Working Time Regulation, Risky Lifetime and Fairness

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Abstract

This paper studies the redistributive impact of working time regulations in an economy with unequal lifetimes. Using a lifecycle model where individuals facing risky lifetime choose their working time profile, we show that uniform working time regulations can, under some conditions, reduce inequalities in realized lifetime well-being between short-lived and long-lived persons. We also characterize the ex post egalitarian optimum, where the realized lifetime well-being of the worst off is maximized, and show that this social optimum involves, unlike the laissez-faire, an increasing age profile in terms of worked hours. Although age-dependent working time regulation alone does not decentralize the social optimum, it can nonetheless reduce inequalities in realized lifetime well-being. We also explore the robustness of our results to allowing for overtime work, and to the extension of our setting to a 3-period OLG economy.

Keywords: Working time regulations, risky lifetime, labor supply, premature deaths, OLG models.

JEL codes: J10, J18, J22.

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1 Introduction

Imposing a maximum limit on the number of hours worked everyday is an old idea, which dates back to the first attempts to characterize what an ideal society could be. In his *Utopia*, Thomas More (1516) described an ideal town where individuals would work 6 hours a day, the rest of time being dedicated to education. In Tommaso Campanella's *The City of the Sun* (1602), each adult would work only 4 hours a day. The main motivation was that individuals should use goods, but not become instruments of those goods.

But working time regulations do not only belong to ancient social utopias. Since the early 19th century, workers' movements have called for reforms involving reductions in the working time. Those social movements have progressively led to the introduction of working time regulations, which have become more and more restrictive over time, as illustrated by Table 1 for the case of France.

Year	Reform fixing the maximum working time to
1848	84 hours per week (12 hours per day)
1900	70 hours per week in the industry (10 hours per day)
1919	48 hours per week (8 hours per day for 6 days)
1936	40 hours per week
1982	39 hours per week
2000	35 hours per week (for firms > 20 workers, in 2002 for all)

Table 1: A brief history of working time regulation in France.

The effects of changes in working time regulations are studied by economists, both at the theoretical and empirical levels. In particular, economists have estimated the impact of recent reductions in working time on several variables including, among other things, unemployment (Crépon and Kramartz 2002, Chemin and Wasmer 2009), firm productivity (Crépon et al 2004), actual hours worked (Hunt 1999), and female labor supply (Goux et al 2011).[1]

An important aspect of changes in working time regulations consists of its redistributive effects. Actually, when a reduction of the working time is carried out at a constant total labor earnings, this implies mechanically a rise in the hourly wage, and, as such, can potentially affect the distribution of income among production factors in the economy (i.e. workers *versus* capital holders).[2]

But beyond the impact of working time regulations on the distribution of income between capital and labor, little attention has been paid so far to the redistributive effects of working time regulations from a lifecycle perspective. Actually, given that, since Jevons (1871), working is regarded both as a source of

[1] See Askenazy (2013) for a synthesis of existing studies on the impact of working time regulations in France during the last 30 years.

[2] Holmlund and Pencavel (1988) and Friesen, (2000) showed that a reduction in the working time is associated with a significant rise in the hourly wage in countries such as Sweden, the Netherlands and Canada.

purchasing power and as a source of disutility, working time regulations impose some restrictions and conditions regarding how individuals can actually solve the trade-offs between consumption and leisure, and on the well-being levels that they can reach along the life cycle.

In a hypothetical world without risky lifetime, and where each individual would enjoy the same duration of life, the distributive impact of working time regulations (i.e. a maximum number of hours worked) would be, from a lifecycle perspective, limited, since all individuals would go through all stages of life, and, hence, would face the same constraints. In that case, working time regulations would be neutral from a life cycle perspective. However, in real life, individuals face risky lifetime, and some persons turn out to die before others. As a consequence, it is not true that working time regulations are neutral from a life cycle perspective, since these regulations, by affecting the number of worked hours and budget constraints of individuals, influence also the level of inequalities in lifetime well-being between short-lived and long-lived persons.

A premature death interrupts a life and a career, and is a major source of deprivation. Given that short-lived individuals can hardly be regarded as responsible for dying prematurely, there is a strong ethical support for public intervention aimed at reducing inequalities in lifetime well-being due to premature deaths.[3] Actually, if one adheres to Fleurbaey's Principle of Compensation (Fleurbaey 2008), according to which inequalities due to circumstances should be abolished, one can find a strong support for policies aimed at reducing inequalities in lifetime well-being between long-lived and short-lived persons.

The goal of this paper is to examine the impact of working time on inequalities between short-lived and long-lived persons, as well as the use of working time regulations as an instrument aimed at reducing those inequalities. For that purpose, we develop a lifecycle model with risky lifetime, where individuals choose how many hours they work at each age of life, as well as their retirement age. We first characterize the laissez-faire, and examine how uniform working time regulations (with or without overtime work) affect inequalities in lifetime well-being between short-lived and long-lived persons. Then, we contrast the laissez-faire with the ex post egalitarian social optimum, that is, the allocation maximizing the realized lifetime well-being of the worst off. That social optimum was shown to minimize inequalities in realized lifetime well-being between short-lived and long-lived persons (see Fleurbaey et al 2014). We then analyse how that social optimum can be decentralized, and the role of age-specific working time regulations in that decentralization.

Anticipating on our results, we first show that imposing uniform working time regulations (with or without overtime work) reduces, under some conditions, inequalities in realized lifetime well-being between the long-lived and the short-lived, but can hardly abolish them completely. Then, we characterize the ex post egalitarian optimum and we show that this social optimum involves, in general, a number of worked hours that is *increasing* with the age. We also

[3] Recent studies such as Christensen et al (2006) emphasized that about 1/4 to 1/3 of longevity inequalities within a cohort are due to differences in the genetic background, on which individuals can have no influence.

show that age-dependent working time regulation (imposing a lower maximum number of hours worked at the young age than at the old age) alone does not allow to fully decentralize the ex post egalitarian optimum. Actually, the decentralization of the social optimum is achieved by both taxing savings returns, so as to induce a decreasing age-profile for consumption and, by introducing lump sum taxation. However, working time regulations can, under some conditions, reduce inequalities in realized lifetime well-being between short-lived and long-lived individuals. Our results are shown to remain valid in an extended 3-period dynamic overlapping generations (OLG) model. Those analytical findings are complemented by numerical simulations showing, in the case of France (1848-2000), that uniform working time reductions (and associated rises in real wages) contributed to improve significantly the situation of the short-lived.

This paper complements the literature on working time regulation and its consequences (see Section 1). This paper is not about the effects of those regulations on unemployment and productivity, but, instead, it is about their consequences on the individuals' well-being in an economy peopled of short-lived and long-lived workers. This paper also complements recent studies on how the government could reduce inequalities in lifetime well-being between short-lived and long-lived persons, as in Fleurbaey and Ponthiere (2013) and Fleurbaey et al (2016). The former paper examines prevention choices, without considering production. The latter paper focuses only on the choice of a retirement age, while assuming that young workers work full time. We complement those papers by considering the impacts of different types of working time regulations on inequalities in lifetime well-being between short-lived and long-lived persons.

The rest of the paper is organized as follows. Section 2 presents the model. The laissez-faire equilibrium is characterized in Section 3. Section 4 considers the impact of uniform working time regulations (with or without overtime work) on inequalities in lifetime well-being between the short-lived and the long-lived. The ex post egalitarian optimum is characterized in Section 5. Its decentralization is studied in Section 6. Section 7 provides numerical illustrations. Section 8 extends our baseline model to a 3-period overlapping generations model (OLG) and examines the robustness of our results to that more general setting. Section 9 explores numerically the impact of working time regulations in France (1848-2000) on the distribution of lifetime well-being. Section 10 concludes.

2 The model

We consider a 3-period economy. Each period has a duration normalized to one. The population is a continuum of agents of size 1.

Period 1 is childhood. Period 2 is young adulthood, during which individuals work $\ell_y \in [0, 1]$ units of time, save and consume. Period 3 is old adulthood, during which individuals work $\ell_o \in [0, 1]$ units of time during a fraction z of that period, and are retired during a fraction $1 - z$. Lifetime is risky: only a fraction $\pi \in [0, 1]$ of the young adults reach the old age.

Preferences are given by a standard time additive utility function:[4]

$$u(c) - v(\ell_y) + \pi[u(d) - zv(\ell_o)] \tag{1}$$

where c denotes consumption at young adulthood, d denotes consumption at mature adulthood, and $u(\cdot)$ satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. We assume that there exists a non negative consumption level $\bar{c} \geq 0$ such that $u(\bar{c}) = 0$. We suppose that the desutility of labor is increasing and convex: $v'(\cdot) > 0$ and $v''(\cdot) > 0$. We have also $v(0) = 0$ and $v'(0) = 0$. Note that the utility function exhibits no pure time preferences. However, the probability to survive to the old age π can be interpreted as a biological discount factor.

We suppose that the labor market is perfectly competitive, with an hourly wage equal to $w > 0$.

We suppose that there exists a perfect annuity market, with actuarially fair return:

$$\hat{R} = \frac{R}{\pi} \tag{2}$$

where R equals one plus the interest rate. For the sake of simplicity, we assume, in the baseline model, that $R = 1$.

The first-period budget constraint is:

$$c + s = w\ell_y \tag{3}$$

where s denotes savings. The second-period budget constraint is:

$$d = \frac{s}{\pi} + z\ell_o w \tag{4}$$

The intertemporal resource constraint is thus:

$$c + \pi d = \ell_y w + \pi z \ell_o w \tag{5}$$

In the rest of the paper, we also assume there is no public pension system.

3 Laissez-faire

As a starting point, let us consider a world without any form of working time regulations, that is, a world where individuals can freely choose how many hours they work at each point in their life (intensive margins of labor) and their retirement age (extensive margin of labor).

At the laissez-faire, individuals choose their consumptions c and d, the quantities of hours worked ℓ_y and ℓ_o, as well as the retirement age z, in such a way as to maximize their expected utility subject to their resource constraint:

$$\begin{aligned} &\max_{c,d,\ell_y,\ell_o,z} u(c) - v(\ell_y) + \pi[u(d) - zv(\ell_o)] \\ &\text{s.t. } \ell_y w + \pi z \ell_o w \geq c + \pi d \end{aligned}$$

[4] As usual, the utility of being dead is normalized to 0.

Denoting by λ the Lagrange multiplier associated to the resource constraint, first order conditions (FOCs) can be rearranged as follows:

$$u'(c) - \lambda = 0 \tag{6}$$
$$u'(d) - \lambda = 0 \tag{7}$$
$$wu'(c) - v'(\ell_y) = 0 \tag{8}$$
$$wu'(d) - v'(\ell_o) \geq 0 \tag{9}$$
$$w\ell_o u'(d) - v(\ell_o) \geq 0 \tag{10}$$

Hence consumption is smoothed along the life cycle: $c = d$. The reason lies in the existence of a perfect annuity market with actuarially fair returns, combined with the absence of"pure" time preferences. The conditions for labor supply coincide with what Jevons (1871) called the "final equivalence of labor and utility". Optimal working time should equalize, at the margin, the utility gain obtained from one additional unit of work (LHS) and the utility loss from this unit (RHS).

Let us now compare (9) and (10). In the appendix, we formally show that given the convexity of $v(\cdot)$, we have $v'(\ell_o) > \frac{v(\ell_o)}{\ell_o}$ and thus, that in equilibrium, the FOC for ℓ_o is binding but not the FOC for z. This implies that the solution for ℓ_o is interior but that the optimal retirement age z is maximum and equal to 1. Moreover, given that consumption is smoothed along the life cycle, we thus also have that labor is smoothed: $\ell_y = \ell_o = \ell$. Finally, since $wu'(c) > v'(0) = 0$ holds, a necessary and sufficient condition for the existence of a unique and interior level of $\ell \in]0,1[$ is $wu'(w) < v'(1)$.

Proposition 1 summarizes our results.

Proposition 1

- *If*

$$wu'(w) < v'(1)$$

there exists a unique laissez-faire equilibrium with an interior working time at each period.

- *At the laissez-faire, the following equalities hold:*

$$\begin{aligned} u'(c) &= u'(d) \\ wu'(c) &= v'(\ell_y) \text{ and } wu'(d) = v'(\ell_o) \\ c + \pi d &= \ell_y w + \pi z \ell_o w \end{aligned}$$

- *Those conditions imply:*

$$\begin{aligned} c &= d = \ell w \\ \ell_y &= \ell_o = \ell \in]0,1[\\ z &= 1 \end{aligned}$$

- *We have also:*

$$\begin{aligned} \frac{d\ell}{d\pi} &= \frac{dc}{d\pi} = \frac{dz}{d\pi} = 0 \\ \frac{d\ell}{dw} & \left\{ \begin{array}{l} > 0 \text{ if } R_c < 1 \\ \leq 0 \text{ if } R_c \geq 1 \end{array} \right. \\ \frac{dc}{dw} &> 0; \frac{dz}{dw} = 0 \end{aligned}$$

where $R_c = -\frac{cu''(c)}{u'(c)}$ *is the coefficient of relative risk aversion.*

Proof. See the Appendix. ■

Thus, at the laissez-faire, individuals perfectly smooth consumption and labor over time (i.e. $c = d$ and $\ell_y = \ell_o = \ell$), and postpone retirement as much as possible (i.e. $z = 1$). As a consequence of late retirement, individuals do not save at all, but consume at each period the product of their labor, that is, $w\ell$.

An interesting feature of the laissez-faire is that the consumption and working time profiles are completely independent from the level of the survival probability to the old age π. In other words, whether individuals expect to have a more or less long life does not impact their consumption profile and labor profile. The intuition behind this somewhat surprising result can be seen from the intertemporal budget constraint. Once we substitute for constant consumption profile and constant labor profile, as well as for the retirement age ($z = 1$), the intertemporal budget constraint becomes:

$$c(1 + \pi) = \ell w(1 + \pi).$$

Thus the life expectancy $1 + \pi$ can be simplified from both sides, making the laissez-faire consumption independent from the level of the life expectancy. But given the "final equivalence of labor and utility", if consumption is independent from the survival conditions, it follows that the marginal utility gain from working is also independent from the survival conditions, implying that the supply of labor is also independent from π. This result is a direct consequence of assuming a perfect annuity market and from having $z = 1$.

Note that Proposition 1 describes not only the laissez-faire equilibrium, but, also the utilitarian social optimum. Actually, a utilitarian social planner maximizing the average realized utility within the population, or, alternatively, the expected utility of a representative agent, would choose consumption and labor profiles that are exactly the same as the ones prevailing at the laissez-faire.

Finally, let us compare the situation of short-lived and long-lived persons at the laissez-faire. We have that short-lived persons are worse off than long lived persons if and only if

$$u(\ell w) - v(\ell) > 0$$

Indeed, in that case, the well-being level associated to the old age is higher than 0. That case depends obviously on the level of the wage, and on the preferences for consumption as well as for labor, i.e. on the functional forms for $u(\cdot)$ and $v(\cdot)$.[5]

4 Uniform working time regulations

4.1 No possibility of overtime work

Having characterized the laissez-faire, let us now examine how the introduction of uniform (i.e. age-independent) working time regulations would affect individual behaviors, as well as the size of well-being inequalities between the long-lived and the short-lived individuals.

For that purpose, let us now suppose that the government fixes a uniform maximum working time $\bar{\ell}$ that individuals must respect. The government also impose mandatory retirement at the age $\bar{z} \leq 1$.

Two cases can arise depending on whether working time regulations are constraining or not. If working time regulations are not constraining, i.e. $\ell_y = \ell_o = \ell < \bar{\ell}$ and $\bar{z} = 1$, we obtain the exact same laissez faire results as before: $\ell_y^R = \ell_o^R = \ell$ and $z^R = 1$ where R stands for regulation. In that case, consumption is given by:

$$c = \frac{w\ell(1+\pi)}{(1+\pi)}$$

and it equals its level at the laissez-faire.

If on the contrary, working time regulations are constraining, i.e. the agent would choose at the laissez faire $\ell > \bar{\ell}$ and $\bar{z} < 1$, the agent will now choose, under working time regulations, $\ell_y^R = \ell_o^R = \bar{\ell}$ and $z^R = \bar{z}$.[6] In that case, consumption is given by:

$$c = \frac{w\bar{\ell}(1+\pi\bar{z})}{(1+\pi)}$$

which is unambiguously *lower* under the working time regulation than under the laissez-faire, since both the working time and the retirement age are lower than in the unconstrained case.

[5] Note that an increase in wage always increases welfare inequalities between the short-lived and the long-lived agents. Indeed

$$u'(c)\frac{dc}{dw} - v'(\ell)\frac{d\ell}{dw} = u'(c)\ell > 0$$

where we made use of the FOC on ℓ.

[6] Note that a solution where $\ell_y^R \neq \ell_o^R$ is not possible as at each period, the agent is constrained to work less than he would wish to. In that case, given that he is constrained to work less in the first period (resp. the second) period, he would like to work even more than the laissez faire level in the second (resp. first) period. However, he cannot do so because of working time regulations, $\bar{\ell}$. Almost the same reasoning holds for the retirement age. Since the agent is constrained to work less at the old age than he would wish, he may want to work longer than at the laissez-faire, but again he is constrained to work less than $\bar{z}$ which itself is smaller than its laissez-faire level, 1. Both at the intensive and the extensive margins, the agent will hit the working time regulations constraints.

Let us now consider the impact of these working time regulations on inequalities between long-lived and short-lived persons. For that purpose, let us first suppose that the uniform working time regulations constraint individuals at all ages.

At the laissez-faire, the inequality is:

$$U^{LL} - U^{SL} = u(\ell w) - v(\ell)$$

When the working time regulation is constraining individuals, so that $\ell_y^R = \ell_o^R = \bar{\ell}$ and $z^R = \bar{z}$, that inequality becomes:

$$U^{LL} - U^{SL} = u\left(\frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)}\right) - \bar{z}v\left(\bar{\ell}\right)$$

Hence, when the working time regulation is constraining individuals, inequalities between long-lived and short-lived persons are reduced when:

$$u\left(\frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)}\right) - \bar{z}v\left(\bar{\ell}\right) < u(\ell w) - v(\ell)$$

That inequality can be rewritten as:

$$v(\ell) - \bar{z}v\left(\bar{\ell}\right) < u(\ell w) - u\left(\frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)}\right)$$

The RHS corresponds the fall in well-being from consumption while the LHS corresponds the fall in the disutility of labor due to the working time regulations which both reduce the duration of the career and the number of hours worked. If the fall in the utility of consumption exceeds the fall in the disutility of labor, then the working time regulation reduces lifetime well-being inequalities between the long-lived and the short-lived. However, when the fall in the disutility of labor due to the regulations exceeds the welfare loss in terms of consumption, working time regulations raise inequalities between short-lived and long-lived with respect to the laissez-faire.

Second, when the working time regulation is not constraining individuals (i.e. $\ell_y = \ell_o = \ell < \bar{\ell}$ and $\bar{z} = 1$), that condition becomes:

$$U^{LL} - U^{SL} = u(\ell w) - v(\ell)$$

It is easy to see that, in that case, inequalities between long-lived and short-lived persons remain the same as at the laissez-faire.

The following proposition summarizes our results.

Proposition 2 *Suppose that the government imposes a maximum number of worked hours $\bar{\ell}$ as well as a mandatory retirement age $\bar{z} \leq 1$.*

- *If the working time regulation constraints individuals at the young age and the old age, we have:*

$$\begin{aligned} \ell_y &= \ell_o = \bar{\ell} \\ z &= 1 < \bar{z} \\ c &= \frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)} \end{aligned}$$

- *If the working time regulation does not constraint individuals, we have:*

$$\begin{aligned} \ell_y^R &= \ell_o^R = \ell < \bar{\ell} \\ z^R &= \bar{z} \leq 1 \\ c &= \frac{w\ell(1+\pi)}{(1+\pi)} \end{aligned}$$

- *The regulation on working time reduces inequalities in lifetime well-being between the short-lived and the long-lived if and only if the working time regulation constraints individuals, and if:*

$$v(\ell) - \bar{z}v\left(\bar{\ell}\right) < u(\ell w) - u\left(\frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)}\right)$$

Proof. See above. ■

Proposition 2 states that, in some cases, imposing uniform working time regulations can reduce inequalities in lifetime well-being between short-lived and long-lived individuals. This reduction is achieved when the regulation is constraining individuals in their choice, and when the loss in well-being due to lower consumption exceeds the gain in well-being due to lower disutility of labor. This depends on the intensity of individual preferences for consumption as well as on the intensity of labor disutility, that is on the specific forms of $u(.)$ and $v(.)$.

However, even when the regulation reduces the extent of lifetime well-being inequalities, nothing guarantees that, in general, such a regulation will suffice to make those inequalities vanish. Actually, the realized lifetime well-being of short-lived persons is still, under the regulation, lower than the one of the long-lived when the following condition holds:

$$u\left(\frac{\bar{\ell}w(1+\pi\bar{z})}{(1+\pi)}\right) - \bar{z}v\left(\bar{\ell}\right) > 0$$

Two effects are here at work. The regulation reduces old-age consumption (first term) with respect to the laissez-faire, which pushes inequalities down. But at the same time, by imposing a reduction in the duration of the working week and of the career, this leads also to lower disutility of old-age labor, which raises inequalities between the long-lived and the short-lived.

4.2 Possibility of overtime work

In this section, we consider another type of working time regulation where overtime work is possible but at an increased wage. To do so, we assume that below some threshold labor supply, $\bar{\ell}$, agents obtain a wage w, while after this threshold, they obtain a wage $w(1+p)$ where p is strictly positive. As in the previous sections, we assume that $\bar{z} \leq 1$. The problem of the agent is now modified to account for working time regulations:

$$\max_{c,d,\ell_y,\ell_o,z} u(c) - v(\ell_y) + \pi [u(d) - zv(\ell_o)]$$
$$\text{s.t. } w\bar{\ell} + (\ell_y - \bar{\ell})w(1+p) + \pi z(w\bar{\ell} + (\ell_o - \bar{\ell})w(1+p)) \geq c + \pi d \text{ if } \ell_o, \ell_y \geq \bar{\ell}$$
$$\text{or } \ell_y w + \pi z \ell_o w \geq c + \pi d \text{ if } \ell_o, \ell_y < \bar{\ell}$$

Whether under this type of regulation, $\ell_y^{RR} = \ell_o^{RR}$ is smaller or greater than $\bar{\ell}$ depends on the specific forms of $u(.)$ and $v(.)$ as well as on the size of s as it becomes clear from the numerical example in Section 7.

If the solution to this problem is such that $\ell_o^{RR}, \ell_y^{RR} < \bar{\ell}$ and $z^{RR} = \bar{z} = 1$ where ^{RR} stands for this second type of working time regulation, we are back to the laissez-faire solution: $\ell_o^{RR} = \ell_y^{RR} = \ell$ and $z^{RR} = 1$. On the contrary, if $\ell_o^{RR}, \ell_y^{RR} \geq \bar{\ell}$ and /or $z^{RR} \geq \bar{z}$, the first order conditions are now:

$$\begin{aligned} u'(c) &= u'(d) = \lambda \\ v'(\ell_y) &= v'(\ell_o) = w(1+p)\lambda \\ -\frac{v(\ell_o)}{\ell_o} + v'(\ell_o) - p\lambda w \frac{\bar{\ell}}{\ell_o} &\geq 0 \end{aligned}$$

In that situation, we therefore obtain the following solution: $c^{RR} = d^{RR}$, $\ell_y^{RR} = \ell_o^{RR} > \ell$ and $z^{RR} = \bar{z}$.[7]

Let us now study how lifetime inequalities are modified by this type of regulation when agents choose to work overtime (i.e. $\ell_O^{RR} > \bar{\ell}$).[8] Inequalities between short-lived and long-lived agents are equal to

$$u\left(c^{RR}\right) - z^{RR} v\left(\ell_o^{RR}\right)$$

under this type of working time regulation, while inequalities at the laissez-faire are

$$u(c) - v(\ell).$$

where $\ell_O^{RR} > \ell$ and $z^{RR} = \bar{z} \leq 1$ so that

$$c^{RR} = \frac{(w\bar{\ell} + w(1+p)(\ell_o^{RR} - \bar{\ell}))(1 + \pi\bar{z})}{1+\pi} \gtrless w\ell.$$

[7] Note that a solution where $FOC_z < 0$ implying $z^{RR} = 0$ is not possible as this would also imply that $\ell_o^{RR} = 0$ consistent with $FOC_z < 0$ but in contradiction with $\ell_o^{RR} > \bar{\ell}$.

[8] Obviously, if the solution is the laissez-faire, inequalities remain identical.

Hence, it is not clear whether the first or the second expression should be higher and inequalities may well increase or decrease with respect to the laissez-faire. Even in the simpler case where $\bar{z} = 1$, we cannot clearly rank inequalities between short-lived and long-lived agents at the laissez faire and under the regulation since $c^{RR} > w\ell$ but $\ell_o^{RR} > \ell$.

5 The ex post egalitarian optimum

Inequalities in realized lifetime well-being between the short-lived and the long-lived can be strongly questioned from an ethical perspective. Actually, in our economy, all agents are perfectly identical *ex ante*, that is, before the duration of life is revealed. No one can do anything to influence his or her survival chances. Longevity inequalities are thus purely arbitrary, and can be regarded as circumstances, on which individuals have no influence at all. Hence it is quite hard to justify that, *ex post*, i.e. once the duration of life of each person is known, some individuals turn out to have a larger realized lifetime well-being than other individuals.

Quite the contrary, if one adheres to the Principle of Compensation (Fleurbaey 2008), one would consider that inequalities in realized lifetime well-being due to unequal exogenous lifetimes should be abolished by the government, since these inequalities are completely unfair.

But in order to do justice to the idea of compensating the short-lived, one cannot, in the present context, rely on a utilitarian social welfare function, since we show above that this social criteria would, in the present context, legitimate the laissez-faire, and the resulting inequalities in realized lifetime well-being between short-lived and long-lived persons.

In the following, we propose to do justice to the compensation of the short-lived by considering an *ex post* egalitarian social welfare function, which takes as objective the maximization of the realized lifetime well-being of the worst off in the society (see Fleurbaey et al 2014). Under an *ex post* egalitarian social objective, the social planner's problem is:

$$\begin{aligned} \max_{c,d,\ell_y,\ell_o,z} \min \quad & \{u(c) - v(\ell_y), u(c) - v(\ell_y) + u(d) - zv(\ell_o)\} \\ \text{s.t.} \quad & \ell_y w + \pi z \ell_o w \geq c + \pi d \end{aligned}$$

The objective function is not differentiable. But we can rewrite that problem as the maximization of first-period utility subject to the constraint that the long-lived is not worst off than the short-lived. The problem becomes:

$$\begin{aligned} \max_{c.d,\ell_y,\ell_o,z} \quad & u(c) - v(\ell_y) \\ & \text{s.t. } \ell_y w + \pi z \ell_o w \geq c + \pi d \\ & \text{s.t. } u(d) - zv(\ell_o) \geq 0 \end{aligned}$$

Denoting the optimal level of variables with *, and denoting by λ the Lagrange multiplier associated to the resource constraint, and by μ the Lagrange multiplier associated to the egalitarian constraint, FOCs yield:

$$u'(c^*) = \lambda \tag{11}$$

$$u'(d^*) = \frac{\pi}{\mu}\lambda \tag{12}$$

$$v'\left(\ell_y^*\right) = \lambda w \tag{13}$$

$$\lambda \pi w - v'(\ell_o^*)\,\mu \geq 0 \tag{14}$$

$$\lambda \pi w \ell_0 - v(\ell_0)\mu \geq 0 \tag{15}$$

as well as the conditions:

$$\begin{aligned} \lambda &\geq 0, \ell_y^* w + \pi z^* \ell_o^* w \geq c^* + \pi d^* \\ \mu &\geq 0, u(d^*) - z^* v(\ell_o^*) \geq 0 \end{aligned}$$

with complementary slackness.

Those conditions can be rewritten as follows:

$$u'(c^*) = \frac{\mu}{\pi} u'(d^*) \tag{16}$$

$$v'\left(\ell_y^*\right) = u'(c^*) w \tag{17}$$

$$u'(d^*)\, w - v'(\ell_o^*) \geq 0 \tag{18}$$

$$u'(d^*)\, w - \frac{v(\ell_o^*)}{\ell_o^*} \geq 0 \tag{19}$$

As before given, the convexity of $v(\cdot)$, it is clear that the FOCs for ℓ_o^* and z^* cannot be both binding. As a solution to this system, we will therefore have that ℓ_0^* satisfies $u'(d^*)\, w = v'(\ell_o^*)$ and $z^* = 1$. Our analysis of those conditions yields the following results, which are summarized in Proposition 3.

Proposition 3

- *At the ex post egalitarian optimum, we have:*

$$\begin{aligned} u'(c^*) &= \frac{\mu}{\pi} u'(d^*) \\ v'\left(\ell_y^*\right) &= u'(c^*) w \\ v'(\ell_o^*) &= u'(d^*)\, w \\ \ell_y^* w + \pi z^* \ell_o^* w &= c^* + \pi d^* \\ u(d^*) &= z^* v(\ell_o^*) \\ z^* &= 1 \end{aligned}$$

- *Those conditions imply*

  – *if* $\frac{\mu}{\pi} \leq 1$,

$$\begin{aligned} c^* &\geq d^* > \bar{c} \\ \ell_o^* &\geq \ell_y^* \text{ and } z^* = 1 \end{aligned}$$

– *if* $\frac{\mu}{\pi} > 1$,

$$\begin{aligned} d^* &> c^* \gtrless \bar{c} \\ \ell_o^* &< \ell_y^* \textit{ and } z^* = 1 \end{aligned}$$

Proof. It directly follows from FOCs (16)-(19). ■

The ex post egalitarian optimum does not, in general, involve flat consumption profiles and flat labor profiles. The social optimum thus differs quite a lot from the laissez-faire (and the utilitarian social optimum). When the shadow price of relaxing the egalitarian constraint is lower than the survival probability to the old age (i.e. the chance that these inequalities are effectively realized), the egalitarian optimum involves a decreasing consumption profile with the age, as well as an increasing labor profile with the age. Hence, according to that social criterion, older, more experienced workers should work more. The underlying intuition is that forcing older workers to work more hours implies that more resources can be transferred to the young age, and, hence, to individuals who may turn out to be short-lived.

Note that, by construction, we have, at the ex post egalitarian optimum, a full equalization of the realized lifetime well-being of the short-lived and the long-lived individuals and no inequalities left between them, since, by the egalitarian constraint, we have

$$u(d^*) = v\left(\ell_o^*\right).$$

6 Age-specific working time regulation and decentralization

Let us now examine how the ex post egalitarian optimum could be decentralized by an appropriate working time regulation. For that purpose, we consider here the impact of introducing *age-specific* working time regulations, which, unlike in Section 4, consists of constraints on working time that differ along the life cycle.

The underlying intuition for introducing age-specific working time regulations is that, at the ex post egalitarian optimum, we have $\ell_y^* < \ell_o^*$ when the survival probability π is big (see Proposition 3).

6.1 The effects of working time regulations ℓ_y^*, ℓ_o^* and z^*

In order to study the impact of age-specific working time regulation, let us first consider a government that imposes individuals to work a maximal time ℓ_y^* at the young age, and ℓ_o^* at the old age. The levels ℓ_y^* and ℓ_o^* are the quantities of labor that prevail at the ex post egalitarian optimum under $\mu < \pi$, so that we have $\ell_y^* < \ell_o^*$. The government also imposes mandatory retirement at age $z^* = 1$, also in line with the ex post egalitarian optimum.

Note that at the laissez faire, the agent always chooses the ex-post egalitarian retirement age. Therefore, only three cases can arise depending on whether the

constraints on labor supply are binding at the young or the old age or at both ages.
Let us first consider the case where the laissez-faire labor supply is lower than the constraint imposed by working time regulation, i.e. $\ell \leq \ell_y^* < \ell_o^*$. In that situation, consumption levels at both periods are identical to their laissez-faire level:

$$c^R = d^R = \ell w$$

If on the contrary, working time regulations are constraining in both periods, i.e. $\ell_y^* < \ell_o^* \leq \ell$, the labor supply chosen by the agent will hit the constraint so that $\ell_y^R = \ell_y^*$ and $\ell_o^R = \ell_o^*$. In that situation, the agent still chooses to retire at $z^* = 1$. Indeed, in this situation, the agent is forced to supply less labor than he would wish to so that he would like to retire even later than at the laissez-faire. Yet working time regulations impose that he retires at the end of the period. Because of the perfect annuity market, consumptions are still smoothed and given by

$$c^R = d^R = \frac{\ell_y^* w + \pi \ell_o^* w}{1+\pi}$$

which is smaller than its laissez-faire level.

Finally, if the working time regulations are constraining only at the young age but not at the old age, i.e. if $\ell_y^* < \ell < \ell_o^*$, we have that under working time regulations, the new labor supply will be $\ell_y^R = \ell_y^*$ and $\ell < \ell_o^R \leq \ell_o^*$. Interestingly, the labor supply at the old age under regulation constraints is likely to be higher than its laissez-faire level since the agent is constrained in the first period to supply less labor than he would have wished to at the laissez-faire. This reduces consumption possibilities, which raises the marginal utility of consumption, and, hence, pushes individuals to work more at the old age as the individual marginal rate of substitution between consumption and labor $wu'(c) = v'(\ell_0^R)$ shows. In that situation, consumptions are given by

$$c^R = d^R = \frac{\ell_y^* w + \pi \ell_o^R w}{1+\pi} \lesseqgtr w\ell$$

since $\ell_y^R = \ell_y^* < \ell$ and $\ell_o^R > \ell$.[9]

As the above results show, age-specific working time regulations cannot alone decentralize the ex-post egalitarian solution. First, in all these three cases, consumption is smoothed along the life-cycle while at the ex post egalitarian optimum, the consumption profile is decreasing. Second, working time regulations enable to obtain the ex-post egalitarian labor supply levels only when working hours *both* at young and old ages are greater at the laissez faire than at the ex-post egalitarian solution.

Nonetheless, one can look at the welfare inequalities between short and long-lived agents and see how these are modified under such regulations. Obviously, in the first case considered above, working time regulations are ineffective so that welfare inequalities remain unchanged as compared to the laissez-faire. In

[9] Note that $z^R = 1$ since we still have $v(\ell_0^R)/\ell_0^R > v_0'^R) = wu'(c)$.

the second case, working time regulations are constraining at both periods and inequalities between short-lived and long-lived become:

$$U^{LL} - U^{SL} = u\left(\frac{\ell_y^* w + \pi \ell_o^* w}{1+\pi}\right) - v\left(\ell_o^*\right)$$

while theses inequalities at the laissez-faire are $u(w\ell) - v(\ell)$ with $c^R = \frac{\ell_y^* w+\pi\ell_o^* w}{1+\pi} < w\ell$ and $\ell_o^* < \ell$. Then depending on the specific forms of $u(.)$ and $v(.)$, i.e. on the intensity of preference for consumption and on the intensity of disutility of work, it may very well be the case that working time regulations enable to decrease lifetime welfare inequalities between short-lived and long-lived agents.

Finally, in the third case, when working time regulations are constraining in the first period but not in the second, inequalities between short-lived and long-lived individuals become

$$U^{LL} - U^{SL} = u\left(\frac{\ell_y^* w + \pi \ell_o^R w}{1+\pi}\right) - v\left(\ell_o^R\right).$$

Working time regulations imply a fall in the inequalities in lifetime well-being between the long-lived and the short-lived when:

$$u\left(\frac{\ell_y^* w + \pi \ell_o^R w}{1+\pi}\right) - v\left(\ell_o^R\right) < u(w\ell) - v(\ell)$$

where $\ell_y^* < \ell < \ell_o^R$. The second term is smaller on the LHS than on the RHS, which goes in the direction of reducing inequalities in lifetime well-being between the short-lived and the long-lived. However, the combination of a rise in the working time of the old and a fall of the working time of the young has ambiguous effects on consumption (first term), so that it is not necessarily the case that working time regulations can reduce inequalities. This reduction occurs only when

$$u\left(\frac{\ell_y^* w + \pi \ell_o^R w}{1+\pi}\right) - u(w\ell) < v\left(\ell_o^R\right) - v(\ell)$$

that is, when the rise in the disutility of working at the old age exceeds the variations in old-age consumption induced by the working time regulation.

To sum up, it appears that merely fixing the maximum working time at each age to the socially optimal levels does not allow to fully decentralize the ex post egalitarian optimum. However, such regulations can, in some cases, reduce inequalities in lifetime well-being between the short-lived and the long-lived in comparison with the laissez-faire.

6.2 Decentralization of the ex post egalitarian optimum

Let us now study the tax-schedule that would allow to fully decentralize the ex-post egalitarian optimum. Comparing the conditions in Proposition 1 with those in Proposition 3, it is clear that one only needs a tax on the return of savings τ so as to induce the optimal decreasing consumption pattern as well as lump sum transfers so as to satisfy the egalitarian constraint. Under such a tax on savings, the FOC for savings obtained from the individual's problem becomes:

$$u'(c) = (1-\tau)u'(d)$$

while the socially optimal profile satisfies:

$$u'(c^*) = \frac{\mu}{\pi}u'(d^*)$$

Hence one needs to set the tax on savings returns equal to:

$$\tau^* = 1 - \frac{\mu}{\pi} = 1 - \frac{u'(c^*)}{u'(d^*)}.$$

Hence, whenever π is big and $\mu < \pi$, agents face a tax on savings. By taxing the return on savings, the government induces agents to have the optimal consumption profile, and reduces consumption at the old age with respect to the laissez-faire. One also needs to impose lump sum transfers so as to ensure that the egalitarian constraint, $u(d^*) = z^* v(\ell_o^*)$ where $z^* = 1$ is satisfied at the decentralized optimum.[10] Proposition 4 summarizes our results.

Proposition 4

- *The ex post egalitarian optimum can be decentralized by means of a tax on savings returns equal to:*

$$\tau^* = 1 - \frac{\mu}{\pi} = 1 - \frac{u'(c^*)}{v'(\ell_o^*)}w$$

as well as lump sum transfers so as to satisfy the egalitarian constraint.

- *The mere imposition of age-specific working time regulations (ℓ_y^*, ℓ_o^*) does not suffice to decentralize the social optimum. However, provided the condition*

$$u\left(\frac{\ell_y^R w + \pi \ell_o^R w}{1+\pi}\right) - u(w\ell) < v(\ell_o) - v(\ell)$$

with $\ell_y^R \leq \ell_y^*$ and $\ell_o^R \leq \ell_o^*$ holds, then age-specific working time regulation contributes to attenuate inequalities in lifetime well-being between short-lived and long-lived agents in comparison with the laissez-faire.

Proof. See above. ■

Proposition 4 states that, if the goal of the government is to abolish inequalities in realized lifetime well-being between the long-lived and the short-lived, the best instruments are a tax on savings return together with lump sum taxation, rather than working time regulations alone. As shown by the tax formula, the tax should be positive, so as to induce the optimal decreasing consumption

[10] One simply needs to set $T = d^* - d$.

profiles and increasing working time profiles. However, in a second best world where a tax on savings return may not be available, age-dependent working time regulations could help reduce inequalities in realized lifetime well-being between long-lived and short-lived persons.

7 Numerical illustration

In order to illustrate our previous findings numerically, this section assumes particular functional forms for the utility function of individuals, and carries out two kinds of comparisons. First, in order to study the impact of introducing uniform working time regulations on welfare inequalities, we compare the laissez-faire equilibrium with a situation where such regulations are imposed. Second, we compare the laissez-faire equilibrium with the *ex post* egalitarian optimum.

Throughout this section, we will assume that annual utility of consumption and annual disutility of work are represented by the functions:

$$\begin{aligned} u(c) &= T_1 \frac{c^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \alpha \\ v(\ell) &= T_2 \beta \frac{\ell^2}{2} + \theta \end{aligned}$$

where T_1 (resp. T_2) is the total number of weeks in a year (resp. of weeks worked in a year). Throughout this section, we set $T_1 = 52$ and $T_2 = 47$, in order to conform to what prevails in France (i.e. 5 weeks of paid holidays).

Regarding the calibration of preference parameters α, β, γ and θ, a major difficulty lies in the fact that the laissez-faire equilibrium is *not observable*, since in contemporary economies the working time is subject to strong regulations. Thus we can hardly take the observed quantities of working time as resulting from a free optimization problem of the agent.

Hence, in order to construct the hypothetical laissez-faire, we will consider two distinct combinations of preference parameters, one under which individuals would, at the laissez-faire, work *more* than under the current regulations, and one under which they would work *less* than under the current regulations. Those two hypothetical laissez-faire equilibria differ regarding the value of the parameter β, i.e. the disutility of labor. In the first case, we fix $\beta = 6$, while in the second case, we fix $\beta = 12$. Regarding other preference parameters, we leave these unchanged under the two calibrations. We assume, following Becker et al. (2005), that $\gamma = 1.250$ and $\alpha = -16.2$. We also set $\theta = 0$.

Assuming that agents devote 8 hours a day to basic daily life activities (like eating, bathing, sleeping), this leaves 16 hours as a maximum to work per day. We set the working time regulation to $\bar{\ell} = 35/80 = 0.4375$ where 35 hours is the working time limit per week in many countries (for example, in France) and $16 \times 5 = 80$ would be the maximum possible length of working time per week.

We assume that the first childhood period lasts 20 years and that the periods of young adulthood and old adulthood last 30 years each, so that agents may live until 80 years old. Assuming, like in many countries that the legal retirement age is 65 years old, this yields that $\bar{z} = (65-50)/(80-50) = 0.5$. Also, since in our model, life expectancy is equal to $50+30\pi$, we set $\pi = 0.9$ which yields a life expectancy of 77 years-old, which is close to OECD statistics on life expectancy. Finally, we set the hourly wage to $w = \$10$, and we assume that extra worked hours are paid with a premium p equal to, respectively, $\{20\%, 50\%, 100\%\}$.

Tables 2 and 3 report the variables of interest under the laissez-faire (LF), under uniform working time regulation (UR) and, under the uniform working time regulation with the possibility of overtime work (UROT).[11] Table 2 reports the case where the intensity of labor disutility β is small (so that agents at the LF would work more than the legal norm), while Table 3 reports the case where β is high (so that agents would work less at the LF than the legal norm).

	LF	UR	$UROT_{20\%}$	$UROT_{50\%}$
c	404	267	397	475
d	404	267	397	475
# of hours worked at young age	40	35	49	53
# of hours worked at old age	40	35	49	53
age at retirement	80	65	65	65
expected lifetime utility	583.84	558.79	572.50	584.36
$U^{LL} - U^{SL}$	307	301	315	324

Table 2: Equilibrium results under the laissez faire and different types of uniform working time regulations, for $\beta = 6$.

	LF	UR	$UROT_{20\%}$	$UROT_{50\%}$	$UROT_{100\%}$
c	275	267	237	303	380
d	275	267	237	303	380
# of hours worked at young age	27	35	31	38	42
# of hours worked at old age	27	35	31	38	42
age at retirement	80	65	65	65	65
expected lifetime utility	538	520	521	521	529
$U^{LL} - U^{SL}$	283	288	286	291	299

Table 3: Equilibrium results under the laissez faire and different types of uniform working time regulations, for $\beta = 12$.

Imposing a uniform working time regulation decreases the agent's lifetime utility (since he is further away from his preferred solution). But a uniform working time regulation has mitigated effects on the extent of inequalities in realized lifetime well-being between long-lived and short-lived persons. Imposing a uniform regulation decreases welfare inequalities when labor disutility is small ($\beta = 6$), but it increases them when the labor disutility is high ($\beta = 12$). Hence, as mentioned in Section 4, whether a uniform working time regulation increases or decreases lifetime welfare inequalities depends on individual preferences.

[11] In any case, when the government regulates the working time duration, it also constrains the retirement age to $\bar{z}$.

The last two columns of Table 2 and the last three columns of Table 3 report our results when overtime work is allowed and remunerated at an extra wage.[12] When the disutility iof labor is low (Table 2), it is only if overtime work is remunerated at a high extra wage that agents are better-off than at the laissez-faire.[13] To the opposite, when the disutility of work is large (Table 3), even doubling the wage is not enough to compensate for the loss in welfare due to extra labor. If the extra-wage compensation is low (i.e. $p = 20\%$), it is even the case that agents supply an amount of labor smaller than 35 hours for a wage w.

Regarding the impact on inequalities, our numerical examples suggest that imposing a regulation with compensation for extra hours worked increases inequalities between short-lived and long-lived in comparison to the laissez-faire. This result is driven by the increase in consumption and by the large reduction in the retirement age from $\bar{z} = 1$ to $\bar{z} = 0.5$, which reduces total disutility of work $\bar{z}v(\ell)$ in the second period, even though labor supply increases.

Let us now turn to the comparison between the laissez faire allocation and the ex post egalitarian social optimum. In the following, we present two cases (i.e. low and high disutility of labor).[14]

	LF	Egalitarian optimum	Diff working time Reg
c	404	923	333
d	404	6	333
# of hours worked at young age	40	21	21
# of hours worked at old age	40	80	47
age at retirement	80	80	80
expected lifetime utility	584	398	573
$U^{LL} - U^{SL}$	307	0	281

Table 4: Laissez-faire and ex post egalitarianism under $\beta = 6$.

	LF	Egalitarian optimum	Diff working time Reg
c	275	724	221.391
d	275	79	221
# of hours worked at young age	27	13	13
# of hours worked at old age	27	74	33
age at retirement	80	80	80
expected lifetime utility	538	381	525
$U^{LL} - U^{SL}$	283	$\rightarrow 0$	255

Table 5: Laissez-faire and ex post egalitarianism under $\beta = 12$.

[12] Note that, under the regulation, agents cannot choose the laissez-faire solution anymore (even though they may be better-off in terms of lifetime welfare), since by law, they are obliged to retire at $\bar{z}$, as well as to be remunerated at $w(1+p)$ for every worked hours above $\bar{\ell}$.

[13] This is due to the fact that under the regulation, they are now constrained to retire earlier than under the laissez faire, $\bar{z} = 0.5 < 1$, which decreases their consumption possibilities. On the other hand, they have the possibility to increase labor time and earn an extra wage to partially or totally compensate for this monetary loss.

[14] In Table 4, we assume $\beta = 6$, and we need to constrain $\ell_o = 1$ (i.e. agents cannot work more than 80 hours a week). In Table 5, $\beta = 12$ ensures that $\ell_o \leq 1$.

The ex post egalitarian optimum involves a strongly decreasing consumption profile with the age, and a strongly increasing labor profile with the age. The optimal number of hours worked is about 20 hours per week for young adults, which is quite close to the 4 hours a day proposed by Campanella (1602) in *The City of the Sun*. However, the optimal number of hours worked is about four times larger for old workers. The intuition behind that differentiated treatment of young and old workers is that, contrary to Campanella (1602), where working time regulations are seen as a way to avoid the alienation of humans, working time regulations are used here to reduce inequalities in lifetime well-being between the short-lived and the long-lived. This is what motivates such a differentiated treatment of the young and the old (the ex post egalitarian optimum actually cancels out those inequalities). Note that the lower the disutility of labor is (i.e. the lower β is), and the lower is the ratio old working time / young working time.[15]

The third column (Diff working time Reg) gives the results when the government only fixes working time regulations at the young and the old ages, as in Section 6.1. As it is clear from Tables 4 and 5, the constraint is binding only at the young age (since $\ell_y^* < \ell < \ell_o^*$), so that the agent re-optimizes and chooses consumption and labor supply at the old age taking into account that he would like to supply more labor at the young age than what he constrained to (this is equivalent to our third case in Section 6.1). The last column of Tables 4 and 5 provide such results. We find that, compared to the laissez-faire, inequalities between short-lived and long-lived agents decrease, but at the expense of decreased expected lifetime utility.[16]

8 Extension: the OLG economy

Let us now consider the dynamic extension of the baseline model studied in the previous sections. For that purpose, we consider now a 3-period OLG economy with risky lifetime, where each young adult has exactly one child (i.e. replacement fertility).

8.1 Production

We suppose that the production process involves capital and labor, and exhibits constant returns to scale. The production function is:

$$Y_t = F(K_t, L_t) \tag{20}$$

where Y_t is the output, K_t is the capital stock, and L_t is the labor quantity at time t. Capital fully depreciates after one period of use.

The total quantity of labor is:

$$L_t = \ell_{yt} + \pi z_t \ell_{ot} \tag{21}$$

[15] The ratio is about 4 when $\beta = 6$, against 5 when $\beta = 12$.

[16] We tried with other values for the β parameter in the range $[4, 20]$. These results were confirmed.

where ℓ_{yt} is the quantity of worked hours per young adult at time t, while ℓ_{ot} is the quantity of worked hours per old adult at time t.

In intensive terms, we can write output per unit of worked hours as:

$$y_t = f(k_t) \tag{22}$$

where $y_t \equiv \frac{Y_t}{\ell_{yt}+\pi z_t \ell_{ot}}$ and $k_t \equiv \frac{K_t}{\ell_{yt}+\pi z_t \ell_{ot}}$ and $f(k_t) \equiv F(k_t, 1)$. As usual, we have $f'(\cdot) > 0$ and $f''(\cdot) < 0$.

Factors are paid at their marginal productivity:

$$w_t = f(k_t) - k_t f'(k_t), \tag{23}$$

$$R_t = f'(k_t). \tag{24}$$

8.2 Temporary equilibrium

As in the baseline model, the first-period budget constraint is:

$$c_t + s_t = w_t \ell_{yt} \tag{25}$$

where variables are indexed with time.

Assuming, as in the baseline model, a competitive market with actuarially fair returns, the second-period budget constraint is:

$$d_{t+1} = \frac{R_{t+1} s_t}{\pi} + z_{t+1}\ell_{ot+1} w_{t+1} \tag{26}$$

Hence the intertemporal budget constraint becomes:

$$w_t \ell_{yt} + \frac{\pi}{R_{t+1}} z_{t+1} \ell_{ot+1} w_{t+1} = c_t + \frac{\pi}{R_{t+1}} d_{t+1} \tag{27}$$

Individuals choose their consumptions, the number of hours worked at each period, and their retirement age, to maximize their expected lifetime well-being. The problem faced by a young adult at time t is:

$$\max_{c_t, d_{t+1}, \ell_{yt}, \ell_{ot+1}, z_{t+1}} \quad u(c_t) - v(\ell_{yt}) + \pi\left[u(d_{t+1}) - z_{t+1} v(\ell_{ot+1})\right]$$

$$\text{s.t.} \quad w_t \ell_{yt} + \frac{\pi z_{t+1} \ell_{ot+1} w^e_{t+1}}{R^e_{t+1}} \geq c_t + \frac{\pi d_{t+1}}{R^e_{t+1}}$$

where R^e_{t+1} and w^e_{t+1} are the expected future levels of the interest factor and the wage rate.

The FOCs are:

$$u'(c_t) = \lambda = R^e_{t+1} u'(d_{t+1}) \tag{28}$$

$$w_t u'(c_t) = v'(\ell_{yt}) \tag{29}$$

$$w^e_{t+1} u'(d_{t+1}) - v'(\ell_{ot+1}) \geq 0 \tag{30}$$

$$w^e_{t+1} u'(d_{t+1}) - \frac{v(\ell_{ot+1})}{\ell_{ot+1}} \geq 0 \tag{31}$$

where λ is the Lagrange multiplier associated to the resource constraint.

We also have the conditions:

$$\lambda \geq 0, w_t \ell_{yt} + \frac{\pi z_{t+1} \ell_{ot+1} w^e_{t+1}}{R^e_{t+1}} \geq c_t + \frac{\pi d_{t+1}}{R^e_{t+1}} \tag{32}$$

with complementary slackness.

As before, the convexity of $v(\cdot)$ implies that $v'(\ell_{ot+1}) > \frac{v(\ell_{ot+1})}{\ell_{ot+1}}$ so that the FOC for ℓ_{ot+1} is binding, the FOC for z_{t+1} is strictly positive, therefore implying that $z_{t+1} = 1$ and $\ell_{ot+1} \in [0, 1]$ at all periods.[17]

8.3 Stationary equilibrium

At a stationary equilibrium with perfect foresight, we have:

$$\begin{aligned}
u'(c) &= Ru'(d) && (33)\\
wu'(c) &= v'(\ell_y) && (34)\\
wu'(d) &= v'(\ell_o) && (35)\\
w &= f(k) - kf'(k) && (36)\\
R &= f'(k) && (37)\\
w\ell_y + \frac{\pi \ell_o w}{R} &= c + \frac{\pi d}{R} && (38)\\
k &= \frac{w\ell_y - c}{\ell_y + \pi \ell_o} && (39)
\end{aligned}$$

In underaccumulation of capital, we have $R > 1$ implying $d > c$. Hence we also have: $\ell_o < \ell_y$. Individuals increase their consumption with their age, and decrease the quantity of hours worked as they become older. Proposition 5 summarizes our results.[18]

Proposition 5 *At the stationary equilibrium with perfect foresight and under-accumulation of capital ($R > 1$), we have:*

$$\begin{aligned}
u'(c) &= Ru'(d) \text{ and } z = 1\\
wu'(c) &= v'(\ell_y) \text{ and } wu'(d) = v'(\ell_o)\\
w &= f(k) - kf'(k) \text{ and } R = f'(k)\\
k &= \frac{w\ell_y - c}{\ell_y + \pi \ell_o}
\end{aligned}$$

This implies $c < d$, $\ell_y > \ell_o$, $z = 1$.

Proof. See above. ■

[17] The formal proof is similar to the static cases.

[18] Note that if $R = 1$, $c = d$, $\ell_y = \ell_o$ implying no capital accumulation, $k = 0$.

In the standard case of underaccumulation of capital ($R > 1$), we observe that the consumption profile is increasing with the age, and that the labor profile is decreasing with the age.

As in the baseline model, it is generally the case that short-lived persons are worst off than long-lived persons. Actually, this is the case if and only if:

$$u(d) - v(\ell_o) > 0.$$

This depends on the functional forms of $u(.)$ and $v(.)$, on the level of consumption, d (and thus the producing capacity of the economy) as well as on the old age labor.

8.4 Utilitarian optimum

Let us now consider the problem faced by a utilitarian social planner, who chooses all variables in such a way as to maximize average lifetime well-being at the stationary equilibrium.

For that purpose, note first that the economy's resource constraint is:

$$F(K_t, L_t) = c_t + \pi d_t + K_{t+1} \tag{40}$$

In intensive terms, and at the steady state, we have:

$$f(k) = \frac{c + \pi d}{\ell_y + \pi z \ell_o} + k \tag{41}$$

The problem of the utilitarian social planner can be written as:

$$\max_{c,d,\ell_y,\ell_o,z,k} u(c) - v(\ell_y) + \pi \left[u(d) - zv(\ell_o)\right]$$
$$\text{s.t. } f(k) \geq \frac{c + \pi d}{\ell_y + \pi z \ell_o} + k.$$

FOCS are:

$$f'(k) = 1 \tag{42}$$
$$u'(c) = \frac{\lambda}{\ell_y + \pi z \ell_o} \tag{43}$$
$$u'(d) = \frac{\lambda}{\ell_y + \pi z \ell_o} \tag{44}$$
$$v'(\ell_y) = \lambda \frac{(c + \pi d)}{\left[\ell_y + \pi z \ell_o\right]^2} \tag{45}$$
$$-v'(\ell_o) + \lambda \frac{(c + \pi d)}{\left[\ell_y + \pi z \ell_o\right]^2} \geq 0 \tag{46}$$
$$-\frac{v(\ell_o)}{\ell_o} + \lambda \frac{(c + \pi d)}{\left[\ell_y + \pi z \ell_o\right]^2} \geq 0 \tag{47}$$

where λ is the Lagrange multiplier of the resource constraint.

We also have the condition:

$$\lambda \geq 0, f(k) \geq \frac{c + \pi d}{\ell_y + \pi z \ell_o} + k \tag{48}$$

with complementary slackness.

Alternatively, this can be rewritten as:

$$\begin{aligned} f'(k) &= 1 &(49)\\ u'(c) &= u'(d) &(50)\\ v'(\ell_y) &= u'(c)\frac{(c+\pi d)}{[\ell_y + \pi z \ell_o]} &(51)\\ v'(\ell_o) &= u'(d)\frac{(c+\pi d)}{[\ell_y + \pi z \ell_o]} &(52)\\ -\frac{v(\ell_o)}{\ell_o} &+ u'(d)\frac{(c+\pi d)}{[\ell_y + \pi z \ell_o]} \geq 0 &(53) \end{aligned}$$

Obviously, the second FOC implies that consumption should be smoothed along the life cycle (i.e. $c = d$). The third and the fourth FOCs also imply that the quantity of hours worked should be smoothed along the life cycle: $\ell_y = \ell_o$. Finally, note that the FOC for ℓ_o and the FOC for z cannot be satisfied together, since $v'(\ell_o) > \frac{v(\ell_o)}{\ell_o}$. As in the preceding sections, we therefore obtain that $z = 1$ and $\ell_o \in [0, 1]$. Proposition 6 summarizes our results.

Proposition 6 *At the long-run utilitarian social optimum, we have:*

$$\begin{aligned} c^u &= d^u \text{ and } \ell_y^u = \ell_o^u = \ell^u \\ v'(\ell^u) &= \frac{c^u u'(c^u)}{\ell^u} \\ z^u &= 1 \text{ and } f'(k^u) = 1 \\ f(k^u) &= \frac{c^u}{\ell_y^u} + k^u \end{aligned}$$

Proof. See above. ■

The long-run utilitarian social optimum is characterized by flat consumption profiles along the life cycle, as well as by flat labor profiles. There is also a postponement of retirement until the end of the period, as well as the standard Golden Rule capital level, $f'(k) = 1$.

The long-run utilitarian optimum differs thus from the laissez-faire on several grounds. First, the level of the capital to labor ratio is lower at the laissez-faire, $k < k^u$ (underaccumulation) which implies that consumption and labor at both periods have different levels at the laissez-faire and at the social optimum. Also, whereas consumption profiles are increasing at the laissez-faire, they should be flat at the utilitarian social optimum. Finally, while the quantity of hours worked is decreasing with the age at the laissez-faire, this should be constant along the life cycle at the utilitarian social optimum.

However, as in the baseline static model, it is likely that short-lived persons, at the long-run utilitarian social optimum, exhibit a lower realized lifetime well-being than long-lived persons. Actually, the necessary and sufficient condition for this inequality is here:

$$u(c^u) - v(\ell^u) > 0$$

That condition is likely to be satisfied, especially when the productive capacity of the economy is large, leading to high (flat) consumption profiles at the utilitarian optimum.

It is easy to see that the introduction of intergenerational lump sum transfers so as to have $f'(k) = 1$ suffices to decentralize the utilitarian social optimum.

Proposition 7 *The long-run utilitarian social optimum can be decentralized by means of intergenerational lumpsum transfers yielding $f'(k) = 1$.*

Proof. Suppose that the intergenerational transfers lead to $f'(k) = 1$. Then, from the FOCs of the agent's problem, we have:

$$u'(c) = u'(d) \iff c = d$$

Given that $R = 1$ and $c = d$, we have from the FOCs $wu'(c) = v'(\ell_y)$ and $wu'(d) = v'(\ell_o)$ so that $\ell_y = \ell_o = \ell$. It satisfies:

$$wu'(c) = v'(\ell)$$

Note that the intertemporal budget constraint becomes:

$$w\ell(1+\pi) = c(1+\pi) \iff w = \frac{c}{\ell}$$

Substituting for the wage, we obtain:

$$v'(\ell) = \frac{cu'(c)}{\ell}$$

Finally, substituting for $w = f(k) - kf'(k)$ in the intertemporal budget constraint of the agent, we obtain:

$$f(k) = \frac{c}{\ell} + k$$

which is the resource constraint faced by the social planner. Thus intergenerational transfers leading to $f'(k) = 1$ suffice to imply all conditions characterizing the utilitarian social optimum. ■

8.5 Ex post egalitarian optimum

Given that inequalities in realized lifetime well-being between short-lived and long-lived agents are hard to justify, it makes sense, as in the baseline model, to consider an alternative social welfare criterion, which would do more justice to the compensation of the unlucky short-lived.

The problem of the ex post egalitarian social planner is to choose the stationary equilibrium that maximizes the well-being of the worst off. That problem can be written as:

$$\max_{c,d,k,\ell_y,\ell_o,z} \min \quad \{u(c) - v(\ell_y), u(c) - v(\ell_y) + u(d) - zv(\ell_o)\}$$

$$\text{s.t. } f(k) \geq \frac{c + \pi d}{[\ell_y + \pi z \ell_o]} + k$$

That problem can be rewritten as:

$$\begin{aligned} \max_{c,d,k,\ell_y,\ell_o,z} \quad & u(c) - v(\ell_y) \\ \text{s.t.} \quad & f(k) \geq \frac{c + \pi d}{[\ell_y + \pi z \ell_o]} + k \\ \text{s.t.} \quad & u(d) - zv(\ell_o) \geq 0 \end{aligned}$$

After some rearrangements, the FOCs can be rewritten as:

$$f'(k) = 1 \tag{54}$$

$$u'(c) = \frac{\mu}{\pi} u'(d) \tag{55}$$

$$v'(\ell_y) = u'(c) \frac{c + \pi d}{\ell_y + \pi z \ell_o} \tag{56}$$

$$v'(\ell_o) = u'(d) \frac{c + \pi d}{\ell_y + \pi z \ell_o} \tag{57}$$

$$-\frac{v(\ell_o)}{\ell_o} + u'(d) \frac{c + \pi d}{\ell_y + \pi z \ell_o} \geq 0 \tag{58}$$

as well as the conditions:

$$\lambda \geq 0, f(k) \geq \frac{c + \pi d}{\ell_y + \pi z \ell_o} + k \tag{59}$$

$$\mu \geq 0, u(d) = zv(\ell_o) \tag{60}$$

Here again, it is not possible that both the FOCs for ℓ_o and z hold. Under the convexity of the function $v(.)$, one obtains that necessarily in equilibrium, $z = 1$ and $\ell_o \in [0, 1]$. We thus have:

$$u'(c) = \frac{\mu}{\pi} u'(d) \tag{61}$$

$$v'(\ell_y) = u'(c) \frac{(c + \pi d)}{[\ell_y + \pi \ell_o]} \tag{62}$$

$$v'(\ell_o) = \frac{\pi}{\mu} u'(c) \frac{(c + \pi d)}{[\ell_y + \pi \ell_o]} \tag{63}$$

We thus have two possible cases: if $\frac{\mu}{\pi} > 1$, $d > c$ and $\ell_o < \ell_y$. But if $\frac{\mu}{\pi} < 1$, $d < c$ and $\ell_o > \ell_y$. Given that μ, which represents the shadow value of relaxing the egalitarian constraint, is likely to be low, and / or that π is likely to be large, the case where $\frac{\mu}{\pi} < 1$ is the most plausible. In that case, we have decreasing consumption profiles with the age and increasing labor profile with the age. The following proposition summarizes our results.

Proposition 8 *At the long-run ex post egalitarian social optimum (e), we have:*

- *if* $\frac{\mu}{\pi} \leq 1$
$$c^e \geq d^e, \ell_o^e \geq \ell_y^e,\ z^e = 1 \text{ and } f'(k^e) = 1$$
- *if* $\frac{\mu}{\pi} > 1$
$$c^e < d^e, \ell_o^e < \ell_y^e,\ z^e = 1 \text{ and } f'(k^e) = 1$$

Proof. See above. ■

The long-run ex post egalitarian optimum exhibits some common points with the utilitarian optimum. In both cases, the optimal capital takes its Golden Rule level. Moreover, in both cases the social optimum involves postponing retirement as much as possible ($z^e = 1$).

However, unlike the utilitarian optimum, the consumption profiles and the abour profiles are not necessarily flat at the ex post egalitarian optimum. When the Lagrange multiplier associated to the egalitarian constraint is smaller than the survival probability to the old age, the consumption profile is decreasing with the age and the labor profile is increasing with the age.

The intuition goes as follows. By forcing the young individuals to work less, and by inducing them to consume more at the young age, the social planner contributes to raise the realized lifetime well-being of those who will turn out to be short-lived, and who will not enjoy the next period.

The following proposition examines the decentralization of the ex post egalitarian social optimum.

Proposition 9 *The long-run ex post egalitarian optimum can be decentralized by means of:*

- *Intergenerational lumpsum transfers yielding* $f'(k^e) = 1$*;*
- *A tax on savings return* τ^e *equal to:*
$$\tau^e = 1 - \frac{\mu}{\pi} = 1 - \frac{u'(c^e)}{v'(\ell_o^e)} \left(f(k^e) - k^e\right)$$
- *Lump sum transfers so as to satisfy the egalitarian constraint,* $u(d^e) = v(\ell_o^e)$.

Proof. See the Appendix. ■

The ex post egalitarian optimum can be decentralized by means of a system of lump sum intergenerational transfers leading to the Golden Rule capital level, as well as by means of a tax on savings returns, which allows to get the optimal consumption profile. Lump sum transfers are also required to satisfy the egalitarian constraint so that no inequality should be left between short-lived and long-lived agents. Under those three instruments, the long-run ex post egalitarian optimum is decentralized, meaning that the realized lifetime well-being of the worst off living at the stationary equilibrium is maximized.

Finally, it should be stressed that, as in the baseline model, imposing only working time regulations could hardly decentralize the long-run ex post egalitarian social optimum. As in the baseline model, such regulations, even in the hypothetical case where these would induce individuals to choose the socially optimal levels of hours worked at all ages, would not suffice to decentralize the socially optimal (decreasing) consumption profiles. Moreover, there is here an additional difficulty, which is that only imposing working time regulations would not solve the problem of underaccumulation of capital at the laissez-faire.

9 Working time reforms in France (1848-2000)

Let us conclude our investigations by going back to history, and to the working time regulation reforms introduced during the last two centuries. How did those reforms affect realized lifetime well-being? Did those reforms reduce inequalities in realized lifetime well-being between the long-lived and the short-lived?

In order to answer those questions, this Section recalibrates our model, taking into account the variations in policy parameters T_2 (number of working weeks), $\bar{\ell}$ (working time) and $\bar{z}$ (retirement age) over the last two centuries in France (1848-2000). We then compute the associated variations in realized lifetime well-being for the short-lived and the long-lived.

Table 6 shows the evolution of parameters T_2, $\bar{\ell}$ and $\bar{z}$. To compute values $\bar{\ell}$, we assumed that agents can work for a maximum length of time of 126 hours a week.[19] The value for $\bar{\ell}$ will therefore be obtained by normalizing the number of hours worked in a week over the maximum length of working time.[20] Regarding $\bar{z}$, we assume a maximum duration of career of 60 years.

Year	T_2	worked hours per week	ℓ	retirement age	$\bar{z}$
1848	52	84	$84/126 = 0.666$	-	1
1900	52	70	$70/126 = 0.555$	-	1
1919	52	48	$48/126 = 0.380$	65	0.500
1936	50	40	$40/126 = 0.317$	65	0.500
1982	47	39	$39/126 = 0.309$	60	0.330
2000	47	35	$35/126 = 0.277$	60	0.330

Table 6 : A brief history of working time regulation in France.

[19] 18 hours at maximum could be devoted to work in a day and 6 hours to activities of daily living.

[20] For instance, in 1848, $\bar{\ell}_{1848} = 84/126$. T_2 is the number of weeks worked in a year and $\bar{z}$, the legal retirement age (before, 1919, we assume agents work until their death).

Over the period considered, there have been substantial variations in all macroeconomic aggregates. We will not try to take all these variations into account. On the contrary, we will focus only on the variation in the real wage, using the recent historical time series computed by Zwart et al. (2014). Zwart et al. (2014) provide estimates, over the last two centuries, of the daily real wage of a building laborer. Table 7 shows the real wage estimates provided by Zwart et al. (2014). Each figure on the second column corresponds to the real wages of building laborers defined as "the number of subsistence baskets that a daily wage buys".[21] The last column reports the full-week-worked-equivalent hourly wage. The hourly wage is computed *as if* agents were working the total number of hours over 7 days (and not over 5 days as today).[22]

Year	daily wage	# worked hours a week	Hourly wage w
1848	14	84	$14/(84/7) = 1.67$
1900	29	70	$29/(70/7) = 2.9$
1919	13	48	$13/(48/7) = 1.90$
1936	15	40	$15/(40/7) = 2.625$
1982	37.9*	39	$37.9/(39/7) = 6.80$
2000	93.5*	35	$93.5/(35/7) = 18.7$

Table 7: Daily real wages (source: de Zwart et al. (2014)),
and hourly wages, France, 1848-2000.

Table 7 shows that, over the period considered, the real wage has grown strongly. This rise in the real wage has several causes, whose study goes beyond the scope of this paper. Note, however, that some part of the rise in the real wage may be due to working time reforms. Indeed, working time reforms involved a reduction in the number of worked hours, while maintaining the total labor income unchanged (which implies mechanically a rise in the hourly wage).

Finally, regarding survival conditions, the survival rate π was calibrated as shown in Table 8.[23]

[21] French real wages are not available for 1982 and 2000. The figures in Table 7 with a * are obtained by linear interpolation assuming a similar trend to the one prevailing in Western Europe.

[22] For example, in 1848, the daily wage is 14 for 12 hours of work a day (over 7 days a week), which gives a hourly wage of $14/(84/7) = 1.67$ while in 1982, the daily wage is 37.9 for 39 hours a week (over 5 days), so that the full-week-worked-equivalent hourly wage is $93.5/(39/7) = 18.7$.

[23] The above values for π are obtained as follows:

$$10 + \text{life expectancy at } 10 = 50 + 30\pi.$$

We use values for life expectancy at 10 as in particular in 1848 and 1900, infant mortality was very high. Note that the value marked with * should be greater than 1 with the way we defined π; we constrain it to 1.

years	life expectancy at age 10	π
1848	47.13	0.24
1900	50.40	0.35
1919	51.76	0.39
1936	57.71	0.59
1982	69.77	0.992
2000	73.24	1*

Table 8: Survival probability at age 50 for France. (source Human Mortality database)

Given that the argument of the utility function does not consist of money any more, we need to reconsider the calibration of preference parameters accordingly. Since the unit of the real wage is a subsistence basket, we can calibrate the intercept of the temporal utility function α in such a way that an individual with a consumption equal to one subsistence basket (i.e. 7 subsistence baskets per week) would be indifferent between life and death. Thus we compute the value for α as the solution to $u(7) - v(\bar{\ell}_{1848}) = 0$, that is, to:

$$52\frac{7^{1-\frac{1}{1.25}}}{1-1.25} + \alpha - T_{2,1848}\beta\frac{(\bar{\ell}_{1848})^2}{2} = 0$$

It yields a value of $\alpha = -314.37$.

Using the parametrisation described in the tables above for $(\pi, w, \bar{z}, \bar{\ell}, T_2)$ as well as the value for α, Table 9 reports the realized lifetime well-being levels for the short-lived, the long-lived, as well as inequalities in lifetime well-being between them, for the period 1848-2000.[24]

Table 9 shows that, as a result of the introduction of working time regulations and of the (potentially induced) rise in the real wage, both the realized lifetime well-being of the short-lived and the long-lived have increased over time (except for the 1919 decrease, which may be mostly attributable to the fall in economic activity after the First World War).

years	$u(c)$	$\bar{z}v(\ell)$	U_{SL}	U_{LL}	$U_{LL} - U_{SL}$	ΔU_{SL} (%)	ΔU_{LL} (%)
1848	384.46	69.33	315.13	630.26	315.13	-	-
1900	438.07	48.15	389.93	779.85	389.93	+ 23.7 %	+ 23.7 %
1919	307.71	11.32	285.07	581.46	296.39	- 26.8 %	- 25.4 %
1936	318.61	7.56	303.49	614.54	311.05	+ 6.4 %	+ 5.6 %
1982	417.86	4.50	404.35	817.70	413.35	+ 33.2 %	+ 33.0 %
2000	562.51	3.63	551.63	1110.51	558.88	+ 36.4 %	+ 35.8 %

Table 9: Realized lifetime well-being for the short-lived and the long-lived, 1848-2000.

[24] We have: $U_{SL} = u(c) - v(\bar{\ell})$; $U_{LL} = 2u(c) - (1+\bar{z})v(\bar{\ell})$, where $c = d = \frac{w\bar{\ell}(1+\pi\bar{z})}{1+\pi}$. Note that when $\bar{z} = 1$ as in 1848 and 1900, $c = d = w\bar{\ell}$, while if $\bar{z} < 1$, $c = d < w\bar{\ell}$.

Turning now to inequalities, it appears that, in absolute terms, the lifetime well-being of the long-lived has, over the period considered, increased more than that of the short-lived, which leads to increased inequalities in realized lifetime well-being between the short-lived and the long-lived.[25]

Note, however, that uniform working time regulations have, in association with the rise in the real wage, contributed to increase the realized lifetime well-being of the worst off, i.e. the short-lived. Thus those reforms implied not only larger inequalities between the short-lived and the long-lived; these also contributed to improve the situation of the unlucky short-lived.

In sum, the evaluation of the effects of past working time regulations on the distribution of well-being in the population is complex. Assuming an invariant lifetime utility function over a period as long as 150 years is a strong assumption. Moreover, many dimensions of standards of living have changed over that period (i.e. rises in the real wage and in life expectancy), and one cannot attribute the entire welfare gains to working time regulations. Having stressed those limitations, our calculations suggest nonetheless that, along with uniform working time regulations, the distribution of realized lifetime well-being within the population has evolved substantially. Inequalities went up, but at the same time the situation of the worst off (the unlucky short-lived) has considerably improved. Thus, although we remain far from the ex post egalitarian optimum, there has been a significant improvement of the situation of the worst-off.

10 Conclusion

This paper examined, from a lifecycle perspective, the redistributive effects of working time regulations in an economy where individuals have unequal longevity. For that purpose, we first examined the impact of labor supply decisions on inequalities in realized lifetime well-being between the long-lived and the short-lived at the laissez-faire, in the absence of any government intervention. Then, we explored the impact of imposing uniform working time restrictions (with or without overtime work) on those inequalities, and showed that, in some cases, such regulations can reduce the size of those inequalities.

In a second stage, we contrasted the laissez-faire with the ex post egalitarian social optimum, and we showed that, under the latter, the consumption profile should be generally decreasing with the age, whereas the working hours profile should be increasing with the age, unlike at the laissez-faire. Our numerical calculations suggest that the ex post egalitarian optimum involves about 20 hours worked per week at the young age. Those numbers are not far from the 4 hours a day proposed by Campanella (1602) in *The City of the Sun*. But for older adults the working time would be about 4 times larger. The reason

[25] The origin of this rise in well-being inequalities depends on the joint evolution of the wage, the labor time and the retirement age. For instance, between 1848 and 1900, both w and π have increased while $\bar{l}$ decreased, which resulted in an increase in consumption (the increase in w compensated the decrease in $\bar{\ell}$ and since $\bar{z} = 1$, π has no impact on the level of c) and a decrease in disutility of work. Both these effects increase welfare inequalities between the short-lived and the long-lived agents.

is that working time regulation is not seen here as an instrument against the alienation of people (as it is in Campanella), but as a way to guarantee some justice between the short-lived and the long-lived.

We also showed that the ex post egalitarian optimum cannot be decentralized by working time restrictions only, even when these are age-specific rather than uniform. Indeed, although such restrictions can, under some conditions, reduce the size of inequalities in lifetime well-being between short-lived and long-lived persons, these regulations do not allow for decentralizing the decreasing consumption profile with the age. Hence, these do not suffice to abolish inequalities in lifetime well-being between the short-lived and the long-lived. We showed also that those results are robust to considering a dynamic OLG model with capital accumulation.

Finally, we complemented those analytical findings by some numerical explorations showing that uniform working time regulations in France (1848-2000) have, over the last two centuries, been associated with a rise in inequalities in realized lifetime well-being between short-lived and long-lived agents, but also with an improvement of the situation of the worst off (the short-lived).

All in all, this paper finds some support for the use of age-specific working time regulations as a way to redistribute from lucky long-lived individuals towards unlucky short-lived individuals, but only as a kind of second-best redistributive device in a world where the taxation of savings return and lump sum taxation are subject to strong political constraints. Note that, in a first-best world without such restrictions, working time regulations would not be necessary, since the tax on savings as as well as lump sum taxation would allow for the full decentralization of the egalitarian optimum. However, once such a tax on savings is not available, or its size is limited by exogenous constraints, age-specific working time restrictions can, under some conditions, help reducing inequalities between long-lived and short-lived persons.

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12 Appendix

12.1 Proof of Proposition 1

Imposing the condition:

$$wu'(w) < v'(1)$$

guarantees that, when there is a flat labor profile with no savings and with late retirement ($z = 1$), this is achieved for a level of labor that is strictly inferior to 1. Note also that the condition:

$$wu'(0) > v'(0) = 0$$

trivially holds. That condition guarantees that, when there is a flat labor profile with no savings and with late retirement ($z = 1$), this is achieved for a level of labor that is strictly superior to 0.

Given those two conditions, we know that, when the labor profile is flat and there is no savings and late retirement, there exists a unique interior level of labor $\ell \in]0, 1[$ that satisfies the FOCs.

Let us now show why it is optimal for individuals to have a flat labor profile with no savings and with late retirement ($z = 1$). Rearranging the FOCs for old age labor and for retirement, we obtain:

$$\begin{aligned} FOC_\ell &= wu'(d) - v'(\ell_o) \geq 0 \\ FOC_z &= wu'(d) - \frac{v(\ell_o)}{\ell_o} \geq 0 \end{aligned}$$

Given that $v(\cdot)$ is convex, it is necessarily the case that: $v'(\ell_o) > \frac{v(\ell_o)}{\ell_o}$. This implies that in equilibrium, $z = 1$ and $\ell \in [0, 1]$. We can show this by contradiction. Assume $z < 1$ so that $FOC_z = 0$. Under the convexity of $v(.)$, this implies that $FOC_\ell < 0$ which is not consistent with FOC_ℓ. Alternatively, assume $FOC_\ell = 0$ for $\ell_o > 0$; this implies that $FOC_z > 0$ consistent with $z = 1$.

From the intertemporal budget constraint, and substituting for $c = d$, $z = 1$ and $\ell_y = \ell_o = \ell$, we obtain:

$$c(1+\pi) = \ell w + \pi \ell w \iff c = \ell w \text{ and } s = 0$$

Given that the conditions $wu'(w) < v'(1)$ and $wu'(0) > v'(0) = 0$ are satisfied, we know that when there is a flat labor profile with no savings and with late retirement ($z = 1$), this is achieved for a unique interior level of labor $\ell \in]0, 1[$. We thus obtain the interiority of the optimal quantities of labor at the laissez-faire.

We can also use the previous conditions to do some comparative statics. Suppose, for instance, that the survival probability goes up. We have:

$$wu'(\ell w) - v'(\ell) = 0$$

so that

$$\begin{aligned} \frac{d\ell}{d\pi} &= = 0 \\ \frac{dc}{d\pi} &= 0 \\ \frac{dz}{d\pi} &= 0. \end{aligned}$$

Consider now a change in the wage rate w. We have:

$$\frac{d\ell}{dw} = -\frac{\frac{d\left(wu'(\ell w) - v'(\ell)\right)}{dw}}{w^2 u''(\ell w) - v''(\ell)} = -\frac{u'(\ell w) + u''(\ell w) w \ell}{w^2 u''(\ell w) - v''(\ell)} \gtrless 0$$

The denominator is negative, but the sign of the numerator is ambiguous and depends on whether the coefficient of relative risk aversion $R_c = -cu''(c)/u'(c)$ is greater or smaller than 1. If $R_c < 1$, $d\ell/dw > 0$ while if $R_c \geq 1$, $d\ell/dw \leq 0$. From the budget constraint, $c = w\ell$, we have that $dc/dw = \ell + w d\ell/dw$ so that together with the expression for $d\ell/dw$, we obtain that $dc/dw > 0$ always. Obviously, $dz/dw = 0$.

12.2 Proof of Proposition 9

Let us start from the FOCs of the individual problem. Once $f'(k) = 1$, and imposing a tax τ on savings return, the FOCs of the agent's problem become:

$$\begin{aligned} u'(c) &= (1-\tau) u'(d) \\ wu'(c) &= v'(\ell_y) \\ wu'(d) &= v'(\ell_o) \\ w &= f(k) - k \end{aligned}$$

We would like the consumption profile to satisfy:

$$u'(c) = \frac{\mu}{\pi} u'(d)$$

Hence we need to fix:

$$\tau = 1 - \frac{\mu}{\pi}$$

Given that

$$v'(\ell_o) = \frac{\pi}{\mu} u'(c) \frac{(c + \pi d)}{[\ell_y + \pi \ell_o]} \iff \frac{v'(\ell_o)}{u'(c)} \frac{\ell_y + \pi \ell_o}{c + \pi d} = \frac{\pi}{\mu}$$

Remind also, from the economy's resource constraint, that:

$$f(k) = \frac{c + \pi d}{\ell_y + \pi \ell_o} + k$$

so that, at the egalitarian optimum:

$$\frac{c^e + \pi d^e}{\ell_y^e + \pi \ell_o^e} = f(k^e) - k^e$$

This tax can be rewritten as:

$$\tau = 1 - \frac{u'(c^e)}{v'(\ell_o^e)} \frac{c^e + \pi d^e}{\ell_y^e + \pi \ell_o^e} = 1 - \frac{u'(c^e)}{v'(\ell_o^e)} \left(f(k^e) - k^e \right).$$