The Business Cycle Implications of Endogenous Schumpeterian Growth in a New Keynesian DSGE Model

Technical Appendix

Preliminary. Comments welcome.

May 17, 2017

1 Detrended equilibrium

Marginal cost of the advanced firm:

$$mc_{A,t} = \frac{w_t^{1-\alpha}\rho_t^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} \tag{1}$$

Marginal cost of the lagging firm:

$$mc_{L,t} = g_{t-1}^{1-\alpha} mc_{A,t} \tag{2}$$

Capital Labor ratio:

$$\frac{k_t}{L_t} = \frac{\alpha w_t}{(1-\alpha)\rho_t} \tag{3}$$

Optimal reset price if the firm is lagging:

$$\upsilon_{L,t} = \frac{\epsilon}{\epsilon-1}\frac{\lambda_t mc_{L,t} y_t + \pi_{t+1}^{\epsilon} f_{1,t+1}}{f_{2,t}} \tag{4}$$

Optimal reset price if the firm is advanced:

$$\upsilon_{A,t} = \frac{\epsilon}{\epsilon-1}\frac{\lambda_t mc_{A,t} y_t + \pi_{t+1}^{\epsilon} f_{1,t+1}}{f_{2,t}} \tag{5}$$

Auxilliary variables:

$$f_{1,t+1} = \xi_p \beta \lambda_{t+1} E mc_{t+1} (\pi_{t,t+1}^{p})^{-\epsilon} y_{t+1} + \xi_p \beta (\pi_{t,t+1}^{p})^{-\epsilon} \pi_{t+2}^{\epsilon} f_{1,t+2} \tag{6}$$

$$f_{2,t} = \lambda_t y_t + \xi_p \beta (\pi^p_{t,t+1})^{1-\epsilon} \pi^{\epsilon-1}_{t+1} f_{2,t+1} \tag{7}$$

Optimal investment in R&D:

$$x_t = \beta \frac{n_t}{1+\eta} \frac{\lambda_{t+1}}{\lambda_t} v_{t+1} \tag{8}$$

Innovation probability:

$$n_t = \left(\frac{x_t}{\omega g_t} \right)^{\frac{1}{1+\eta}} \tag{9}$$

Frontier growth rate:

$$g_t = 1 + \sigma_t n_{t-1} \tag{10}$$

Value of the firm to the innovator:

$$v_{t+1} = \xi_p \Psi_{I,t+1} + (1-\xi_p) \Psi_{A,t+1} + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} \Psi_{t+2} \tag{11}$$

Profits at $t+1$ if the intermediate firm indexed in t:

$$\Psi_{t+1}(\upsilon_{I,t+1}) = \zeta^{1-\epsilon}_{1,t+1}(y_{t+1} + \frac{\pi^{\epsilon-1}_{t+2}}{\lambda_{t+1}} x_{1,t+2}) - \zeta^{-\epsilon}_{2,t+1}(mc_{t+1}(A^{max}_t) y_{t+1} + \frac{\pi^{\epsilon}_{t+2}}{\lambda_{t+1}} x_{2,t+2}) \tag{12}$$

$$\zeta^{1-\epsilon}_{1,t+1} = (1-\xi_p) \left(n_{t-1} (\upsilon_{A,t} \pi^p_{t,t+1})^{1-\epsilon} + (1-n_{t-1})(\upsilon_{L,t} \pi^p_{t,t+1})^{1-\epsilon} \right) + \xi_p (\zeta_{1,t} \pi^p_{t,t+1})^{1-\epsilon} \tag{13}$$

$$\zeta^{-\epsilon}_{2,t+1} = (1-\xi_p) \left(n_{t-1} (\upsilon_{A,t} \pi^p_{t,t+1})^{-\epsilon} + (1-n_{t-1})(\upsilon_{L,t} \pi^p_{t,t+1})^{-\epsilon} \right) + \xi_p (\zeta_{2,t} \pi^p_{t,t+1})^{-\epsilon} \tag{14}$$

Profits at $t+1$ if the intermediate firm reoptimized in t:

$$\Psi_{t+1}(\upsilon_{A,t+1}) = \upsilon^{1-\epsilon}_{A,t+1}(y_{t+1} + \frac{\pi^{\epsilon-1}_{t+2}}{\lambda_{t+1}} x_{1,t+2}) - \upsilon^{-\epsilon}_{A,t+1}(mc_{t+1}(A^{max}_t) y_{t+1} + \frac{\pi^{\epsilon}_{t+2}}{\lambda_{t+1}} x_{2,t+2}) \tag{15}$$

Auxilliary variables:

$$x_{1,t+2} = \xi_p \beta \lambda_{t+2} (1-n_{t+1}) (\pi^p_{t+1,t+2})^{1-\epsilon} y_{t+2} + \xi_p \beta (1-n_{t+1}) \pi^{\epsilon-1}_{t+3} (\pi^p_{t+1,t+2})^{1-\epsilon} x_{1,t+3} \tag{16}$$

$$x_{2,t+2} = \xi_p \beta \lambda_{t+2} (1-n_{t+1}) (\pi^p_{t+1,t+2})^{-\epsilon} mc_{t+2}(A^{max}_t) y_{t+2} + \xi_p \beta (1-n_{t+1}) \pi^{\epsilon}_{t+3} (\pi^p_{t+1,t+2})^{-\epsilon} x_{2,t+3} \tag{17}$$

Profits after $t+2$:

$$\Psi_{t+2} = (1-\xi_p)(x_{3,t+2} - x_{4,t+2}) + \beta \frac{\lambda_{t+3}}{\lambda_{t+2}} (1-n_{t+1}) \Psi_{t+3} \tag{18}$$

Auxilliary variables:

$$x_{3,t+2} = \upsilon^{1-\epsilon}_{L,t+1} (1-n_{t+1}) y_{t+2} + \xi_p \beta \frac{\lambda_{t+3}}{\lambda_{t+2}} \left(\frac{\upsilon_{L,t+2}}{\upsilon_{L,t+3}} \right)^{1-\epsilon} \pi^{\epsilon-1}_{t+3} (1-n_{t+1}) (\pi^p_{t+2,t+3})^{1-\epsilon} x_{3,t+2} \tag{19}$$

$$x_{4,t+2} = \upsilon_{L,t+1}^{-\epsilon}(1-n_{t+1})mc_{L,t+2}y_{t+2} + \xi_p\beta\frac{\lambda_{t+3}}{\lambda_{t+2}}\left(\frac{\upsilon_{L,t+2}}{\upsilon_{L,t+3}}\right)^{-\epsilon}\pi_{t+3}^{\epsilon}(1-n_{t+1})(\pi_{t+2,t+3}^{p})^{-\epsilon}x_{4,t+2} \tag{20}$$

Consumption:

$$\frac{1}{c_t - hc_{t-1}g_{t-1}^{-1}} - \lambda_t + \beta\frac{-h}{c_{t+1}g_t - hc_t} = 0 \tag{21}$$

Physical capital:

$$\beta\lambda_{t+1}g_t^{-1}\left(\rho_{t+1}u_{t+1} - a(u_{t+1})\right) + \beta\phi_{t+1}g_t^{-1}\mu_{t+1}(1-\delta) = \phi_t\mu_t \tag{22}$$

Investment:

$$\lambda_t = \phi_t\mu_t(-\frac{i_t}{i_{t-1}}g_{t-1}S'(\frac{i_t}{i_{t-1}}g_{t-1}) + 1 - S(\frac{i_t}{i_{t-1}}g_{t-1})) + \beta\phi_{t+1}\mu_{t+1}g_t^{-1}\left(\frac{i_{t+1}}{i_t}g_t\right)^2 S'(\frac{i_{t+1}}{i_t}g_t) \tag{23}$$

Capital utilization:

$$\rho_t = a'(u_t) \tag{24}$$

Bond holdings:

$$\beta\lambda_{t+1} = \frac{\lambda_t}{1+r_t}g_t\pi_{t+1} \tag{25}$$

Optimal reset wage:

$$w_t^{\#\,-\gamma\nu-1} = \frac{\gamma-1}{\phi\gamma}\frac{f_{3,t}}{f_{4,t}} \tag{26}$$

Auxilliary variables:

$$f_{3,t} = \lambda_t w_t^{\gamma} L_t + \xi_w\beta{\pi_{t,t+1}^{w}}^{1-\gamma}(g_t\pi_{t+1})^{\gamma-1}f_{3,t+1} \tag{27}$$

$$f_{4,t} = w_t^{\gamma(1+\nu)}L_t^{1+\nu} + \xi_w\beta{\pi_{t,t+1}^{w}}^{-\gamma(1+\nu)}(g_t\pi_{t+1})^{\gamma(1+\nu)}f_{4,t+1} \tag{28}$$

Aggregation:

$$y_t = c_t + i_t + a(u_t)\bar{k}_t + x_t \tag{29}$$

$$y_t = (n_{t-1} + (1-n_{t-1})g_{t-1}^{\alpha-1})\frac{k_t^{\alpha}L_t^{1-\alpha}}{\zeta_t} \tag{30}$$

$$\zeta_t = (1-\xi_p)(n_{t-1}\upsilon_{A,t}^{-\epsilon} + (1-n_{t-1})\upsilon_{L,t}^{-\epsilon}) + \xi_p\zeta_{t-1}\pi_t^{\epsilon} \tag{31}$$

$$w_t^{1-\gamma} = (1-\xi_w){w^{\#}}_t^{1-\gamma} + \xi_w\left(\frac{w_{t-1}{\pi^{w}}_{t-1,t}}{g_{t-1}\pi_t}\right)^{1-\gamma} \tag{32}$$

$$1 = (1-\xi_p)(n_{t-1}\upsilon_{A,t}^{1-\epsilon} + (1-n_{t-1})\upsilon_{L,t}^{1-\epsilon}) + \xi_p \left(\frac{\pi_{t-1,t}^p}{\pi_t}\right)^{1-\epsilon} \tag{33}$$

Inflation:

$$\pi_{t,t+s} = \prod_{i=1}^{s} \pi_{t+i} \tag{34}$$

Wage indexation:

$$\pi_{t,t+s}^w = \prod_{i=1}^{s} \pi_{t+i-1}^{\iota_w} \pi^{1-\iota_w} \tag{35}$$

Price indexation:

$$\pi_{t,t+s}^p = \prod_{i=1}^{s} \pi_{t+i-1}^{\iota_p} \pi^{1-\iota_p} \tag{36}$$

2 Steady State

Capacity utilization FOC yields the following real rate of return on capital:

$$\boldsymbol{\rho = a'(1)} \tag{37}$$

The FOC on investment gives us the following relation between lagrangian multipliers:

$$\boldsymbol{\lambda = \phi} \tag{38}$$

I substitute equations 37 and 38 in the physical capital FOC:

$$\beta\lambda g^{-1}\rho + \beta\phi g^{-1}(1-\delta) = \phi \tag{39}$$

$$\beta\lambda g^{-1}\rho + \beta\lambda g^{-1}(1-\delta) = \lambda \tag{40}$$

$$\beta\rho + \beta(1-\delta) = g \tag{41}$$

$$\boldsymbol{g = \beta(\rho + 1 - \delta)} \tag{42}$$

The steady state interest rate is given by the bonds FOC:

$$\beta\lambda = \frac{\lambda}{1+r} g\pi \tag{43}$$

$$\beta = \frac{1}{1+r} g\pi \tag{44}$$

$$1 + r = \frac{g\pi}{\beta} \tag{45}$$

$$\boldsymbol{r = \frac{g\pi}{\beta} - 1} \tag{46}$$

I, then, use the wage index to write the optimal reset wage as a function of the aggregate wage:

$$w^{1-\gamma} = (1-\xi_w){w^{\#}}^{1-\gamma} + \xi_w \left(\frac{w\pi}{g\pi}\right)^{1-\gamma} \tag{47}$$

$$w^{1-\gamma} = (1-\xi_w){w^{\#}}^{1-\gamma} + \xi_w \left(\frac{w}{g}\right)^{1-\gamma} \tag{48}$$

$$w^{1-\gamma} = (1-\xi_w){w^{\#}}^{1-\gamma} + \xi_w g^{\gamma-1} w^{1-\gamma} \tag{49}$$

$$w^{1-\gamma} - \xi_w g^{\gamma-1} w^{1-\gamma} = (1-\xi_w){w^{\#}}^{1-\gamma} \tag{50}$$

$$(1-\xi_w g^{\gamma-1}) w^{1-\gamma} = (1-\xi_w){w^{\#}}^{1-\gamma} \tag{51}$$

$$w^{1-\gamma} = \frac{1-\xi_w}{1-\xi_w g^{\gamma-1}} {w^{\#}}^{1-\gamma} \tag{52}$$

$$w^{1-\gamma} = \frac{1-\xi_w}{1-\xi_w g^{\gamma-1}} {w^{\#}}^{1-\gamma} \tag{53}$$

$$\boldsymbol{w = \left(\frac{1-\xi_w}{1-\xi_w g^{\gamma-1}}\right)^{\frac{1}{1-\gamma}} w^{\#}} \tag{54}$$

The optimal reset wage can be rewritten as function of labor, aggregate wages and λ:

$${w^{\#}}^{-\gamma\nu-1} = \frac{\gamma-1}{\phi\gamma}\frac{f_3}{f_4} \tag{55}$$

$$f_3 = \lambda w^{\gamma} L + \xi_w \beta \pi^{1-\gamma} (g\pi)^{\gamma-1} f_3 \tag{56}$$

$$f_3 - \xi_w \beta g^{\gamma-1} f_3 = \lambda w^{\gamma} L \tag{57}$$

$$(1-\xi_w \beta g^{\gamma-1}) f_3 = \lambda w^{\gamma} L \tag{58}$$

$$\boldsymbol{f_3 = \frac{\lambda w^{\gamma} L}{1-\xi_w \beta g^{\gamma-1}}} \tag{59}$$

$$f_4 = w^{\gamma(1+\nu)} L^{1+\nu} + \xi_w \beta \pi^{-\gamma(1+\nu)} (g\pi)^{\gamma(1+\nu)} f_4 \tag{60}$$

$$(1-\xi_w \beta g^{\gamma(1+\nu)}) f_4 = w^{\gamma(1+\nu)} L^{1+\nu} \tag{61}$$

$$\boldsymbol{f_4 = \frac{w^{\gamma(1+\nu)}L^{1+\nu}}{1-\xi_w\beta g^{\gamma(1+\nu)}}} \tag{62}$$

I subsitute $f3$ and $f4$ in the optimal reset wage

$$w^{\#^{-\gamma\nu-1}} = \frac{\gamma-1}{\phi\gamma}\frac{\lambda w^{\gamma}L}{1-\xi_w\beta g^{\gamma-1}}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{w^{\gamma(1+\nu)}L^{1+\nu}} \tag{63}$$

$$w^{\#^{-\gamma\nu-1}} = \frac{\gamma-1}{\phi\gamma}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}\frac{\lambda w^{\gamma}L}{w^{\gamma(1+\nu)}L^{1+\nu}} \tag{64}$$

$$\boldsymbol{w^{\#^{-\gamma\nu-1}} = \frac{\gamma-1}{\phi\gamma}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}\lambda w^{-\gamma\nu}L^{-\nu}} \tag{65}$$

I, now, use the price index to derive a relationship between steady state optimal reset prices:

$$1 = (1-\xi_p)(n\upsilon_A^{1-\epsilon} + (1-n)\upsilon_L^{1-\epsilon}) + \xi_p \tag{66}$$

$$1-\xi_p = (1-\xi_p)(n\upsilon_A^{1-\epsilon} + (1-n)\upsilon_L^{1-\epsilon}) \tag{67}$$

$$\boldsymbol{1 - n\upsilon_A^{1-\epsilon} = (1-n)\upsilon_L^{1-\epsilon}} \tag{68}$$

Optimal reset prices can be rewritten as function of marginal costs, which are themselves functions of wages, parameters and known steady state variables:

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}\frac{\lambda mc_L y + \pi^{\epsilon}f_1}{f_2} \tag{69}$$

$$\upsilon_A = \frac{\epsilon}{\epsilon-1}\frac{\lambda mc_A y + \pi^{\epsilon}f_1}{f_2} \tag{70}$$

$$mc_L = g^{1-\alpha}mc_A \tag{71}$$

$$f_1 = \xi_p\beta\lambda Emc\pi^{-\epsilon}y + \xi_p\beta(\pi)^{-\epsilon}\pi^{\epsilon}f_1 \tag{72}$$

$$f_1 - \xi_p\beta f_1 = \xi_p\beta\lambda Emc\pi^{-\epsilon}y \tag{73}$$

$$(1-\xi_p\beta)f_1 = \xi_p\beta\lambda Emc\pi^{-\epsilon}y \tag{74}$$

$$f_1 = \frac{\xi_p\beta\lambda Emc\pi^{-\epsilon}y}{1-\xi_p\beta} \tag{75}$$

$$f_2 = \lambda y + \xi_p\beta f_2 \tag{76}$$

$$(1-\xi_p\beta)f_2 = \lambda y \tag{77}$$

$$f_2 = \frac{\lambda y}{1-\xi_p\beta} \tag{78}$$

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}\frac{\lambda mc_L y + \pi^{\epsilon}\frac{\xi_p\beta\lambda Emc\pi^{-\epsilon}y}{1-\xi_p\beta}}{\frac{\lambda y}{1-\xi_p\beta}} \tag{79}$$

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}\frac{(1-\xi_p\beta)\lambda mc_L y + \xi_p\beta\lambda Emcy}{\lambda y} \tag{80}$$

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)mc_L + \xi_p\beta Emc) \tag{81}$$

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}mc_A + \xi_p\beta Emc) \tag{82}$$

The expected marginal cost can be rewritten as follows:

$$Emc = nmc_A + (1-n)mc_L \tag{83}$$

$$Emc = nmc_A + (1-n)g^{1-\alpha}mc_A \tag{84}$$

$$Emc = mc_A(n + (1-n)g^{1-\alpha}) \tag{85}$$

I substitute it in the steady state optimal reset price for lagging firms:

$$\upsilon_L = \frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}mc_A + \xi_p\beta mc_A(n+(1-n)g^{1-\alpha})) \tag{86}$$

$$\boldsymbol{\upsilon_L = \frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha} + \xi_p\beta(n+(1-n)g^{1-\alpha}))mc_A} \tag{87}$$

Similarly the optimal reset price for advanced firms can be written as:

$$\boldsymbol{\upsilon_A = \frac{\epsilon}{\epsilon-1}(1-\xi_p\beta + \xi_p\beta(n+(1-n)g^{1-\alpha}))mc_A} \tag{88}$$

I substitute equations 87 and 88 into equation 68

$$1 - n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))mc_A\right)^{1-\epsilon} = (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))mc_A\right)^{1-\epsilon} \tag{89}$$

$$1 - n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon} mc_A^{1-\epsilon}$$
$$= (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon} mc_A^{1-\epsilon} \quad (90)$$

$$mc_A^{\epsilon-1} - n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$= (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon} \quad (91)$$

$$mc_A^{\epsilon-1} = n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+ (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon} \quad (92)$$

$$mc_A = \Bigg[n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+ (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}\Bigg]^{\frac{1}{\epsilon-1}} \quad (93)$$

I know that the marginal cost of advanced firms is:

$$\boldsymbol{mc_A = \frac{w^{1-\alpha}\rho^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}} \quad (94)$$

The marginal cost of advanced firms is substituted in 94

$$\frac{w^{1-\alpha}\rho^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} = \Bigg[n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+ (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}\Bigg]^{\frac{1}{\epsilon-1}} \quad (95)$$

$$w^{1-\alpha} = \rho^{-\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha}\Bigg[n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+ (1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}\Bigg]^{\frac{1}{\epsilon-1}} \quad (96)$$

$$w^{1-\alpha} = \rho^{-\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha}\Bigg[n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+(1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}\Bigg]^{\frac{1}{\epsilon-1}} \tag{97}$$

I now have the aggregate wage as a function of known steady state variables ans parameters:

$$w = \Bigg\{\rho^{-\alpha}(1-\alpha)^{1-\alpha}\alpha^{\alpha}\Bigg[n\left(\frac{\epsilon}{\epsilon-1}(1-\xi_p\beta+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}$$
$$+(1-n)\left(\frac{\epsilon}{\epsilon-1}((1-\xi_p\beta)g^{1-\alpha}+\xi_p\beta(n+(1-n)g^{1-\alpha}))\right)^{1-\epsilon}\Bigg]^{\frac{1}{\epsilon-1}}\Bigg\}^{\frac{1}{1-\alpha}} \tag{98}$$

Equation 98 allows me to obtain the optimal reset wage as well as the optimal reset prices for both lagging and advanced firms. Thanks to equation 65, I can derive a relationship between the aggregate labor and the lagrange multiplier of the households problem:

$$\lambda^{-1} = \frac{\gamma-1}{\phi\gamma}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}w^{-\gamma\nu}w^{\#\gamma\nu+1}L^{-\nu} \tag{99}$$

The first order condition of the household with respect to consumption combined with equation 99 allows me to write steady state consumption as a function of labor:

$$\frac{1}{c-hcg^{-1}} - \lambda + \beta\frac{-h}{cg-hc} = 0 \tag{100}$$

$$\frac{1}{c-hcg^{-1}} + \beta\frac{-h}{cg-hc} = \lambda \tag{101}$$

$$\frac{g}{gc-hc} + \beta\frac{-h}{cg-hc} = \lambda \tag{102}$$

$$\frac{g-\beta h}{c(g-h)} = \lambda \tag{103}$$

$$c = \frac{1}{\lambda}\frac{g-\beta h}{(g-h)} \tag{104}$$

$$\boldsymbol{c = \frac{\gamma-1}{\phi\gamma}\frac{g-\beta h}{(g-h)}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}w^{-\gamma\nu}w^{\#\gamma\nu+1}L^{-\nu}} \tag{105}$$

Investment can be written as function of labor as well:

$$kg = i + (1-\delta)k \tag{106}$$

$$i = kg - (1-\delta)k \tag{107}$$

$$i = (g - 1 + \delta)k \tag{108}$$

The labor capital ratio is constant for all firms:

$$\frac{k}{L} = \frac{\alpha w}{(1-\alpha)\rho} \tag{109}$$

Investment can be rewritten as a function of it and leave steday state labor as the only remaining unknown:

$$\boldsymbol{i = (g - 1 + \delta)\frac{\alpha w}{(1-\alpha)\rho}L} \tag{110}$$

The value of the firm to the innovator is the last remaining block needed to derive steady state labor. Therefore, it must be written as a function of labor. I will begin by deriving all 6 steady steady state variables including 2 price indices.

$$\zeta_1^{1-\epsilon} = (1-\xi_p)\left(n(\upsilon_A\pi)^{1-\epsilon} + (1-n)(\upsilon_L\pi)^{1-\epsilon}\right) + \xi_p(\zeta_1\pi)^{1-\epsilon} \tag{111}$$

$$\zeta_1^{1-\epsilon} - \xi_p(\zeta_1\pi)^{1-\epsilon} = (1-\xi_p)\left(n(\upsilon_A\pi)^{1-\epsilon} + (1-n)(\upsilon_L\pi)^{1-\epsilon}\right) \tag{112}$$

$$\zeta_1^{1-\epsilon}(1-\xi_p\pi^{1-\epsilon}) = (1-\xi_p)\left(n(\upsilon_A\pi)^{1-\epsilon} + (1-n)(\upsilon_L\pi)^{1-\epsilon}\right) \tag{113}$$

$$\boldsymbol{\zeta_1^{1-\epsilon} = \frac{(1-\xi_p)\left(n(\upsilon_A\pi)^{1-\epsilon} + (1-n)(\upsilon_L\pi)^{1-\epsilon}\right)}{1-\xi_p\pi^{1-\epsilon}}} \tag{114}$$

$$\zeta_2^{1-\epsilon} = (1-\xi_p)\left(n(\upsilon_{A,t}\pi)^{-\epsilon} + (1-n)(\upsilon_L\pi)^{-\epsilon}\right) + \xi_p(\zeta_2\pi)^{-\epsilon} \tag{115}$$

$$\boldsymbol{\zeta_2^{-\epsilon} = \frac{(1-\xi_p)\left(n(\upsilon_A\pi)^{-\epsilon} + (1-n)(\upsilon_L\pi)^{-\epsilon}\right)}{1-\xi_p\pi^{-\epsilon}}} \tag{116}$$

$$x_1 = \xi_p\beta\lambda(1-n)(\pi)^{1-\epsilon}y + \xi_p\beta(1-n)\pi^{\epsilon-1}(\pi)^{1-\epsilon}x_1 \tag{117}$$

$$x_1 - \xi_p\beta(1-n)x_1 = \xi_p\beta\lambda(1-n)\pi^{1-\epsilon}y \tag{118}$$

$$(1-\xi_p\beta(1-n))x_1 = \xi_p\beta\lambda(1-n)\pi^{1-\epsilon}y \tag{119}$$

$$x_1 = \frac{\xi_p\beta\lambda(1-n)\pi^{1-\epsilon}}{1-\xi_p\beta(1-n)}y \tag{120}$$

$$x_2 = \xi_p\beta\lambda(1-n)\pi^{-\epsilon}mc_Ly + \xi_p\beta(1-n)x_2 \tag{121}$$

$$x_2 - \xi_p\beta(1-n)x_2 = \xi_p\beta\lambda(1-n)\pi^{-\epsilon}mc_Ly \tag{122}$$

$$(1-\xi_p\beta(1-n))x_2 = \xi_p\beta\lambda(1-n)\pi^{-\epsilon}mc_Ly \tag{123}$$

$$x_2 = \frac{\xi_p \beta \lambda (1-n) \pi^{-\epsilon} mc_L}{1 - \xi_p \beta (1-n)} y \tag{124}$$

$$x_3 = \upsilon_L^{1-\epsilon}(1-n)y + \xi_p \beta \frac{\lambda}{\lambda} \left(\frac{\upsilon_L}{\upsilon_L} \right)^{1-\epsilon} \pi^{\epsilon-1} (1 - n_{t+1}) (\pi)^{1-\epsilon} x_3 \tag{125}$$

$$x_3 = \upsilon_L^{1-\epsilon}(1-n)y + \xi_p \beta (1-n) x_3 \tag{126}$$

$$x_3 - \xi_p \beta (1-n) x_3 = \upsilon_L^{1-\epsilon}(1-n)y \tag{127}$$

$$(1 - \xi_p \beta (1-n)) x_3 = \upsilon_L^{1-\epsilon}(1-n)y \tag{128}$$

$$x_3 = \frac{\upsilon_L^{1-\epsilon}(1-n)}{1 - \xi_p \beta (1-n)} y \tag{129}$$

$$x_4 = \upsilon_L^{-\epsilon}(1-n) mc_L y + \xi_p \beta \frac{\lambda}{\lambda} \left(\frac{\upsilon_L}{\upsilon_L} \right)^{-\epsilon} \pi^{\epsilon} (1-n) (\pi)^{-\epsilon} x_4 \tag{130}$$

$$x_4 = \upsilon_L^{-\epsilon}(1-n) mc_L y + \xi_p \beta (1-n) x_4 \tag{131}$$

$$x_4 - \xi_p \beta (1-n) x_4 = \upsilon_L^{-\epsilon}(1-n) mc_L y \tag{132}$$

$$(1 - \xi_p \beta (1-n)) x_4 = \upsilon_L^{-\epsilon}(1-n) mc_L y \tag{133}$$

$$x_4 = \frac{\upsilon_L^{-\epsilon}(1-n) mc_L}{1 - \xi_p \beta (1-n)} y \tag{134}$$

$$\zeta = (1 - \xi_p)(n \upsilon_A^{-\epsilon} + (1-n) \upsilon_L^{-\epsilon}) + \xi_p \zeta \pi^{\epsilon} \tag{135}$$

$$\zeta - \xi_p \zeta \pi^{\epsilon} = (1 - \xi_p)(n \upsilon_A^{-\epsilon} + (1-n) \upsilon_L^{-\epsilon}) \tag{136}$$

$$(1 - \xi_p \pi^{\epsilon}) \zeta = (1 - \xi_p)(n \upsilon_A^{-\epsilon} + (1-n) \upsilon_L^{-\epsilon}) \tag{137}$$

$$\zeta = \frac{1 - \xi_p}{1 - \xi_p \pi^{\epsilon}} (n \upsilon_A^{-\epsilon} + (1-n) \upsilon_L^{-\epsilon}) \tag{138}$$

Steady state profits if the innovator is not allowed to reoptimize:

$$\Psi_I = \zeta_1^{1-\epsilon} (y + \frac{\pi^{\epsilon-1}}{\lambda} x_1) - \zeta_2^{-\epsilon} (mc_A y + \frac{\pi^{\epsilon}}{\lambda} x_2) \tag{139}$$

$$\Psi_I = \zeta_1^{1-\epsilon} (y + \frac{\pi^{\epsilon-1}}{\lambda} \frac{\xi_p \beta \lambda (1-n) \pi^{1-\epsilon}}{1 - \xi_p \beta (1-n)} y) - \zeta_2^{-\epsilon} (mc_A y + \frac{\pi^{\epsilon}}{\lambda} \frac{\xi_p \beta \lambda (1-n) \pi^{-\epsilon} mc_L}{1 - \xi_p \beta (1-n)} y) \tag{140}$$

$$\Psi_I = \zeta_1^{1-\epsilon}(y + \frac{\xi_p\beta(1-n)}{1-\xi_p\beta(1-n)}y) - \zeta_2^{-\epsilon}(mc_A y + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)}y) \tag{141}$$

$$\Psi_I = \zeta_1^{1-\epsilon}(1 + \frac{\xi_p\beta(1-n)}{1-\xi_p\beta(1-n)})y - \zeta_2^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})y \tag{142}$$

$$\Psi_I = \zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)})y - \zeta_2^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})y \tag{143}$$

$$\Psi_I = \left\{\zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \zeta_2^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right\}y \tag{144}$$

Steady state profits if the innovator is allowed to reoptimize, the profits can be similarly written:

$$\Psi_A = \upsilon_A^{1-\epsilon}(y + \frac{\pi^{\epsilon-1}}{\lambda}x_1) - \upsilon_A^{-\epsilon}(mc_A y + \frac{\pi^{\epsilon}}{\lambda}x_2) \tag{145}$$

$$\Psi_A = \left\{\upsilon_A^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \upsilon_A^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right\}y \tag{146}$$

The remaining steady state profits can be written as a double recursion:

$$\Psi_2 = (1-\xi_p)(x_3 - x_4) + \beta\frac{\lambda}{\lambda}(1-n)\Psi_2 \tag{147}$$

$$\Psi_2 = (1-\xi_p)(x_3 - x_4) + \beta(1-n)\Psi_2 \tag{148}$$

$$\Psi_2 - \beta(1-n)\Psi_2 = (1-\xi_p)(x_3 - x_4) \tag{149}$$

$$(1-\beta(1-n))\Psi_2 = (1-\xi_p)(x_3 - x_4) \tag{150}$$

$$\Psi_2 = \frac{1-\xi_p}{1-\beta(1-n)}(x_3 - x_4) \tag{151}$$

$$\Psi_2 = \frac{1-\xi_p}{1-\beta(1-n)}\left(\frac{\upsilon_L^{1-\epsilon}(1-n)}{1-\xi_p\beta(1-n)}y - \frac{\upsilon_L^{-\epsilon}(1-n)mc_L}{1-\xi_p\beta(1-n)}y\right) \tag{152}$$

$$\Psi_2 = \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)}\left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon}mc_L}{1-\xi_p\beta(1-n)}\right)y \tag{153}$$

Therefore, the steady state value of the firm to the innovator is:

$$v = \xi_p\Psi_I + (1-\xi_p)\Psi_A + \beta\frac{\lambda}{\lambda}\Psi_2 \tag{154}$$

$$
\begin{aligned}
v = \xi_p \Bigg\{ \zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \zeta_2^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)}) \Bigg\} y \\
+ (1-\xi_p) \Bigg\{ \upsilon_A^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \upsilon_A^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)}) \Bigg\} y \\
+ \beta \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)} \left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon} mc_L}{1-\xi_p\beta(1-n)} \right) y
\end{aligned}
\tag{155}
$$

$$
\begin{aligned}
v = \Bigg\{ \xi_p \left[\zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \zeta_2^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)}) \right] \\
+ (1-\xi_p) \left[\upsilon_A^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)}) - \upsilon_A^{-\epsilon}(mc_A + \frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)}) \right] \\
+ \beta \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)} \left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon} mc_L}{1-\xi_p\beta(1-n)} \right) \Bigg\} y
\end{aligned}
\tag{156}
$$

The steady state aggregate output supply can be written as a follows:

$$
y = (n + (1-n)g^{\alpha-1}) \frac{k^\alpha L^{1-\alpha}}{\zeta} \tag{157}
$$

$$
y = \frac{n + (1-n)g^{\alpha-1}}{\zeta} \left(\frac{k}{L} \right)^\alpha L \tag{158}
$$

$$
y = \frac{n + (1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^\epsilon}(n\upsilon_A^{-\epsilon} + (1-n)\upsilon_L^{-\epsilon})} \left(\frac{k}{L} \right)^\alpha L \tag{159}
$$

$$
y = \frac{n + (1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^\epsilon}(n\upsilon_A^{-\epsilon} + (1-n)\upsilon_L^{-\epsilon})} \left(\frac{\alpha w}{(1-\alpha)\rho} \right)^\alpha L \tag{160}
$$

I can now write the elements of the steady state aggregate demand as function of labor. Consumption:

$$
\boldsymbol{c = \frac{\gamma - 1}{\phi\gamma} \frac{g - \beta h}{(g-h)} \frac{1 - \xi_w \beta g^{\gamma(1+\nu)}}{1 - \xi_w \beta g^{\gamma-1}} w^{-\gamma\nu} w^{\#\,\gamma\nu+1} L^{-\nu}} \tag{161}
$$

Investment:

$$
\boldsymbol{i = (g - 1 + \delta) \frac{\alpha w}{(1-\alpha)\rho} L} \tag{162}
$$

Investment in $R\&D$:

$$
x = \frac{\beta}{1+\eta} n v \tag{163}
$$

$$x = \frac{\beta}{1+\eta} n \Bigg\{ \xi_p \left[\zeta_1^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \zeta_2^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + (1-\xi_p) \left[\upsilon_A^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \upsilon_A^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + \beta \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)} \left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon} mc_L}{1-\xi_p \beta (1-n)} \right) \Bigg\} y \quad (164)$$

$$\boldsymbol{x = \frac{\beta}{1+\eta} n \Bigg\{ \xi_p \left[\zeta_1^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \zeta_2^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + (1-\xi_p) \left[\upsilon_A^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \upsilon_A^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + \beta \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)} \left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon} mc_L}{1-\xi_p \beta (1-n)} \right) \Bigg\} \frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p \pi^{\epsilon}} (n\upsilon_A^{-\epsilon} + (1-n)\upsilon_L^{-\epsilon})} \left(\frac{\alpha w}{(1-\alpha)\rho} \right)^{\alpha} L} \quad (165)$$

To find the steady state hours worked, I must solve for L when equating aggregate demand and supply:

$$\frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p \pi^{\epsilon}} (n\upsilon_A^{-\epsilon} + (1-n)\upsilon_L^{-\epsilon})} \left(\frac{\alpha w}{(1-\alpha)\rho} \right)^{\alpha} L = \frac{\gamma-1}{\phi\gamma} \frac{g-\beta h}{(g-h)} \frac{1-\xi_w \beta g^{\gamma(1+\nu)}}{1-\xi_w \beta g^{\gamma-1}} w^{-\gamma\nu} w^{\#\gamma\nu+1} L^{-\nu} + (g-1+\delta) \frac{\alpha w}{(1-\alpha)\rho} L + \frac{\beta}{1+\eta} n \Bigg\{ \xi_p \left[\zeta_1^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \zeta_2^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + (1-\xi_p) \left[\upsilon_A^{1-\epsilon} \left(\frac{1}{1-\xi_p \beta (1-n)} \right) - \upsilon_A^{-\epsilon} \left(mc_A + \frac{\xi_p \beta (1-n) mc_L}{1-\xi_p \beta (1-n)} \right) \right] + \beta \frac{(1-\xi_p)(1-n)}{1-\beta(1-n)} \left(\frac{\upsilon_L^{1-\epsilon} - \upsilon_L^{-\epsilon} mc_L}{1-\xi_p \beta (1-n)} \right) \Bigg\} \frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p \pi^{\epsilon}} (n\upsilon_A^{-\epsilon} + (1-n)\upsilon_L^{-\epsilon})} \left(\frac{\alpha w}{(1-\alpha)\rho} \right)^{\alpha} \quad (166)$$

$$
\begin{aligned}
&\frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^{\epsilon}}(n\upsilon_A^{-\epsilon}+(1-n)\upsilon_L^{-\epsilon})}\left(\frac{\alpha w}{(1-\alpha)\rho}\right)^{\alpha} \\
&\qquad = \frac{\gamma-1}{\phi\gamma}\frac{g-\beta h}{(g-h)}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}w^{-\gamma\nu}w^{\#\gamma\nu+1}L^{-\nu-1} \\
&+(g-1+\delta)\frac{\alpha w}{(1-\alpha)\rho}+\frac{\beta}{1+\eta}n\Bigg\{\xi_p\left[\zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)})-\zeta_2^{-\epsilon}(mc_A+\frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right] \\
&\qquad +(1-\xi_p)\left[\upsilon_A^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)})-\upsilon_A^{-\epsilon}(mc_A+\frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right] \\
&\qquad +\beta\frac{(1-\xi_p)(1-n)}{1-\beta(1-n)}\left(\frac{\upsilon_L^{1-\epsilon}-\upsilon_L^{-\epsilon}mc_L}{1-\xi_p\beta(1-n)}\right)\Bigg\} \\
&\qquad \frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^{\epsilon}}(n\upsilon_A^{-\epsilon}+(1-n)\upsilon_L^{-\epsilon})}\left(\frac{\alpha w}{(1-\alpha)\rho}\right)^{\alpha}
\end{aligned} \tag{167}
$$

$$
\begin{aligned}
L^{-\nu-1} = &\Bigg\{\frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^{\epsilon}}(n\upsilon_A^{-\epsilon}+(1-n)\upsilon_L^{-\epsilon})}\left(\frac{\alpha w}{(1-\alpha)\rho}\right)^{\alpha} \\
&(g-1+\delta)\frac{\alpha w}{(1-\alpha)\rho}-\frac{\beta}{1+\eta}n\Bigg\{\xi_p\left[\zeta_1^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)})-\zeta_2^{-\epsilon}(mc_A+\frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right] \\
&\qquad +(1-\xi_p)\left[\upsilon_A^{1-\epsilon}(\frac{1}{1-\xi_p\beta(1-n)})-\upsilon_A^{-\epsilon}(mc_A+\frac{\xi_p\beta(1-n)mc_L}{1-\xi_p\beta(1-n)})\right] \\
&\qquad +\beta\frac{(1-\xi_p)(1-n)}{1-\beta(1-n)}\left(\frac{\upsilon_L^{1-\epsilon}-\upsilon_L^{-\epsilon}mc_L}{1-\xi_p\beta(1-n)}\right)\Bigg\} \\
&\frac{n+(1-n)g^{\alpha-1}}{\frac{1-\xi_p}{1-\xi_p\pi^{\epsilon}}(n\upsilon_A^{-\epsilon}+(1-n)\upsilon_L^{-\epsilon})}\left(\frac{\alpha w}{(1-\alpha)\rho}\right)^{\alpha}\Bigg\}\left[\frac{\gamma-1}{\phi\gamma}\frac{g-\beta h}{(g-h)}\frac{1-\xi_w\beta g^{\gamma(1+\nu)}}{1-\xi_w\beta g^{\gamma-1}}w^{-\gamma\nu}w^{\#\gamma\nu+1}\right]^{-1}
\end{aligned} \tag{168}
$$