policy change so that the response of the enrollment rate is not yet transpired in the fraction of schooled workers in the economy. In the medium run, there is a disproportionate response of working hours and savings of schooled individuals and those of unschooled individuals. This difference is due to (i) redistribution of income under progressive income taxation and the incumbent pension system, and (ii) the change in demand for schooled hours due to the change in capital accumulation (stemming from capital-skill complementarity).

In the long run, the fraction of educated persons adjusts to the policy change. Our policy experiments will demonstrate that the long run reallocation of total hours from unschooled to schooled workers often has a dampening effect on wage inequality and an amplifying effect on the level of output.

## 5.1 Income tax reforms

Our first set of experiments centers around tax system reforms.

In the first experiment, the progressive income tax function is replaced with a (revenue-neutral) proportional income tax rate. This reform is akin to the "fundamental" flat tax reform proposed by Hall and Rabushka (1995), in which the marginal tax rate on income above a certain threshold is flat. Life cycle models of wage and income inequality typically

predict positive effects of such a reform on the level of output and capital accumulation, small effect on average working hours, as well as ambiguous effects on wage and income inequality.<sup>22</sup>

In the experiment, the level of the flat tax rate is equal to the tax rate faced by the mean income individual in the 2002 benchmark model, so that the tax reform is revenue neutral (see Table 9). The marginal tax rates are lower for both income groups, and more so for schooled workers. The reduction of the marginal tax rates has positive effects on labour effort and savings. The average working time of unschooled workers goes up by 1.5 hours per week. Schooled workers also work more but by only an hour per week. Smaller effect on schooled workers, whose time is compensated at a higher wage rate, is due to the income effect. Furthermore, lower future marginal tax rates on capital part of individual's income encourage savings: capital-output ratio increases from 2.68 to 2.74. Higher level of capital equipment, in addition to increasing output by 4%, boosts the demand for schooled hours. The wage rate of schooled workers goes up by almost a dollar, whereas wages of unschooled workers experience almost no change. This drives up the wage premium from 1.76 to 1.78 in the medium run.<sup>23</sup> Income inequality also rises as the Gini coefficient applied to taxable income increases from 0.304 to 0.324.

Positive income effect of the tax cut and higher wages for schooled hours increase benefits of going to school. They induce a larger fraction of the population to enroll in school. In the long run, the fraction of schooled workers goes up by 2 percentage points, whereas the workweek length stays virtually unchanged. Higher supply of total hours of schooled workers and shorter supply of unschooled hours, in turn, reduce compensation of schooled workers relative to unschooled workers, bringing the education premium from 1.78 down to 1.72. Income inequality also falls from 0.324 in the medium run to 0.318. Hence, the wage inequality after the reform is lower than before the reform with schooled wages unchanged and unschooled wages higher by about half a dollar than before the reform. The long run income inequality remains higher than before the reform, reflecting faster accrual of the capital income induced by lower marginal tax rates. Finally, the increase in the fraction of more productive (schooled) workers boosts output by an additional 2%. To sum up, in the long run changes in the skill composition of the workforce decrease wage inequality, mitigate

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<sup>22</sup>Positive effects on output and savings range from small ( $\sim 1\%$ ) in Heckman et al. (1998b) to large ( $\sim 10\%$ ) in Ventura (1999) and Altig et al. (2001). The flat income tax reform decreases the education premium in Heckman et al. (1998b), and Gini earnings inequality in Castaneda et al. (1998). The reform increases the Gini income inequality in Heckman et al. (1998b), Altig and Carlstrom (1999). In Ventura (1999) the distributions of earnings and income become more concentrated after the reform. In Diaz-Gimenez and Pijoan-Mas (2006) the effects on earnings and income inequality are small.

<sup>23</sup>Note that the level of the stock of capital structures is irrelevant for education premium in the model, see equation (26).

the increase in income inequality and augment positive effect on output.

In the second experiment, we lower the tax schedule proportionally by 19% (that is, average and marginal tax rates are multiplied by 0.81).<sup>24</sup> This reform is comparable to the flat tax reform in the sense that after the reform the mean income individual faces the same marginal income tax rate of 19.5%. The overall results turn out to be very close to those for the flat tax reform (see Table 9). Lower level of marginal tax rates encourage individual labor supply and savings, leading to the increase in output, capital-output ratio and, via capital-skill complementarity, to higher wage inequality in the medium run. The incentives of attending school increase, but not as much as in case of the flat tax, since the income tax after the cut is still progressive. In the long run, the fraction of schooled workers goes up by 1.3% points, wage and income inequality measures are close to their respective levels prior the reform, 1.74 and 0.309.

We conclude that proportional 19% tax cut yields implications for inequality and potential output that are quantitatively very similar to those after the switch from progressive to flat income tax.

WHAT ABOUT OTHER UTILITY FUNCTIONS, DIFFERENT FROM LOG?

## 5.2 Change in retirement system

Many European countries as well as the United States, Canada, and Japan experience population aging. Especially for countries with a pay-as-you-go retirement system, PAYG, like the United States, this can lead to severe problems in the future. Various ways out of the retirement fund imbalances have been suggested and in some cases tried: increasing the full benefit retirement age (this has been done in the United States, Germany, Italy among others), increasing contributions to the pension fund or cutting retirement benefits. More drastic reforms of the pension system aim at reducing or eliminating the redistribution of income from income rich to income poor retirees. Examples of such reforms include decreasing the redistribution of retirement benefits from higher to lower income retirees, or switching from a PAYG to a fully-funded retirement system (as suggested, for example, in Prescott (2004)).

In the model, life expectancy increases by 4 years from 78 years in 1961 steady state, to 82 years in 2002 steady state. As a result, the fraction of retired workers in population goes

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<sup>24</sup>A similar in magnitude tax cut was instituted in the U.S. by the Tax Reform Act of 1986. See Altig and Carlstrom (1999) for the analysis in the general equilibrium lifecycle framework. In our model, the tax cut is financed by higher lump sum taxes on workers.

up from 23.0% to 27.7%.<sup>25</sup> This increase captures ??? of the increase in the ??? data. The remaining rise in the share of retired workers in population is due to higher proportion of population born after the World War II ("baby-boomers"), a feature that is not modelled here for simplicity.

The size of the retirement benefits per a retiree in 2002 steady state (expressed as a fraction of the average lifetime wage income) equals to 0.15 for schooled and 0.31 for unschooled retired individuals. If population had not aged, the benefits would have been higher at 0.19 and 0.41 respectively. We consider two reforms that are intended to offset the decrease in the size of retirement benefits due to population aging. Specifically, the revenues of the pension fund are increased by raising the social security tax rate or by increase in the mandatory retirement age (see Table 10).

First, we increase contributions to the pension fund by 3.5% points. This reform increases the average tax rates by 3.5% points without affecting the marginal tax rates. The resulting income effect somewhat discourages savings and labor effort as the capital-output ratio and average hours go down slightly. Wage and income dispersion change very little, whereas potential output falls by 2%. This reform also decreases incentives to educate by increasing the share of retirement benefits to wage income to unschooled workers faster than those for schooled workers (0.31 to 0.41 for unschooled, 0.15 to 0.19 for schooled workers). In the long run, the fraction of schooled employed falls to 27.1.

In the alternative experiment, we increase the mandatory retirement age from 64 to 67. Mechanically, this brings the fraction of retired workers in population back to 1961 level of 23.0%. Hence, as in the previous case, benefits per a retired person increase to the same level as in 2002 steady state with life expectancy at 1961 level (78 years) and retirement after 64 years old. Longer working age and shorter life after retirement reduce incentives to save and work. Capital per output goes down from 2.68 to 2.63 and the workweek length declines by 1 hour for schooled and 1.5 hours for unschooled workers. This leads to a medium run 5% decline in output. The fraction of schooled employed falls to 27.1 just as in the first experiment. The overall effect on wage inequality is close to zero. Income distribution is more concentrated since the growth of individual income over the lifetime is smaller.

The next two experiments are aimed at eliminating the redistribution of retirement bene-

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<sup>25</sup>We use life expectancy at the age 65 from Arias (2004). Life expectancy at the age of 15 is 69 years in 1961, and it is 75 years in 2002. We assume that life expectancy at the age 65 captures the length of life that individuals entering the workforce in the 1960's and 2000's expected to have.

fits from schooled (income rich) retirees to unschooled (income poor) retired individuals (see Table 11). In the first experiment, lump sum pension benefits as a fraction of wage income are equalized for all retirees, keeping total retirement payments the same. After the reform the retirement benefits to wage income ratio increases from 0.15 to 0.24 for schooled retirees, and decreases from 0.31 to 0.24 for unschooled retired workers.<sup>26</sup> The reform has a very small effect on working time and savings, since the the redistribution of lump sum benefits generates a negligible income effect. For example, the ratio of lifetime retirement payments to lifetime wage income increases by only 1.5% points for schooled and decreases by 3.5% points for unschooled individuals. The reform only marginally improves the incentives of attending college as the fraction of schooled workers goes down to 27.1% in the long run. The effects on output and wage and income inequality are also small.

Finally, in our last experiment, the PAYG retirement system is substituted with the fully-funded system, in which each retired individual saves for his own retirement. The reform affects the economy via two distinct channels. First, it stops the redistribution of the retirement income from schooled to unschooled workers. We know from the previous experiment that this channel is weak. Second, a switch to a fully-funded retirement system implies a positive income effect on average hours worked and savings due to the elimination of the social security part of income taxes. All workers under the fully-funded system work by about 1 hour more per week than under the PAYG system, and the capital-output ratio boosts to 2.86. As shown in Table 11, the model implies a large long run increase in the fraction of schooled workers to 29.3, and a slightly lower education premium, 1.74. As in the case of income reforms, higher savings rate and labor effort lead to higher lifetime dispersion of income, with Gini coefficient increasing from 0.304 to 0.320 in the long run. Most importantly, output level rises by 6% in the medium run, and by 8% in the long run.

To summarize, the reforms of the retirement system aiming at maintaining the viability of the pension fund, discourage incentives to work, save and educate, leading to output losses. They typically do not affect wage inequality and slightly decrease income inequality. Quite opposite results are expected from eliminating the PAYG retirement system altogether and letting agents save for their own retirement. This reform influences the economy not through the abolition of redistribution of pension benefits, but through lower average tax rates.

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<sup>26</sup>The benefit-earnings ratios estimated by the social security agency at <http://www.socialsecurity.gov/cgi-bin/benefit6.cgi> for persons with pre-retirement income of 20,000 and 40,000 U.S. dollars in 2006 (roughly corresponding to earnings of unschooled and schooled individuals in 2002) are 0.42 and 0.31. The level of respective benefits in the model, 0.31 and 0.15, is lower (partially due to not modelling the employers contribution) and the redistribution is higher than in the data. Both discrepancies imply the conservative approach in the redistribution experiments.

Long after the reform wage inequality decreases, income dispersion may rise.

### 5.3 Efficiency of the schooling system

In the final part of our social policy section we consider policies that directly affect the incentives to educate. In our analysis, we will focus on the effects of such measures without spelling out precisely how they are implemented.

In the first experiment, college tuition is fully subsidized. Many European countries had full subsidization of higher education for many decades. The aim was a society with equal opportunities for all its members and a compact income distribution. In the model, this reform increases disposable income of schooled individuals by removing the necessity to pay tuition costs. Higher income implies lower effort as average hours worked per week by a schooled worker decrease by 1 hour (see Table 12). In the medium run, lower supply of total hours of schooled workers leads to the increase in their wages relative to wages of the unschooled workers, and also to the slight increase in hours worked by unschooled workers. Thus in the medium run, subsidizing tuition decreases inequality in hours worked and increases wage inequality, whereas the effects on aggregate output and savings are very small. In the long run, the fraction of schooled workers goes up to 30.5%, a swing of 2 million male workers. A surge in the supply of schooled workers overturns the initial increase in wage and income inequality as education premium and Gini income coefficient fall relative to the baseline case, 1.69 and 0.299 respectively. Higher number of more productive workers increases potential output by 3% in the long run.<sup>27</sup>

In the second experiment, we consider the consequences of a policy that leads to a decrease in college study time from 4 to 3 years. It is assumed that college graduates are able to complete the 4-year college in three years. Hence, by decreasing the number of years of forgone labor earnings, such a policy implies smaller costs of attending college. In this sense, it is similar to the experiment with tuition subsidy. Indeed, as Table 12 shows, the results between the two policy experiments are quantitatively very similar: in the medium run, working hours are redistributed from schooled to unschooled workers; in the long run, the redistribution of total hours is reversed as the fraction of college graduates rises. The

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<sup>27</sup>In the model, the effect of the change in tuition cost on the enrollment rate is small. This implication of the model is consistent with the general equilibrium analysis in Heckman et al. (1998a). They find that the increase in tuition cost by about a third decreases the average probability of going to college by 0.08. In our model the corresponding decrease in probability is even smaller, 0.02. We conjecture that the difference in the results is due to the redistribution of hours worked across workers in our model, whereas in Heckman et al. (1998a) hours are supplied inelastically.

economy achieves lower wage and income inequality and higher long run output.

In our last experiment, we consider policies that improve the efficiency of the schooling system and lead to a higher performance on the cognitive ability test. We focus on policies that do not require the increase in public spending. Examples of such policies include designing a system of graduation tests, advancing homework and grading standards, and facilitating higher competition among schools.<sup>28</sup> In the experiment, we assume that realized raw AFQT scores in the model are 1 point (0.04 of standardized AFQT score) higher than in the benchmark 2002 model. As a result, more individuals qualify to go (and finish) college, so that the fraction of schooled workers increases from 27.6% to 28.5%. Thus this effect represents a pure shift in the supply of schooled workers. As Table 12 demonstrates, even such a small increase in test performance has non-trivial effects on output and inequality. The wage premium goes down from 1.76 to 1.73 and output increases by 1%.

In sum, more effective schooling systems shift out the supply of schooled hours leading to unambiguous decrease in wage and income inequality and increase in long run output.

## 6. Conclusion

In recent years macroeconomic theory has had great success in matching the historical secular increases in relative wages and hours of schooled and unschooled male workers. This paper undertakes the next natural step in economic analysis and asks: What are the effects of various government policies on economy-wide differences in wages, income, working hours and savings? In answering this question we build a framework suitable for policy analysis, which we then use to conduct some of the proposed policy experiments. One of the main roles in driving the demand in the labor market in the data has been attributed to the skill-biased technological change. This paper fills the gap in that literature by building an empirically sound general equilibrium model of aggregate supply in the labor and investment markets.

Our battery of policy experiments highlights the importance of modeling education choice and labor supply for gauging the effects of policy change on wage inequality and output. Table 13 summarizes medium and long run responses in the number of college graduates, wage inequality and output level to policy reforms in our model. Even though the reforms are quite different, two regularities stand out. First, the long run reallocation of the workforce

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<sup>28</sup>Betts and Ferrall (1997) argue that, given little evidence on the benefits from increasing school spending, the more promising type of reforms for public schools in the U.S. and Canada may be changing the incentive structure for public schools.

between unschooled and schooled jobs can be quite substantial, reaching on several occasions a million or more workers. Such response is caused by the differential effect of policy change on the lifetime income, consumption and leisure streams of schooled and unschooled individuals. Typically a policy change operates through the redistribution of income or through its effect on the accumulation of skill intensive capital stock.

The second common feature across the reforms is the effect of the workforce reallocation on wage inequality and output level. Here we distinguish between the medium and long run effects of the policy change, depending on whether the change in the overall fraction of schooled workers in the employed population adjusts. As Table 13 demonstrates higher supply of schooled worker-hours and lower supply of unschooled worker-hours most of the time reverses the medium run effect on wage inequality. The output level receives an additional change of 1 to 2% due to different number of workers in more and less productive jobs.

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Table 1: Working time decomposition for 1961 and 2002

	1961	2002	change <sup>1</sup>
Working hours per capita	33.0	29.9	-3.1
Fraction of employed in population, %	81.7	75.3	-6.4
Fraction of schooled in employed population, %	12.3	29.3	17.0
Workweek length, schooled <sup>2</sup>	44.5	43.2	-1.3
Workweek length, unschooled <sup>2</sup>	39.8	38.3	-1.5
Education premium	1.43	1.80	26.1

Note: <sup>1</sup>change is given as a difference from 1961 to 2002, except for education premium (in %).

<sup>2</sup>Workweek length is calculated assuming 100 hours a week available for work.

Table 2: Changes in labor supply and earnings inequality for 1961, 1980, and 2002 by gender and education group.

	Males		Females	
	1961-1980	1980-2002	1961-1980	1980-2002
Working hours per capita <sup>1</sup>	-3.4	0.3	4.5	5.2
Fraction of schooled in employed population <sup>2</sup>	9.4	7.6	6.8	12.7
Wage, schooled <sup>3</sup>	32.4	30.3	20.0	41.5
Wage, unschooled <sup>3</sup>	37.0	0.10	38.9	23.1
Education premium <sup>3</sup>	-3.3	30.1	-13.6	15.0

Note: <sup>1</sup>in hours per week, <sup>2</sup>in percentage points, <sup>3</sup>in percent.

Table 3: Calibrated parameters

Parameter	Value	Source or Moment
Elasticity of substitution between unschooled hours and equipment, $\eta$	0.693	Wage-bill ratio growth, 1961-2002
Elasticity of substitution between schooled hours and equipment, $\kappa$	-0.536	Labor share growth, 1961-2002
Share of unschooled hours, $\theta_u$	0.467	Wage-bill ratio, 1961
Share of schooled hours, $\theta_k$	0.479	Labor share, 1961
Share of structures, $\alpha$	0.230	NIPA Structures-to-Equipment, 1961
Labour-efficiency, $A_S$	118	Real GDP, 1961
Depreciation rate of structures, $\delta_s$	0.05	Krusell et al. (2000)
Depreciation rate of equipment, $\delta_{eq}$	0.125	
Discount factor, $\beta$	0.96	Real interest rate is 4%
Elasticity of labor supply, $\phi$	1.777	Workweek length of schooled, 1961
Education cost, $\{d_{\max}, \delta_d\}$	{34.11 , 6.10}	Model fits quality-adjusted equipment stock, given fraction of schooled in employed population, in 1961 and 2002

Table 4: Tax function: parameter estimates

	Parameter	1961	2002
Federal	$a_0$	0.057	0.007
Income	$a_1, \times 10^{-5}$	0.231	0.306
Taxes	$a_2, \times 10^{-10}$	-0.257	-0.414
	$a_3, \times 10^{-15}$	0.246	0.348
	$a_4, \times 10^{-20}$	-0.107	-0.134
	$a_5, \times 10^{-25}$	0.016	0.019
	$R^2$	0.33	0.52
State taxes and	$\bar{tx}_{st}$	0.04	0.05
Social Security taxes	$\bar{tx}_{ss}$	0.06	0.12

Note: The federal tax function is approximated by a polynomial of degree five:

$$\tau_{fed}(y) = a_0 + a_1y + a_2y^2 + a_3y^3 + a_4y^4 + a_5y^5$$

The state tax function is of the following form:

$$\tau_{st}(y) = \begin{cases} \frac{y}{20,000} \bar{tx}_{st} \\ \bar{tx}_{st} \end{cases} \text{ for } y \begin{cases} \leq \\ > \end{cases} 20,000 .$$

The social security tax function follows:

$$\tau_{ss}(y) = \begin{cases} 0.7 (\bar{tx}_{Retirement} + \bar{tx}_{other SS}) \\ 0.7 (\bar{tx}_{Retirement} \times \frac{\bar{y}_{Retirement}}{y} + \bar{tx}_{other SS}) \end{cases} \text{ for } y \leq y \begin{cases} \leq \\ > \end{cases} \bar{y}_{Retirement} .$$

Table 5: Estimating the effect of ability on college completion

Parameter	Value	St. dev.
$h_0$	-1.22	0.07
$h_1$	0.62	0.11
$h_2$	0.02	0.08
$h_3$	0.08	0.07

Note: We restrict the NLSY79 sample to white males who took the ASVAB tests and who were not in college in 1979. For each individual in this sample we document her highest grade completed as of May 1, 1988. Let  $HGR_i$  be equal to 1 if individual  $i$ 's highest grade completed was 4 or more years in college, and 0 otherwise. We estimate the parameters of the following weighted probit equation:

$$HGR_i = h_0 + h_1 Q_i + h_2 Q_i^2 + h_3 Q_i^3$$

where  $Q_i$  is the standardized AFQT score for individual  $i$ :

$$Q_i = \frac{Q_i^{AFQT}}{std(Q_i^{AFQT})}$$

and  $Q_i^{AFQT}$  is the raw (demeaned by age) AFQT score.

Table 6: Exogenous factors in the model

	1961	2002	change
Capital embodied progress, $q$	0.96	0.26	-73%
TFP, $A$	1	1.50	50%
Average effective tax rate, $\tau(\cdot)$	see Figure 8		
Consumption taxes, $\tau_c$	6.5%	4.8%	-1.7%pt
Life expectancy	78	82	4 years
Government purchases, $G/Y$	0.25	0.16	-37%
Tuition cost, $B/(w^U l_U)$	0.16	0.37	131%

Source data: National Income and Product Accounts, the College Board's publication "Trends in College Pricing 2005".

Table 7: Benchmark model vs data in 1961 and 2002

	1961		2002	
	Data	Model	Data	Model
Workweek length, schooled <sup>1</sup>	44.5	(*)44.5	43.2	42.8
Workweek length, unschooled <sup>1</sup>	39.8	39.4	38.3	36.1
Education premium	1.43	(*)1.43	1.80	1.76
Capital-output ratio	2.30	2.69	2.43	2.68
Fraction of schooled workers in employed population, %	12.3	(*)12.3	29.3	27.6
Output <sup>2</sup>	1.00	1.04	2.26	2.08

Note: \* denotes moments matched directly by calibration. <sup>1</sup>Workweek length is calculated assuming 100 hours in a week available for work. <sup>2</sup>Output is normalized to 1.00 for 1961.

Table 8: Effect of technological change and education choice in the model

	Model	Model	Experiments with 2002 Model	
	1961	2002	A. No technological progress	B. No change in schooled fraction
Workweek length, schooled <sup>1</sup>	44.5	42.8	44.3	43.0
Workweek length, unschooled <sup>1</sup>	39.4	36.1	38.1	36.9
Wages, schooled	17.3	37.7	17.7	48.0
Wages, unschooled	12.1	21.4	12.1	19.0
Education premium	1.43	1.76	1.46	2.53
Gini coefficient	0.267	0.304	0.292	0.372
Capital-output ratio	2.69	2.68	2.71	2.55
Fraction of schooled workers in employed population, %	12.3	27.6	11.5	12.3
Output <sup>2</sup>	1.00	2.00	0.97	1.64

Note: In experiment A, we simulate a steady state equilibrium for the year 2002 keeping the level of embodied and neutral technological change at 1961 level. In experiment B, the fraction of schooled in employed population is kept at its 1961 level.

<sup>1</sup>Workweek length is calculated assuming 100 hours in a week available for work. <sup>2</sup>Output is normalized to 1.00 in 1961 model.

Table 9: Income tax reforms

	Model	Experiments with 2002 Model			
	2002	A. Switch to proportional tax		B. Income tax cut by 19%	
		Medium run	Long run	Medium run	Long run
Mean marginal tax rate, schooled <sup>1</sup>	24.4	19.5	19.5	19.9	19.8
Mean marginal tax rate, unschooled <sup>1</sup>	23.8	19.5	19.5	19.3	19.4
Workweek length, schooled <sup>2</sup>	42.8	43.5	43.5	43.5	43.5
Workweek length, unschooled <sup>2</sup>	36.1	37.7	37.6	37.3	37.3
Wages, schooled	37.7	38.4	37.6	38.5	38.0
Wages, unschooled	21.4	21.6	21.9	21.6	21.8
Education premium	1.76	1.78	1.72	1.78	1.74
Gini taxable income	0.304	0.324	0.318	0.314	0.309
Capital-output ratio	2.68	2.74	2.75	2.76	2.77
Fraction of schooled workers in employed population, %	27.6	27.6	29.6	27.6	28.9
Output <sup>3</sup>	1.00	1.04	1.06	1.04	1.05

Note: In experiment A, the progressive income tax function is replaced with a (revenue neutral) proportional income tax rate. In experiment B, average and marginal tax rates are multiplied by 0.807. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.

<sup>1</sup>The marginal tax rate of the mean income individual in the group. <sup>2</sup>Workweek length is calculated assuming 100 hours in a week available for work. <sup>3</sup>Output in the 2002 model is normalized to 1.00.

Table 10: Increasing viability of the pension fund

	Model	Experiments with 2002 Model			
	2002	A. Increase in SS tax by 3.5% points		B. Increase in retirement age from 64 to 67	
		Medium run	Long run	Medium run	Long run
Ret. benefits to wage income <sup>1</sup> ,					
schooled	0.15	0.20	0.19	0.20	0.19
unschooled	0.31	0.41	0.41	0.41	0.41
Workweek length, schooled <sup>2</sup>	42.8	42.6	42.6	41.4	41.4
Workweek length, unschooled <sup>2</sup>	36.1	35.8	35.8	34.5	34.6
Wages, schooled	37.7	37.2	37.3	37.1	37.3
Wages, unschooled	21.4	21.3	21.2	21.3	21.3
Education premium	1.76	1.75	1.76	1.74	1.76
Gini coefficient	0.304	0.297	0.298	0.278	0.279
Capital-output ratio	2.68	2.63	2.62	2.63	2.63
Fraction of schooled workers in employed population, %	27.6	27.6	27.1	27.6	27.1
Output <sup>3</sup>	1.00	0.98	0.98	0.95	0.95

Note: In experiment A, the social security tax rate is increased by 3.5%. In experiment B, the retirement age is increased from 64 to 67 years of age. In both experiments, the ratio of the retirement benefits to the mean wage income is the same as in the 2002 model without population aging (0.19 - schooled, 0.41 - unschooled).

<sup>1</sup>The ratio of the mean annual retirement benefits to the mean annual wage income for a retiree in the group. <sup>2</sup>Workweek length is calculated assuming 100 hours in a week available for work.

<sup>3</sup>Output in the 2002 model is normalized to 1.00.

Table 11: Eliminating redistribution of retirement benefits

	Model 2002	Experiments with 2002 Model			
		A. Equal retirement payments		B. Switch to fully-funded SS system	
		Medium run	Long run	Medium run	Long run
Ret. benefits to wage income <sup>1</sup> ,					
schooled	0.15	0.24	0.24	0	0
unschooled	0.31	0.24	0.24	0	0
Workweek length, schooled <sup>2</sup>	42.8	42.6	42.6	43.6	43.6
Workweek length, unschooled <sup>2</sup>	36.1	36.0	36.0	37.5	37.4
Wages, schooled	37.7	37.8	37.7	39.4	38.7
Wages, unschooled	21.4	21.5	21.5	21.9	22.2
Education premium	1.76	1.76	1.75	1.80	1.74
Gini coefficient	0.304	0.304	0.303	0.325	0.320
Capital-output ratio	2.68	2.69	2.69	2.86	2.87
Fraction of schooled workers in employed population, %	27.6	27.6	27.9	27.6	29.3
Output <sup>3</sup>	1.00	1.00	1.00	1.06	1.08

Note: In experiment A, retirement benefits (as a share of wage income) are the same for all retirees and equal to those for mean wage income individual in 2002 model. In experiment B, the social security tax rate and retirement benefits are zero. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.

<sup>1</sup>The ratio of the mean annual retirement benefits to the mean annual wage income for a retiree in the group. <sup>2</sup>Workweek length is calculated assuming 100 hours in a week available for work.

<sup>3</sup>Output in the 2002 model is normalized to 1.00.

Table 12: Adjustments of the schooling system

	Model	Experiments with 2002 Model				C. Improvement in ability by +1 AFQT point
	2002	A. Full tuition subsidy		B. Decrease in schooling time from 4 to 3 years		
		Med. run	Long run	Med. run	Long run	
Workweek length, schooled <sup>1</sup>	42.8	41.8	41.6	41.4	41.3	42.8
Workweek length, unschooled <sup>1</sup>	36.1	36.6	36.5	36.4	36.3	36.0
Wages, schooled	37.7	38.1	37.0	38.2	37.0	37.4
Wages, unschooled	21.4	21.4	21.8	21.4	21.9	21.6
Education premium	1.76	1.78	1.69	1.79	1.69	1.73
Gini coefficient	0.304	0.309	0.299	0.303	0.293	0.302
Capital-output ratio	2.68	2.69	2.71	2.69	2.71	2.69
Fraction of schooled workers in employed population, %	27.6	27.6	30.5	27.6	30.7	28.5
Output <sup>2</sup>	1.00	1.00	1.03	0.99	1.02	1.01

Note: In experiment A, college tuition is fully subsidized. In experiment B, the number of years it takes to graduate from college is decreased from 4 to 3 years. In experiment C, the realized raw AFQT scores in the model are 1 point (or 0.04 of standardized AFQT score) higher than in the benchmark 2002 model. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.

<sup>1</sup>Workweek length is calculated assuming 100 hours in a week available for work. <sup>2</sup>Output in the 2002 model is normalized to 1.00.

Table 13: The effect government policies on education choice, wage inequality and output

	Fraction of schooled workers in employed population <sup>1</sup>	Education premium <sup>2</sup>	Output <sup>2</sup>
Switch to proportional income tax	+2.0 (0.0)	-2.1 (+1.5)	+6.2 (+4.2)
Income tax cut by 19%	+1.4 (0.0)	-1.1 (+1.4)	+5.4 (+4.0)
Increase in SS tax by 3.5% points	-0.4 (0.0)	+0.2 (-0.7)	-2.0 (-1.5)
Increase in retirement age from 64 to 67	-0.4 (0.0)	-0.1 (-0.9)	-5.0 (-4.6)
Equal retirement payments	+0.3 (0.0)	-0.4 (+0.2)	+0.1 (-0.2)
Switch to fully-funded SS system	+1.7 (0.0)	-0.7 (+2.3)	+7.7 (+5.9)
Full tuition subsidy	+3.0 (0.0)	-3.6 (+1.4)	+2.6 (-0.2)
Decrease in schooling time from 4 to 3 years	+3.1 (0.0)	-3.6 (+1.7)	+2.0 (-0.8)
Improvement in ability by +1 AFQT point	+0.8	-1.4	+0.7

Note: The table compares the effect of government policies on the fraction of schooled workers in the employed population, education premium and output in the long run and (in parentheses) in the medium run. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.

<sup>1</sup>percentage points change, <sup>2</sup>percent change.

Figure 1: Earnings per hour of schooled and unschooled, U.S. 1961 to 2002.

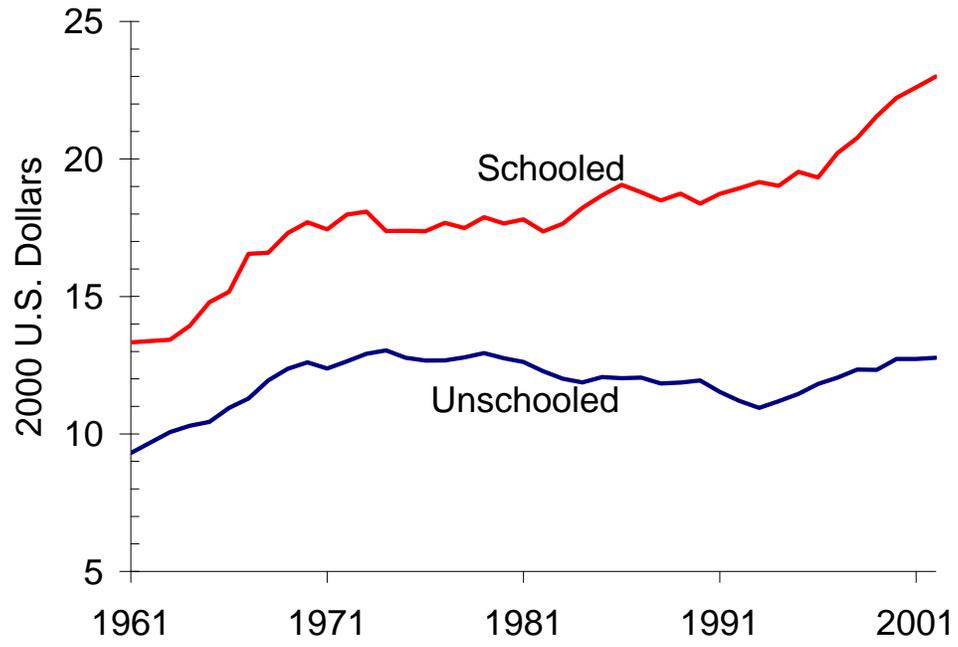


Figure 2: Education premium, U.S. 1961 to 2002.

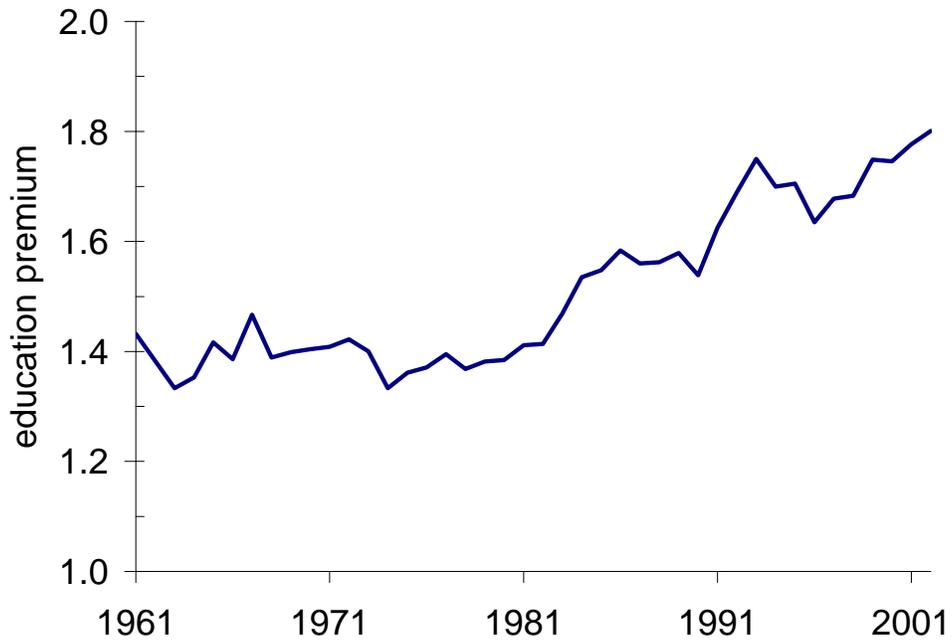


Figure 3: Change in working time contribution of schooled, U.S. 1961 to 2002.

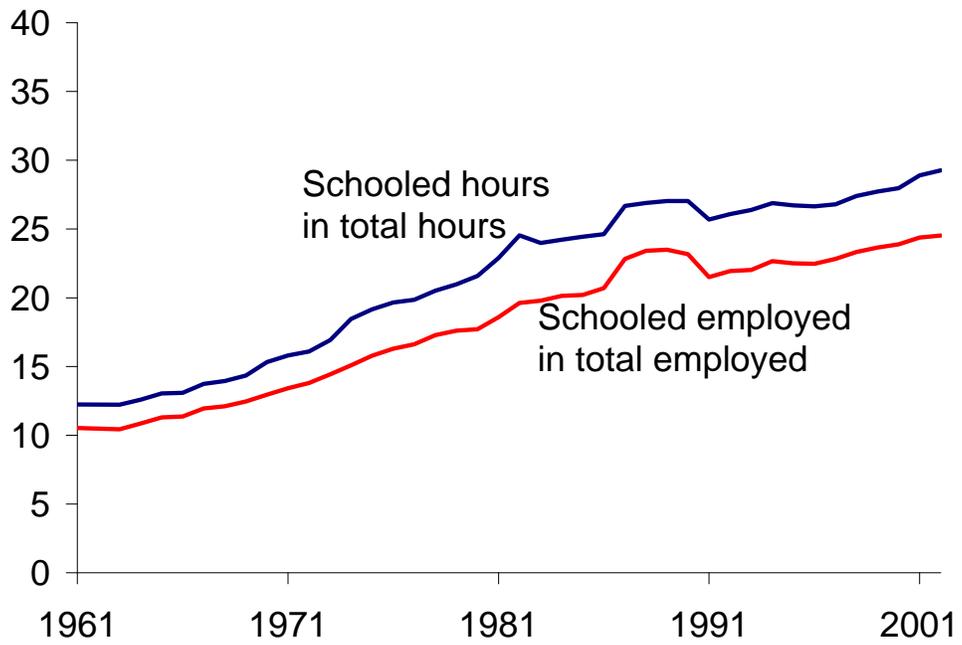


Figure 4: Workweek by group, U.S. 1961 to 2002.

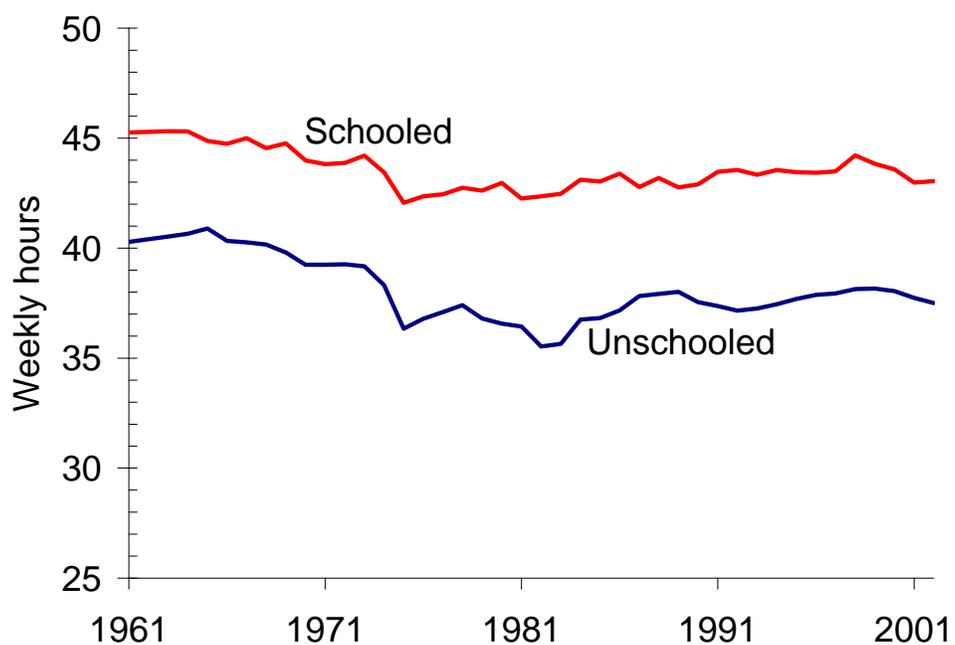


Figure 5: Measures of capital stocks and prices, U.S. 1961 to 2002.

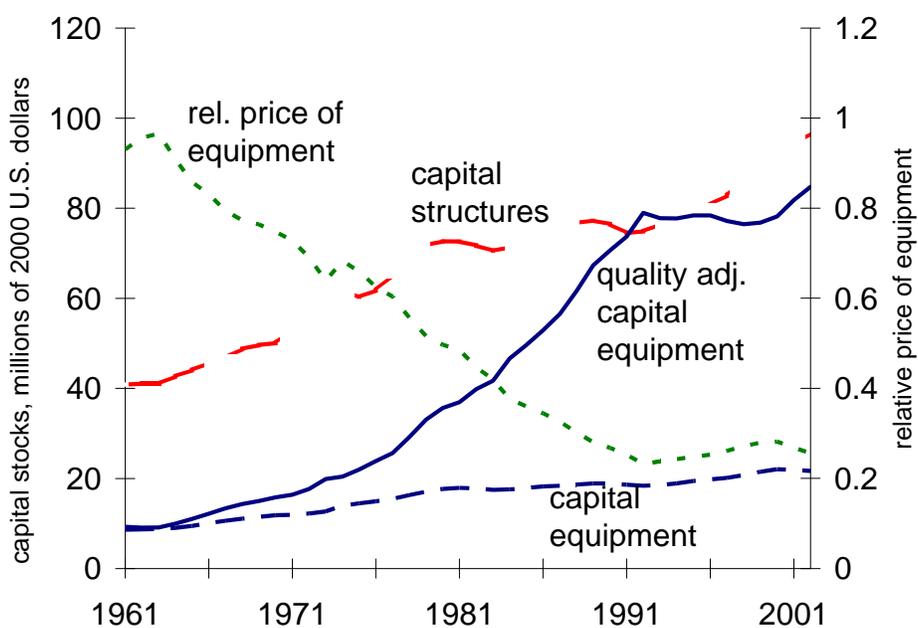


Figure 6: Life cycle of an individual in the model.

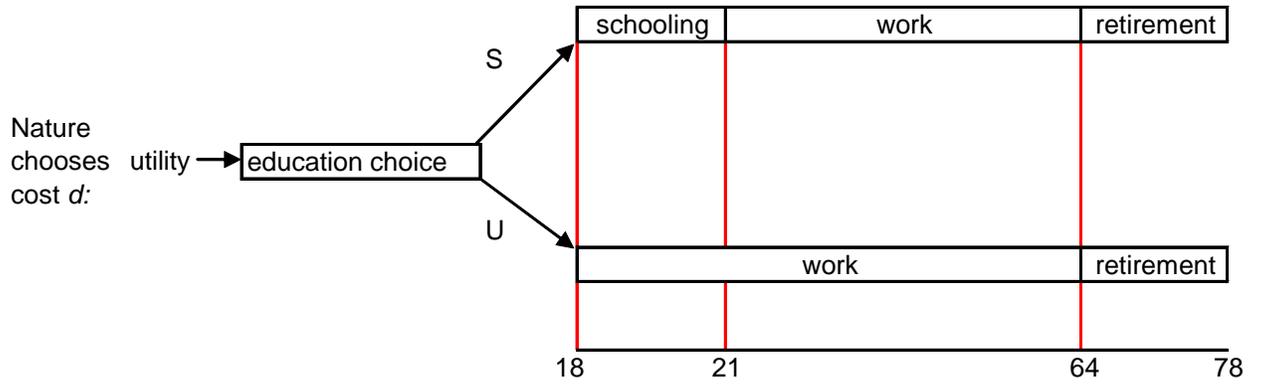


Figure 7: Probability of completing college, US 1979.

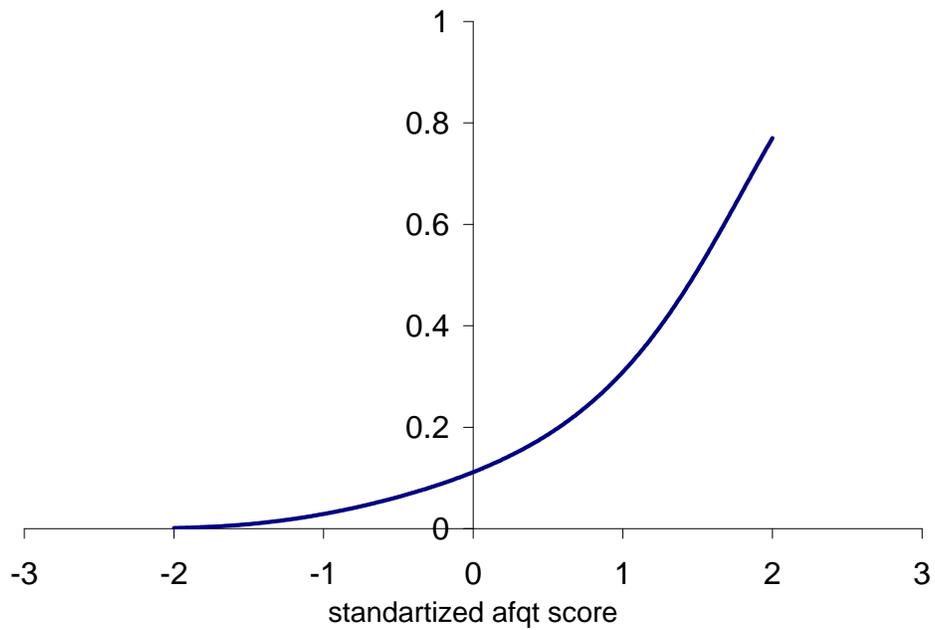
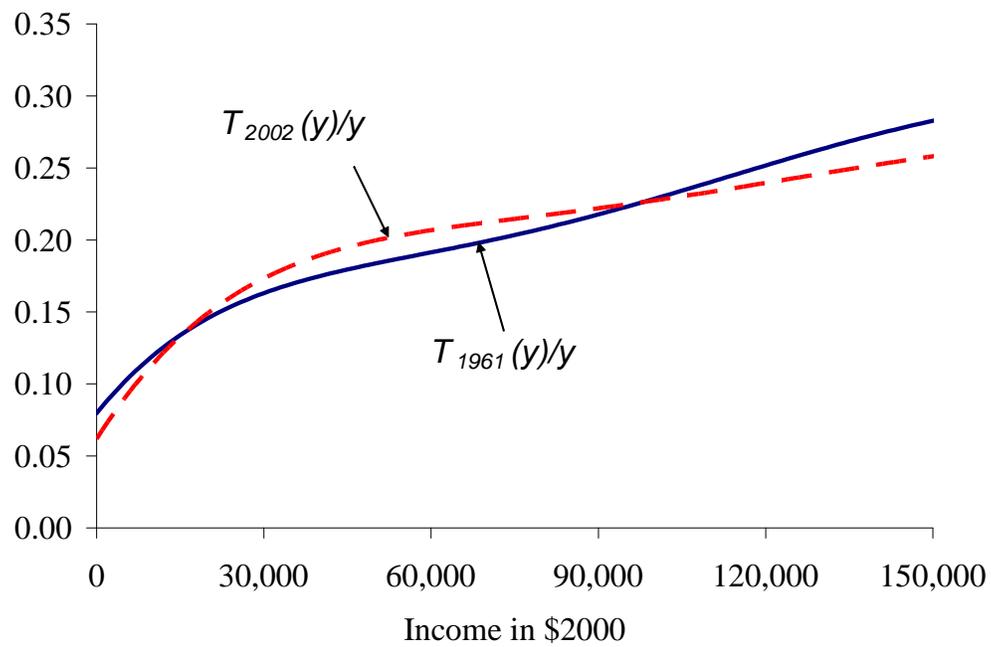


Figure 8: Average effective tax rate functions for the U.S. in 1961 and in 2002.



## Appendix A: Data sources

All the labor data, including wage data, are taken from the March Supplement of the Current Population Survey (CPS). We also used CPS data for federal, state, and social security taxes and income for the year 2003 to determine coefficients of the tax function.

We downloaded the following variables from the IPUMS-CPS home at [www.ipums.org/cps](http://www.ipums.org/cps) for the 1962 to 2003 surveys (Miriam King, Steven Ruggles, Trent Alexander, Donna Leicach, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Version 2.0. [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004).

Table A1: CPS variables used.

Variable	Content	Restrictions used
LABOR and INCOME data		
perwt	frequency weight	
year	year of survey	
age	age	16-64
sex	gender	male
educrec	educational attainment record	grades $\geq 1$
empstat	employment status	employed / employed at work
classwkr	class of worker	control for self employment
wkswork1	weeks worked last year	1976 to 2003
wkswork2	weeks worked last year (intervalled)	1962 to 1975
hrswork	hours worked last week	
inctot	total income last year	
incwage	total wage income last year	
TAX data		
adjginc	adjusted gross income	$0 < \text{adjginc} < 99980^1$
fedtax	federal income tax liability	$\text{fedtax} > 0$
fica	social security retirement payroll deduction	$\text{fica} > 0$
statetax	state income tax liability	$\text{statetax} > 0$

To determine the relevant capital stock, output and income data we used the following

tables from the National Income and Product accounts as supplied by the Bureau of economic analysis (from <http://www.bea.gov/> downloaded between January and March 2006). We downloaded the tables for the year 1960 to 2002 at the annual rate.

Aside from this we have to adjust our wage income data for wage supplements. To get the right adjustment factor we make use of both the income side of the national accounts and a detailed listing of the wage supplements as provided by the BEA. The income side is also used to determine the capital income share in GDP.

Table A2: NIPA tables used.

Table	Content	
National GDP accounts		
1.1.5.	Gross Domestic Product	Nominal
1.10	Gross Domestic Income	Nominal
1.1.4.	Price Indices for GDP	Base 2000
2.7.	Investment in Private Fixed Assets, Equipment and Software, and Structures by Type	Historical-Cost
2.8.	Investment in Private Fixed Assets, Equipment and Software, and Structures by Type	Chain-Type Quantity Indices
Fixed assets accounts		
2.1.	Private Fixed Assets, Equipment and Software, and Structures by Type	Current-Cost Net Stock
2.2.	Price indices for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type	Chain-Type Quantity Indices
Wage supplements		
7.8	Supplements to wages and salaries by type	Nominal

For capital equipment from 1963 to 1992 we make use of the time series provided by Krusell et al. (2000). We use a suggestion by Robert Gordon to extend the series from 1992 to 2000.

Concerning the tax data:

Federal taxes: *Statistics of Income, 1961 (2002)*; *Individual Income Tax Returns for 1961 (2002)*, Internal Revenue Service, U.S. Treasury Department Publication.

State taxes: *State Tax Handbook 1964 (2002)*, CCH Editorial Staff Publication, Commerce Clearing House Inc., New York, Chicago, Washington, Boston, Philadelphia, Los An-

geles, San Francisco.

Consumption taxes and social security taxes for 1961 and 2002: We used the SourceOECD, OECD database to get the expenditure and income side of the National Accounts as well as the consumption taxes and social security contributions from the Revenue Statistics. The data were downloaded in March 2006 at [http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch\\_oecd\\_stats](http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch_oecd_stats).

The data on tuition and fees of full time undergraduate students was taken from the College Board's web page. In particular it was taken from their publication "Trends in College Pricing 2005". We took a weighted average of the private and public universities student tuition and fees series ( $0.2 \text{ tuition}_{private} + 0.8 \text{ tuition}_{public}$ ). To extend the series back prior to 1976, we used the growth trend from 1976 to 1980.

## Appendix B: Data transformation

### B.1 Labor variables

For all the variables generated we restricted our sample to the male age 16 to 64 population. Furthermore, we defined college graduates (CG) to be all persons in the sample, that has at least 4 years of college completed. We define as less than college graduates (LC) all students that have either 1 to 12 years of highschool or 1 to 3 years of college. All the time series generated are generated for the respective groups.

For the work related variables, we also restrict our attention to the persons actually at work during the last week.

For the artificial cohort generation we make use of the age variable by looking at all the work variables for the respective groups and subdivide the education groups by age in 1 or 5 year steps depending on the context.

From the IPUMS samples, we determined the following for all years from 1961 to 2002:

Table B1: CPS data generated.

Name	Content	Variables used	Operation on sample
<i>wap</i>	Working age population	perwt	sum
<i>emp</i>	Employment	empstat, perwt	sum
<i>hours(ave)</i>	Average annual hours <sup>1</sup>	empstat, perwt, wksworkX, hrswork	average(wksworkX*hrswork)
<i>hours(med)</i>	Median annual hours	empstat, perwt, wksworkX, hrswork	median(wksworkX*hrswork)
<i>wph(ave)</i>	Average wage per hour <sup>2</sup>	empstat, perwt, incwage, wksworkX, hrswork	average(incwage/wksworkX*hrswork)
<i>wph(med)</i>	Median wage per hour	empstat, perwt, incwage, wksworkX, hrswork	median(incwage/wksworkX*hrswork)
<i>Inc</i>	Total income before taxes	inctot	median for each group

For the *hours* calculation from 1962 to 1975, we had to transform the intervalled weeks

worked per year into representative weeks worked. We did this with the following mapping:

wks=9 if weeks worked in 1 to 13

wks=22 if weeks worked in 14 to 26

wks=34 if weeks worked in 27 to 39

wks=43 if weeks worked in 40 to 47

wks=48 if weeks worked in 48 to 49

wks=52 if weeks worked in 50 to 52

We choose this mapping based on the post 1975 data. For the late period we have both the actual number of weeks worked and the intervalled weeks worked. To find the right weights we determine the median hours worked in each interval.

Based on these primary results we determined the following objects for each education group:

$$\begin{aligned}
 \text{emp fraction with college graduates} &= \frac{\text{emp}[S]}{\text{emp}[S + U]} \\
 \text{total hours fraction of college graduates} &= \frac{\text{emp}[S] \times \text{hours}[i]}{\text{emp}[S + U] \times \text{hours}[S + U]} \\
 \text{workweek}[i] &= \frac{\text{hours}[i]}{\text{emp}[i] \times 5200}
 \end{aligned}$$

where the square brackets represent the restriction to a subgroup (college graduates or less than college graduates). The adjustment factor 5200 in average hours worked is used to normalize the annual hours worked to the interval zero one.

To make the wage data comparable, we used the consumption deflator from NIPA table 1.1.4.

## B.2 Capital stocks

For the construction of the structures capital stocks we used the nominal private and residential fixed assets from table 2.1 and deflated it using the consumption deflator from table

1.1.4.

$$K_{structures} = \frac{\text{nominal structures}}{\text{consumption deflator}}$$

As a final step we transformed the variable into per working age terms by deviding the resulting capital stock through the number of persons age 16 to 64.

$$k_{structures} = \frac{K_{structures}}{wap}$$

For the construction of equipment capital stocks we are starting under the assumption that Gordon (1990) and subsequently KORV did a good job when determining the quality component of capital equipment and we could use their quality adjustment factor from 1963 to 1992. For the period before we assumed that their was little to no quality adjustment necessary. For the period after 1992 we followed Grodon's suggestion that the BEA did a good job on capturing the quality adjustment for computers and that for the other equipment the missed quality part amounted to 1.5%. We made the appropriate adjustments to the deflators and spliced the resulting  $q$  series with that from KORV.

As first step, we calculated the shares in private equipment using the following groupings: Information processing equipment and software (Computers and peripheral equipment, Software, Communication equipment, Medical equipment and instruments, Nonmedical instruments, Photocopy and related equipment, Office and accounting equipment), Industrial equipment, Transportation equipment, Other equipment.

The only bigger cathergory we split is Information processing equipment and software. This is necessary since the computer part includes quality adjustment in its deflator while the other sub-cathergories don't.

As the next step we perform the following transformation:

$$\begin{aligned}\frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}} &= \frac{d_{i,t+1}}{d_{i,t}} - 0.015; \quad i = \text{all except computer} \\ \frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}} &= \frac{d_{i,t+1}}{d_{i,t}}; \quad i = \text{computer} \\ \frac{d_{Thoern,t+1}}{d_{Thoern,t}} &= \sum_i \log \left( \frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}} \right) \left( \frac{s_{i,t+1} + s_{i,t}}{2} \right) \\ q_{NIPA} &= \frac{d_{Thoern}}{d_{consumption}}\end{aligned}$$

here  $s_i$  stands for the share in total non-residential investment,  $d_i$  stands for the NIPA investment deflator (or if stated the Thoern / consumption deflator), and  $\tilde{d}_i$  stands for the adjusted deflator.

The NIPA  $q$  is different from the KORV  $q$  so that we have to make an adjustment. I splice the two series together assuming, that KORV is correct and get the adjusted  $\tilde{q}_{NIPA}$ . Then I determine the equipment deflator the following way:  $\tilde{d}_{Thoern} = \tilde{q}_{NIPA} \times d_{consumption}$ .

Given all these steps we find the capital equipment in the following way:

$$\begin{aligned}K_{equipment} &= \frac{\text{nominal equipment}}{\tilde{d}_{Thoern}} \\ k_{equipment} &= \frac{K_{equipment}}{wap}\end{aligned}$$

One implicit assumption we are making is that the residential equipment exhibits the same quality change as the non-residential equipment.

### B.3 Taxes

The micro regression results for the income tax functions for 1961 and 2002 are provided by the Statistics of Income division of the Internal Revenue Agency. They used the respective years tax return forms to regress the effective income tax rate paid on the respective taxable income. We restricted the regression such that the income was between 3000 and 300000 year 2002 US\$. The lower bound takes the tax exempted income into account and the upper bound reflects the fact that the median person does not exceed that income in any given

year of his life. Furthermore we restricted the average income tax rate to be between zero and 70%, excluding the boundary points.

The consumption taxes were taken from Mendoza, Rasin, and Tesar as provided on Mendoza's web page: <http://www.bsos.umd.edu/econ/mendoza/pdfs/newtaxdata.pdf>. We used SourceOECD data on expenditures and tax revenues to extend the series forward to 2004. The data were downloaded in March 2006 at [http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch\\_oecd\\_stats](http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch_oecd_stats). Finally, to get coverage of the early period for the consumption tax we had to extend the series using the trend of a consumption tax rate series estimated based on consumption expenditures from the BEA-NIPA together with Indirect Business Taxes net of Subsidies.

## Appendix C: Armed Forces Qualification Test in NLSY79

ASVAB Administration: During the summer and fall of 1980, NLSY79 respondents participated in an effort of the U.S. Departments of Defense and Military Services to update the norms of the Armed Services Vocational Aptitude Battery (ASVAB)<sup>1</sup>. The Department of Defense and Congress, after questioning the appropriateness of using the World War II reference population as the primary basis for interpreting the enlistment test scores of contemporary recruits, decided in 1979 to conduct this new study. NLSY79 respondents were selected since they comprised a pre-existing nationally representative sample of young people born during the period 1957 through 1964. This testing, which came to be referred to as the “Profile of American Youth,” was conducted by NORC representatives according to standard ASVAB procedural guidelines; respondents were paid \$50 for their participation. Groups of five to ten persons were tested at more than 400 test sites, including hotels, community centers, and libraries throughout the United States and abroad. A total of 11,914 civilian and military NLSY79 respondents (or 94% of the 1979 sample) completed this test: 5,766 or 94.4% of the cross-sectional sample, 4,990 or 94.2% of the supplemental sample, and 1,158 or 90.5% of the military sample.

The ASVAB consists of a battery of 10 tests that measure knowledge and skill in the following areas: (1) general science; (2) arithmetic reasoning; (3) word knowledge; (4) paragraph comprehension; (5) numerical operations; (6) coding speed; (7) auto and shop information; (8) mathematics knowledge; (9) mechanical comprehension; and (10) electronics information. The following variables are available for each youth tested: raw scores, scale scores, standard errors, sampling weight, high school graduation status, and whether the test was completed under normal or altered testing conditions.

A composite score derived from select sections of the battery can be used to construct an approximate and unofficial Armed Forces Qualifications Test score (AFQT) for each youth. The AFQT is a general measure of trainability and a primary criterion of enlistment eligibility for the Armed Forces. Two methods of calculating AFQT scores, developed by the U.S. Department of Defense, have been used by CHRR to create two percentile scores, an AFQT80 and an AFQT89, for each Profiles respondent. To construct AFQT80, the raw scores from the following four sections of the ASVAB are summed: Section 2 (arithmetic reasoning), Section

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<sup>1</sup>This excerpt is taken from BLS’ “NLYS79 User’s Guide 1979-2002”, Chapter 4, pages 95-96, <<http://www.bls.gov/nls/79guide/2002/nls79g4a.pdf>>.

3 (word knowledge), Section 4 (paragraph comprehension), and one half of the score from Section 5 (numerical operations). Beginning in January 1989, the Department of Defense began using a new calculation procedure. Creation of this revised percentile score, called AFQT89, involves (1) computing a verbal composite score by summing word knowledge and paragraph comprehension raw scores; (2) converting subtest raw scores for verbal, math knowledge, and arithmetic reasoning; (3) multiplying the verbal standard score by two; (4) summing the standard scores for verbal, math knowledge, and arithmetic reasoning; and (5) converting the summed standard score to a percentile.

## Appendix D: Finding the equilibrium

The steady state equilibrium consists of 393 variables: allocations  $\{c_a^X, l_a^X\}_{a=1, X=\{S,U\}}^{63}$ ,  $\{k_a^X\}_{a=1, X=\{S,U\}}^{64}$ ,  $Y, K^{st}, K^{eq}, \varepsilon, L^S, L^U$ , transfers to retirees  $TR_{ss}$ , residual transfers  $TR$ , and prices  $r, r^{st}, r^{eq}, w^S, w^U$  that satisfy the system of 393 equations below, given relative price of equipment  $q$ , productivity  $A$ , tuition cost  $N$ , income tax function  $T(\cdot)$ , and government purchases  $G$ .

First introduce the following additional variables and notation ( $X = \{S, U\}$  whenever applicable):

- taxable income of age  $a$  individual:  $y_a^X = w^X l_a^X - N^X + (r^s - \delta_s) k_a^X, \quad a = 1, \dots, 63$

- marginal tax rate of age  $a$  individual:  $\tau_a^X = T'(y_a^X), \quad a = 1, \dots, 63$

- average tax rate of age  $a$  individual:  $\bar{\tau}_a^X = T(y_a^X)/y_a^X, \quad a = 1, \dots, 63$

- Euler equations:

$$c_{a+1}^X = c_1^X \beta^a \prod_{k=1}^a [1 + (r^{st} - \delta_{st}) (1 - \tau_k^X)], \quad a = 2, \dots, 63$$

- labor market equations:

$$l_a^S = 1 - \phi \frac{(1 - \tau_c) c_a^S}{(1 - \tau_a^S) w^S}, \quad a = 5, \dots, 47$$

$$l_a^U = 1 - \phi \frac{(1 - \tau_c) c_a^U}{(1 - \tau_a^U) w^U}, \quad a = 1, \dots, 47$$

- budget constraints of age  $a$  individual:

$$\begin{aligned} k_2^X &= -(1 - \tau_c) c_1^X + (1 - \bar{\tau}_1^X) (w^X l_1^X - N^X) + TR \\ k_{A+1}^X &= \left\{ \begin{array}{l} -(1 - \tau_c) c_1^X + (1 - \bar{\tau}_1^X) (w^X l_1^X - N^X) + TR - \\ \sum_{a=2}^A [1 + (r^{st} - \delta_{st}) (1 - \bar{\tau}_a^X)]^{-a+1} ((1 - \tau_c) c_a^X - (1 - \bar{\tau}_a^X) (w^X l_a^X - N^X) - TR) \end{array} \right\} \\ &\times [1 + (r^{st} - \delta_{st}) (1 - \bar{\tau}_A^X)]^{A-1}, \quad A = 2, \dots, 63 \end{aligned}$$

- arbitrage conditions:

$$\begin{aligned} r^{st} - \delta_{st} &= \frac{r^{eq}}{q} - \delta_{eq} \\ r &= r^{st} - \delta_{st} \end{aligned}$$

- initial and terminal conditions on individual  $a$ 's asset holdings:

$$\begin{aligned} k_1^X &= 0 \\ k_{64}^X &= 0 \end{aligned}$$

- working time constraints for individuals at college:

$$l_a^S = 0, \quad a = 1, \dots, 4$$

- working time constraints for individuals at college:

$$l_a^X = 0, \quad a = 48, \dots, 63$$

- education choice equation:

$$\frac{\varepsilon}{63} = \int_{-\infty}^{\infty} F \left( \frac{\Delta V(0) - d_{\max}}{\delta_d} + \iota(Q) \right) dF(Q)$$

where  $\Delta V(0) = \sum_{a=1}^{63} \beta^{a-1} \left[ \ln \frac{c_a^S}{c_a^U} + \phi \ln \frac{1-l_a^S}{1-l_a^U} \right]$

- aggregate output:

$$Y = A (K^{st})^\alpha \left\{ \theta_u (A_S L^U)^\eta + (1 - \theta_u) \left( \theta_K (K^{eq})^\kappa + (1 - \theta_K) (A_S L^S)^\kappa \right)^{\eta/\kappa} \right\}^{(1-\alpha)/\eta}$$

- factor shares in aggregate production:

$$\frac{r^{st} K^{st}}{Y} = \alpha$$

$$\frac{r^{eq} K^{eq}}{Y} = \frac{(1-\alpha) \Theta \left( \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\frac{\eta-\kappa}{\kappa}}}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}} \cdot \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa$$

$$\frac{w^S L^S}{Y} = \frac{(1-\alpha) \Theta \left( \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\frac{\eta-\kappa}{\kappa}}}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}}$$

$$\frac{w^U L^U}{Y} = \frac{(1-\alpha) \left( \frac{L^S}{L^U} \right)^\eta}{\left( \frac{L^S}{L^U} \right)^\eta + \Theta \left( \frac{\theta_K}{1-\theta_K} \left( \frac{K^{eq}}{A_S L^S} \right)^\kappa + 1 \right)^{\eta/\kappa}}$$

where  $\Theta \equiv \frac{(1-\theta_u)(1-\theta_K)^\frac{\eta}{\kappa}}{\theta_u}$ ;

- government's budget constraint:

$$\begin{aligned} & \tau_c \frac{1}{63} \sum_{a=1}^{63} (\varepsilon c_a^S + (1-\varepsilon) c_a^U) + \frac{1}{63} \sum_{a=1}^{63} \varepsilon \bar{\tau}_a^S y_a^S + \frac{1}{63} \sum_{a=1}^{63} (1-\varepsilon) \bar{\tau}_a^U y_a^U \\ &= G + TR_w + \frac{16}{63} TR_{ss} \end{aligned}$$

- social security distribution constraint:

$$\frac{1}{63} \sum_{a=1}^{63} \varepsilon \tau_{ss} y_a^S + \frac{1}{63} \sum_{a=1}^{63} (1-\varepsilon) \tau_{ss} y_a^U = \frac{16}{63} TR_{ss}$$

- market clearing conditions:

$$\begin{aligned} \frac{1}{63} \sum_{a=1}^{63} \varepsilon k_a^S + \frac{1}{63} \sum_{a=1}^{63} (1-\varepsilon) k_a^U &= K^{st} + qK^{eq} \\ \frac{1}{63} \sum_{a=1}^{63} \varepsilon l_a^S &= L^S \\ \frac{1}{63} \sum_{a=1}^{63} (1-\varepsilon) l_a^U &= L^U \end{aligned}$$