The Business Cycle Implications of Endogenous Schumpeterian Growth in a New Keynesian DSGE Model

Preliminary. Comments welcome.

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Abstract

Despite the development of modern endogenous growth models, commonly-used DSGE models are based on an exogenous neoclassical long-term growth and focus only on the fluctuations around that trend growth. In contrast, we propose a model of economic fluctuations with New Keynesian nominal rigidities and common shocks that builds in the features of a Schumpeterian endogenous growth model to capture the interactions between an economy's trend growth and its cyclical fluctuations. The innovation process is embedded in the intermediate production sector thus generating a dynamic not commonly seen in New Keynesian models, while conserving nominal wage and price stickiness and studying the impact of productivity, spillover and monetary shocks.

We show that endogenous decisions to invest in R&D have implications that impact the likelihood of innovating and pushing the technological frontier, while adding a significant transmission channel. The implications of the model on the business cycle characteristics (such as volatility, co-movements and persistence in real variables and inflation) are highlighted. Moreover, the mechanics of innovation being considered brings some support and microfoundations to the monopolistic competition de facto introduced in New Keynesian models. Finally, our hybrid model highlights and addresses new challenges at the modelling and simulation stages.

KEY WORDS: Schumpeterian endogenous growth; Business cycles; New Keynesian dynamic stochastic general equilibrium (DSGE) model.

JEL CODE: E32, E52, O31, O33, O42

1 Introduction

Investing in research and development (R&D) leads to innovation that is at the core of a knowledge-based economy and of an endogenous foundation for secular growth, as shown in the second generation of growth models by Romer (1986, 1990), Lucas (1988), Rebelo (1991), and Aghion and Howitt (1992), just to name a few. However, most general equilibrium models simply ignore this aspect with respect to business cycles. Yet, this may not be innocuous.

Indeed, Comin and Gertler (2006) show that R&D effects do matter, not only for the long run as previously thought, but also at business cycle frequencies. The exclusion of the innovation process from modern general equilibrium models deprives them from a propagation mechanism. For instance, Barlevy (2007) and Fatas (2000) both show that investments in innovation are sensitive to monetary policy shocks. The introduction of labour and capital augmenting innovation process would also partially endogenise technology. In conjunction with the monetary policy effect on R&D, this could even lower the contribution to business cycles that is attributed to the neutral technology shock.

Until recently, relatively few papers had attempted to combine a business cycle model with the features of a Schumpeterian growth model. From the early RBC models to the most recent New-Keynesian (NK) macroeconomic models, the usual dynamic-stochastic general equilibrium (DSGE) model has indeed been constructed around a classical exogenous growth model. Indeed, seminal articles such as Smets and Wouters (2007) and Justiniano et al. (2010) feature calibrated exogenous growth. Nuño (2011) studies a real business cycle model with Schumpeterian growth, using specific functional forms and no nominal rigidity. In a NK model with both Calvo staggered prices and wages, as well as endogenous growth operating through non-rival access to knowledge, Annicchiarico et al. (2011) analyse monetary volatility and growth in relation with nominal rigidities, and the persistence of monetary shocks on a Taylor monetary policy rule. Amano et al. (2012) have studied the implications of endogenous growth with horizontal innovations in the variety of intermediate goods, for the welfare costs of inflation in NK economies with nominal rigidities being modelled as Taylor (1980) staggered price and wage contracts. Annicchiarico and Rossi (2013) look at optimal monetary policy in a NK economy characterized by Calvo staggered prices that embeds endogenous growth induced by knowledge externalities with increasing returns-to-scale. Annicchiarico and Pelloni (2014) examine how nominal rigidities affect uncertainty on long-term growth, when prices and wages are preset with a one-period lag, and assuming constant returns to the level of technology in the innovation activity, in a model with only labour as an endogenous input that is divided between producing output or R&D. Using Bayesian estimation, Cozzi et al. (2017) show the implication of financial conditions for innovation dynamics in a NK model with price and wage rigidities arising from specific adjustment costs in a Solow-neutral technology and Schumpeterian growth. Finally, to evaluate the sources of the productivity slowdown following the Great Recession, Anzoategui et al. (2017) construct an estimate a DSGE model with staggered Calvo contracts driving sluggish adjustments of wages and final-good prices, that also features endogenous growth *via* the expanding variety of intermediate goods resulting from public learning-by-doing in the R&D process and an endogenous pace of technology adoption.

In this paper, we show that the inclusion and treatment of Schumpeterian growth features matter for several reasons. First, the endogenous choice of investing in R&D has implications for the likelihood of advancing or not the technological boundary and to vary the entry and exit of firms. Therefore, the Schumpeterian dimension of our model, with Harrod-neutral technical progressing in the production function for goods and a decreasing return-to-scale innovation production function, adds a relevant transmission channel for understanding economic fluctuations and the impact of both real and monetary disturbances. In particular, we consider its implications for the volatility, comovements and persistence of real variables. We believe this to be a prerequisite, prior to study the policy implications of such a model in future work. Second, this dimension provides some microfoundation to monopolistic competition that is introduced *de facto* in NK models, as differing levels of technological advancement in the intermediate sector bring a justification for existing market power. Third, our hybrid model highlights and addresses new challenges at the modelling and simulation stages when considering the implications of price rigidities on R&D investments, as the sluggish dynamics of prices impact directly on the expected discounted value derived from innovations.

Hence, we aim at jointly accounting for endogenous growth through creative destruction and business cycle in an extended NK model. The proposed model is composed of the following groups of agents: the households, the final good producers, employment agencies, the intermediate good producers, the entrepreneurs/innovators, a monetary authority and the government. Forward-looking households maximize their expected utility with respect to their sequence of budget constraints, by making optimal decisions regarding their time-paths for consumption, labour, utilization of physical capital, private investment, and net bond holdings. Final good producers operate in a perfectly competitive market and use intermediate goods as input. The employment agencies aggregate the households' specialized labour into homogeneous labour used by the intermediate good producers. The latter evolves in a monopolistically competitive setting that allows them to set prices. Operating within the intermediate sector, entrepreneurs/innovators invest final goods to increase their odds of pushing the technological frontier, so that an intermediate good producer that implements the new technology takes over the incumbent producer in his respective intermediate sector. Finally, prices and wages are subjected to nominal rigidities through contracts *à la* Calvo (1983). In particular, we show that sluggish price adjustments interact directly with the innovation process, as the discounted expected value of investing in R&D matters for the rate of innovation over the business cycles.

While monetary policy is set according to a Taylor rule, and deviations thereof. The impacts of the followings shocks are also considered: a transitory technological shock, a knowledge-spillover shock, and an investment shock.

By including an explicit innovation process and its interaction with the producers of intermediate goods, that serve as inputs to the production of the final consumption good, we draw attention to an additional propagation mechanism for shocks. As expected, investment in R&D is sensitive to monetary policy shocks. A positive shock increases investments in R&D which, in turn, have a positive effect on marginal productivities of labour and capital. This ultimately affect the growth rates and levels of macroeconomic prices and quantities, including factor demand and wages.

In Section 2, we present in details the features of the model. We begin with those that are typically encountered in modern NK-DSGE setup, with the necessary adjustments needed to account for the presence of innovating firms. We then direct our attention to the key elements derived from the endogenous Schumpeterian considerations linked to innovations. Finally, we discuss aggregation and the model's general equilibrium. Section 3 presents the steady-states properties of the model, while Section 4 deals with calibration and characterisation of the various disturbances that affect our model economy. Section 5 displays the impulse response functions and statistical moments implied by the model and proceeds with the business cycle analysis. A summary of our findings and possible extensions make up the conclusion in Section 6.

2 The Model

In this section, we describe the environment and the problems faced by each types of agents. We first describe the characteristics of the final good producer, the employment agency, and the households' problems. To a large extent, these are very similar to the standard setup encountered in the modern dynamic stochastic general equilibrium literature. As needed, we introduce features arising from the existence of innovating firms ultimately owned by households. Second, we focus on the specificities brought by Schumpeterian considerations for the innovators and the intermediate good producers, especially with sluggish adjustment in prices. Finally, we address the aggregation issues and we present the monetary authority's policy function.

2.1 A presentation of the common features of a New Keynesian DSGE model

2.1.1 The final good producer

The final consumption good is produced by a representative firm that operates in a perfectly competitive setting and that aggregates a continuum of intermediate goods $i \in (0,1)$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad , \tag{1}$$

where Y_t is total final output, the input $Y_t(i)$ is the good produced by an intermediate level firm i, and $0 \leq \epsilon < \infty$ is the elasticity of substitution between intermediate goods.

Because of perfect competition, the final-good producer takes as given the price of its final output, P_t, and the prices of the intermediate goods, $P_t(i)$. Hence, its profit maximization problem

$$\underset{Y_t(i)}{\text{Max}} \; \Pi_{FG} = P_t \, Y_t - \int_0^1 P_t(i) \, Y_t(i) \, di \tag{2}$$

yields the demand for the i^{th} intermediate good as a negative function of its relative price, namely

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \; . \tag{3}$$

Since economic profits are zero in perfect competition, total nominal output is given by the sum of the nominal value of all intermediate goods i

$$P_t \, Y_t = \int_0^1 P_t(i) \, Y_t(i) \, di \; , \tag{4}$$

which, using equation (3), yields the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \; . \tag{5}$$

2.1.2 The employment agency

In our model economy, a continuum of households possess different skills and offer specialized labour $L_{Ht}(j)$ for $j \in (0,1)$, that gives them some degree of market power in setting wages. Since intermediate firms use a combination of specialized labour, we can think of a representative employment agency which aggregates specialized labour and turns it into the combined labour input L_t employed by the intermediate firms, namely

$$L_t = \left(\int_0^1 L_{Ht}(j)^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{\gamma}{\gamma-1}} \; , \tag{6}$$

where $0 \leq \gamma < \infty$ is the elasticity of substitution between each labour type.

Operating in perfect competition, the employment agency maximizes its profits with respect to $L_{Ht}(j)$ while taking as given the aggregate wage rate W_t and the prevailing labour compensation specific to each labour type j.

The solution of its optimization problem

$$\underset{L_{Ht}(j)}{\text{Max}} \;\; \Pi_{EA} = W_t \, L_t - \int_0^1 W_t(j) \, L_{Ht}(j) \, dj \tag{7}$$

yields the demand for specialized labour j as a negative function of its relative wage rate

$$L_{Ht}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\gamma} L_t \ . \tag{8}$$

From equation (8), and the competitive equilibrium for the employment agency, the aggregate wage rate is

$$W_t = \left[\int_0^1 W_t(j)^{1-\gamma}\, dj\right]^{\frac{1}{1-\gamma}} \ . \tag{9}$$

2.1.3 Households

The Budget Constraint

Each period t, a representative type j household faces the following budget constraint

$$P_t\, C_t + P_t\, I_t + P_t\, a(u_t)\bar{K}_t + \frac{B_t}{1+R_t} \leq W_t(j)\, L_{Ht}(j) + q_t\, u_t\bar{K}_t + B_{t-1} + D_t - T_t \ . \tag{10}$$

As of date t, the household's nominal value for its uses of funds is the sum of its nominal value of consumption in the final good, denoted as $P_t\, C_t$, of its desired level of investment in capital goods, $P_t\, I_t$, the resources devoted to adjust the utilization rate of physical capital (if applicable), $P_t\, a(u_t)\bar{K}_t$, and its end-of-period net holdings of a one-period discount bond $\frac{B_t}{1+R_t}$, where R_t the nominal interest rate between t and $t+1$.[1] It is assumed that the price of consumption, private investment in physical capital, and varying utilization of capital is that of the aggregate price level P_t.

Our setup allows that the utilization of the existing stock of physical capital, $\bar{K}_t$, may be less than 100% and be time-varying, provided that the household bears a cost of varying capital utilization u_t. This real cost is captured by a convex function $a(u_t)$ that is increasing in u_t.[2]

[1] The net bond holdings may be positive or negative whether the household is either creditor or debtor.

[2] It is also assumed that, in the special case where the capital stock is used at full capacity with $u_t = 1$, this real cost function takes a value of zero, i.e. $a(1) = 0$.

Type j household's nominal after-tax sources of funds arise from its labour income, i.e. the product of its nominal wage rate $W_t(j)$ and the hours worked $L_{Ht}(j)$, its nominal payments received from supplying capital services to intermediate firms, from renting some of its existing physical capital, $u_t\ \bar{K}_t$, at a gross capital rental rate q_t, the nominal face value of the net discount bond holdings carried from the previous period, and the nominal dividends, D_t, received from its ownership of shares in the intermediate production sector that operates in monopolistic competition, from which is subtracted the value of lump-sum taxes, net of government transfers.[3]

We, therefore, need to assess the dividends stemming from the economic rent, as implied in part by investments in R&D. Using aggregate labour and capital, a firm i, belonging to a continuum defined over $i \in (0, 1)$, produces intermediate good i in a monopolistically competitive market, thus generating positive economic profits

$$\Pi_{i,t} = P_t(i)\, Y_t(i) - w_t\, L_t(i) - r_t\, K_t(i)\ , \tag{11}$$

that are in turn paid as dividends among households.

The investment in R&D has to be accounted for in each period, while being treated as a sunk cost since it is irrelevant whether or not an innovator is successful *ex post*. We will use two examples to illustrate how we account for the innovation process in the representative household's budget constraint.

Having invested $P_t\, X_t$ to reach the frontier, a failed innovator generates no profits, as depicted on the timeline of his cash flow in Figure 1.

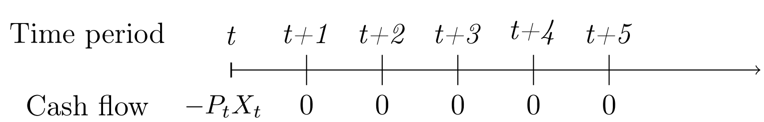

Figure 1: A failed innovator's timeline for cashflows

In this case, the household's budget constraint needs only to include his initial investment. The *ex-post* value of engaging in the innovation process is $-P_t X_t$.

[3]It is assumed that the government follows a Ricardian fiscal policy.

By comparison, a successful innovator invests $P_t\,X_t$ in R&D, which turns into a successful endeavour that allows him to collect each period monopoly profits π_t for τ periods, until it is replaced by a new innovator. Figure 2 illustrates the corresponding flow timeline.

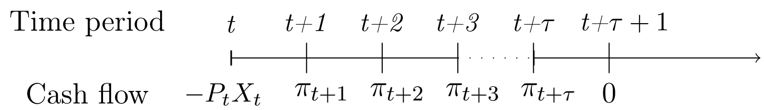

Figure 2: A succesfull innovator's timeline for cashflows

Here, the initial investment as well as future profits should be included in their respective budget constraints. It is important however to highlight that the profits included in the timeline above do not exclusively result from the innovation process. Indeed, an intermediate firm is already generating profits prior to an innovator taking over. Hence, the profits generated by the intermediate firm after the takeover includes both monopoly profits and innovation profits.

Accordingly, the overall dividends paid to households are therefore defined as:

$$D_t = \int_0^1 \Pi_{i,t} - P_t\,X_t(i)\,di\ . \tag{12}$$

Utility maximization

Similarly to Christiano et al. (2005), household j maximizes its utility function with respect to the sequence of its budget constraints for each period, while taking into account the law of movement of capital. Its preferences for consumption embed habit formation, with an intensity parameter $h > 0$, which generates some additional intrinsic dynamics and persistence on both the demand and supply sides of the economy following various shocks. The subjective discount factor is $0 < \beta < 1$, the parameter $\theta > 0$ induces disutility of labour, and the parameter $\nu \geq 0$ implies that the Frisch elasticity of labour supply is $1/\nu$. Furthermore, we assume that the household incurs some cost of adjusting investment $S(\cdot)$, that is an increasing concave function of the growth rate in investment.

The representative household must decide how much to consume C_t, how many hours $L_{Ht}(j)$ to work, how much capacity to use u_t, how much physical capital they want next period $\bar{K}_{t+1}$, how much to invest I_t in physical capital and the size of their net bond holdings B_t, by solving the following optimisation problem, where E_t^j is the expectation operator conditioned of known information as of the beginning of period t.

$$\underset{C_t, L_{Ht}(j), u_t, \bar{K}_{t+1}, I_t, B_t}{\text{Max}} E_t^j \sum_{s=0}^{\infty} \beta^s \left(ln\big(C_{t+s} - h\, C_{t+s-1}\big) - \theta\, \frac{L_{Ht+s}(j)^{1+\nu}}{1+\nu} \right) \tag{13}$$

subject to

$$\begin{aligned} P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\bar{K}_{t+s} + \frac{B_{t+s}}{1+i_t} \leq W_{t+s}(j)\, L_{Ht+s}(j) + q_{t+s}\, u_{t+s}\bar{K}_{t+s} \\ + B_{t+s-1} + D_{t+s} - T_{t+s}\ , \end{aligned} \tag{14}$$

$$\bar{K}_{t+s+1} = \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right] I_{t+s} + (1-\delta)\bar{K}_{t+s}\ , \tag{15}$$

$$K_{t+s} = u_{t+s}\bar{K}_{t+s}\ . \tag{16}$$

In addition, having assumed that a household j possesses some specialised skills underlying some market power over its wage rate, we also assume the existence of wage rigidities modelled with Calvo contract arrangements, with a constant proportion $1-\xi_w$ being allowed to reoptimise their wage each period. Hence, household j maximizes its expected utility weighed by the probability ξ_w of not being allowed to reoptimise with respect to wages subject to the labour demand function:

$$\underset{W_{t+s}(j)}{\text{Max}}\, E_t^j \left(\sum_{s=0}^{\infty} \xi_w^s\, \beta^s \left(-\theta\, \frac{L_{Ht+s}^{1+\nu}}{1+\nu} \right) + \Lambda_{t+s}\, W_{t+s}(j)\, L_{Ht+s}(j) \right) \tag{17}$$

subject to

$$L_{Ht+s}(j) = \left(\frac{W_t(j)\, \pi_{t,t+s}^w}{W_{t+s}} \right)^{\gamma} L_{t+s}\ , \tag{18}$$

where γ is type-j labour demand elasticity to the relative wage, and $\pi_{t,t+s}^w$ is the gross rate of backward wage indexation from t and $t+s$, built in the wage-setting rule.

2.2 The Schumpeterian add-ons in a New-Keynesian DSGE model and their implications

The Schumpeterian growth paradigm is similar to that of Aghion and Howitt (1992) and Nuño (2011). However, the existence of price stickiness adds an additional level of complexity that is absent in typical Schumpeterian growth models, as well as in recent business cycles models with Schumpeterian features that abstract from including price rigidity. Namely, since a firm's investments in R&D depends on the discounted expected profits arising from innovating, if successful, then the monopolistic rent depends also on the prices that a firm will be allowed to set (with some probability that prices may be fix, and some probability that they may be adjusted). This is important because...

2.2.1 The patent structures

A key element of endogenous growth models is the path followed by the state of technology. We introduce a patent system in which an innovating firm is the sole proprietor of any innovation for one period. After one period, patents expire. Then, subsequently as of next period, competitors can costlessly adopt that technology. Hence each period an intermediate producer may either be lagging or advanced. Figure 3 illustrates the technological advancement tree that summarizes this system, assuming that the firm's level of technology was initially $\bar{A}$.

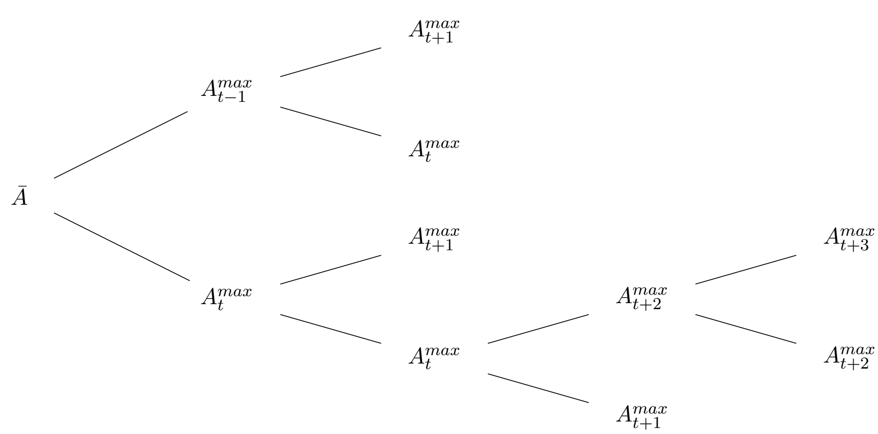

Figure 3: The technological advancement tree

This assumption will play an important role when analysing both future expected profits of intermediate firms and the expected value of a firm to the innovator.

2.2.2 The optimal reset price

Operating in a monopolistically competitive market, intermediate firms hold market power from both their diversification and the technology used in production. Moreover, prices are fixed through Calvo contracts and set as to maximize their expected profits conditional on not being allowed to reoptimise. Since firms are infinitely lived in our setup, the Calvo contracts are binding regardless of who is at the helm of a monopoly.

Hence, given an initial level of technological advancement $A_t(i)$, an intermediate firm faces the following constrained minimization of their cost:

$$\underset{K_t(i),L_t(i)}{\text{Min}} \; W_t L_t(i) + q_t K_t(i) \tag{19}$$

subject to

$$Y_t(i) = Z_t \, A_t(i)^{1-\alpha} \, K_t(i)^{\alpha} \, L_t(i)^{1-\alpha} \ , \tag{20}$$

and

$$A_t(i) = \begin{cases} A_{t-1}^{max} & \text{with probability } n_{t-1} \ , \\ A_{t-2}^{max} & \text{with probability } 1 - n_{t-1} \ . \end{cases} \tag{21}$$

where n_{t-1} is the probability that an innovation arising from investment in R&D in period $t-1$ pushed the technology level at A_{t-1}^{max} for the intermediate firm operating at date t, making it an advanced firm. Otherwise, there is a $1-n_{t-1}$ probability that the intermediate firm is lagging, while still using the previous technology level A_{t-2}^{max},

In traditional New-Keynesian models, all firms operate at the same level of technological advancement, so that all firms set the same optimal reset price. In contrast, in our setup, there are two different levels of technological advancement at any given time, as shown in Figure 3. Therefore, there are two optimal reset prices: a lagging optimal reset price and an advanced optimal reset price.

Consequently, given their respective marginal cost, intermediate firms maximize their profits with respect to their price $P_t(i)$:

$$\begin{aligned}\underset{P_t(i)}{\text{Max}}\, E_t \Bigg\{ \Big(P_t(i) - MC_t\big(A_t(i)\big)\Big)\, Y_t \\ + \sum_{s=1}^{\infty} \xi_p^s\, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Big\{ n_{t+s-1} \Big(P_t(i)\pi^p_{t,t+s} - EMC_{t+s}\Big)\, Y_{t+s}(i) \Big\} \Bigg\}\end{aligned} \tag{22}$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t(i)\,\pi^p_{t,t+s}}{P_{t+s}}\right)^{-\epsilon} Y_{t+s}\ , \tag{23}$$

and

$$EMC_{t+s} = n_{t+s-1}\, MC_{t+s}(A^{max}_{t+s-1}) + (1 - n_{t+s-1})\, MC_{t+s}(A^{max}_{t+s-2})\ , \tag{24}$$

where $\pi^p_{t,t+s}$ is the gross rate of backward price indexation from t and $t+s$, built in the price-setting rule. the gross rate of backward price indexation, built in the price-setting rule. To solve this problem, it must be noticed that initial technology is known and can differ from firm to firm, while future technology levels depend on future levels of investment in R&D. This is why we use the expected marginal cost of production to account for both the possibility that the intermediate firm may be technologically advanced or lagging as seen in Figure 3.

Thus, it can be shown that the optimal reset price is a function of initial technology $A_t(i)$:

$$P^{\#}_t(i) = \frac{\epsilon}{\epsilon - 1}\, \frac{MC_t\big(A_t(i)\big)\, P^{\epsilon}_t\, Y_t + \sum\limits_{s=1}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}\, EMC_{t+s}}{\sum\limits_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}}\ . \tag{25}$$

Given the patent system introduced in the previous subsection, at any given time, there will only be two optimal reset prices. Technologically advanced firms will set their prices at:

$$P^{\#}_{A,t} = \frac{\epsilon}{\epsilon - 1}\, \frac{MC_t(A^{max}_{t-1})\, P^{\epsilon}_t\, Y_t + \sum\limits_{s=1}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}\, EMC_{t+s}}{\sum\limits_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}}\ , \tag{26}$$

while lagging firms will set their price at:

$$P^{\#}_{L,t} = \frac{\epsilon}{\epsilon - 1} \; \frac{MC_t(A^{max}_{t-2})\, P^{\epsilon}_t\, Y_t + \sum\limits_{s=1}^{\infty} \xi^s_p\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}\, EMC_{t+s}}{\sum\limits_{s=0}^{\infty} \xi^s_p\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P^{\epsilon}_{t+s}\, Y_{t+s}} \,. \tag{27}$$

2.2.3 The innovation process

R&D activities are conducted by entrepreneurs/innovators. If they lead to an innovation, the implementation of this technology by an intermediate good producer conveys some additional market power from producing an improved version of the intermediate good, as it reaches the new technological frontier. Hence, the innovation process unfolds within the intermediate sector as the mechanism that pushes the technological frontier onward.

An entrepreneur/innovator invests some amount of final goods to raise the probability of innovating. Outside researchers or a new successful innovator supplants or "leapfrogs" an incumbent entrepreneur.[4] This prospects is however uncertain as the probability to innovate is n_t, and that of not discovering is $1 - n_t$. Yet, n_t is endogenous as it is linked to the intensity of R&D effort $\frac{X_t}{\omega\, A^{max}_t}$, where X_t is the real amount of final goods invested in R&D, A^{max}_t is the prevailing state of the technology, or technological frontier, prior to new innovations, and $\omega > 1$ is the extent of productivity improvement derived from the innovation. In particular, when $\omega\, A^{max}_t$ is bigger, a given amount of resources X_t in R&D is associated with a lower intensity, thus capturing the increasing complexity of further progress. Finally, we consider the following innovation production function that exhibits diminishing marginal returns, with $\eta > 0$:

$$n_t = \left(\frac{X_t}{\omega\, A^{max}_t} \right)^{1/(1+\eta)} \,. \tag{28}$$

[4]In our model, we therefore abstract from step-by-step technological progress, that would imply both Schumpeterian and escape-competition effects. This can also be justified by assuming that engaging in R&D is prohibitively costly to develop perfectly substitutable technology that can be used to produce a cheap and faked copy of an existing intermediate good, i.e. a knockoff, given the duration of the patent.

Hence, to choose the amount of final good to be invested in R&D that maximizes expected discounted profits, the entrepreneur faces the following constrained optimization problem

$$\underset{X_t}{\text{Max}}\ \beta \frac{\Lambda_{t+1}}{\Lambda_t} n_t\, E_t V_{t+1}(A_t^{max}) - P_t\, X_t \tag{29}$$

subject to equation (28), where $E_t V_{t+1}(A_t^{max})$ is the expected discounted value of future profits contingent on the entrepreneur remaining at the helm of the monopoly. If successful, the innovator will collect monopoly profits as long as no further innovation occurs in his sector.

For convenience and in a way that is compatible with complete markets, we assume that entrepreneurs invest in a diversified form of R&D. Strictly speaking, this is as if an entrepreneur, provided one is successful, does not know in which sector, one may end up. This implies that all entrepreneurs will invest the same amount of final good in R&D. Consequently, if an innovation occurs, the technology jumps to the frontier and the expected value of the intermediate firm will be the same regardless of the intermediate sector. In accordance with the problem in equation (29), the optimal investment in R&D is given by

$$X_t = \beta \frac{\Lambda_{t+1}}{\Lambda_t} n_t \frac{E_t V_{t+1}(A_t^{max})}{P_t}\ . \tag{30}$$

To complete the solution of equation (30), we still need to write explicitly the expected value of the firm to the entrepreneur. This happens to be more challenging than it may look at first, as it relies on the answers to three questions:

- Which path will technology follow?
- Which path will prices follow?
- What if the innovator is not allowed to reset its price at its optimal value when he takes over?

Time period	t	t+1	t+2	t+3	t+τ	t+$\tau+1$
Technology		A_t^{max}	A_t^{max}	A_{t+1}^{max}	$A_{t+\tau-3}^{max}$	$A_{t+\tau-2}^{max}$

Figure 4: The timeline for technology

The first hurdle is to ensure that the assessment of future profits follows the correct technological path. For instance, as shown in Figure 4, an innovator may reach the frontier A_t^{max} in t+1, holds a patent on this innovation, and remains there in t+2. Then, the patent on the $t+2$ frontier, A_{t+1}^{max}, expires in $t+3$ and therefore can be adopted by anyone. The subsequent adoption of the new technology is automatic as a more advanced technology decreases the marginal cost of operation, and any rational agent will decide to adopt it. This addresses the issue.

The second difficulty springs from price rigidities as they play a crucial role in determining future profits since they condition both the profit margin and the conditional demand for that specific intermediate good. Profits will differ based on whether the innovator is allowed to reoptimise.In addition, all intermediate firms face an identical probability ξ_p of not being allowed to reoptimise their price in a Calvo-contract setup. If this contingency occurs, the monopolist is stuck at charging a certain price that may not be the optimal reset one. However, because we assume that investments in R&D are diversified by the entrepreneurs/innovators, even though, *ex-ante*, an entrepreneur ignores the sector in which he will be innovating, he can evaluate the discounted expected profits from a potential discovery using the individual prices of the continuum of intermediate goods.

If the innovator is allowed to reoptimize upon taking over, he will choose the advanced optimal reset price. The technological path followed will be that of Figure 4 and will yield the following profits:

$$\Psi_{A,t+1} = P_{A,t+1}^{1-\epsilon}(Y_{t+1} + \frac{X_{1,t+2}}{\Lambda_{t+1}}) - P_{A,t+1}^{-\epsilon}(MC_{t+1}(A_t^{max})y_{t+1} + \frac{X_{2,t+2}}{\Lambda_{t+1}}), \tag{31}$$

with $X_{1,t+2}$ and $X_{2,t+2}$, two recursive auxilliary variables:

$$X_{1,t+2} = \xi_p \beta \Lambda_{t+2}(1 - n_{t+1})(\pi^p_{t+1,t+2})^{1-\epsilon} Y_{t+2} + \xi_p \beta (1 - n_{t+1})(\pi^p_{t+1,t+2})^{1-\epsilon} X_{1,t+3}, \quad (32)$$

$$X_{2,t+2} = \xi_p \beta \Lambda_{t+2}(1 - n_{t+1})(\pi^p_{t+1,t+2})^{-\epsilon} MC_{t+2}(A^{max}_t) Y_{t+2} \\ + \xi_p \beta (1 - n_{t+1})(\pi^p_{t+1,t+2})^{-\epsilon} X_{2,t+3} \quad (33)$$

As can be seen in these in both $X_{1,t+2}$ and $X_{2,t+2}$, we only weigh the future revenue or cost by the probability of entrepreneurs failing to innovate since if they are successful, the monopolist is wiped out and collects no profits.

If the innovator is not allowed to reoptimize upon taking over, he will inherit the price set by the latest Calvo contract. Our diversified R&D hypothesis allows for a straightforward solution to this problem. If the innovator does not know in which sector he will end up, he has to take into account all possible prices set in the past in the continuum of intermediate sectors:

$$\Psi_{I,t+1} = \zeta^{1-\epsilon}_{1,t+1}(Y_{t+1} + \frac{X_{1,t+2}}{\Lambda_{t+1}}) - \zeta^{-\epsilon}_{2,t+1}(MC_{t+1}(A^{max}_t) Y_{t+1} + \frac{X_{2,t+2}}{\Lambda_{t+1}}), \quad (34)$$

with ζ_1 and ζ_2, two auxilliary variables who through bakward recursion account for the previously set Calvo prices:

$$\zeta^{1-\epsilon}_{1,t+1} = (1 - \xi_p)\left(n_{t-1}(P_{A,t}\pi^p_{t,t+1})^{1-\epsilon} + (1 - n_{t-1})(P_{L,t}\pi^p_{t,t+1})^{1-\epsilon}\right) + \xi_p(\zeta_{1,t}\pi^p_{t,t+1})^{1-\epsilon}, \quad (35)$$

$$\zeta^{-\epsilon}_{2,t+1} = (1 - \xi_p)\left(n_{t-1}(P_{A,t}\pi^p_{t,t+1})^{-\epsilon} + (1 - n_{t-1})(P_{L,t}\pi^p_{t,t+1})^{-\epsilon}\right) + \xi_p(\zeta_{2,t}\pi^p_{t,t+1})^{-\epsilon}. \quad (36)$$

After $t+2$, the only price that can be set by the monopolist is the lagging optimal reset price and given that there is an optimal reset price per period, we will use a double recursion:

$$\Psi_{t+2} = (1-\xi_p)(X_{3,t+2} - X_{4,t+2}) + \beta \frac{\Lambda_{t+3}}{\Lambda_{t+2}}(1-n_{t+1})\Psi_{t+3}, \tag{37}$$

with

$$\begin{aligned} X_{3,t+2} = {} & P_{L,t+1}^{1-\epsilon}(1-n_{t+1})Y_{t+2} \\ & + \xi_p \beta \frac{\Lambda_{t+3}}{\Lambda_{t+2}} \left(\frac{P_{L,t+2}}{P_{L,t+3}}\right)^{1-\epsilon} (1-n_{t+1})(\pi^p_{t+2,t+3})^{1-\epsilon} X_{3,t+3}, \end{aligned} \tag{38}$$

and

$$\begin{aligned} X_{4,t+2} = {} & P_{L,t+1}^{-\epsilon}(1-n_{t+1})MC_{L,t+2}Y_{t+2} \\ & + \xi_p \beta \frac{\Lambda_{t+3}}{\Lambda_{t+2}} \left(\frac{P_{L,t+2}}{P_{L,t+3}}\right)^{-\epsilon} (1-n_{t+1})(\pi^p_{t+2,t+3})^{-\epsilon} X_{4,t+3}. \end{aligned} \tag{39}$$

$X_{3,t+2}$ and $X_{4,t+2}$ are the sum of, respectively, the revenue and the costs, generated by the monoply if the firm is allowed to reoptimize at $t+2$ and does not reoptimize thereafter. The double recursion ensures that Ψ_{t+2} accounts for all the possible reoptimization after $t+3$.

Therefore, the value of the firm to the innovator is:

$$V_{t+1} = (1-\xi_p)\Psi_{A,t+1} + \xi_p \Psi_{I,t+1} + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} \Psi_{t+2} \tag{40}$$

2.3 Aggregation and the monetary authority's policy function

The aggregation of the budget constraint over the continuum of hourseholds yields the following aggregate resource constraint which accounts for the investment in R&D:

$$P_tC_t + P_tI_t + P_ta(u_t)\overline{K_t} + P_tX_t + \frac{B_t}{1+i_t} = P_tY_t + B_{t-1} \tag{41}$$

The monetary authority follows a Taylor rule with a smoothing component, inflation targeting and output growth:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r}\right)^{\rho_r}\left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha_y}\right]^{1-\rho_r}. \tag{42}$$

2.3.1 The specification of monetary policy

The central bank's policy function is modelled as a Taylor-type rule, as it sets the nominal interest rate according to the following equation:

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{Y_{t-1}}g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R}\epsilon_{m,t}. \tag{43}$$

The ρ_i parameter represents the degree of smoothing of interest rate changes, as the monetary authority seeks to avoid too large changes with respect to its one-period-lag value, R_{t-1}, and adjusts it somewhat gradually following demand and technology shocks. The parameters α_π and α_y are the monetary authority's weights attached to deviations from its inflation target, π, and its output growth targets.

2.3.2 The aggregate economy and its general equilibrium

In this model, output, consumption, physical capital, investments in physical capital and investments in research and development fluctuate around a balanced growth path because of the endogenous nature of technological growth. The variables need to be detrended before simulating the model around the steady state. The investment in R&D, X_t is a function of the technological frontier, as can be seen in equation (30), we have taken the party of detrending by the technological frontier. We could also have detrended by the average technology, which we will do, in the following weeks, to ensure that the detrending does not affect our results. This question is irrelevant in the context of an exogenous growth model since the frontier and the average technology are one and the same. The steady state of the detrended model is then computed. The details of the steps can be found in the technical appendix. The model is, then, loglinearly approximated around its steady state.

3 The Calibration of the Parameters and the Characteristics of the Various Shocks

3.1 The standard parameters

Table 1 presents the values used for calibration of key parameters of the model.

The share of capital in the production function α, the discount rate β, the depreciation rate of physical capital δ as well as the monetary policy are set to standard values in the literature. The steady-state gross trend inflation π is set at 1 (i.e. the inflation rate is zero in the steady-state). We also assume full capacity utilization in steady-state, i.e. $u = 1$. Market power in labour and intermediate goods leads to, generally agreed upon, wage and price markups of around 20% which is the benchmark in Christiano et al. (2005). It is equivalent to an elasticity of substitution of 6 between intermediate goods, as well between labour types.

The Calvo parameters ξ_p and ξ_w are consistent with microeconomic data on price and wage adjustments. They are set at 0.5, thus corresponding to the average duration of contracts, for both prices and wages, of 2 quarters. This value is in line with Bils and Klenow (2004) and the priors in Smets and Wouters (2007).

The Frisch elasticity of labour supply ν determines the response of hours worked to changes in wages, holding constant the marginal utility of wealth. Disutility of labour is impacted by the preference parameter θ. Both parameters are set so that steady state worked hours are approximately $L = .3$. The value of Frisch elasticity is set to fall between both macroeconomic estimates (that lies between 2 and 4), and microeconomic estimates (generally under 0.5). In particular, Peterman (2016) shows that the Frisch elasticity is sensitive to estimation methods. For our purposes, we use $\theta = 5$ and $\nu = 1$.

Parameter	Value	Meaning
α	1/3	Share of capital
β	.99	Discount rate
δ	.025	Depreciation rate of physical capital
α_π	2	Taylor rule inflation
α_y	.2	Taylor rule output
ρ_R	.8	Taylor rule smoothing
u	1	Steady state capacity utilization
ϵ	6	Elasticity of substitution of intermediate goods
γ	6	Elasticity of substitution of labor
ξ_p	.5	Calvo parameter of prices
ξ_w	.5	Calvo parameter of wages
θ	5	Weight on the disutility of labour
ν	1	Utility parameter that determines the Frisch elasticity of labour, $(\frac{1}{\nu})$

Table 1: Key parameters

3.2 The parameters pertaining to the Schumpeterian features of the model

Calibrating Schumpeterian models is a relatively new endeavour in economic research. We will base our calibration on the statistical moments of the variables related to research and development and technological advancement.

As shown in Figure 5, from 1960 and 2016, the share of GDP dedicated to R&D investment has varied between 2.3% ans 2.9%, averaging a 2.56%-share of the GDP over the period. We use this sample average as the steady state to reproduce for investment in R&D.

We also target the growth rate of technology. Fernald (2014, 2017)'s estimation of U.S. Total Factor Productivity (TFP) yields an average quarterly growth rate of 1.21% over the same sample. In our model, we look at technology from two different angles. Part of technology can be explained through a transitory technological shock, whose calibration will be discussed in the next section, the rest of it is explained through a Schumpeterian innovation process. When innovations occur, they represent a permanent technological shift, that permanently pushes the frontier forward. In this respect, the frontier growth rate is

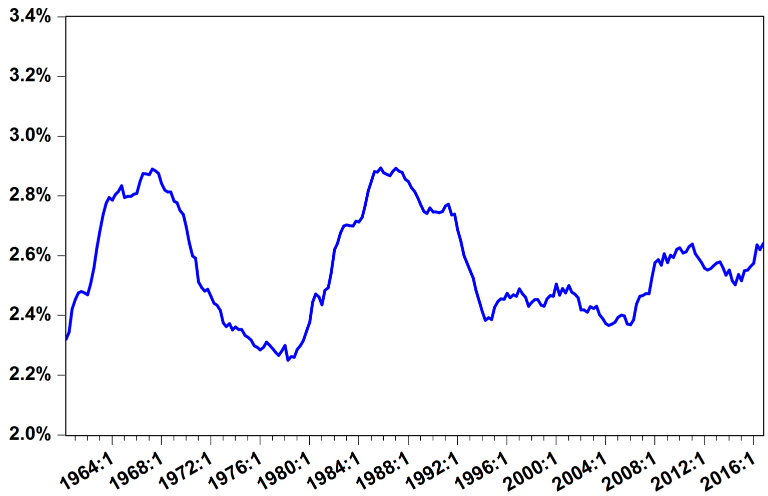

Figure 5: R&D investment-to-GDP ratio in the United States(1960q2-2016q4)

akin to the growth rate of the trend in TFP. We therefore target a steady state growth rate of the frontier of 1.06%, in accordance with the Hodrick-Prescott trend component of U.S. TFP. Assigning values to the innovation probability and the spillover effect at the steady state allows to mimic the above steady-state growth rate.

Given the relationship between the frontier growth rate, the spillover and the innovation probability, there are additional degrees of freedom left as the variation along one dimension can be compensated by a change in another. This problem can be overlooked when analysing steady states, but will deserve further consideraion when discussing volatility and comovements of aggregate variables.

3.3 Persistence and variance of shocks

3.3.1 The technological, monetary and investment specific shocks

Our model includes a common transitory technological shock which temporarily affects the intermediate firms' production function, regardless of their individual level of technological advancement. The shock, by definition, is transitory. Its persistance was set at $\rho_z = .2$ and its variance at $\sigma_z = .005$. An investment specific shock affects the efficiency with which investment is transformed into capital, has a persistence of $\rho_\mu = .6$ and a variance of $\sigma_\mu = .005$. Finally, the monetary policy shock is an innovation to the Taylor rule as specified in the previous section and has a persistence of $\rho_m = .6$ and its variance at $\sigma_m = .005$.

3.3.2 The spillover shock

A knowledge-spillover is the positive externality derived from an innovation,since it permanently pushes the technological frontier forward. We apply an HP filter on Fernald (2014, 2017)'s quarterly utilization-adjusted TFP series to extract the cycle from the trend. We, then, compute the growth rate of the trend as can be shown in Figure 6.

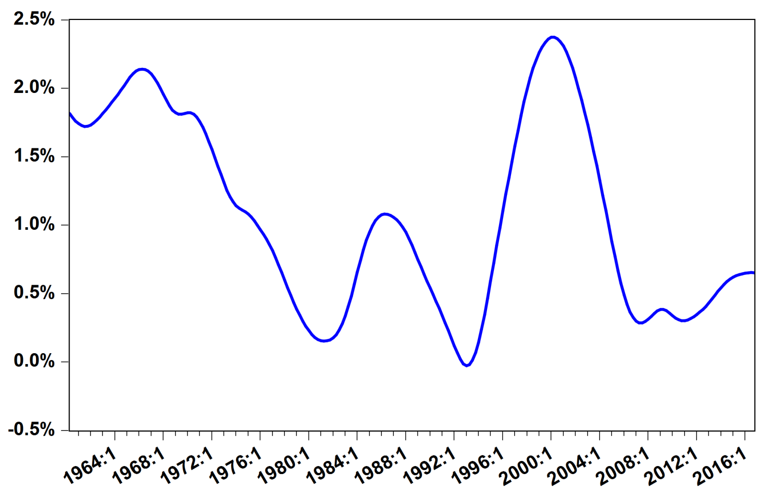

Figure 6: TFP trend growth in the United States (1960q2-2016q4). Source: Fernald (2017)

For our model to exhibit such persistence, it needs to include a somewhat persistent spillover shock. We set the persistence parameter $\rho_\sigma = .9$, with a volatility $\sigma_\sigma = .01$, that allows to match the observed autocorrelation and volatility of TFP.

4 The Business Cycle Analysis

The business cycle analysis revolves around the study of various impulse responses [5] and the corresponding variance decompositions analysis. In both cases, we compare the endogenous growth model to the exogenous growth model. For these preliminary results, both model include sticky prices and wages, trend inflation, investment adjustment cost and variable capacity utilization, but abstract from both price and wage indexation.

4.1 Impulse response functions

As far as monetary, technological and investment shocks are concerned, the impulse response functions are, as expected, very similar with and without endogenous growth. Some slight differences in amplitude can be explained by the additional transmission channel arising from endogenous growth. The monetary policy shock is an increase of one standard deviation of the interest rate. As can be seen in Figure 7, the interest rate increase generates decreases in inflation, output, consumption and investment. The decrease in the rate of return on capital occurs as there is an increase in R&D which comparatively becomes a more attractive alternative. The R&D increase pushes up the frontier growth rate and underlies the deviations relative to the exogenous growth impulse responses.

[5] The impulse responses are that of real, detrended variables. It shall be noted that they are expressed as deviations from the trend so they should, therefore, be analyzed as such. For instance, a decrease of the output would not necessary entail a decrease of the aggregate production, it may rather represent as smaller rise in output when taking the trend into account

Similarly the positive efficiency of investment shock in Figure 8 makes the conversion from investment to physical capital mote efficient. Physical capital, output, consumption and investments increase before returning to their steady states around 10 quarters after the shock. The technological shock's impulse responses, shown in Figure 9 are in line with what is commonly found in the litterature. In both cases, the spread between the exogenous and endogenous growth impulse responses can be traced back to an arbitrage between the rate of return on physical capital and R&D.

As shown in Figure 10, impulse responses for a spillover shock are specific to the endogenous growth model. An exogenous growth framework does not allow for the inclusion of such a shock. An increase in spillover increases the growth rate of the frontier. Consequently, output and inflation increases. Given the specification of monetary policy, interest rates go up because of deviations from both output growth and inflation targets. Moreover, the persistent increase in the interest rate increases the discount rate which depreciates the discounted value of an innovation, which, in turn, drives investment in R&D down.

4.2 Variance decomposition

Table 2 summarizes the variance decomposition which allows us to analyze the relative contribution of each shock to the volatility of selected variables. Two conclusions can be drawn from these results. First, the spillover shock, that it specific to the endogenous growth model, explains a significant share of variability. Indeed, the spillover shock explains, respectively, 15% and 24% of changes in R&D investment and physical capital. The second conclusion is that the monetary policy shock doeshave an impact on R&D investment as it is responsible for 19% of its variance. This is consistent with Comin and Gertler (2006).

Variance decompositions at different horizons are reported in the appendix in tables 5, 6 and 7 confirm the conclusion drawn earlier. They also illustrate the spillover's role in aggregate fluctuations, with the size of its contribution increasing with the horizon. It is due to the inherent longer term nature of R&D and its effects.

	Technology	Monetary	Investment	Spillover
Output	2%	16%	77%	5%
Consumption	2%	11%	80%	7%
Investment	1%	34%	62%	3%
R&D	36%	19%	31%	15%
Physical Capital	0%	40%	35%	24%
Capacity Utilization	19%	7%	72%	2%
Wages	30%	28%	21%	20%
Labor	18%	7%	75%	0%
Rate of return on capital	19%	7%	72%	2%
Interest Rate	12%	14%	70%	4%

Table 2: Variance decomposition at t=15 quarters

4.3 Contemporaneous correlations

In this section, we confront contemporaneous correlations generated by the models with the data.

	$\rho(\hat{Y},\hat{C}))$	$\rho(\hat{Y},\hat{I}))$	$\rho(\hat{C},\hat{I}))$
Data	0.85492	0.75337	0.57955
Exogenous growth	0.39616	0.14439	-0.05816
Endogenous growth	0.8464	0.88807	0.50802

Table 3: Key contemporaneous correlations
Note: $\hat{Y}$, $\hat{C}$ and $\hat{I}$ are percentage deviations from the trend.

Table 3 shows selected correlations from both an exogenous growth New Keynesian model, and an endogenous growth New Keynesian as well as data retrieved from the Federal Reserve Bank of St Louis F.R.E.D. database. We apply a logarithmic transformation to the data before extracting the cyclical component using the Hodrick-Prescott filter. The simulated data being already detrended, we use the logarithmic difference to compute percentage deviations from the trend. Moments in the data are computed for the 1960q1-2016q4 sample.

According to the reseults in Table 3, the endogenous growth model outperforms the exogenous growth model. The correlation between detrended consumption and investment is -0.05816 for the exogenous growth model. This compares to a value of 0.50802 in our endogenous growth model which does significantly better and is much closer to the 0.57955 in the data.

	$\rho(\hat{X}, \hat{I}))$	$\rho(\hat{X}, \hat{C}))$	$\rho(\hat{Y}, \hat{X}))$
Data	0.53291	0.51521	0.57955
Endogenous growth	0.61245	0.97237	0.89619

Table 4: Contemporaneous correlations with respect to R&D
Note: $\hat{Y}$, $\hat{X}$, $\hat{C}$ and $\hat{I}$ are percentage deviations from the trend.

Table 4 shows selected correlations from an endogenous growth New Keynesian(which includes trend inflation, variable capacity utilization and no price or wage indexation) and data from the Federal Reserve Bank of St Louis. There is no comparable data that can be generated from the exogenous growth model since it devoid of R&D. The model does very well when looking at correlations of investment R&D and investment in physical capital. It however overstates the correlations of R&D investment with output and consumption. We offer some conjectures as to why and how this issue can be resolved in the next section that will warrant further verification.

5 Preliminary conclusions

We have developed a rich model of economic fluctuations with New Keynesian nominal rigidities and common shocks that builds in the features of a Schumpeterian endogenous growth model to capture the interactions between an economy's trend growth and its cyclical fluctuations.

The next step in our work is to introduce infinitely-lived patents. Technology which could have been previously rendered obsolete will remain productive. Any shock that had affected past levels of investment in R&D will still, partly, affect aggregate output because a fraction of the intermediate firms remain stuck with an obsolete technology. Further work will also consider a patent lasting some intermediate time period between 1 and an infinite horizon.

The added mechanism of endogenous growth provides a new channel through which shocks can be propagated. In addition, it does not only allow to analyze the effects of well identified shocks, it also enables us to include a spillover shock. So far, the variance decompositions have been done solely based on calibrated shocks. We plan to conduct a Bayesian estimation of the model to inquire further as to the role, size and importance of the entire R&D channel. In the current version model, the probability of innovating is fairly high (with a steady state value of .6). A reduction in the innovation probability compensated by an increase in steady state spillover may well allow a larger fraction of the intermediate firms to use older technology and further increase persistence. The estimation is likely to allow these questions, which are yet to be addressed in the litterature, to be settled.

In future research, we intend to exploit the rich structure of the hybrid model built in this paper to investigate the respective impacts of fiscal and monetary policies over the cycle and economic growth. An interesting research avenue would be to analyze the interaction between R&D and monetary policy. In the case of a spillover shock, the interest rate increases abruptly and remains high because of the joint increase in output and inflation. The detrimental effects of persistently high interest rate, in this case, begs the question of what an optimal monetary policy should be. In particular, we may consider whether the monetary authority should adjust its reaction depending on the nature of the shock that underlies the aggregate fluctuations?

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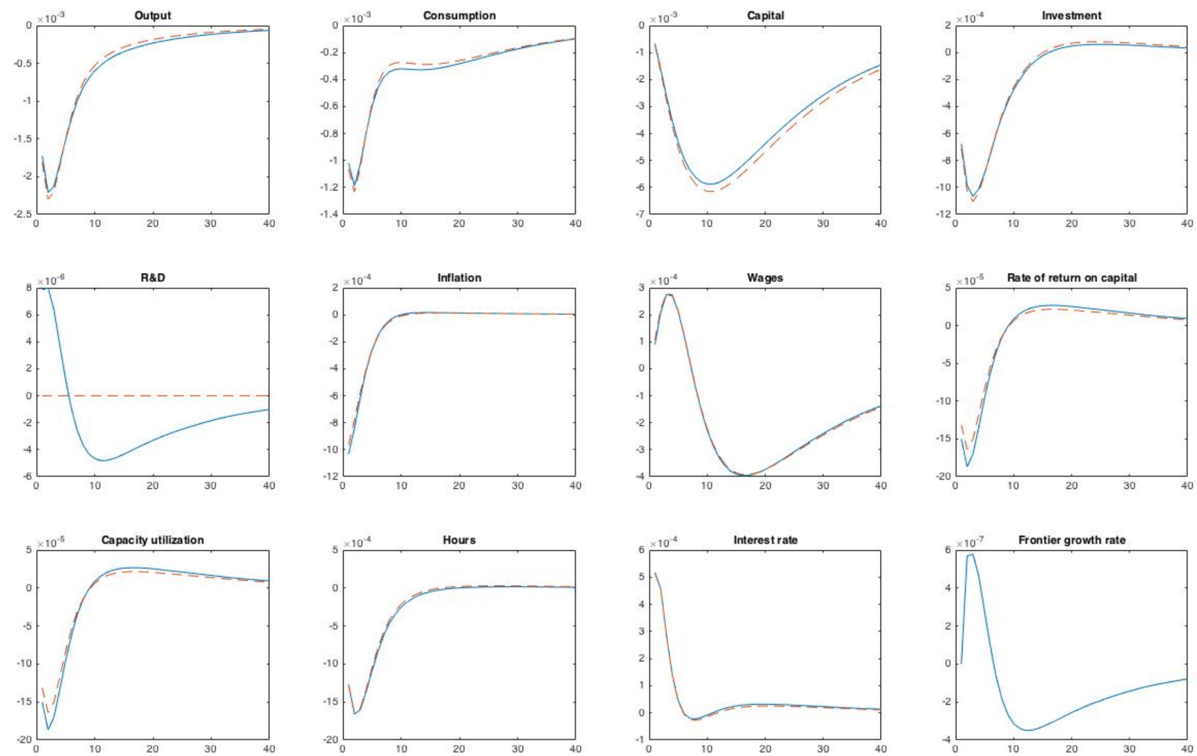

Figure 7: Impulse responses to a one standard deviation monetary policy shock. The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

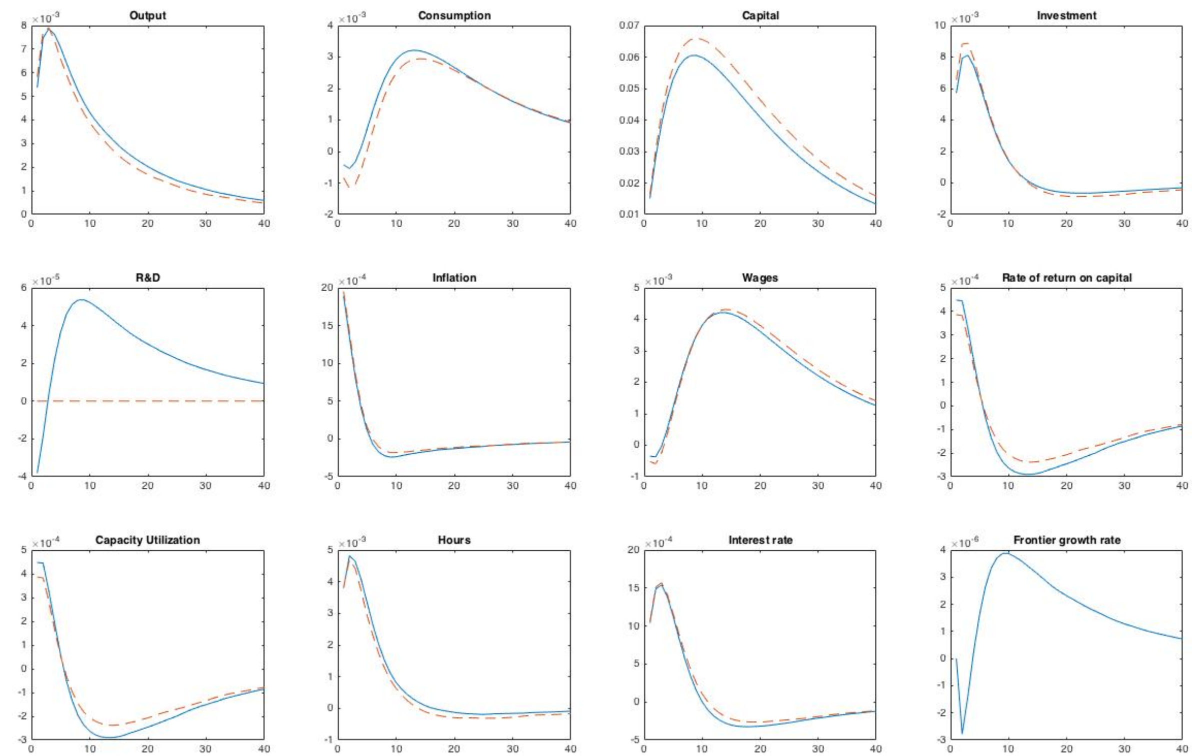

Figure 8: Impulse responses to a one standard deviation investment shock. The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

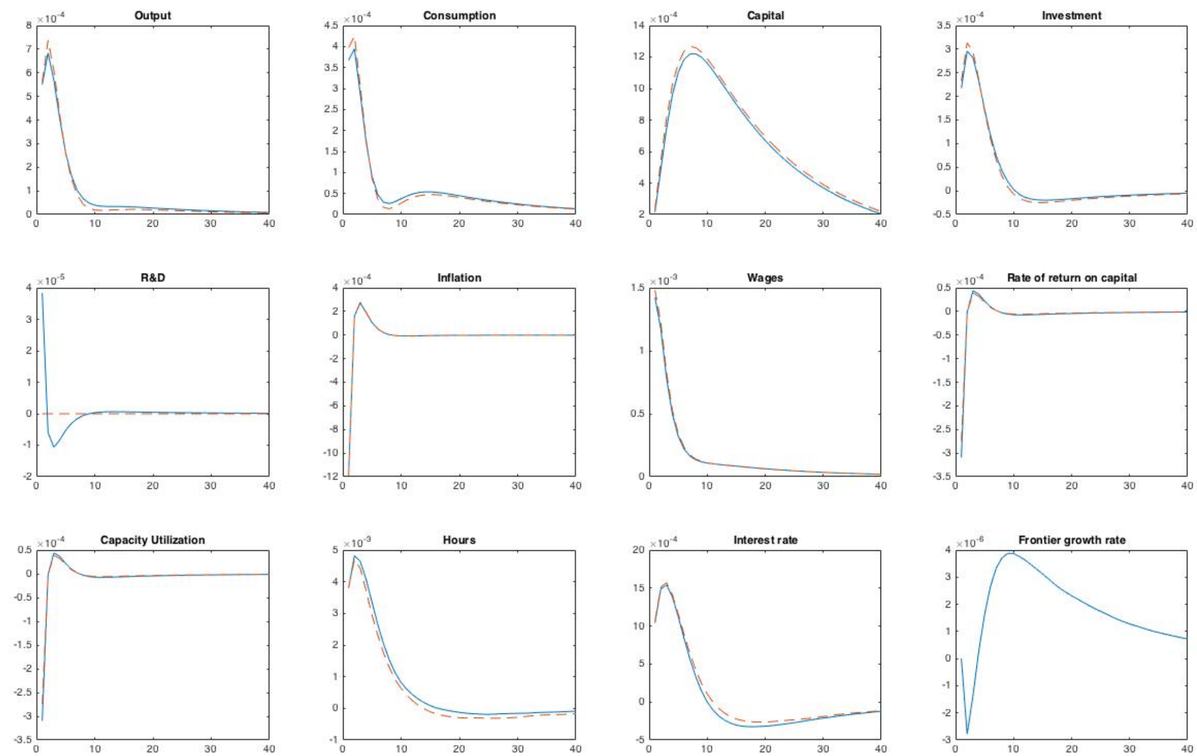

Figure 9: Impulse responses to a one standard deviation technology shock. The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

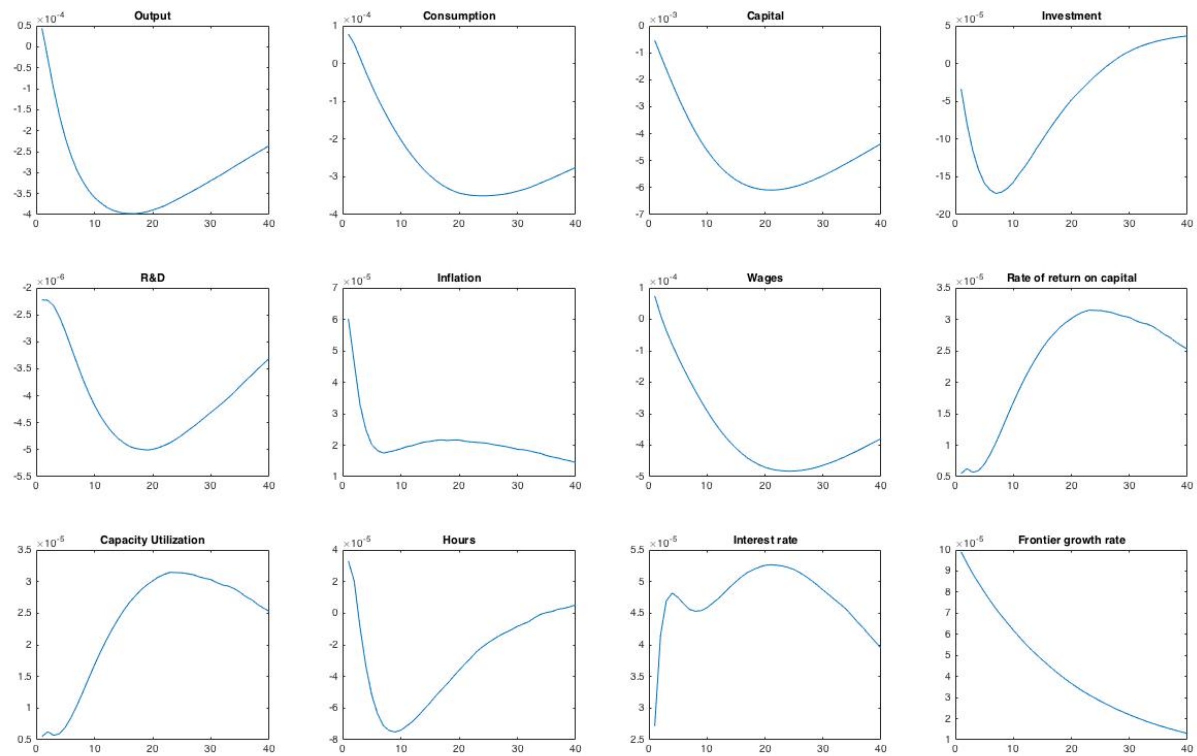

Figure 10: Impulse responses to a one standard deviation spillover shock. The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

	Technology	Monetary	Investment	Spillover
Output	2%	12%	85%	0%
Consumption	3%	0%	97%	0%
Investment	1%	37%	61%	1%
R&D	60%	5%	32%	4%
Physical Capital	1%	56%	31%	12%
Capacity Utilization	21%	4%	75%	0%
Wages	79%	1%	18%	2%
Labor	21%	6%	73%	0%
Rate of return on capital	21%	4%	75%	0%
Interest Rate	13%	12%	74%	1%

Table 5: Variance Decomposition t=5 quarters

	Technology	Monetary	Investment	Spillover
Output	2%	15%	81%	2%
Consumption	2%	5%	90%	3%
Investment	1%	35%	62%	2%
R&D	46%	15%	30%	9%
Physical Capital	1%	46%	36%	17%
Capacity Utilization	20%	5%	75%	1%
Wages	55%	18%	16%	11%
Labor	19%	7%	75%	0%
Rate of return on capital	20%	5%	75%	1%
Interest Rate	13%	13%	71%	2%

Table 6: Variance Decomposition t=10 quarters

	Technology	Monetary	Investment	Spillover
Output	2%	16%	75%	7%
Consumption	2%	14%	72%	12%
Investment	1%	34%	62%	3%
R&D	31%	19%	30%	20%
Physical Capital	0%	36%	34%	30%
Capacity Utilization	18%	8%	70%	4%
Wages	20%	29%	24%	27%
Labor	18%	7%	75%	0%
Rate of return on capital	18%	8%	70%	4%
Interest Rate	12%	14%	69%	5%

Table 7: Variance Decomposition t=20 quarters