Foreign Firms and the Development of Domestic Entrepreneurial Skills*

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Abstract

Foreign firms may enhance a developing country's formation of know-how by exposing or directly transferring local entrepreneurs the productive ideas of developed countries. However, foreign firms may also reduce the domestic entrepreneurs' incentive to accumulate know-how by increasing their competition and reducing the returns to entrepreneurial skills. It is shown that if externalities drive the formation of skills, after openness, initial conditions determine if a country converges to one of two steady states or to exhibit non'monotone dynamics. If instead, the costs and benefits of skill formation are fully internalized, openness gradually removes the pre-existing sector, generates a new sector of domestic firms, and the country catches up with developed countries. In both models, convergence requires the destruction of pre-existent firms. The implications for empirical work are also discussed.

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1. Introduction

Would hosting foreign firms lead a developing country to catch up with developed countries? Can this form of openness instead lead the country to lag further behind? Does the presence of foreign firms enhance or impair the accumulation of productive know-how by domestic firms? What is the impact of this openness on the overall welfare of a country? Should the governments of developing nations promote the entry of foreign firms? In this paper I use simple general equilibrium growth models to answer these questions.

Entrepreneurial and managerial know-how can be the limiting factor in a country's aggregate productivity. They determine the technologies and market opportunities that local firms can efficiently operate and access.[1] Broadly defined, these "skills" can be as simple as knowing key individuals and conventions of a particular market and as sophisticated as the scientific and technological training to coordinate the development, selection and marketing of a new slow-release drug. Like other skills, entrepreneurial know-how is self-productive. Countries with an initial poor supply may never, by themselves, accumulate the amounts required to access the world's technological frontier.

A country can import entrepreneurial and managerial skills from abroad by allowing foreign firms to operate in the country. Foreign firms can combine their control of domestic labor with know-how that is only available elsewhere in the world. Indeed, this form international trade of skills seems to have gained importance with the increasing multinational activity and foreign direct investment of recent years. Burstein and Monge-Naranjo (2009) quantify significant output and consumption gains for developing countries that host firms from developed countries.[2] Moreover, Antras, Garicano and Rossi-Hansberg (2006) show that the distributional impact of foreign skills go well beyond those implied by standard factor endowment models because they alter the organization of production and the within occupations distribution of income. Yet, by taking skills as fixed endowments, these papers are silent about their accumulation over time and across countries.

Aside from static output gains, the presence of foreign firms can impact the acquisition of the country's own entrepreneurial and managerial know-how. Diffusion of productive know-how can take place via externalities, as the exposition to new and possibly more advanced ideas from abroad facilitates domestic entrepreneurs in their accumulation of know-how. Diffusion of productive ideas can also take place via implicit or explicit fully internalized transactions as a foreign expert trains a local future firm manager. However, the presence of foreign firms increase the competition faced by local managers and reduce their total and marginal returns to entrepreneurial skills. This paper studies the aggregate dynamics and welfare consequences for a developing country that opens up to foreign firms.

[1] There is an extensive literature that links the productivity of firms to the quality of their management. See, for example, Kaldor (1934), Lucas (1978), Oi (1983), Prescott and Visscher (1980), and Rosen (1982).

[2] Ramondo (2008) and McGrattan and Prescott (2008) show that the gains would be even larger if, instead of skills, productivity is driven by non-rival factors, i.e. that can be used simultaneously in multiple locations.

In the model, production is carried out by teams of workers led by an entrepreneur.[3] The skills of the manager determines the productivity of the team. The model is an OLG economy in which some of the young individuals invest in skills to become the manager –and residual claimant– of a firm when old. Over time, the equilibrium skill formation of young future managers is determined by the set of productive ideas currently implemented by active managers operating in the country. Workers are fixed in their country of origin but managers can move across countries.[4] In the context of the model, a "closed" country is one in which only national managers can lead firms while an "open" country allows free entry of foreign firm leaders.[5] Entry of foreign managers impacts they country by increasing the domestic price of labor, reducing the marginal return to entrepreneurial skills and increasing the set of productive ideas upon which the young can form their future skills.

The presence of externalities is a standard assumption in models of human capital formation, e.g. Romer (1986), Lucas (1988) and Stokey (1991). I use a variant of Stokey (1991) in which the aggregate skills level of old managers impacts the cost of accumulating skills of young potential managers. In this variant of the model, the set of ideas that surround individuals during their formative years is a national public good shaped by all skills effectively used within the country. There is an externality because firm leaders only receive returns for their production activities, and not for their contribution to the ideas circulating in the country. Equally, the presence of foreign firms has an externality that impacts the national public good in an open economy. Hence, the framework integrates quite naturally the mechanism in Findlay (1978), in which foreign firms have positive technology spillovers on domestic firms.

Regardless of initial conditions, closed economies always follow a balanced-growth-path (BGP). A small open economy exhibits a significantly more complex dynamics. Openness to foreign firms has countervailing forces as it reduces both the marginal costs and the marginal returns for the accumulation of skills of domestic entrepreneurs. On one hand, the current entry of foreign firms enhances the set of productive ideas surrounding the forming crop of young managers. On the other hand, the foreseen future entry of foreign firms bids up the domestic cost of labor and squeeze total and marginal returns to entrepreneurial skills. As a result, open economies exhibit a form of predatory-prey dynamics because entry is an inverse function of the relative level of domestic entrepreneurial skills. Indeed, an open country can exhibit non-monotone dynamics and there might be two different steady states to which the country can converge depending on initial conditions. In one steady state, the country converges to the skill levels of developed countries (and foreign firms no longer enter). In the other steady state, the country remains forever behind

[3] In the model entrepreneurial and managerial know-how are equivalent. For models in which they are different see Holmes and Schmitz (1991) and Chari, Golosov, and Tsyvinski (2004).

[4] See Klein and Ventura (2004) for an analysis of cross country labor mobility.

[5] The emphasis on the cross-border reallocation of management conforms with the observation that multinational firms heavily rely on home expatriates –and home trained individuals– to manage their operations, specially in developing countries (see Chapters 5 and 6 of UNCTAD 1994). It also conforms with the emphasis of the existent literature on firm specific intangible assets for multinational activity (e.g. Barba-Navarretti 2004 and Markusen 2004).

(and foreign firms are always present). It is shown that, regardless of the path followed by the country, it catches up with developed countries if and only if for one period domestic firms are shut-down and the entire production of the country is led by foreign firms. The model also predicts leapfrogging, as the most backward countries not only are more likely to converge to the high steady state, but also to do it more quickly. Finally, it is shown that the extent in which a country gains with openness depends on initial conditions.

Another leading hypothesis for the formation of know-how is that it results from transactions that fully internalize the costs and benefits of all parties involved, e.g. Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991) and Jovanovic and Nyarko (1995).[6] I consider a variant of the Boyd-Prescott model in which the skill formation of a young manager depends on the skills and actions of the leader for whom he works. Both trainers and apprentices must be purposely involved and a well functioning market for skill formation operates.[7] No externalities are present, since the skill formation of each young future manager depends solely on the skills of his manager/trainer. Instead of an externality, foreign firms disseminate skills to the host country by directly training the workers under their control.

The internalization of the returns of skills in future production of skills can easily lead to dynamic increasing returns and degenerate solutions.[8] I provide sufficient conditions for existence, uniqueness and efficiency of a BGP in a closed economy.[9] In an open economy, the entry of more advanced foreign firms introduces heterogeneity in the population of trainers, which in turn creates heterogeneity of domestic firms. Specifically, openness leads to the emergence of a sector of new-domestic entrepreneurs all of which have the same skill levels of foreign managers. This new sector grows over time and eventually overtakes the entire economy. Along the transition, the progeny of the pre-existing domestic managers shrinks in size and level of skills and then disappears. Regardless of its initial relative backwardness, an open country fully catches up with developed countries a finite number of periods after opening. Also, regardless of initial conditions, openness is always welfare improving.

The implications of both models complement the negative empirical results on the positive effect of foreign firms on the productivity of existing domestic firms. As in the model with externalities, with internalized diffusion a developing country catches up with developed countries if and only when domestic pre-existent firms (or their progeny) are entirely removed and replaced by a new sector of domestic firms. In both models, the country as a whole can catch up, even if pre-existing firms respond *reducing*, not increasing, the investments that drive their productivities. This implication is in line with ample evidence (e.g. Aitken and Harrison [1999]) on the absence of positive effects of foreign firms on the productivity of existing domestic firms.[10] But even if

[6] Agarwal, R. et al (2004) and Filson and Franco (2006) extend the Chari-Hopenhayn model.

[7] Abstracting from private information frictions that lead to inefficiency as.in Jovanovic and Nyarko (1995).

[8] Recall that the profit function in a Lucas (1978) is strictly convex. The self-productivity of skills adds to the convexity of the value function.

[9] The conditions in Boyd and Prescott (1987) and Prescott and Boyd (1987) are not enough for this result.

[10] See Xu (2000) and Alfaro et al. (2006) for a discussion of the empirical findings. While there is some evidence

spillovers drive aggregate productivity, it is shown that it is quite likely that the negative impact of foreign competition overdoes the positive effect of spillovers.

The remarkable differences in the implications of the two models highlight the importance of distinguishing between externalities and fully internalized transfers of know-how. Some authors (e.g. Javorcik [2004] and Kugler [2005]) have argued for the existence of inter-industry spillovers, specifically, from foreign firms to local suppliers. However, productivity gains might be driven by internalized transfers, not spillovers, since as Javorcik herself reports, foreign firms in her sample were directly involved providing training, equipment and know-how to the local suppliers. At the level of domestic industries, skill formation at the interior of the firm seems to be a major mechanism for aggregate skill formation and dissemination, as indicated by the empirical evidence that links the characteristics and the outcomes of parent firms with their spin-offs.[11] Unfortunately, with the exception of the recent work of Malchow-Møller et. al (2007) and Poole (2006) on multinational firms in Denmark and Brazil, respectively, there has been little empirical work on internalized skill formation by multinational firms.[12]

The paper proceeds as follows. Section II lays out the model and equilibrium preliminaries. Sections III considers the equilibrium when geography defines the scope of learning and Section IV when the case of intrafirm learning. Section V developes the general case. Section VI concludes.

of positive spillovers for developed countries (e.g. Griffith et al 2002), for developing countries most authors come back empty-handed when trying to revert the negative results of Aitken and Harrison (1999).

[11] For the U.S. car industry, Keppler (2001, 2002, 2006) documents that the genesis of the most successful car makers can be traced to former employees of other car makers. Agarwal et al (2004), Filson and Franco (2006) and Franco (2005) show the same for the rigid disk drive industry.

[12] There is however, ample anecdotal. The best known case may be the emergence of a textile sector in Bangladesh after the seed planted by a Korean firm (see Easterly [2001]). Also, some multinationals spend significant resources in training their workers (see UNCTAD [1994]).

2. The Model

Consider a discrete time, infinite horizon OLG economy. Each individuals lives for two periods. There is a single consumption good. An agent born in period t that consumes c_t^t and c_{t+1}^t in periods t and $t+1$ attains an utility level

$$U^t = c_t^t + \beta c_{t+1}^t,$$

where $0 < \beta < 1$.

The consumption good is produced by teams of *one* leader (or manager) and n units of labor. If the firm leader has entrepreneurial skills z the team produces

$$y = zn^\alpha,$$

units of the good. As in Lucas (1978) the person-specific skills z of the leader determines the productivity of the firm. The span-of-control parameter $\alpha \in (0,1)$ is the degree of decreasing returns to the amount of labor n. I follow Lucas (1978) calling these teams "firms" even if they can equally be seen as parts of a conglomerate of teams within the boundaries of the same firm.[13] Firm leaders are interchangeably called "entrepreneurs" or "managers" because they set up and control firms and are also their residual claimant.

The size (measure) of all generations is normalized to one. All individuals have a labor endowment of one in each period of their lives. Within each generation, however, there is a fraction $\omega \in (0,1]$ of individuals who have the *potential* to be firm leaders or entrepreneurs. These individuals are workers when young, but have the option to allocate their time endowment to be entrepreneurs when old. Thus, every period there is a maximum mass of ω of active (old) firm leaders and a minimum mass $2-\omega$ of workers (all the young plus the $1-\omega$ non-entrepreneurial old).

Every old potential entrepreneur chooses between remaining a worker or becoming a manager. As in Lucas (1978), these occupation choices are made on the basis of each individual's entrepreneurial skills relative to the rest. The key addition of our analysis is to endogeneize the formation of skills, which Lucas (1978) take as exogenously given. In this framework entrepreneurial skill are determined by the investment decisions made in previous periods, on the basis of the cost of acquiring and the rate of return of skills of those skills.

Each potential entrepreneur $i \in [0,\omega]$ is exposed to a subset of the productive ideas implemented in the country at the time when he was young. In general, this platform of productive ideas, which I denote z_i^E, will be a function of the skills z_i of the particular manager(s) for whom the potential entrepreneur worked as well as the overall average of the skills operated in the economy when the agent was young. Formally, let the pair of vectors (z^*, m^*) denote of skills levels and the *incidence* (of subsets of homogeneous) of managers *actively operating* in the country. That is,

[13] See Garicano (2000), Oi (1983) and Rosen (1982) for related issues.

each entry of z^* indicates, respectively a skill level and the corresponding entry in m^* indicates the share of labor controlled by entrepreneurs wich such skill level. Obviously, all entries in z^* and m^* are non-negative. Scale effects are ruled out since the vector m^* adds up to one.

I assume that the platform of ideas z_i^E is a function $z_i^E = F(z_i, z^*, m^*)$ that satisfies some basic admissibility properties: (i) it is non-negative and increasing in (z_i, z^*); (ii) F is linearly homogeneous in (z_i, z^*); (iii) if z_i and all entries in z^* are equal to a scalar $Z > 0$, then $z_i^E = Z$ for all m^*; (iv) for any entry j of (z^*, m^*), $\partial z_i^E/\partial m_j^* \geq 0$ if $z_j^* \geq z_i^E$ and $\partial z_i^E/\partial m_j^* \leq 0$ if $z_j^* \leq z_i^E$; (v) for any entry j of (z^*, m^*), $\partial^2 z_i^E/\partial z_j^* \partial m_j^* \geq 0$ and $\partial^2 F/\partial Z_h \partial m < 0$. Conditions (i)-(iii) are natural compatibility restrictions in the units of measurement. Conditions (iv) and (v) simply indicate that m^* is a set of weights.

Given the function F, the value of z^E in each point in time is determined by equilibrium investment and entry decisions of domestic and foreign entrepreneurs. In terms of the consumption good, each young potential entrepreneur $i \in [0, \omega]$ bears a cost $z_i^E \phi\left(z_i'/z_i^E\right)$ of acquiring skills any level of skills $z_i' \geq 0$ for the next period. Here $\phi(\cdot)$ is continuously differentiable, positive and strictly convex, with $\lim_{x\to 0} \phi(x) = \phi'(x) = 0$ and $\lim_{x\to\infty} \phi(x) = \phi'(x) = \infty$. These assumptions imply that the total and marginal cost of investing any level z' is strictly decreasing in the level the platform of productive ideas z_i^E, i.e. $\phi_{z_i^E} < 0$. I will keep using $\phi(\cdot)$ as a shorthand, but will focus on the case in which, for $z^E > 0$, the cost function is

$$\phi\left(\frac{z'}{z^E}\right) = \frac{v_0}{1+v}\left(\frac{z'}{z^E}\right)^{1+v}, \tag{2.1}$$

where v_0, $v > 0$.

I consider perfect foresight competitive equilibrium. Given prices, all the following decisions must be individually rational: old potential entrepreneurs must decide whether to be workers or managers, in which case they hire workers optimally. Young potential entrepreneurs invest in skills foreseeing their occupation choice when old. Workers maximize the earnings of their labor. Since the discount factor β pins down the one-period interest for all t, i.e. $R_t = \beta^{-1}$, the key equilibrium prices is the wage sequence $\{w_t\}_{t=0}^{\infty}$, where w_t is the price of one unit of labor in period t.

Consider an old individual with entrepreneurial skills $z > 0$. Should he decide to be an active entrepreneur when facing a wage w for labor, then he would earn a payoff

$$\begin{aligned} \pi(z, w) &= \max_{\{n\}} \{z n^\alpha - w n\} \\ &= \theta z^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}. \end{aligned} \tag{2.2}$$

where $\theta \equiv \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) > 0$. Given w, $\pi(z, w)$ is strictly increasing and convex in z, and strictly decreasing in w. His optimal demand for labor would also be increasing in z and decreasing in w:

$$n^*(z, w) = \left[\frac{\alpha z}{w}\right]^{\frac{1}{1-\alpha}}. \tag{2.3}$$

Recall that the individual has the option of earning the wage w instead. Hence, he will become an active entrepreneur only if $\pi(z, w) \geq w$, i.e. only if relative to the wage, his skills are high enough so that $z/w > 1/\left(\alpha^\alpha [1-\alpha]^{1-\alpha}\right)$.

Now consider the problem of a young potential entrepreneur. Should he set out to be an active entrepreneur when old, then his optimal investment in entrepreneurial skills solves

$$\max_{z'} \left\{ \beta \pi \left(z', w' \right) - z^E \phi \left(z'/z^E \right) \right\}. \tag{2.4}$$

For his decision, the two key variables are z^E and w', his exposure to productive ideas and the wages that he foresees he would have to pay. The first reduces total and marginal costs of investment z'. The second reduces total and marginal returns to investment. Obviously, if future wages are high enough, then the potential entrepreneur would be better off remaining a worker and investing $z' = 0$. In equilibrium, a young potential entrepreneur invests $z' > 0$ in skills only if z^E is high enough and w' is low enough as we develop below.

I will consider and compare two different scenarios. First, I develop the equilibrium in a closed economy where only domestic entrepreneurs can hire domestic labor. Then, I analyze the impact in a small and less developed country of allowing free entry of foreign managers.

3. Local Externalities and the Accumulation of Know-How

Consider an environment in which the stock of ideas z^E is given by a local (or national) public good, Z. The level of this public good is closely pinned down by the actual skills and know-how implemented by firms operating in the country. Such assumptions are standard in models of growth, e.g. Romer (1986). In the context of intergenerational models, this assumption would be closest to Stokey (1991), where the level of skills of older generations impact the cost of younger generations to accumulate skills. The value of Z evolves over time as entrepreneurs accumulate skills to maximize their net rents. However, there is an externality because entrepreneurs are not compensated from their contribution to the stock of ideas surrounding the future generations. I first explore the case of a close economy in which Z is entirely pinned down by the skills of domestic entrepreneurs. Then, I explore the case in which the entry of foreign entrepreneurs impact the level of Z. [14]

Assume that the level of entrepreneurial skills of the old generation is equal to Z. The variable Z has a dual role. On the one hand, it defines the average productivity of the country and the market-clearing wage w. On the other hand, it delineates the level of productive ideas and know-how z^E to which potential future entrepreneurs are exposed to during their formative years. Under the assumption that Z is a local public good, then $z^E = Z$ for *all* the young potential individuals. Under this form of learning, all members of the current and future generations will make the same investments, so any initial heterogeneity will be dissipated within one period.[15]

[14] It might be helpful to think of this model as a happy hour diffusion: Imagine that everyday, after work, all the young workers go to a bar for a happy hour. Among more other things –and with objectives different than training– they talk about their work and the ideas they confront every day. After the many happy hours of a typical young person, the set of ideas in the brain of each and every one is Z.

[15] Relaxing the assumption of initial homogeneity is trivial. Entrepreneurs heterogeneity remains for at most one period. Regardless of the occupation choices of theinitial old, the initial level z_0^E would be strictly positive and the same for all potential entrepreneurs.

Consider the first role of Z. Commanding skills z and facing wages w, an entrepreneur would hire $(\alpha z/w)^{\frac{1}{1-\alpha}}$ workers. Assuming all potential entrepreneurs are active and command equal skills $z = Z$, then, in equilibrium, each one of them hires an amount of labor equal to the ratio $\eta \equiv (2-\omega)/\omega$ since there are ω potential entrepreneurs and $2-\omega$ workers. Equating individual labor demand $(\alpha Z/w)^{\frac{1}{1-\alpha}}$ with average individual supply η, the market-clearing w

$$w = \alpha Z \eta^{\alpha-1}. \tag{3.1}$$

In equilibrium, the wages of workers are proportional to the entrepreneurs' level of skills.

An entrepreneur with skills z competing with an economy-wide level Z would earn $\pi = (1-\alpha)\eta^{\alpha}\left(\frac{z}{Z}\right)^{\frac{1}{1-\alpha}} Z$, a function that is strictly increasing and convex in the entrepreneur's skills z and strictly decreasing in the other entrepreneurs' skills Z. If all potential entrepreneurs are homogeneous, $z = Z$ and then the ex-post earnings of all entrepreneurs would also be proportional to Z

$$\pi = (1-\alpha)\eta^{\alpha} Z. \tag{3.2}$$

For equations (3.1) and (3.2) to be equilibrium outcomes, entrepreneurs must be better off than workers. This condition has two components. First, *ex-post*, already formed (old) entrepreneurs must be better off than workers, i.e. $\pi > w$. This inequality holds as long as $\omega < 2(1-\alpha)$, i.e. if the supply of managerial talent is scarce relative to its productivity, an assumption to be sustained throughout the paper.[16] Second, it must be *ex-ante* optimal for young potential entrepreneur to invest in skills and become active entrepreneurs, i.e.

$$\max_{z'} \left\{\beta\pi(z', w') - z^E \phi\left(z'/z^E\right)\right\} > \beta w'. \tag{3.3}$$

It is here where the second role of Z enters, because it defines the the platform z^E of ideas to which each young entrepreneur is exposed to. Let us start by characterizing the maximization in the left-hand-side. Notice that both $\pi(\cdot)$ and $\phi(\cdot)$, are strictly increasing and strictly convex in z'. Given w' and z^E, for the maximizer to be different from the trivial $z' = 0$, the marginal cost $\partial\phi/\partial z'$ must cross the marginal return $\partial\pi/\partial z'$ from below at a positive value z'. Indeed, the former is steeper than the latter if $v/(1+v) > \alpha$.[17] I shall assume this inequality throughout. Then, the necessary first order condition for optimal skill investment, $(\beta\eta^{\alpha}/v_0)(z'/Z')^{\frac{\alpha}{1-\alpha}} = \left(z'/z^E\right)^v$, is also sufficient.

Investments z' are always increasing in the platform z^E of ideas to which each young entrepreneur is exposed to. Individual investments z', however, are decreasing in the average investments Z' of peers. However, since all of them face the same $z^E = Z$, they end up choosing the same

[16] If this condition does not hold, then some of the potential entrepreneurs would optimally choose to be workers. In such equilibrium, the ex-post payoffs for entrepreneurs and workers would be the same, and $w = \pi = \theta^{1-\alpha} Z$ and each active manager hires $n = \alpha/(1-\alpha)$ workers.

[17] The power (elasticity w.r.t z') in $\phi'(\cdot)$ is equal to v and in $\pi'(\cdot)$ it is $\alpha/(1-\alpha)$. The condition then is $v > \alpha/(1-\alpha)$ which is equivalent to $v/(1+v) > \alpha$.

investment $z' = (\beta\eta^{\alpha}/v_0)^{\frac{1}{v}} \times z^E$. For future references, define $G \equiv (\beta\eta^{\alpha}/v_0)^{\frac{1}{v}}$ the gross rate of skill accumulation, so the aggregate law of motion is

$$Z' = GZ, \tag{3.4}$$

I will assume that the net return of accumulating skills is high, i.e. $\beta\eta^{\alpha} > v_0$, leading to $G > 1$. Moreover, I will also impose $v_0 > \beta\left[\beta/\left(1+v\right)\right]^{v}\left[\left(2-\omega\right)/\omega\right]^{\alpha}$ which rules out the possibility that $\phi\left(G\right) > \eta^{\alpha}$, i.e. it precludes the case when skill investments are so high that aggregate consumption becomes negative.

Condition (3.4) indicates that each young entrepreneur augments the knowledge z^E received from the previous generation. For this accumulation process to be an equilibrium outcome, the ex-ante individual optimality condition (3.3) must hold, given that all other entrepreneurs accumulate skills according to (3.4), and equilibrium wages follow (3.1). This is guaranteed by the simple condition $v/\left(1+v\right) > \alpha/\left(1-\omega/2\right)$.[18] Obviously, since $\alpha/\left(1-\omega/2\right) > \alpha$, if it is ex-ante optimal to be an entrepreneur, so it is ex-post.

The following proposition summarizes the results:

Proposition 3.1. *Assume the parameter restrictions $\omega < 2\left(1-\alpha\right)$, $v/\left(1+v\right) > \alpha/\left(1-\omega/2\right)$ and $v_0 > \beta\left[\beta/\left(1+v\right)\right]^{v}\left[\left(2-\omega\right)/\omega\right]^{\alpha}$. Suppose a closed economy starts at period $t = 0$ with an old generation of homogeneous potential entrepreneurs with skills $Z_0 > 0$. Then, there exist an unique equilibrium in which, for all $t \geq 0$: (i) all old potential entrepreneurs become active entrepreneurs, hire $n_t = \eta$ units of labor and attain profits equal to $\pi_t = \left(1-\alpha\right)\eta^{\alpha}Z_t$; (ii) labor markets clear, and equilibrium wages are $w_t = \alpha\eta^{\alpha-1}Z_t$; and (iii) all young potential entrepreneurs invest the same amount as the aggregate level, $Z_{t+1} = GZ_t$, in entrepreneurial skills.*

Aggregation is straightforward. The country's output of the consumption good is given by

$$Y_t = Z_t\left[\omega^{1-\alpha}\left(2-\omega\right)^{\alpha}\right]. \tag{3.5}$$

Netting the aggregate resources to invest in skills $\omega Z_t\phi\left(G\right)$, the aggregate consumption is

$$C_t = Y_t\left(1 - \frac{\beta G}{1+v}\right). \tag{3.6}$$

C_t is positive, because $v_0 > \beta\left[\beta/\left(1+v\right)\right]^{v}\left[\left(2-\omega\right)/\omega\right]^{\alpha}$, which implies $\beta G/\left(1+v\right) < 1$.

In sum, regardless of initial conditions, a closed economy always embarks in a balance growth path (BGP). The engine of growth is entrepreneurial skill accumulation, the process of young generations improving upon the knowledge that the old generation implemented for production.

I now ask whether foreign entrepreneurs (firms) disrupt or boost the domestic process of accumulating skills.

[18] To see this, notice that imposing all the other equilibrium conditions, the LHS of the inequality becomes $\beta\left(1-\alpha\right)\eta^{\alpha}GZ - Z\frac{v_0}{1+v}\left[G\right]^{1+v} = GZ\beta\eta^{\alpha}\left[\frac{v}{1+v} - \alpha\right]$ where I have used the formula for G and factorized. On the other hand, the RHS is $\beta w' = \alpha GZ\eta^{\alpha-1}$. Both sides are proportional to GZ, so condition (3.3) boils down to $v/\left(1+v\right) > \alpha\left(1+1/\eta\right)$, which given the definition of η, is the same as $v/\left(1+v\right) > \alpha/\left(1-\omega/2\right)$.

3.1. Entry of Foreign Firms

Consider now a country that allows free entry of foreign firms, i.e. foreign entrepreneurs can freely enter the domestic labor market and hire workers inside the country.[19] I shall focus on the case where the home country is *small* (i.e. does not affect the equilibrium of the foreign country), and *less developed* (home entrepreneurs have a lower level of skills Z_h than their foreign peers' Z_f.) The subindices f and h will denote aggregate variables of the foreign and home countries, respectively.

The presence of foreign entrepreneurs has two different impacts in the home country. First, being more advanced, foreign entrepreneurs bring and implement technological knowledge and productive know-how that otherwise would be absent in the home country. Second, foreign entrepreneurs bid up the local market clearing wage.

Consider the first impact. By bringing skills Z_f, foreign entrepreneurs enhance the set of ideas z^E that surround the cohort of young potential entrepreneurs. When learning is geographic, z^E is a local public good, i.e. the same value z^E accrues for all, regardless of whether they work for foreign or domestic firms. The value of z^E would be given by $F(Z_h, Z_f, m)$, an average of Z_f and Z_h. weighted by the incidence m of foreign firms in the country. I will take m to be the *fraction* of aggregate domestic labor units employed by foreign firms.

I assume that $F(Z_h, Z_f, m)$ is a generalized (or Hölder) weighted mean of Z_h and Z_f:

$$z^E = \left[(1-m)(Z_h)^\rho + m(Z_f)^\rho\right]^{\frac{1}{\rho}}, \tag{3.7}$$

where the parameter ρ can be any real number. This formulation encompasses many different ways of averaging Z_h and Z_f: minimum (Leontieff) if $\rho \to -\infty$; maximum if $\rho \to \infty$; arithmetic mean if $\rho = 1$ and harmonic mean if $\rho = -1$. When $\rho = 0$, z^E is the geometric (Cobb-Douglas). average with shares $1-m$ and m. I will also consider the case of complete home bias $z^E = Z_h$, i.e. when domestic entrepreneurs can only only learn from older individuals from the same country.

However, m is endogenously determined in equilibrium by the entry decisions of foreign entrepreneurs and the occupation choices of domestic entrepreneurs. Both of those decisions are driven by the impact of foreign firms in the local labor market and ultimately depend on the ratio of domestic-to-foreign skills Z_h/Z_f as I proceed to describe.

First of all, foreign entrepeneurs will enter the home country until they are indifferent between operating in the foreign country or in the home country. Therefore, in both cases he must attain profits $\pi_f = (1-\alpha)\eta^\alpha Z_f$, and this can only happen when the cost of each unit of labor is the same in both countries. Therefore, the new domestic market-clearing wages is

$$w_h = w_f = Z_f \alpha \eta^{\alpha-1}.^{20} \tag{3.8}$$

[19] I will assume that individuals from the home country cannot move. This is without loss of generality for workers and old entrepreneurs, since, in equilibrium they will be indifferent between moving to foreign or remaining in home. However, ruling out the possibility for domestic young potential entrepreneurs to move and "grow up" in the developed country is crucial. I will discuss further below the factual and analytical relevance of this assumption.

Then, foreign entrepreneurs hire the same number of workers when they operate at home as when they operate at foreign.[21][22]

Now consider the decisions of old potential entrepreneurs. The higher wages due to the presence of foreign firms makes the option of being a worker more attractive. Moreover, with higher wages the payoff of being an entrepreneur falls

$$\pi_h = (1-\alpha)\,\eta^{\alpha} Z_h \left(\frac{Z_h}{Z_f}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3.9}$$

For old domestic entrepreneurs to remain active it is needed that $\pi_h > w_f$, which holds if the ratio of domestic-to-foreign skills is above a threshold R_S:

$$\frac{Z_h}{Z_f} \geq R_S \equiv \left[\frac{\alpha}{(1-\alpha)\,\eta}\right]^{1-\alpha} < 1, \tag{3.10}$$

where the inequality follows from previous parameter restrictions.

If domestick skills are above that threshold, each domestic entrepreneur hires

$$n_h = \eta \left(\frac{Z_h}{Z_f}\right)^{\frac{1}{1-\alpha}}, \tag{3.11}$$

units of labor. Aggregate domestic hiring is $\omega n_h = (2-\omega)\left[\frac{Z_h}{Z_f}\right]^{\frac{1}{1-\alpha}}$. The labor market clearing condition implies that all remaining workers must be hired by foreign firms. However, if the home country is so far behind that Z_h/Z_f falls below that R_S, domestic entrepreneurs switch to supplying (not demanding) labor. Therefore, solving for m

$$m = \begin{cases} 1 & \text{if } Z_h/Z_f < R_S, \\ 1-(Z_h/Z_f)^{\frac{1}{1-\alpha}} & \text{if } Z_h/Z_f \geq R_S. \end{cases} \tag{3.12}$$

If $Z_n/Z_f = 1$, then $m = 0$. Foreign firms do not operate in the country because domestic firms are productive enough to bid up the price of labor to the foreign level.

A slightly more complex decision is faced by the young crop of home entrepreneurs. On one hand, they are exposed to more and better ideas than before. On the other hand, once the country is open they foresee that they will have to pay the higher wages $w_h' = \alpha\eta^{\alpha-1} G Z_f$, which reduces both the entrepreneurial rents and the marginal return of investing in skills and also increases the attractiveness of remaining a worker. Plugging the equilibrium expression (3.12) for m inside the function F in (3.7), then:

$$z^E = \begin{cases} Z_f & \text{if } Z_h/Z_f < R_S, \\ Z_f\left[1+\left(\frac{Z_h}{Z_f}\right)^{\rho+\frac{1}{1-\alpha}} - \left(\frac{Z_h}{Z_f}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}} & \text{if } Z_h/Z_f \geq R_S. \end{cases} \tag{3.13}$$

[21] Again, the equality is in terms of effective units of labor. If workers in the home country have fewer units of effective labor per unit of physical labor, foreign firms will be larger than the domestic firms in the foreign country. If $Z_f > Z_h$, then foreign firms at the home country will be larger also than domestic firms in the home country.

[22] Notice that since the price of labor is the same at home and foreign, neither workers nor old entrepreneurs have the incentive to move from home to foreign.

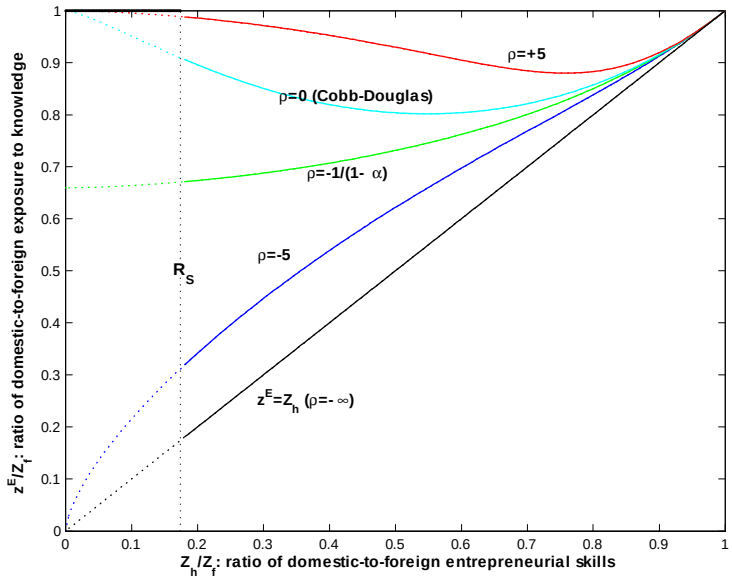


Figure 3.1: z^E/Z_f: Ratio of Domestic-to-Foreign Exposure to Knowledge.

The ratio z^E/Z_f indicates the exposure of ideas for local young entrepreneurs relative to foreign ones and is key in determining the dynamic consequences of openness to foreign firms for the home country. The ratio z^E/Z_f subsumes the equilibrium contributions of foreign and domestic know-how levels and can be non-increasing in the ratio Z_h/Z_f. While a higher Z_h/Z_f implies that local entrepreneurs expose more knowledge to their own young, it also implies lower entry of foreign firms, and hence, a lower exposure of foreign knowledge inside the home country. Indeed, as the first branch of (3.13) indicates, if the home country is so underdeveloped that $Z_h/Z_f < R_S$, then all economic activity inside the country is on the basis of foreign entrepreneurial skills. The sacrifice of the current generation of domestic entrepreneurs allows the future generations to be exposed to the same level (frontier) knowledge as their foreign peers.

Figure 1 illustrates the behavior of z^E/Z_f as a function of Z_h/Z_f for alternative values of ρ. Unless $\rho = -\infty$, it is always the case that $z^E/Z_f > Z_h/Z_f$ and the presence of foreign firms enhances the learning platform of the country, relative to the close economy. Indeed, if $\rho = +\infty$, then $z^E/Z_f = 1$ and any positive presence of foreign firms suffices to disseminate the superior foreign knowledge into the home country and put the new crop of domestic entrepreneur at par with the foreign peers. With intermediate levels $-\infty < \rho < \infty$, the relationship is non-linear. It is non-monotone as long as $\rho > -1/(1-\alpha)$; at the low end, a reduction in the local knowledge Z_h/Z_f is more than compensated by the increased entry of foreign firms. In the high end, the impact of local knowledge more than compensates its negative effect on foreign entry. The latter effect is always dominant when $\rho \leq -1/(1-\alpha)$ because of the strong complementarity between the two sources of knowledge. Regardless of ρ, $z^E/Z_f = 1$ whenever $Z_h/Z_f < R_S$ because only foreign knowledge is active in the country. Finally, in the home-biased case when $z^E = Z_h$, foreign

firms do not contribute to local learning and $z^E/Z_f = Z_h/Z_f$ (the 45^0 line) even for $Z_h/Z_f < R_S$.[23] Appendix A provides further details on the bheavior of z^E/Z_f.

Over time, the evolution of domestic entrepreneurial skills is driven by the investment decisions of young potential entrepreneurs, who, on the basis of z^E and w' must decide whether to be an entrepreneur and if so, how much to invest. Young potential entrepreneurs in the home country correctly predict that the foreign country stays in a BGP, that the next period skills follow $Z'_f = GZ_f$ and that, by being open, workers' wages in the home country will be given by $w'_h = \alpha\eta^{\alpha-1}GZ_f$. If in effect, they opt to become entrepreneurs, their optimal investment in skills solves

$$\max_{\{z'\}}\left\{\beta\left(1-\alpha\right)\eta^{\alpha}\left(z'\right)^{\frac{1}{1-\alpha}}\left(GZ_f\right)^{\frac{-\alpha}{1-\alpha}}-\frac{v_0}{1+v}z^E\left(\frac{z'}{z^E}\right)^{1+v}\right\}.$$

Under the assumed parameter restrictions, the first order condition $\left(\beta\eta^{\alpha}/v_0\right)\left(z'/Z'_f\right)^{\frac{\alpha}{1-\alpha}} = \left(z'/z^E\right)^{v}$ uniquely determines the optimal investment z'. Of course, in equilibrium all young entrepreneurs would invest the same amount. Therefore, the national level Z'_h (relative to Z'_f) would evolve according to

$$\left(\frac{Z'_h}{Z'_f}\right) = \left(\frac{z^E}{Z_f}\right)^{\mu}, \tag{3.14}$$

where $\mu \equiv v/\left[v-\alpha/\left(1-\alpha\right)\right]$. Because of our parameter restrictions, $\mu > 1$, implying that the next period relative domestic level of entrepreneurial skills levels is a convex function of the relative exposure of knowledge. Moreover, the home country would only catch up, i.e. $Z'_h/Z'_f = 1$ when $z^E/Z_f = 1$.

For (3.14) to actually define the transition function for the relative skill levels of the home country, young potential entrepreneurs should in fact opt to entrepreneurs and not workers. As before, this requires that: (i) ex-post, the entrepreneurial payoff must exceed the wages of workers, i.e.$Z'_h/Z'_f > R_S$; and that (ii) the ex-ante maximized (discounted) entrepreneurial rents net of learning costs more than compensate the foregone (discounted) wages of workers, i.e. that $\max_{\{z'\}}\left\{\beta\pi\left(z',w'_f\right)-z^E\phi\left(\frac{z'}{z^E}\right)\right\} > \beta w'$. It turns out that condition (ii) takes a very simple form, and also that whenever it holds, condition (i) also holds.

Define

$$R_M \equiv \left[\frac{\alpha}{\left(\frac{v}{1+v}-\alpha\right)}\frac{\omega}{2-\omega}\right]^{1-\alpha}. \tag{3.15}$$

Evidently $R_M > R_S$. Using this definition, the following result (whose proof is in Appendix A) characterize the optimal decisions of young domestic entrepreneurs.

Lemma 3.2. *If $z^E/Z_f \geq \left(R_M\right)^{1/\mu}$, then young domestic entrepreneurs in an open economy invest skills according to (3.14) and become active entrepreneurs next period. Otherwise, if $z^E/Z_f < \left(R_M\right)^{1/\mu}$, then, they do not invest in entrepreneurial skills ($Z'_h = 0$) and remain workers in both periods.*

[23] In this case, if we insist that old entrepreneurs must be active in order for the young to be learn from their skills, then $z^E/Z_f = 0$ for $Z_h/Z_f < R_S$.

Clearly, there is minimum relative level of knowledge required for domestic entrepreneurial activity to subsist. Whenever the access to knowledge falls below the threshold $(R_M)^{1/\mu}$, it becomes too costly for local entrepreneurs to invest in skills in the amount required to compete with the foreign entrepreneurs. Instead, working for those foreign entrepreneurs becomes the dominant option.

When $\rho \leq -1/(1-\alpha)$, this simple threshold for z^E/Z_f leads to a simple threshold condition in terms of Z_h/Z_f, because z^E/Z_f is strictly increasing in Z_h/Z_f. However, when $\rho > -1/(1-\alpha)$ the function z^E/Z_f is non-monotone and the set of values of Z_h/Z_f for which $z^E/Z_f \geq (R_M)^{1/\mu}$ is not necessarily an upper sub-interval of $[0,1]$. Regardless of ρ, however, the restricitons for Z'_h/Z'_f are quite simple: When following (3.14) Z'_h/Z'_f would fall below R_M then Z'_h/Z'_f is set all the way to zero.

The transition function $\Gamma : [0,1] \to [0,1]$, i.e. the function that defines the value of next period's Z'_h/Z'_f in terms of this period's Z_h/Z_f, is

$$Z'_h/Z'_f = \Gamma(Z_h/Z_f) \equiv \begin{cases} 1 & \text{if } Z_h/Z_f < R_S, \\ 0 & \text{if } Z_h/Z_f \geq R_S \text{ and } z^E/Z_f < (R_M)^{1/\mu}, \\ \left[1 + \left(\frac{Z_h}{Z_f}\right)^{\rho + \frac{1}{1-\alpha}} - \left(\frac{Z_h}{Z_f}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}} & \text{otherwise.} \end{cases} \tag{3.16}$$

The first branch is when the current ratio of domestic-to-foreign skills is so low that old domestic entrepreneurs opt to become workers for foreign firms. Then, all production inside the country uses foreign know-how, which, as a by-product, puts the forming crop of domestic entrepreneurs at par with their foreign peers. The second branch is when already formed domestic entrepreneurs are good enough so they remain active, but the exposure of knowledge for the young crop falls below the minimum necessary. The third branch is when both generations of domestic entrepreneurs, the young and the old, are active.

The function $\Gamma(\cdot)$ contains all the information needed to understand the dynamics of the home country after it opens up to foreign firms. In particular, the fixed points of $\Gamma(\cdot)$ define the balance-growth-path (BGP) $Z'_h/Z'_f = Z_h/Z_f$ of an open economy. As a first step, let us examine the BGP when the occupation choices of all generations are freezed (i.e. all potential entrepreneurs become active entrepreneurs.) If so, the transition function would be $Z'_h/Z'_f = \Gamma_C(Z_h/Z_f) \equiv \left[1 + \left(\frac{Z_h}{Z_f}\right)^{\rho + \frac{1}{1-\alpha}} - \left(\frac{Z_h}{Z_f}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}}$, i.e. the third branch of Γ.

Lemma 3.3. *Assume that in all periods, all potential entrepreneurs must become active entrepreneurs, and therefore $Z'_h/Z'_f = \Gamma_C(Z_h/Z_f)$ as defined above. Then:* ***(i)*** *regardless of the value of ρ, $Z_h/Z_f = 1$ is a BGP of the open economy. However, that BGP is unstable;* ***(ii)*** *if $-\infty < \rho < \infty$, there exists a unique $0 < R_I < 1$ such $R_I = \Gamma_C(R_I)$. Moreover, the BGP with $Z_h/Z_f = R_I$ is locally stable;* ***(iii)*** *if $\rho = +\infty$, $Z_h/Z_f = 1$ is the unique BGP and an open economy converges to it in just one period;* ***(iv)*** *if $\rho < -1/(1-\alpha)$, then $Z_h/Z_f = 0$ is a BGP. However, it is unstable unless $\rho = -\infty$, i.e. $Z'_h/Z'_f = (Z_h/Z_f)^{\mu}$.*

The most interesting result is the existence of a stable BGP in which the home country does not fully catch-up. Moreover, if the home country is initially not too far behind, openess can lead it to fall further behind. In this BGP, the reduction in the cost of accumulating entrepreneurial know-how is exactly compensated, in the margin, by the reduction in the rate of return of those skills. The stability of this BGP results from the fact that when a country's Z_h/Z_f is below R_I, the home country accumulates skills at a faster pace than the foreign country because the "diffusion effect" dominates the "competition effect", i.e. the entry of foreign firms boosts z^E/Z_f by more than what is necessary to compensate the lower return reduction in the rate of return of entrepreneurial skills that arises from $Z_h'/Z_f' < 1$. The opposite holds when $Z_h/Z_f > R_I$. Finally, the differences between Z_h and Z_f persist over time since $z^E/Z_f < 1$, i.e. young foreign entrepreneurs have access to superior platform of knowledge than their counterparts from the home country.

A BGP when the home country is fully at par with the foreign one always exists. In this BGP, no foreign firms operate in the home country and home is neither subject nor in need of knowledge spillovers from foreign entrepreneurs. From their own domestic firms, young generations from both countries have access to exactly the same level of knowledge and make exactly the same level of investments. As indicated in the lemma, in the absence of occupation decisions, this BGP is unstable and cannot be reach from any other point, unless, of course $\rho = +\infty$, which represents a case of extreme diffusion. Finally, $Z_h/Z_f = 0$ is also a BGP when there is strong complementarity between the foreign and domestic sources of knowledge, i.e. $\rho < -1/(1-\alpha)$ This BGP is only stable when the complementarity is extreme, since it is the limit of the intermediate R_I when $\rho = -\infty$.

Occupation choices drastically change the dynamic behavior of skill accumulation, including the existence and stability of BGPs. First of all, notice that R_I may be lower than R_S. In this case, domestic old entrepreneurs would not be active since they would rather work for a foreign firm. If so, all the knowledge to which domestic youth are exposed to is the know-how of foreign firms and $z^E/Z_f = 1$. Therefore, R_I would no longer be a BGP, since instead of being stuck at R_I, the home country will converge to the levels of the more advanced country within only one period. Alternatively, it may be the case that $R_S < R_I \leq R_M$. In such case, the occupation and investment choices of the domestic young are the ones ruling out R_I as a BGP. This is because instead of investing $Z_h'/Z_f' = R_I$, the young entrepreneurs would not invest and would remain workers their entire lives. The subsequent generation of domestic young entrepreneurs would face $z^E/Z_f = 1$, and, being at par with their foreign peers in access of knowledge, they will fully catch up with them in terms of skills. It is only when $R_I > R_M$ that the intermediate BGP remains. When this happens, R_I is locally stable and initial conditions Z_h/Z_f determine the long-run behavior of the home country when it opens up to foreign firms.

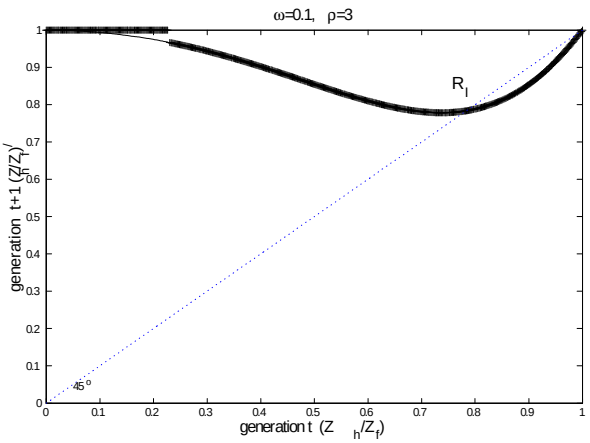
Proposition 3.4. *Full convergence, i.e. $Z_h/Z_f = 1$, is always a BGP for the home country when it is open. Moreover, if $\Gamma_C(R_M) \leq R_M$, such BGP is the unique BGP and it is attained in finite time regardless of initial conditions. However, if $\Gamma_C(R_M) > R_M$, another, locally stable BGP*

exists with $Z_h/Z_f = R_I < 1$. *In this case, the open economy always converges to a BGP, but initial conditions determine whether the home country converges to* $Z_h/Z_f = R_I$ *or to* $Z_h/Z_f = 1$.

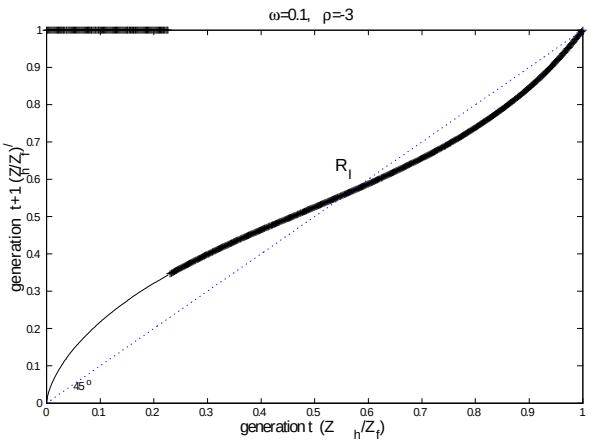
The most interesting aspect of this proposition is the result that occupation choices can result in the global convergence and stability of a BGP with full convergence of the home country to the foreign one. Also, notice the stark process by which such convergence would occur: It requires that for *one period*, the old domestic entrepreneurs become workers and all economic activity in the home country is led by foreign firms. In this way, the inferior local know-how remains dormant giving way to the more advanced foreign know-how.to be disseminated to the forming crop of domestic entrepreneurs. From then on, the countries will be forever at par in know-how.

Unless this abrupt destruction of domestic entrepeneurial activity takes place at some point, the home country would be forever stuck behind the foreign country. There are two additional and crucially important implications of this model. First, *openness can be detrimental.*for the productivity, and income of the home country. If home is initially not too far behind the foreign country, i.e. initial $Z_h/Z_f \in (R_I, 1)$, openness leads to a monotone decline towards R_I. The negative impact is also on welfare as discussed below in Section 5.

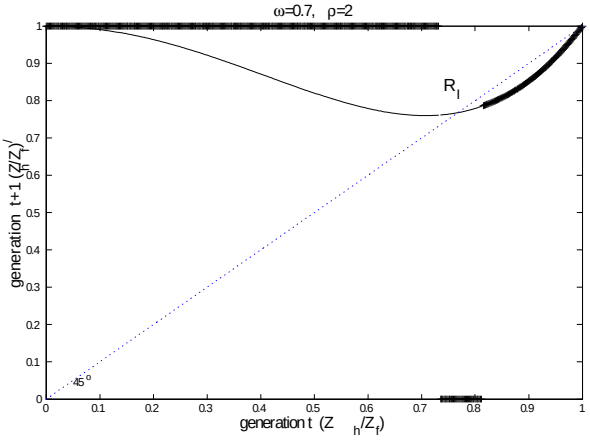
Second, *openness can lead to leapfrogging* among developing countries. If two developing countries allow free entry from developed country(ies), the less advanced country could surpass the more advanced one. This can happen in two forms: (i) after one period, the less advanced jumps ahead of the most advanced and stays ahead until both converge to a common BGP, or, even more strongly, when (ii) the initially less advanced converges to the income and productivity levels of the developed country and the initially more advanced converges to the laggard BGP R_I. Case (i) can happen when the transition $\Gamma(\cdot)$ is non-monotone, while case (ii) happens since $\Gamma(\cdot)$ can take the values 1 or 0, which, respectively, lead to convergence to $Z_h/Z_f = 1$ after one or two periods, while higher values lead to convergence to $Z_h/Z_f = R_I$.



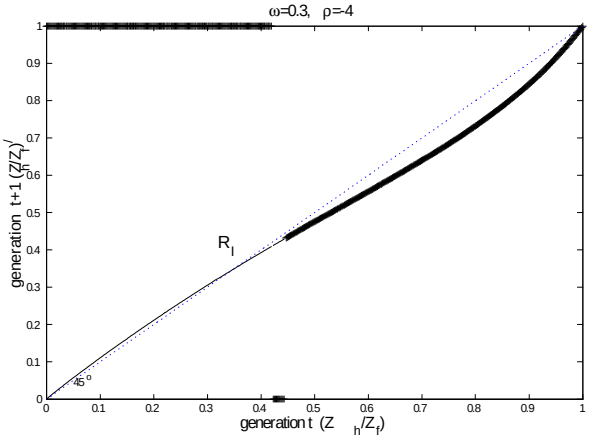
(a) A laggard BGP exists and is stable.



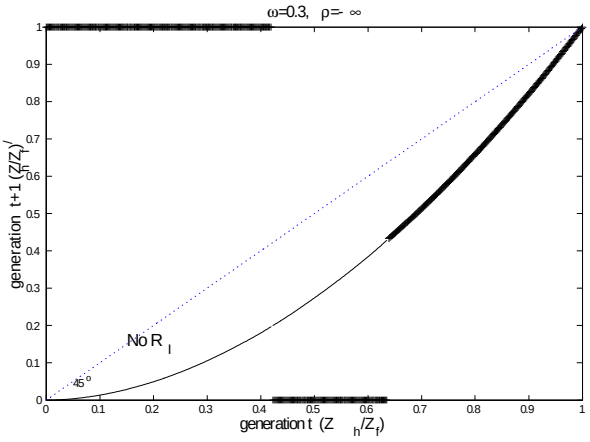
(b) A laggard BGP exists and is stable.



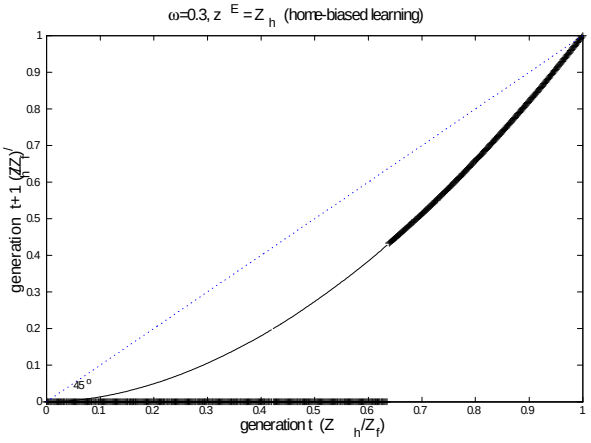
(c) Level convergence led by young.



(d) Level convergence led by old.



(e) Strong complementarity I.



(f) Extreme Complementarity II.

Figure 2: Transition functions for alternative cases.

Figure 2 illustrates these and other aspects of the dynamics of an open economy. Each of

the panels shows the transition function $\Gamma(\cdot)$ (thick lines) and the conditional transition $\Gamma_C(\cdot)$ (thin lines) that result from different combinations of ω and ρ. In each panel, the horizontal axis indicates today's ratio of domestic-to-foreign entrepreneurial know-how and the vertical axis indicates the same ratio for the next generation. The key features do not depend on the other parameters since they can be replicated with other values of α,v, v_0 and β as long as the assumed restrictions hold. For illustration purposes I set $\alpha = 0.5$, $\beta = (1.025)^{20}$, $v = 2.1$, while the value of v_0 is set so that countries grow at a rate $G = (1.02)^{20}$ once they have settle in a BGP.

Panels (a) and (b) in Figure 2 illustrate those cases in which the BGP with $R_I < 1$ persist. This can happen when foreign and domestic knowledge are highly substitutable, e.g. $\rho = +3$ as in panel (a), or when they are strongly complementary, e.g. $\rho = -3$ as in panel (b). Panel (c) shows the case in which R_I falls inside the range where the old generation of entrepreneurs while panel (d) shows the case in which it falls in the range where the old domestic entrepreneurs remain active but the young generation finds it optimal not to invest in skills. In both cases, R_I cannot be a BGP and, regardless of initial conditions, home catches up with foreign in finite time. persist because the occupation choices of local entrepreneurs in the home country.

Finally, panels (e) and (f) show two distinct cases when the presence of foreign firms have a minimal impact on the exposure of local youth to knowledge and their ability to form skills. Panel (e) represents the case when $\rho \to -\infty$, so that z^E is the minimum of the know-how actively implemented in the country. Even in this case, when foreign firms completely overtake production in the country, the minimum becomes Z_f, and therefore, after one generation of complete foreign control, the home country catches up as the ensuing generations will have the same know-how as their foreign counterparts. Panel (f) presents a very different result. When $z^E = Z_f$, young domestic potential entrepreneurs cannot learn anything from foreign entrepreneurs. Therefore, Z_h eventually converges to zero and stays there. Domestic firms will disappear forever and the aggregate income of the home country equals the wages. Wages grow as the foreign country accumulates skills and knowledge over time.

4. Internalized Accumulation of Know-How

I now consider the opposite extreme when $z_i^E = z^i$. This is, regardless of the overall know-how implemented in the country, the knowledge z^E upon which each young potential entrepreneur builds up his skills is entirely determined by the know-how of the manager for which he works when young. Such is precisely the assumption in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995), and others.[24] This alternative assumption fundamentally changes the relationship between young and old entrepreneurs since the costs and benefits of accumulating skills are fully internalized and efficient exchange.[25] From now on, I will assume

[24] See also the extensions of the Chari-Hopenhayn model by Agarwal, R. et al (2004) and Filson and Franco (2006) as well as Dasgupta (2010).

[25] I abstract from private information issues such as adverse selection that introduces inefficiencies in the equilibrium of Jovanovic and Nyarko (1995).

that all the young are potential entrepreneurs, i.e. $\omega = \eta = 1$. The analytical complexities that arise when $\omega < 1$ are not central for skill accumulation and are deferred to Appendix B. That appendix which also contains the proofs and other analytical aspects related to this section,

Besides consumption goods y, firms produce the entrepreneurial skills z' that young will command in the next period. The production function of goods and the cost of producing skills is exactly the same as in the previous section. A team of one entrepreneur and n workers units of labor and procuring z' units of skills for each of the n future managers produces goods equal to

$$y = z\left[n^{\alpha} - n\frac{v_0}{1+v}\left(\frac{z'}{z}\right)^{1+v}\right]. \tag{4.1}$$

To simplify the exposition, (4.1) incorporates the equilibrium result that all workers in a firm receive the same training.[26]

Each active manager chooses the amount of labor hired n and the level of training z' provided to each worker. The manager or entrepreneur is the residual claimant of the production team. Denote, by $\pi_t^I(z)$ the (optimized) net payoff an entrepreneur with skills z operating at time t. In equilibrium, since $\omega = 1$, all young workers are ex-ante identical and attain the same expected life-time utility. Let W_t denote the discounted utility attained by each young person at time t. The same value of W_t, which includes the first period wages, cost of training –if any– and next period payoffs –in the form of wages or entrepreneurial net rents– must be attained by all young workers, regardless of the manager that they work for.

From the point of view of an entrepreneur with skills z, the cost of each unit of labor is $W_t - \max_{z'}\left[\beta\pi_{t+1}^I(z') - z\phi\left(\frac{z'}{z}\right)\right]$. To see this, fix any level of training z' provided to the young workers. Then, the old entrepreneur only needs to transfer $\left[W_t - \beta\pi_{t+1}^I(z')\right]$ to the young future entrepreneur for him to attain W_t. On the other hand, training z' has a cost $z\phi(z'/z)$ in terms of consumption goods. By assumption, the unitary cost of training is not affected by the number of workers being trained, and thus z' will be chosen to maximize the difference between the total returns and the total cost of training for each worker $\beta\pi_{t+1}^I(z') - z\phi\left(\frac{z'}{z}\right)$.

Therefore, a manager maximizes his net payoff $\pi_t(z)$ by choosing n and that z':

$$\pi_t^I(z) = \max_{\{n,z'\}\geq 0}\left\{zn^{\alpha} - n\left[W_t + z\phi\left(\frac{z'}{z}\right) - \beta\pi_{t+1}^I(z')\right]\right\}. \tag{4.2}$$

For this problem to be well defined, it is needed that $\max_{z'}\left\{\beta\pi_{t+1}^I(z') - z\phi(z'/z)\right\} < W_t$, otherwise, the effective cost of labor would be non-positive and the old entrepreneur would attain infinite profits. More interestingly, as long as the previous inequality holds, workers fully internalize the costs and benefits of skill formation.

Lemma 4.1. *Assume that $\pi_{t+1}^I(z')$ is (strictly) bounded above by $[W_t + z\phi(z'/z)]/\beta$. Then, the optimal transfer $z_t'^*(z)$ of skills is independent of the number of workers $n^*(z)$, and the payoff to*

[26] Uniform training levels for all workers results from the same restriction on $v > \alpha/(1-\alpha)$ which makes cost function $\phi(x)$ convex enough so that randomizing over the skill levels is not efficient. See Appendix B.

the old manager is

$$\pi_t^I(z) = (1-\alpha)\, z\, [n^*(z)]^{\alpha}.$$

Moreover, if $\pi_{t+1}^I(z')$ is strictly increasing, strictly convex and differentiable, then $n^*(z)$ is strictly increasing in z.

Proof. See appendix. ■

It is important to notice that the first part of the lemma holds for any arbitrary future valuation of skills, $\pi_{t+1}^I(\cdot)$, not only along the equilibrium path. The key aspect is is that $\pi_{t+1}^I(\cdot)$ is commonly perceived by both, old and young entrepreneurs. Therefore, regardless of the form of $\pi_{t+1}^I(\cdot)$, the current payoff $\pi_t^I(z)$ equals his marginal contribution to the production of goods. The net gains of skill accumulation are captured by the young workers. Notice, however, that the second part is also quite important, since in equilibrium $\pi_{t+1}^I(\cdot)$ will certainly be strictly increasing and strictly convex. Therefore $n^*(z)$ is strictly increasing in z, which, in turn, implies that $\pi_t^I(z)$ is also strictly convex.

Since young workers capture the net gains of skill accumulation, $A_t(z) \equiv \max_{z'} \left\{\beta \pi_{t+1}^I(z') - z\phi(z'/z)\right\}$, then, in equilibrium, entrepreneurs must pay the difference $w_t(z) \equiv W_t - A_t(z)$ to hire a young worker. Notice that $w_t(z)$ depends on the particular skill level z of the manager, since it defines the exposure to knowledge and learning opportunities for the young worker. For instace, if two managers have different levels $z_0 < z_1$, then $w_t(z_0) > w_t(z_1)$ because $A_t(z_1) > A_t(z_0)$. Low skill managers must compensate their workers with higher wages because of the inferior learning possibilities offered.

The first order condition for the accumulation of skills z' of trainees under a manager with skills z equates the marginal cost to the marginal revenue:

$$\phi'\left(\frac{z'}{z}\right) = \beta \frac{\partial \pi_{t+1}^I(z')}{\partial z'}. \tag{4.3}$$

On the other hand, the envelope condition is

$$\frac{\partial \pi_{t+1}^I(z')}{\partial z'} = (n')^{\alpha} + n'\left[\left(\frac{z''}{z'}\right)\phi'\left(\frac{z''}{z'}\right) - \phi'\left(\frac{z''}{z'}\right)\right], \tag{4.4}$$

which indicates that higher the amount of labor n' that young workers project to control in the next period, the higher the returns to investing in skills. Likewise, the higher the skill accumulation z'' in the subsequent period, the higher the returns of z' because a higher z' reduces the cost of accumulating z''. These forces are important in determining the aggregate dynamics of skill accumulation. As in the previous section, if a young potential entrepreneurs expects to be a worker then $n' = 0$, and the the returns of investing in skills are zero. However, contrary to that model, the individual internalizes the (dynamic) value of skills in their productivity of future skills.

I now look for a BGP in which, in every point in time, all potential entrepreneurs are active and homogeneous skills levels. I shall use specific functional form $\phi(\cdot)$, and check for parameter

conditions for which all old entrepreneurs are active and all young entrepreneurs accumulate a same level of skills that is proportional to that of the previous generation.

When all managers are active and have the same level of skills $z = Z$, each entrepreneur commands one unit of labor and trains one young manager. Combining the first order condition (4.3) and the envelope condition (4.4), impossing a constant growth rate $G = z'/z = z''/z'$, and using the functional form $\phi(\cdot)$, the constant growth rate in a BGP must satisfy:

$$v_0 G^v = \beta \left[1 + \frac{v v_0}{1+v} G^{1+v}\right]. \tag{4.5}$$

A growth rate G that solves equation (4.5) is an equilibrium BGP if it satisfies conditions: (1) it is a "maximization", i.e. the marginal cost (left-side) crosses the marginal benefit (right-side) from below; (2) the growth rate is below β^{-1}, so that the value of $\pi_t^I(\cdot)$ remains finite and net output per period is positive; and (3) all potential managers prefer to accumulate skills and becoming active entrepreneurs than to be workers in both periods. Appendix B contains the proof for the following result.

Proposition 4.2. *(Existence and Uniqueness of a BGP.) Assume that (v_0, v, β) satisfy $\beta < \left(\frac{v_0}{1+v}\right)^{\frac{1}{1+v}}$. Then there exists a unique BGP with growth rate $G \in \left(0, \beta^{-1}\right)$, i.e. solves (4.5), and the LHS crosses the RHS from below. Moreover, if $\beta > \left[1/v_0 + v/(1+v)\right]^{-1}$, then $G > 1$.*

Therefore, under Condition 6, the lowest root of equation (4.5) is an equilibrium BGP if it also satisfies conditions (2) and (3) as laid out above. If so, managerial skills evolve accordingly to $Z_n' = G Z_n$. In such BGP, the wage rate of laborers, the payoff of active managers and the discounted utility of young potential managers are, respectively

$$\begin{aligned} w &= Z\alpha, \\ \pi &= Z(1-\alpha), \\ W &= Z\left[\alpha - \phi(G) + \beta(1-\alpha)\right]. \end{aligned}$$

In this BGP, respectively, entrepreneurs are ex-post and ex-ante better-off than workers if, $\alpha < 1/2$ and $\frac{\nu_0}{1+\nu} G^\nu \leq \beta[1-2\alpha]$.

It is instructive to consider the social planner's allocation in this economy. Given a cohort of old managers, all with the same expertise Z_n, the planner must decide the units of labor to assign to each manager and the skills Z_n' to invest in each of the young workers. Because of decreasing returns, each old manager will end up commanding the same amount of labor and aggregate output of goods is $Z_n \omega \eta^\alpha$. On the other hand, forming Z_n' skills for each of the young potential managers entails an aggregate cost of $\omega Z_n \phi(Z_n'/Z_n)$. The value function $S(Z_n)$ for the planner is defined by the Bellman Equation (BE):

$$S(Z_n) = \max_{\{Z_n' \geq 0\}} \left\{ Z_n \left[\omega \eta^\alpha - \omega \phi(Z_n'/Z_n)\right] + \beta S(Z_n') \right\}. \tag{4.6}$$

Notice that the period return function $Z_n\left[\omega\eta^{\alpha}-\omega\phi\left(Z_n'/Z_n\right)\right]$ is linearly homogeneous and jointly concave in (Z_n, Z_n') and that the feasible set for Z_n' does not depend on Z_n. These properties lead to the following result:

Proposition 4.3. *Assume Condition 6 holds. Then, there is a unique value function that solves (4.6) and it has the form* $S\left(Z_n\right)=S_0Z_n$ *with* $0<S_0<\infty$ *that solves* $S_0=\max_{G\in[0,\infty]}\left\{\omega\eta^{\alpha}-\omega v_0\left(G\right)^{1+v}/\left(1+\right.\right.$ *Moreover, the value* G *that solves this maximization coincides with the* G *in the previous proposition.*

4.1. A Small Open Economy.

Consider now an initially closed country that is in a BGP but unexpectedly and permanently allows entry of foreign firms. As before, openness to foreign firms means that foreign managers can hire workers in the country. I assume that the technology frontier of both countries is the same, but that the foreign country is more advanced in the sense that its local managers have a higher level of skills, i.e. $Z_f > Z_n$. In particular, foreign managers can use also transfer skills to the workers under their control. I restrict attention to the case in which the home country is small and has not impact on the equilibrium of the foreign country, which I assume moves along a BGP.

In the absence of frictions, foreign managers must be indifferent between operating in the home country or staying abroad. Hence, if they enter the home country, it is because they earn also $\pi_f = Z_f\left(1-\alpha\right)\eta^{\alpha}$. With openness, the equilibrium is achieved by a mass of foreign managers that pushes the domestic prices of labor w_n and of young potential managers to be equal to the international prices, i.e. $w_n = w_f = Z_f\alpha\eta^{\alpha-1}$ and $W_n = W_f = Z_f\left[\alpha\eta^{\alpha-1}-\phi\left(G\right)+\beta\left(1-\alpha\right)\eta^{\alpha}\right]$. Foreign managers face the same problem and market prices as in the foreign country and hence, choose they same values $n=\eta$ and $z'=GZ_f$ units of skills. Domestic managers, however, have lower levels of skills $z=Z_d$ and might instead opt to be a worker. Otherwise, they might hire only laborers.

For simplicity I look directly at the social planner's problem, i.e. the maximization the present value of aggregate consumption of goods of the country by allocating domestic labor to domestic and foreign managers and choosing the skill formation of the country's young potential managers. The country collects the entire output of domestic firms, i.e. the sum of payments to managers, workers and young potential managers. However, for each foreign manager the country must pay out π_f. In either case, the country gathers the future returns and takes on the costs of skill accumulation.

The initial heterogeneity between domestic and foreign managers in an open economy makes it necessary to determination of different firms. However, in our case, in each period there can be at most three 'types' of managers. The first are the foreign managers, whose skills evolve exogenously to the country. The second type is "new domestic sector," which are those who were directly trained by a foreign manager, by someone who was or by some who was trained by

someone who was, etc. The third type is "deep-rooted domestic sector" which is the progeny of the domestic managers that existed in the country before it opened.

Managers in the new domestic sector have the same skills as foreign managers. To see this, recall that after the country opens, in all period $w_n = w_f$ and $W_n = W_f$. Hence, a domestic trainee receives $z' = GZ_f$, i.e. identical to foreign peers. Next period he will transfer $z'' = G^2 Z_f$ to the group of young trainees. And so on. Hence, the "state" for the optimal allocation problem is the triplet (X, Z_n, Z_f) which indicates, respectively, the fraction $X \in [0,1]$ of the mass ω of domestic managers in the old-domestic sector and Z_n their level of skills. The variable Z_f is the skill level of foreign and new-domestic managers, which grows at the exogenous rate G of the BGP. Given (X, Z_n, Z_f), a planner would first allocate labor optimally across sectors.

Given the state (X, Z_n, Z_f), the country has two options: employ the deep-rooted managers as managers or as workers. In the first case, if all X are active managers, then foreign and new-domestic firms command η units of labor and old-domestic managers command $n_n = \eta \left(Z_n/Z_f\right)^{\frac{1}{1-\alpha}}$. With this, it is easy to show that if next period the country chooses $X' \leq X$ and $Z'_n \geq 0$, then aggregate output flows, net of training costs and foreign remittances, are as follows: $Z_n \omega \left\{ X \left(Z_n/Z_f\right)^{\frac{\alpha}{1-\alpha}} \eta^\alpha - X' \phi \left(Z'_n/Z_n\right) \right\}$ from old-domestic firms, $Z_f \omega \left(1-X\right) \left\{\eta^\alpha - \phi\left(G\right)\right\}$ and $Z_f \omega \left(X - X'\right) \left\{\alpha \eta^\alpha - \phi\left(G\right)\right\}$ from new-domestic is and foreign sector are, respectively. Adding the three sources, the flow of consumption goods for the home country is:

$$\begin{aligned} & C^M\left(Z_f, Z_n, X, Z'_n, X'\right) \\ = \; & \omega Z_f \left[\eta^\alpha - \phi\left(G\right)\right] - \omega Z_f \left(X\eta^\alpha \left[1 - \alpha - \left(\frac{Z_n}{Z_f}\right)^{\frac{1}{1-\alpha}}\right] - X' \left[\alpha \eta^\alpha + \left(\frac{Z_n}{Z_f}\right) \phi\left(\frac{Z'_n}{Z_n}\right) - \phi\left(G\right)\right] \right) \end{aligned} \tag{4.7}$$

where the superscript M indicates that the country maintains active the deep-rooted sector. Notice that C^M is equal to the consumption of a country with all domestic managers with the leading Z_f skill levels, minus the net-output gap of the deep-rooted sector, minus the payout of foreign profits.

Alternatively, the country can liquidate that sector and release the labor units of old potential managers. Notice that in this case, the country has a supply of 2 units of labor. In this case, the $\omega\left(1-X\right)$ new-domestic managers demand an aggregate amount of labor equal to $\omega\left(1-X\right)\eta$. The remaining $\left[2 - \omega\left(1-X\right)\eta\right]$ units of labor must be hired by foreign managers, and since each one of them hires η, the country must pay a total $\left[2/\eta - \omega\left(1-X\right)\right]\pi_f$ of foreign profits. Using the value of π_f and simplifying, the aggregate consumption flow of the country is,

$$C^L\left(Z_f, X\right) = Z_f \left\{\eta^\alpha \left[2\alpha/\eta + \omega\left(1-\alpha\right)\left(1-X\right)\right] - \omega\phi\left(G\right)\right\}, \tag{4.8}$$

where the superscript L liquidates that the country's deep-rooted sector is being liquidated. Observe that when the country scraps this sector, the country's output might be higher than if it had all domestic managers with the leading level Z_f. However, $C^L\left(\cdot\right)$ indicates that domestic consumption might be significantly lower. On one hand, much of this output might be used to

train the country's future crop of managers. On the other hand, much of this output might flow out of the country as foreign profits.

With the functions $C^M(\cdot)$ and $C^L(\cdot)$, it is straightforward to write the Bellman Equation for the social planner's problem. First, notice that if the country liquidates the domestic deep-rooted sector, next period it would converge to the BGP of developed countries. The value $V^L(Z_f, X)$ of this option is:

$$V^L(Z_f, X) = C^L(Z_f, X) + \beta S(GZ_f), \tag{4.9}$$

where $S(\cdot)$ is the value function defined by (4.6). If the country opts to maintain the deep-rooted sector active this period, it has to choose its size and skill level for the next period. Since each foreign and new-domestic manager in the country trains one young potential manager, the value $V^M(Z_f, Z_n, X)$ of this option is:

$$V^M(Z_f, Z_n, X) = \max_{\{Z'_n \geq 0, 0 \leq X' \leq X\}} \left\{ C^M(Z_f, Z_n, X, Z'_n, X') + \beta V(GZ_f, Z'_n, X') \right\}, \tag{4.10}$$

where the value function $V(Z_f, Z_n, X)$ is defined by

$$V(Z_f, Z_n, X) = \max \left\{ V^M(Z_f, Z_n, X), V^L(Z_f, X) \right\}. \tag{4.11}$$

Direct inspection reveals that $C^M(\cdot)$ and $C^L(\cdot)$ are linearly homogeneous in $\left(Z_f, Z_n, Z'_f, Z'_n\right)$. This property simplifies the proof for the following result:

Proposition 4.4. *Assume Condition 6 holds. Then, there is a unique value function V that solves the Bellman Equation defined by (4.7), (4.8), (4.9), (4.10) and (4.11). Moreover: (a) If $Z_n < Z_f$ then $V(Z_f, Z_n, X)$ is strictly decreasing in X; (b) if $X > 0$, there is a $r_0 > 0$ st. $V(Z_f, Z_n, X) = V^L(Z_f, X)$ for $0 \leq Z_f, Z_n \leq r_0$, while $V(Z_f, Z_n, X) = V^M(Z_f, Z_n, X)$ for $Z_f/Z_n > r_0$. In the latter case, $V(Z_f, Z_n, X)$ is strictly increasing in Z_n.*

The intuition of this result is straightforward. The farther behind is a country with respect to developed countries, i.e. the lower Z_n, the lower its welfare. Even if the country choose to scrap the old domestic sector and temporarily exhibits a large output level. Notice that the lower is Z_n/Z_f, the more likely a country is to liquidate the laggard deep-rooted sector and catch up next period with developed countries in terms of skills and income levels. On the other hand, given $Z_n/Z_f < 1$, the higher the fraction of managers X that are lagging behind, the lower the welfare of the country.

[Insert Figure 5: opening up: fraction, relative productivity, aggregate output]

Figure 5 considers the evolution of an closed economy that opens up permanently to hosting foreign firms. In the initial period, $X = 1$ and $Z_n/Z_f < 1$. The optimal response is for the deep-rooted sector to shrink over time and fully disappear in finite time. The optimal response is to gradually reduce Z_n/Z_f. As can be expected, in the period before the country opts to scrap

the deep-rooted sector, the social planner sets $Z'_n = 0$, since, obviously, it is pointless to form managerial skills in individuals that would be workers in the next period. The figure shows that the mass of new-domestic firms grows over time. Eventually this sector controls the supply of domestic workers and train the entire crop of young potential entrepreneurs. When a country reaches that point, the country has converged and foreign managers no longer step in.

It is important to notice that the transfer of skills from multinational firms materialize in a new sector of firms, not in the pre-existing sector of firms. This is indeed, somewhat in line with empirical findings. The model implies that the presence of foreign firms should hurt the productivity of pre-existing firms –because of the competition effect and the absence of spillovers. However, the economy as a whole fully catches up. When facing competition, the optimal behavior of foreign firms is to use their ability to form national skills in order to hire local workers.

As with the model with spillovers, with internalized diffusion, a developing country catches up with developed countries only when the less productive domestic know-how is fully replaced by more advanced foreign know-how in the formation of skills of future generations. Moreover, the two models are compatible with leapfrogging, since laggard countries converge more quickly than more advanced developing countries. However, there are important differences. First, with internalized diffusion all developing countries eventually catch up. This is far from being the case in the model with spillovers. Indeed, in the latter model, leapfrogging is not only in terms of speed of convergence, but also whether such convergence takes place at all. Second, the impact of openness is drastically different in the two models. With national spillovers, the ideas of foreign firms impact the formation of skills for all the agents in the economy. The dynamics is governed by the skill ratio of domestic-to-foreign managers. With internalized diffusion, the presence of foreign firms lead to the emergence of a new sector of domestic firms. This new sector is at par with the firms in developed countries, and eventually overtakes the whole economy. Along the adjustment, the deep-rooted group of firms decline in size and in relative (and absolute) skills and eventually disappear. Third, by construction, with internalized diffusion the allocation are optimal. There is no room left for government policy. On the contrary, many interesting issues for government policy may arise in the model of spillovers. In particular, since the current (future) presence of foreign firms enhances (impairs) the accumulation of domestic skills, time-consistency might limit the ability to implement the optimal national policy. This issue should be examined further.

5. The General Case: Externalities and Internalized Learning

To be added.

6. Concluding Remarks.

In this paper I used simple general equilibrium growth models to study the impact of multinational firms on the formation of skills and the long run behavior of output a small developing country.

Within the context of a simple model environment, I examined three different growth models: (a) an exogenous growth model, (b) an endogenous growth model with an externality in the formation of skills and (c) an endogenous growth model in which skills are internally produced in the firm. The impact of multinational firms on the host country in models (a) and (b) is via spillovers. These two models are rather standard in the growth literature and the existence and measurement of spillovers have been the subject of a vast empirical literature. Spillovers are also the tenet underlying much debate and policy proposals and programs. I show that spillovers are not sufficient to propel the country to catch up with developed countries.

In model (c) there are no spillovers and any transfer of skills is the result of a market transaction. In a competitive environment, firms will optimally transfer a level of skills that gradually obliterate the existent sector and creates a new sector of domestic firms that are at par with the ones in developed countries. In finite time, the small country converges to the income level of developed countries. Spillovers are not necessary.

These results are very suggestive about the role of government policy. In model (a), a benevolent government would definitely want to subsidize foreign firms. In model (b) optimal policy can be quite kinky. For instance, if local skills are very low, a subsidy would be pointless since the country will converge next period. For higher initial level of skills, the government may want to subsidize foreign firms fully obliterating the local firms for one period and converge to the developed country level in the next. The competition effect in this model can also introduce interesting issues of time consistency.

In model (c) the equilibrium is efficient and the government must not subsidize. However, if there are obstacles to the transmission of skills from foreign firms to local agents, a government may want to undo the obstacle by providing a subsidy. But it is necessary to go beyond these somewhat speculative arguments. Optimal government policy in models with obstacles to the flow of firms and/or the transmission of skills deserve a rigorous and comprehensive consideration.

A. Proofs

Proof of Lemma XXX (Transition Function for Endogenous Growth with Spillovers)

First, notice that if the conditions for (a) hold, then necessarily the economy will reach the set $[0, R_S]$ in a finite number of steps and therefore converge to 1. On the other hand, if the conditions for (b) hold, given that the function Γ_0 crosses the 45^0 line from above and that it is continuous, there is an $\epsilon > 0$ such that the ball $B(R_L, \epsilon)$ is such that $B(R_L, \epsilon) \subset (R_L, 1)$, $\Gamma_0[B(R_L, \epsilon)] \subset B(R_L, \epsilon)$, and $\Gamma_0'(x)$ all $x \in B(R_L, \epsilon)$. Therefore, R_L is locally stable. Finally, if $\Gamma_0(x) > R_S$ and $\Phi(x) > \alpha$, then if the economy starts in a position where old agents become managers, it will always remain there, and in this case the limiting point is R_L.■

Characterization of the Manager's problem. To prove (1) notice that the maximization can be factorized, i.e.:

$$\begin{aligned}
\pi_t(z) &= \max_{\{(n_1,n_2,z')\geq 0\}} \left\{ z(n_1+n_2)^{\alpha} - n_1 \left[W_t + z\phi\left(\frac{z'}{z}\right) - \beta\pi_{t+1}(z') \right] - wn_2 \right\} \\
&= \max_{(n_1,n_2)\geq 0} \left\{ z(n_1+n_2)^{\alpha} - W_t n_1 - wn_2 + n_1 \left[\max_{\{z'\geq 0\}} \left\{ \beta\pi_{t+1}(z') - z\phi\left(\frac{z'}{z}\right) \right\} \right] \right\}.
\end{aligned}$$

Since the inner maximization is independent of the choice of n_1 and n_2, the result follows. To prove (2), take the (or one of, if there are multiple ones) optimal choice of $z'^*(z)$. Then, the remaining problem is to choose n_1, n_2 optimally. This is a maximization of a concave objective function with a convex feasible set, and under Condition 5, $\pi_t(z)$ is bounded. The first order conditions with respect to n_1 and n_2 are sufficient and are respectively given by:

$$\begin{aligned} z\alpha\left[n_1^*(z)+n_2^*(z)\right]^{\alpha-1} &\leq W-\beta\pi_{t+1}\left[z'^*(z)\right]+z\phi\left[\frac{z'^*(z)}{z}\right],\ n_1\geq 0,\\ z\alpha\left[n_1^*(z)+n_2^*(z)\right]^{\alpha-1} &\leq w,\ n_2\geq 0, \end{aligned}$$

and in both cases at least one inequality holds with equality. Given z, the manager can (a) only hire laborers, (b) only hire young potential managers and (c) hire both. Assume the latter is the case, i.e. $n_1>0$ and $n_2>0$. This can only happen if $w=W-\beta\pi_{t+1}\left[z'^*(z)\right]+z\phi\left[z'^*(z)/z\right]$. The net payoff of the manager is

$$\begin{aligned} \pi_t(z) &= z\left[n_1^*(z)+n_2^*(z)\right]^{\alpha}-wn_2^*(z)-\left[W-\beta\pi_{t+1}\left[z'^*(z)\right]+z\phi\left[z'^*(z)/z\right]\right]n_1^*(z),\\ &= z\left[n_1^*(z)+n_2^*(z)\right]^{\alpha}-z\alpha\left[n_1^*(z)+n_2^*(z)\right]^{\alpha-1}\left[n_2^*(z)+n_1^*(z)\right], \end{aligned}$$

where the second lines uses the F.O.C.s with equality. From here, the result is straightforward. The same argument applies to cases (a) and (b). To prove part (3) notice that in case (a) $n_1^*(z)+n_2^*(z)=n_2^*(z)=\left[\alpha z/w\right]^{1/(1-\alpha)}$, proving the statement for this case. Now, consider case (b). Rearranging the first order condition,

$$n_1^*(z)=\left\{\alpha z/\left[W-\beta\pi_{t+1}\left[z'^*(z)\right]+z\phi\left[z'^*(z)/z\right]\right]\right\}^{1/(1-\alpha)}.$$

Using the envelope condition and the functional form of $\phi(\cdot)$ it can be shown that,

$$\frac{\partial \ln n_1^*(z)}{\partial z}=\left(\frac{1}{1-\alpha}\right)\left\{\frac{1}{z}+\frac{\left(\frac{\nu\nu_0}{1+\nu}\right)\left[z'^*(z)/z\right]^{1+\nu}}{W-\beta\pi_{t+1}\left[z'^*(z)\right]+z\phi\left[z'^*(z)/z\right]}\right\}>0,$$

which, of course, implies that $\partial n_1^*(z)/\partial z>0$. Case (c) follows from cases (a) and (b).■

Existence and Uniqueness of a BGP. As a short hand, let $L(G)\equiv v_0G^v$ and $R(G)\equiv\beta\left[\eta^{\alpha}+\frac{vv_0}{1+v}G^{1+v}\right]$. Obviously, $L(0)=0<R(0)=\beta\eta^{\alpha}$. Moreover, since the curvatures of L and R are ν and $1+\nu$, respectively, and $L'(0)=R'(0)=0$, then, the derivatives cross only once, i.e. $\exists!\ \bar{G}\in(0,\infty)$ implicitly defined by $L'\left(\bar{G}\right)=R'\left(\bar{G}\right)$, $L'(G)>R'(G)$ for $G<\bar{G}$ and $L'(G)<R'(G)$ for $G>\bar{G}$. It so happens that $\bar{G}=\beta^{-1}$. Then, if $L\left(\beta^{-1}\right)>R\left(\beta^{-1}\right)$, then L crosses R twice, once for below and once from above. If $L\left(\beta^{-1}\right)=R\left(\beta^{-1}\right)$, then, the unique crossing is $G=\beta^{-1}$. Finally, $L\left(\beta^{-1}\right)<R\left(\beta^{-1}\right)$, then $L(\cdot)<R(\cdot)$ in all the positive reals. It can be directly shown that $\beta<\left(v_0/\left[\eta^{\alpha}(1+v)\right]\right)^{\frac{1}{1+v}}$ implies that $L\left(\beta^{-1}\right)>R\left(\beta^{-1}\right)$, and hence that there exists a single $G<\beta^{-1}$ where L crosses R from below, i.e. satisfies the maximization condition. On the other hand, it can be directly verified that if $\beta>1/\left[\eta^{\alpha}/v_0+v/(1+v)\right]$, then $L(1)<R(1)$, and hence, the lowest crossing is above 1.■

The Social Planner's Problem in a Closed Economy. First, notice that the problem can be equivalently written as the optimal choice of the rate of growth $G=Z_n'/Z_n$, i.e.:

$$S(Z_n)=\max_{\{G\geq 0\}}\left\{Z_n\left[\omega\eta^{\alpha}-\omega\phi(G)\right]+\beta S(GZ_n)\right\}.$$

Since the return function is homogeneous of degree one in Z_n and the feasible set is clearly convex, then, following Alvarez and Stokey (1998), it can be shown the functional operator defined by this BE maps linearly homogeneous functions into linearly homogeneous functions. Hence, the unique fixed point of this BE is of the form $S(Z_m)=S_0Z_m$ for a constant $S_0>0$. Then, plugging $S(Z_m)=S_0Z_m$, in the objective function, the constant S_0 must solve (**??**). This is a maximization of a concave function in a convex set, so the first order conditions are sufficient and given by

$$\omega v_0G^v=\beta S_0, \tag{A.1}$$

which implies that $G = [\beta S_0/(\omega v_0)]^{\frac{1}{\nu}}$ and, then the constant S_0 satisfies

$$\begin{aligned} S_0 &= \omega\eta^\alpha - \omega v_0 \frac{\left(\left[\frac{\beta S_0}{\omega v_0}\right]^{\frac{1}{\nu}}\right)^{1+v}}{1+v} + \beta\left[\frac{\beta S_0}{\omega v_0}\right]^{\frac{1}{\nu}} S_0 \\ &= \omega\eta^\alpha + \left(\frac{1}{\omega v_0}\right)^{\frac{1}{\nu}}\left(\frac{\nu}{1+v}\right)(\beta S_0)^{1+\frac{1}{\nu}} \end{aligned}$$

Since the first order condition implies $S_0 = \omega v_0 G^v/\beta$, which plugged in the previous equation leads to (4.5). Therefore, under Condition 6, there is a unique fixed point and it coincides with the equilibrium G defined in the previous proposition.■

The Social Planner's Problem in an Open Economy. Define $r \equiv Z_n/Z_f$, and since $C^M(\cdot)$ and $C^L(\cdot)$ are HD1 in $\left(Z_f, Z_n, Z_f', Z_n'\right)$, then $C^M(\cdot) = Z_f c^M(r, X, r', X')$ and $C^L(\cdot) = Z_f c^L(X)$ where

$$\begin{aligned} c^L(X) &= \eta^\alpha\left[2\alpha/\eta + \omega(1-\alpha)(1-X)\right] - \omega\phi(G), \\ c^M(r, X, r', X') &= \omega\left\{\eta^\alpha - \phi(G) - X\eta^\alpha\left[1-\alpha-(r)^{\frac{1}{1-\alpha}}\right] - X'\left[\alpha\eta^\alpha + r\phi\left(\frac{r'}{r}G\right) - \phi(G)\right]\right\}. \end{aligned}$$

With these functions, the BE can be written in normalized terms as:

$$\begin{aligned} v^L(X) &= c^L(X) + \beta S_0 G, \\ v^M(r, X) &= \max_{\{r'\geq 0, 0\leq X'\leq X\}}\left\{c^M(r, X, r', X') + \beta G v(r', X')\right\}, \\ v(r, X) &= \max\left\{v^M(r, X), v^L(X)\right\}, \end{aligned}$$

where $S_0 > 0$ is as defined for the closed economy. The first equation uniquely defines the function $v^L(X)$ and since Condition 6 implies that $\beta G < 1$, this Bellman Equation defines a contraction mapping $T : \Upsilon \to \Upsilon$ on the set of bounded and continuous functions Υ defined over $[0,1]\times[0,1]$. This establishes existence and uniqueness of v. Notice that $c^M(r, X, r', X')$ is strictly increasing in r and the feasible set for $\{L, M\}$ and, if M, for $\{r', X'\}$ are independent of r, then, if v is weakly increasing in r, v^M is strictly increasing in r. Notice that for any $X \in (0,1)$, $v^M(0, X) < v^L(X)$. Because of the theorem of the maximum, both v^M and v^L are continuous functions, then, for all X there must exist an r_0 st. $v^M(r_0, X) = v^L(X)$. This proves (b). Part (a) follows directly from the fact that $c^L(\cdot)$ and $c^M(\cdot)$ are strictly decreasing in X. ■

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