

# Rehabilitated or Not?: To Release (?) is the Question\*

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## Abstract

When parole boards learn whether inmates are rehabilitated by observing their behavior in prison, we show why they would release one inmate, while continuing to incarcerate another with a longer sentence, but who is otherwise observationally identical. This reflects that the longer a parole board has discretion, the more valuable is additional information gleaned from observing behavior. A consequence is that an increase in sentence length can lead to even greater increases in expected time served. We also consider the effect of increased sentences on inmates' incentives to undertake rehabilitative effort. To encourage effort, sentences cannot be too short, but when inmates discount the future sufficiently, long sentences may also be undesirable. We show how different parole board priors can support multiple equilibria in rehabilitation effort, and investigate the effects of discretion restrictions like parole eligibility.

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# 1 Introduction

In this paper, we consider the simple problem of a public authority that must decide whether to release prisoners before the end of their terms, and, if so, when. The state does not value punishment for punishment's sake, and in contrast to many economic models of crime,<sup>1</sup> it ignores deterrence as a motive for incarceration. The sole reason for incarceration in our model is crime prevention through the incapacitation of criminals,<sup>2</sup> and hopefully through their rehabilitation. The state gains from releasing inmates who have been rehabilitated and will become productive contributors to society, and loses from releasing recidivists who return to a life of crime.<sup>3</sup>

The choice of whether to grant early release is not blind. Common sense dictates that public authorities should release inmates early only if they believe that inmates will be successful on parole—in other words, they believe inmates are rehabilitated and will not commit more crimes.<sup>4</sup> However, a formal model describing such a process, and its implications for prisoners' behavior has yet to be delivered.<sup>5</sup>

Our central premise is that parole authorities use institutional behavior as a predictor of future parole success. The parole authorities have prior beliefs about an inmate's potential parole success or likelihood of recidivism that they update on the basis of behavior observed in prison. Parole authorities acquire information by observing over whether the inmate engages in incidents of bad behavior. 'Bad' behavior is correlated with being a recidivist, so an absence of incidents signals likely rehabilitation, and therefore, parole success. The incidents we have in mind can range from

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<sup>1</sup>For example, Becker (1968) and much of the subsequent literature consider deterrence the primary objective of incarceration.

<sup>2</sup>Barbarino and Mastrobuoni (2007) estimate that the incapacitation effect of prison on crime is large. Incarceration rates for men in 1993 were about 2% of male labor force participation, and including parole and probated men, comprised 7% of the male labor force.

<sup>3</sup>Behind our model, one could imagine a conflict between different state actors. We model only the decisions of a parole board concerned with incapacitation. Other actors, for example, legislators and judges who determine overall sanctions, may be also motivated by deterrence and punishment.

<sup>4</sup>Avio (1973) provides an early discussion of the potential benefits of parole.

<sup>5</sup>Miceli (1994) and Garoupa (1996) view parole as a principal-agent problem where early release is used to promote good behavior in prison. Other studies like Fabel and Leung (1995) or Meier (1999) look at the deterrence aspect of parole, while Ehrlich (1981) takes the rehabilitation process as given, and analyzes recidivism in a general equilibrium context. Scoones (2007) looks at the deterrence value and the cost reduction benefits of statutory releases that are almost entirely independent of an individual's characteristics and behavior.

violation of prison rules to non-compliance with a training program.

This modeling assumption mirrors practice: Pogrebin et al.(1986) cite four primary predictors of recidivism—the nature of the offense, the inmate’s prior record, personal history, and institutional behavior<sup>6</sup>—and while authorities take all of these factors into account in their decisions, Carroll and Mondrick (1976) find that the best predictor of parole decisions is inmates’ institutional behavior. Kuziemko (2007) also finds that parole boards do indeed assign longer terms to those with higher initial risks, and that inmates who no longer had a chance at early release were more likely to accumulate disciplinary infractions, less likely to complete courses and more likely to return to prison after release.

We begin by investigating the state’s optimal release policy when rehabilitation is completely exogenous, so that nothing inherent about the inmate changes over his time in prison. We consider two types of inmates: those who will re-commit and those who will not. It is desirable to release inmates who will not recommit since incapacitation (and not deterrence) is the sole goal of imprisonment. We then extend our analysis to consider the possibility that prisoners can invest effort in the hope of rehabilitation. The empirical evidence suggests that incarceration alone is ineffective at inducing rehabilitation (Myers (1983), Grogger (1991)). However, other studies find success when inmates fully participate in rehabilitation programs such as vocational training programs – Witte (1977), Lattimore et al. (1990). Together these two facts suggest that effort on the part of inmates matters for rehabilitation.

Our paper begins by developing the following simple proposition about release times. Given two inmates about whom the authorities have identical beliefs about their prospects for rehabilitation, the state will release first the inmate with the shorter sentence. A simple option argument explains why even though the state holds identical beliefs, it should release first the inmate with the shorter sentence. For each inmate, the state essentially has an option with time until expiry equal to the length of the sentence. The state exercises the option by releasing the inmate and collecting the positive dividends if the individual is rehabilitated, and the negative dividends if he is a recidivist. The state retains the option by keeping the inmate in prison.

*Ceteris paribus*, the longer the sentence, the more valuable it is to hold onto the

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<sup>6</sup>Criminologists and economists such as Carr-Hill and Carr-Hill (1972), Schmidt and Witte (1989), or Worthington et al. (2000) have tried to predict recidivistic probabilities with limited success, using different methodologies and combinations of the aforementioned characteristics.

option rather than exercise it. The expected annual benefit earned from releasing an inmate is not affected by the sentence length, but longer sentences increase the value of information gleaned from observing an inmate's behavior in prison. This is because the benefits from decisions based on additional observations can be exploited for longer. To see this most clearly, consider an inmate whose sentence is nearly completed: there is no value to learning about such an inmate's prospects for rehabilitation, because the inmate must then be released. If, instead, the sentence is longer, then this information has value because the state can condition its release decision on whether the inmate appears reformed.

This analysis suggests that a move toward longer sentences will be associated with a greater reluctance by the state to grant parole. Indeed, we show that the average increase in time served may *exceed* the increase in sentence. This reflects that an increase in sentence not only delays release directly, but this delay may also lead the state to observe behavior inconsistent with rehabilitation.

Within this environment, we consider the incentive for an inmate to undertake costly effort to rehabilitate themselves. We show that to encourage rehabilitative effort, (i) sentences must not be too short, but (ii) excessively long sentences may also discourage rehabilitation. If sentences are short, the gains from early release are too small relative to the cost. As the sentence becomes increasingly long however, inmates may also not wish to undertake rehabilitative efforts, since the state's optimal response to a longer sentence is to delay release dates following good behavior. While maximal sentences only reduce rehabilitative efforts when inmates are sufficiently impatient, we view this scenario as particularly plausible, as impatience and/or impulse are required to reconcile much criminal behavior. For example, Levitt and Venkatesh (2000) provide evidence that while the hourly wages from drug activities slightly exceed the legitimate labor market alternative, "the enormous risks of drug dealing more than offset this small wage premium"<sup>7</sup>, indicating that economically-motivated criminals must discount heavily. There is also strong evidence that much crime is impulsive in nature,<sup>8</sup> with a physiological basis (see Krakowski (2003)), suggesting that individuals who engage in criminal activity, weight the present heavily relative

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<sup>7</sup>Over a four year period, 28% died, with an average of 2.4 wounds, 6 arrests, and an average wage of about \$7.75 in 1995 dollars; Freeman (1996) estimates hourly wages of \$10 for criminal activity in Boston in 1989.

<sup>8</sup>Tonry and Farrington (1995) observe that "...much crime is committed on impulse. . . by offenders who live from moment to moment."

to distant future outcomes.

These results point naturally to the observation that with prisoner decisions at the aggregate level affecting parole board beliefs and these beliefs affecting the decision of whether to undertake costly rehabilitative effort, multiple prison equilibria may exist; we refer to this phenomena as different prison cultures. There is a substantial sociology literature that discusses the “informal structure” or culture of prisons (see, e.g., Camp et al. (2003)). In economics, Katz et al. (2003) and Chen and Shapiro (2007) have explored the relationship between prison conditions and recidivism and found conflicting results. In this paper, we show that a prison culture that generates optimistic beliefs about prisoners’ rehabilitation statuses reinforces incentives to exert rehabilitative effort, and a prison culture with pessimistic beliefs has the opposite effect. In this way, similar conditions could generate multiple equilibria featuring very different patterns of rehabilitation and recidivism.

Our paper suggests that sentencing discretion is important in eliciting rehabilitative effort. However, legislators concerned with deterrence may have an incentive to restrict judicial discretion in order to guarantee punishment severity; for example, restrictions have been widely imposed in the United States in recent years in the form of both sentencing guidelines and mandatory minimum sentences. We show how restricting parole board discretion in order to increase deterrence could significantly alter incentives to undertake rehabilitation. We show that although an increase in sentences through discretion restrictions may reduce crime, it can also imply increased prison population and hence higher expenditures by the state, and possibly less rehabilitation, with correspondingly higher rates of recidivism.

In the next section, we develop the basic model of parole decisions. In Section 3, we consider the decision of an inmate to undertake rehabilitative effort, show how the sentence length affects rehabilitative incentives, and characterize when different parole board beliefs about rehabilitative effort can support multiple equilibria. In Section 4, we look at the impact of judicial discretion and parole eligibility. Following our concluding remarks, we provide an appendix containing all proofs.

## 2 The Basic Model

Consider an individual who has been convicted of a crime and sentenced to a maximum term of  $T$  years in prison. There are two types of prisoners: “rehabilitated”

prisoners who have learned their lesson and will not return to crime upon release, and “non-rehabilitated” prisoners who will continue down the road of crime. The cost of holding any prisoner is  $B$ . The cost of releasing a non-rehabilitated inmate who will return to a life of crime is  $C'$ , where  $C' > B > 0$ . Without loss of generality, we normalize the cost of releasing a rehabilitated inmate to zero. Thus, instantaneous costs are given by

$$\begin{array}{c} \text{IN} \\ \text{OUT} \end{array} \begin{array}{cc} \text{Non - Rehabilitated} & \text{Rehabilitated} \\ \left( \begin{array}{cc} B & B \\ C' & 0 \end{array} \right) \end{array}.$$

It is useful to re-examine these payoffs by looking at the costs and benefits of release. There is a benefit,  $B$ , of releasing a rehabilitated inmate, thereby avoiding the costs of imprisonment; and a cost  $C = C' - B$  of releasing a non-rehabilitated inmate.

The parole board has a prior belief  $\pi_0$  that a given inmate is rehabilitated. We assume that  $\pi_0 B < (1 - \pi_0)C$ ; otherwise it is optimal for the state to release the inmate immediately at time zero. If the inmate has not been reformed, then with instantaneous probability  $\lambda_n > 0$ , he will be involved in a prison incident at any given point in time. If rehabilitated, the inmate is less likely to misbehave in prison: the instantaneous probability that a rehabilitated inmate is involved in a prison incident is  $\lambda_r < \lambda_n$ . Throughout, except where explicitly noted, we assume that the same parameters characterize all inmates. In particular, differences in sentence lengths reflect the capricious nature of the legal system, rather than different priors over the prospects for rehabilitation. To ease presentation, we assume that the parole board does not discount the future. We use  $\Delta t$  to indicate a short period of time, over which the probability of multiple incidents is second order.

As prison incidents are generated by a Poisson distribution, the probability an inmate of type  $i \in \{r, n\}$  is involved in exactly  $j$  incidents by time  $t$  is given by  $\lambda_i^j e^{-\lambda_i t}$ . If an inmate has been involved in  $j$  incidents by time  $t$ , the state then places probability

$$\pi_t(j) = \frac{\pi_0 \lambda_r^j e^{-\lambda_r t}}{\pi_0 \lambda_r^j e^{-\lambda_r t} + (1 - \pi_0) \lambda_n^j e^{-\lambda_n t}} \quad (1)$$

on the possibility that the inmate is rehabilitated.

Over the very last  $\Delta t$  moments of an inmate’s sentence, it is optimal for the state to release the inmate if and only if the expected benefits from doing so are positive, i.e., if and only if  $\pi_{T-\Delta t}(j)B \geq (1 - \pi_{T-\Delta t}(j))C$ . Information gathered over the final

stretch  $\Delta t$  of the sentences is irrelevant as the inmate must be released anyway, so that the option value of keeping the inmate longer is zero. More generally, the state weighs the expected benefits from release against the option value of continued incarceration.

The expected payoff to society of releasing an inmate with a  $T$ -year sentence and  $j$  prison incidents at time  $t$  of his sentence, with  $T - t$  remaining, is then<sup>9</sup>

$$V_T^t(j) = (T - t)[\pi_t(j)B - (1 - \pi_t(j))C] = (T - t)[\pi_t(j)(B + C) - C]. \quad (2)$$

Define  $W_T^t(j)$  to be the option value of keeping such an inmate in prison longer, and let  $Z_T^t(j) = \max\{V_T^t(j), W_T^t(j)\}$  be the expected value to society of taking the more attractive alternative—release or continued imprisonment. Here,

$$\begin{aligned} W_T^t(j) = & [\pi_t(j)e^{-\lambda_r\delta} + (1 - \pi_t(j))e^{-\lambda_n\delta}] Z_T^{t+\delta}(j) \\ & + [\pi_t(j)\lambda_r e^{-\lambda_r\delta} + (1 - \pi_t(j))\lambda_n e^{-\lambda_n\delta}] Z_T^{t+\delta}(j + 1). \end{aligned} \quad (3)$$

Note that  $W_T^t(j)$  incorporates the value of decisions based on additional observation: the future decision to release will be based in part on whether the inmate had an additional incident during the time interval  $[t, t + \delta]$ . Moreover, in the last moment of a sentence, the option value of incarceration is 0:  $W_T^T(j) = Z_T^{T+\delta}(j) = 0, \forall j$ .

We begin with two useful lemmas. The first shows the more likely an inmate is to be rehabilitated, the more attractive it is for the state to release him. The second shows that if it is not optimal to release an inmate at some time  $t$ , then the state should not release the inmate if he is involved in an incident in the next short  $\delta$  period of time.

**Lemma 1** *Consider two inmates, Upperbar and Underbar, about whom the state has beliefs for rehabilitation of  $\bar{\pi}_t > \underline{\pi}_t$ . Then if it is optimal to release inmate Underbar, it is also optimal to release inmate Upperbar, about whom the state is more optimistic.*

**Lemma 2** *If an inmate is not released at time  $t$  and has an incident in  $[t, t + \delta]$  of his sentence, then he will not be released at  $t + \delta$ .*

The second lemma reflects that if an inmate were released after an incident, then his release would be non-contingent and would generate positive expected benefits to society. But, then it would be optimal to release the inmate earlier in order to exploit

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<sup>9</sup>For simplicity, we assume that a recidivist is never returned to prison. Allowing for this possibility does not alter our results qualitatively, and therefore, we abstract from this possibility.

further these expected benefits. Proposition 1 below states that if two inmates have the same number of prison incidents and time remaining in the sentence, the one with the shorter sentence is released only if the one with the longer sentence is.

**Proposition 1** *Suppose that at time  $t$  an inmate with a sentence of length  $T$  and  $j$  prison incidents would be released. Then, an otherwise identical inmate with a sentence length  $T + \Delta T$  and  $j$  incidents would be released before time  $t + \Delta T$ ,  $\Delta T > 0$ .*

Intuitively, the state has more optimistic and tighter beliefs about the chances for rehabilitation of the inmate with the longer sentence who has had fewer incidents per unit of time served; and because both have the same time remaining in their sentence, it is more attractive to release the inmate who has served more time, about whom the state is more optimistic.

The next proposition captures the crux of the option argument. *Ceteris paribus*, the expected immediate benefit earned from releasing any inmate is the same, but the value of the information gleaned from observing an inmate's behavior in prison is greater for a prisoner with a longer sentence. This is because the benefits from decisions based on the better information can be exploited for longer.

**Proposition 2** *Consider two inmates who face sentences of different lengths, but are otherwise identical. If the inmate with the longer sentence is released, then so will the inmate with the shorter sentence.*

Propositions 2 imply that a  $\Delta T$  increase in legislated sentences causes an inmate with  $j$  incidents to serve an additional  $\Delta t(j) > 0$  without new prison incidents before being released. With an increase in the length of sentences, comes an increase in the value of observing inmates for longer. Because  $\lambda_r < \lambda_n$ , a parole board will want to wait longer before releasing an inmates with a given number of prison incidents. Proposition 1 implies that for inmates receiving early release, each  $\Delta t(j) < \Delta T$ , meaning that were sentences to increase by six months, then a individual would need to serve less than six more months without new incidents before being granted an early release: waiting the full six months before releasing the inmate would only make sense were the parole board's beliefs the same after six months, but, in fact, the board now holds more optimistic beliefs.

Obviously, this change in early release threshold increases the overall expected time served. Moreover, rehabilitated and non-rehabilitated inmates are affected differently by this change in threshold. To illustrate this most transparently, we consider



a setting in which rehabilitated inmates always have clean prison records ( $\lambda_r = 0$ ). In this special case, a single reported prison incident causes the prisoner to serve the full sentence.

**Impact of  $\Delta T$  increase in prison sentence on time served when  $\lambda_r = 0$ .** When  $\lambda_r = 0$ , Proposition 1 implies that a  $\Delta T$  increase in prison sentence only causes a rehabilitated prisoner to remain incarcerated for the additional  $\Delta t(0) < \Delta T$  period of time, after which he or she is released with certainty. In contrast, the increase in sentence length causes a non-rehabilitated criminal to serve on average an extra

$$[e^{-\lambda_n t} \Delta t(0) + (1 - e^{-\lambda_n t}) \Delta T] + [1 - e^{-\lambda_n \Delta t(0)}] (T + \Delta T - t),$$

before being released. Under the original sentence  $T$ , a non-rehabilitated inmate involved in an incident prior to  $t$ , would have to serve the full sentence. Now, the same individual must serve an additional  $\Delta T$ . Similarly, inmates who are released early must serve the extra  $\Delta t(0)$  before being released. The first bracketed expression captures what we refer to as the “incapacitation effect” of longer sentences. The incapacitation effect simply accounts for the fact that longer sentence causes inmates to be imprisoned longer, whether they are released early or have to serve the full sentence.

In addition to the incapacitation effect, inmates are also exposed to what we refer as the “sorting effect” of longer sentences. Imagine a non-rehabilitated inmate who is on the cusp of being released at time  $t$ , and now has to serve an additional  $\Delta t(0)$ . Any incident during this additional period of time would force this inmate, who would have been released otherwise, to serve the full sentence  $T$  instead. This sorting effect, represented by the second term, is simply the product of the probability of being involved in an incident during the additional  $\Delta t(0)$  period of time, and the increment in incarceration period ( $T + \Delta T - t$ ).

Note that because rehabilitated inmates are never involved in prison incidents, they are not subject to the sorting effect. This difference between inmate types in sorting effects is important. Quite generally, non-rehabilitated inmates suffer more from an increase in sentence simply because rehabilitated inmates are less likely to be involved in prison incidents. As we will see in the next section, this will influence the choice of rehabilitation effort by inmates. ■

We next show that an increase in legislated sentences can lead to an even longer expected incarceration period: A one month increase in legislated sentences could

raise the expected time served by more than one month. The larger is the sorting effect, the greater is the increase in expected incarceration period.

To formalize this intuition, we first need to derive the optimal release date following any given number of incidents. Define  $\hat{t}(j)$  to be the optimal release date for an inmate with  $j$  prison incidents. In particular, the number of incidents is a sufficient statistic, as the posterior at any time  $t$  following  $j$  incidents does not depend on the *timing* of the  $j$  incidents (even though the fact that someone has had  $j$  incidents reveals information about the timing of those incidents — the inmate must have had  $k + 1$  incidents by  $\hat{t}(k)$ ,  $k = 0, \dots, j - 1$ , else he would have been released prior to  $\hat{t}(j)$ ). The following guarantees the existence and uniqueness of  $\hat{t}(j)$ .

**Lemma 3** *An inmate with  $j$  incidents is released at  $\hat{t}(j)$ , where  $\hat{t}(j)$  solves  $V_T^{\hat{t}(j)}(j) = W_T^{\hat{t}(j)}(j)$  if such a  $\hat{t}(j)$  exists, and  $\hat{t}(j) = T$  otherwise.*

Early release dates  $\hat{t}(j)$  depend on the number of incidents an inmate experiences. Lemma 2 already revealed that incidents postpone release, i.e.,  $\hat{t}(j)$  is strictly increasing in  $j$ , for  $\hat{t}(j) < T$ . Quite generally, an inmate is released at  $\hat{t}(j)$  if he had  $j$  incidents at time  $\hat{t}(j)$ , and at least  $y + 1$  incidents at  $\hat{t}(y)$ ,  $y = 0, 1, \dots, j - 1$ . Moreover, Proposition 2 implies that  $\hat{t}(j)$  is an increasing function of the sentence  $T$ , i.e., longer sentences delay early release. Let  $E[P_s]$  be the expected stay in prison for a individual of rehabilitation status  $s \in \{r, n\}$ , and define  $q_s(t)$  to be the probability an individual of rehabilitation status  $s$  is released at time  $t$ . Then we can write the expected stay in prison for an inmate of type  $s$  as

$$E[P_s] = \sum_{j=0}^{\infty} q_s(\hat{t}(j)) \hat{t}(j), \quad (4)$$

where  $\hat{t}(j) = T$  whenever the number of incidents  $j$  is high enough that the inmate will not gain early release. Because non-rehabilitated individuals are more likely to be involved in prison incidents, they expect longer incarceration periods, i.e.,  $E[P_n] > E[P_r]$ . The impact on an inmate's expected incarceration period due to a marginal increase in sentence  $T$  is

$$\frac{\partial E[P_s]}{\partial T} = \sum_{j=0}^{\infty} q_s(\hat{t}(j)) \frac{\partial \hat{t}(j)}{\partial T} + \sum_{j=0}^{\infty} \frac{\partial q_s(\hat{t}(j))}{\partial t} \frac{\partial \hat{t}(j)}{\partial T} \hat{t}(j). \quad (5)$$

The first term represents the incapacitation effect of longer sentences. For a given probability of being released after  $j$  incidents, inmates must spend more time in prison

because each  $\hat{t}(j)$  is pushed farther into the future including  $T$  itself. The second term represents the sorting effect of longer sentences. For each release date  $\hat{t}(j)$ , the probability of being released at that date changes. More specifically, the probability that an inmate is released after  $j$  incidents falls, favoring later release dates. To be released at time  $\hat{t}(0)$ , an inmate must have no incidents up to that point. When  $\hat{t}(0)$  increases, the inmate is less likely to make it to  $\hat{t}(0)$  without incident. Obviously, this increase in  $\hat{t}(0)$  raises the likelihood of being released after  $\hat{t}(0)$ . With longer sentences, both the probability of early release is reduced, and early release dates are pushed further into the future. Overall, an inmate expects to spend more time in prison.

We now illustrate numerically the interaction between sentences and expected prison stays. To do this we return to the setting in which rehabilitated inmates are never involved in prison incidents. Then, an inmate will be released after spending  $\hat{t}$  in prison without incident, where  $\hat{t}$  solves  $V_T^{\hat{t}}(0) = W_T^{\hat{t}}(0)$ . A single incident forces an inmate to serve the full sentence.

**Example: Impact of increase in prison sentence on time served when  $\lambda_r = 0$ .** Consider an individual who is arrested and found guilty of auto theft in the United States. According to the Bureau of Justice Statistics, more than two-thirds of released prisoners accused of property crimes are re-arrested within three years, so we set  $\pi_0 = 30\%$ .<sup>10</sup> According to the Federal Sentencing Commission the median prison sentence for auto theft in 1995 was eighteen months; it increased to thirty months by 2005. Consequently, we look at the impact of an increase in  $T$  from  $T = 18$  to  $T = 30$ .<sup>11</sup> The U.S. Courts Office estimates a monthly imprisonment cost of around \$1900, while the probation supervision cost is approximately \$300.<sup>12</sup> Consequently, we set  $B = 1600$ . The FBI estimates that in 2000, the average value of a stolen motor vehicle was \$6,682, and that the recovery rate of stolen motor vehicles is 62.2%.<sup>13</sup> This corresponds to an average loss of \$4000 per theft. If a non-rehabilitated individual steals a car every month, then  $C = 4000$ .<sup>14</sup>

**Uninformative environment.** First consider a relatively un-informative environment in which  $\lambda_n = 0.2$ ; this low value of  $\lambda_n$  implies that non-rehabilitated inmates

<sup>10</sup>See <http://www.ojp.usdoj.gov/bjs/reentry/recidivism.htm>.

<sup>11</sup>See <http://www.uscc.gov/linktojp.htm> for references.

<sup>12</sup>See <http://www.uscourts.gov/ttb/may04ttb/costs/index.html> for references. The imprisonment cost is consistent with Levitt (1996).

<sup>13</sup>See <http://www.fbi.gov/pressrel/pressrel01/cius2000.htm> for references.

<sup>14</sup>Levitt (1996) uses similar numbers with a cost of auto theft of \$4000, and an average of 15 crimes a year.

are not involved in prison incidents significantly more frequently than rehabilitated inmates who have  $\lambda_r = 0$ . An inmate with no prison incidents would be released after 12.0 months under the initial 18 month sentence, while the same individual would be released after 15.1 months under a 30 month sentence. A rehabilitated inmate would spend an additional 3.1 months in jail, while a non-rehabilitated inmate would spend an additional 11.8 months on average. More precisely, a non-rehabilitated individual would expect to serve on average 17.5 months of a 18 months sentences, and 29.3 months of a 30 months sentence.

**Informative environment.** Now consider a relatively informative environment with  $\lambda_n = 0.8$ ; non-rehabilitated prisoners are sufficiently likely to be involved in incidents that even a short period of time without incidents strongly suggests rehabilitation. In this informative environment, an inmate would only need to serve 4.6 months of an 18 month sentence without any transgressions before being released. If the sentence were increased to 30 months, the same inmate would need to spend 5.5 months. A rehabilitated inmate would only have to spend an additional 0.9 months in prison following this 12 month increase in sentence. Non-rehabilitated inmates would expect to remain in prison for an average of 17.6 months out of a 18 months sentences, and 29.7 months of a 30 months sentences. This implies an additional 12.1 months in prison on average, which exceeds the 12 month increase in the sentence. Large  $\lambda_n$  facilitates substantial sorting, and the extra scrutiny forces non-rehabilitated inmates to suffer from significant increase in expected time served.

In this example, even though the twelve month increase in legislated sentence raises the expected time served by non-rehabilitated inmates by more than twelve months, the average prison stay across all inmates only increased by 8.5 months. This is because rehabilitated individuals only spend an additional 0.9 months in prison. However, a slight modification of the example reveals that the aggregate expected incarceration period can increase by more one-for-one with the sentence increase.

**Informative environment with few rehabilitated inmates.** Imagine now that only one percent of all inmates are rehabilitated. Then, with the inmate population dominated by non-rehabilitated individuals, substantial screening will occur. Suppose further that the cost of releasing a non-rehabilitated inmate is only  $C = 10$ . Then with an 18 month sentence, rehabilitated individuals spend 2.1 months in prison; and with a 30 month sentence, rehabilitated inmates must spend 2.8 months in prison, an increase of 0.7 months. In contrast, a non-rehabilitated inmate expects to serve 15 months of an 18 month sentence, and 27.3 months of a 30 month sentence, a 12.3

month expected increase in time served. Overall, the expected prison stay for all inmates increases by 12.2 months, which exceeds the 12 month sentence increase. ■

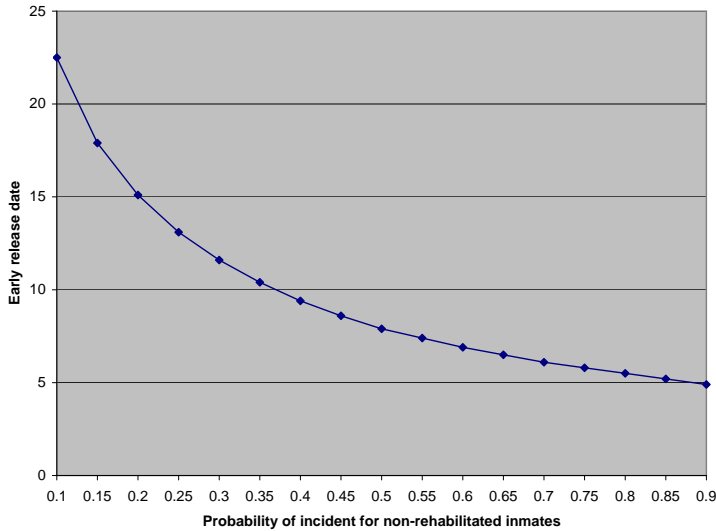
The expected time served depends on the ease with which the state can distinguish a recidivist. In these examples, non-rehabilitated individuals always suffer more from an increase in sentence, but the difference is less pronounced when  $\lambda_n$  is only 0.2, so that it is harder for the state to distinguish between rehabilitated and non-rehabilitated inmates from their behavior in prison. In most cases, as  $\lambda_n - \lambda_r$  falls, the time an inmate must serve rises. This is because as  $\lambda_n - \lambda_r$  decreases, the option of gaining more information becomes more valuable because information is harder to extract from a prison incident history.

Figure 1 illustrates the relationship between  $\lambda_n$  and minimal time served when  $\lambda_r$  is zero, and the sentence is 30 months. To see that this relationship does not always hold, consider the limit as  $\lambda_n \rightarrow \lambda_r$ . In the limit, information cannot be screened and the parole board will rely only on its prior and either release immediately or hold all inmates until  $T$ . When there is very little information available, screening interacts with prior beliefs. When  $\lambda_n$  is close to  $\lambda_r$  and the parole board has an optimistic prior, they will release everyone immediately. From that point as we consider increasing  $\lambda_n$  the parole board may want to hold inmates longer, as the value of screening rises. In this case, better information actually increases the time inmates must serve. In contrast, in the same situation save that the state has a pessimistic prior, increasing  $\lambda_n$  may encourage the parole board to offer early release. This makes it clear that there is no general result. However, in most situations, increasing the informativeness of the signal reduces the time required to be served without incident before release as in the figure. In particular, this relationship always holds when  $\lambda_r = 0$  and  $\hat{t}(0)$  is interior.

### 3 Rehabilitative Effort

We now look at an inmate's incentive to work on becoming rehabilitated, and how prison culture can affect equilibrium outcomes. Rehabilitation is obviously a long and time-consuming process, but modeling it this way would complicate analysis, without providing much additional compensating insight. Accordingly, we assume that immediately after sentencing, inmates choose whether or not to engage in costly effort to rehabilitate themselves. To make the analysis most transparent, we again

Figure 1



suppose that rehabilitated inmates are never involved in prison incidents ( $\lambda_r = 0$ ). It follows that inmates who are involved in an incident prior to  $\hat{t}(0)$  are revealed to be non-rehabilitated, and hence are forced to serve their full sentence  $T$ , while all other inmates are released at  $\hat{t}(0)$ .

Imagine that there are three types of inmates. Fraction  $a > 0$  of inmates were “scared straight” by their prison experience, and know that they do not have to exert effort to be rehabilitated (equivalently, their effort costs of rehabilitation are low enough that they always exert effort). Fraction  $b > 0$  of inmates face a non-trivial rehabilitative choice problem: at a private cost of  $\theta > 0$ , these inmates can choose effort rehabilitative effort  $e = 1$ , which, with probability  $\mu$ , results in their rehabilitation. If such an inmate chooses  $e = 0$  instead, he is never rehabilitated.<sup>15</sup> These

<sup>15</sup>We could “endogenize” the sizes of  $a$ ,  $b$  and  $1 - a - b$  by introducing heterogeneity in  $\theta$  and solving for the cutoff agent who was indifferent between exerting effort and not. We abstract from this for simplicity.

inmates are risk neutral, and suffer a disutility  $D > 0$  per unit of time incarcerated.<sup>16</sup> An individual released at time  $\tau$  would suffer a total disutility from incarceration of

$$\int_0^\tau e^{-\rho t} D dt = \frac{[1 - e^{-\rho\tau}]}{\rho} D, \quad (6)$$

where  $\rho \geq 0$  is the rate of time preference, and  $\rho = 0$  represents an perfectly patient inmate with total disutility  $\tau D$ . The remaining fraction  $c = 1 - a - b \geq 0$  of inmates know that efforts by them to rehabilitate themselves will be unsuccessful (equivalently, their private costs of rehabilitation are high enough that it is not optimal for them to exert rehabilitative effort).

The parole board's beliefs about whether these discretionary effort types  $b$  engage in rehabilitative effort influence the early release date  $\hat{t}(0)$ . To understand the role of the parole board's initial beliefs, suppose that the parole board is optimistic, and believes that the fraction  $b$  of inmates with non-trivial choices will exert rehabilitative effort. Then after serving time  $t$  of a sentence, the board holds the belief

$$\pi_t^1(0) = \frac{a + b\mu}{a + b\mu + ce^{-\lambda_n t}} \quad (7)$$

that an individual with no incidents is rehabilitated. Let  $\hat{t}_1(0)$  (determined by Lemma 3) be the associated optimal release date.

If, instead, the parole board holds pessimistic beliefs that only those who were “scared straight” will be rehabilitated, then after serving time  $t$  of a sentence, the board holds the belief

$$\pi_t^0(0) = \frac{a}{a + (b + c)e^{-\lambda_n t}} < \pi_t^1(0) \quad (8)$$

that an individual with no incidents is rehabilitated. Let  $\hat{t}_0(0)$  (determined by Lemma 3) be the associated optimal release date, and note that Proposition 2 implies that when the board holds more pessimistic beliefs, it delays release— $\hat{t}_0(0) > \hat{t}_1(0)$ .

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<sup>16</sup>Note that we are assuming that the only benefit from rehabilitation is the reduction in expected prison time. Of course, rehabilitated agents may also experience a post release benefit as well. In this model, this would act like a reduction in the cost of effort. When the time of release increases, it pushes this additional benefit further into the future reducing the relative overall benefit of rehabilitation. With finitely-lived agents, post release benefits would vary with age, and our model would suggest that it is easier to induce rehabilitation among younger agents, so we would expect to find greater leniency.

For both optimistic and pessimistic board beliefs to be consistent with equilibrium outcomes, it must be that early release date  $\hat{t}_0(0)$  induces inmates to exert rehabilitative effort, while the deferred early release date,  $\hat{t}_1(0)$ , associated with pessimistic parole board beliefs, discourages inmates from rehabilitation.

An increase in  $\hat{t}(0)$  has two opposing effects on the incentives to undertake effort. The first one is associated with the incapacitation effect. Inmates benefit less from any early release, so effort provision is less attractive. However, inmates are subject to more scrutiny, so providing effort is more important. As long as the incapacitation effect dominates the sorting effect, the increase in  $\hat{t}(0)$  reduces the gains from exerting effort. In particular, multiple equilibria can arise.

An inmate who decides not to provide effort suffers a total expected cost  $\Omega(0)$ , where:

$$\Omega(0) = e^{-\lambda_n \hat{t}(0)} \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} D + (1 - e^{-\lambda_n \hat{t}(0)}) \frac{1 - e^{-\rho T}}{\rho} D. \quad (9)$$

Alternatively, the total expected disutility for an inmate who provides effort is

$$\Omega(1) = \left[ \mu + (1 - \mu) e^{-\lambda_n \hat{t}(0)} \right] \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} D + (1 - \mu) (1 - e^{-\lambda_n \hat{t}(0)}) \frac{1 - e^{-\rho T}}{\rho} D + \theta. \quad (10)$$

An inmate will provide effort if and only if  $\Omega(0) - \Omega(1) \geq 0$ , or equivalently, if and only if:

$$\begin{aligned} \theta &\leq \mu (1 - e^{-\lambda_n \hat{t}(0)}) \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} \right] D \\ \Leftrightarrow \theta &\leq \mu (1 - e^{-\lambda_n \hat{t}(0)}) \left[ \frac{e^{-\rho \hat{t}(0)} - e^{-\rho T}}{\rho} \right] D. \end{aligned} \quad (11)$$

As anticipated, Lemma 4 shows that one can expect less rehabilitation with more impatient inmates.

**Lemma 4** *As inmates become more impatient,  $\Omega(0) - \Omega(1)$  falls, i.e., rehabilitation becomes less attractive.*

To begin, we investigate the condition under which a “good” equilibrium can be supported — i.e. when it is optimal for those inmates who face a non-trivial optimization problem to invest in rehabilitation. In particular, we derive the qualitative impact of varying the parameters of the economy on the incentives to invest in rehabilitative



effort. We first draw an important and intuitive conclusion about the interaction between sentence length and effort provision. When sentences are too short, the gain from providing effort is small relative to its cost. In essence, gains from earlier release are small when sentences are short. The next proposition captures this intuition.

**Proposition 3** *If  $T$  is sufficiently short, then inmates do not exert rehabilitative effort.*

For perfectly patient inmates,  $\rho = 0$ , it is easy to solve implicitly for  $\bar{T} = \hat{t}(0) + \frac{\theta}{\mu D(1 - e^{-\lambda_n \hat{t}(0)})}$ , such that inmates do not exert rehabilitative effort if  $T \leq \bar{T}$ . When inmates are impatient and discount the future,  $\rho > 0$ , “solving” for  $\bar{T}$  becomes more difficult because  $\hat{t}(0) + \frac{\theta}{\mu D(1 - e^{-\lambda_n \hat{t}(0)})}$  may not be monotonic in  $T$ , as we will show. However, if inmates are sufficiently impatient, no effort is provided.

Surprisingly, longer sentences can have an ambiguous impact on effort provision. As the legislated sentences increase, the benefit of being released earlier increases. But at the same time, early release  $\hat{t}(0)$  is pushed farther into the future. Longer sentences can potentially make effort less attractive because, independently of their effort choices, all inmates will spend more time in prison before being able to enjoy an early release. More precisely, the direction and strength of the effect that longer sentences have on effort is determined by:

$$\begin{aligned} \frac{\partial [\Omega(0) - \Omega(1)]}{\partial T} &= \mu \lambda_n e^{-\lambda_n \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T} \left[ \frac{e^{-\rho \hat{t}(0)} - e^{-\rho T}}{\rho} \right] D \\ &\quad + \mu \left( 1 - e^{-\lambda_n \hat{t}(0)} \right) \left[ e^{-\rho T} - e^{-\rho \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T} \right] D. \end{aligned} \quad (12)$$

The first term on the right-hand side represents the difference in sorting effects, and is always positive. Because rehabilitated inmates are less likely to be involved in prison incidents (here, with  $\lambda_r = 0$ , they are never involved), they are less affected by the increase in sorting: Increased scrutiny hurts non-rehabilitated individuals more. The second term describes the difference in incapacitation effects. When  $\rho$  is strictly positive,  $e^{-\rho \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T}$  may exceed  $e^{-\rho T}$ . When  $\rho \rightarrow 0$  however, we get clear cut results:

**Proposition 4** *When inmates do not discount the future ( $\rho \rightarrow 0$ ), an increase in legislated sentence  $T$  increases the incentive to provide rehabilitative effort.*

The intuition for Proposition 5 is that with fully patient prisoners, only the total expected incarceration period dictates effort choices. Longer sentences raise the time non-rehabilitated inmates expect to be incarcerated relative to their rehabilitated counterpart. The additional scrutiny is simply more costly for those who are not rehabilitated; as a result, longer sentences encourage the provision of effort.

However, the same cannot be said when prisoners discount the future. Even if longer sentences keep non-rehabilitated inmates in prison for longer on average, this does not ensure that longer sentences encourage rehabilitation. The cornerstone of the argument rests on the fact that although longer sentences increase the benefit of being released early, they also delay early releases, causing all inmates, whether rehabilitated or not, to be incarcerated for longer. If inmates care more about this setback than the increase in the threat of serving a longer full sentence, they may be discouraged from providing effort.

To see this most transparently, first consider a setting in which the rate at which inmates discount payoffs from more distant outcomes grows, i.e., where  $\rho(t)$  grows with  $t$ . For example, imagine that  $\frac{d\rho(t)}{dt}$  is very large for all  $t > \tau$  and small for  $t < \tau$ . Then it follows directly that longer sentences that push  $\hat{t}(0)$  past  $\tau$  lower the incentives to exert rehabilitative effort.

More generally, Proposition 5 shows that increasing sentence length can discourage rehabilitative effort even when inmates' discount factor is time invariant.

**Proposition 5** *Increasing sentence length can discourage rehabilitative effort when inmates' discount factor is positive and time invariant.*

**Constructive Proof:** Using the same parameters as in the example in the last section, we show that an increase in sentence length from 18 to 30 months can eliminate the willingness to provide rehabilitative effort. We now have to specify the parameters describing rehabilitative effort. We suppose that at an effort cost of  $\theta = 2$ , an inmate obtains a probability  $\mu = 1/2$  of being rehabilitated, and that the instantaneous cost of incarceration is  $D = 1000$ . Inmates are impatient, with a time discount factor of  $\rho = 0.5$ . We focus on the less informative environment with  $\lambda_n = 0.2$ . With an 18 month sentence, plugging these parameter values and  $\hat{t} = 12$  months into (11) reveals that the benefit of providing effort exceeds the costs with a net benefit equal to 0.2, but with the 30 month sentence, plugging in  $\hat{t} = 15.1$ , the net benefit of rehabilitative effort becomes negative, -1.5. That is, the longer sentence makes rehabilitation effort

less attractive. Two factors contribute to this reduction in the incentive to undertake effort. First, because non-rehabilitated individuals are only slightly more likely to be involved in prison incidents, the difference in sorting effects is small. Second, because information is hard to disentangle, the early release date  $\hat{t}(0)$  increases significantly with the longer sentence, and the value of this later release is discounted heavily.

### 3.1 Prison Culture

We now investigate how “prison culture” can give rise to multiple equilibria in rehabilitative outcomes. The existence of multiple equilibria in crime rates has received considerable attention in the literature,<sup>17</sup> but the idea of different “prison cultures” has not to our knowledge been explored by economists. It seems natural to imagine that in certain prisons or even within a prison with different wards, different attitudes toward rehabilitation and participation in training programs could exist and be self-enforcing. In our simple environment, a good prison culture would be characterized by optimistic beliefs, and consistent good rehabilitative behaviors; while a bad prison culture would be characterized by pessimistic beliefs and little rehabilitative effort. We now show that this notion of prison culture can give rise to multiple equilibria, even in the absence of exogenous externalities or endogenous social sanctions or norms among inmates. In our environment, multiple equilibria arise endogenously from the interaction between beliefs of the parole board and inmates’ effort decisions.

**Proposition 6** *There exist effort costs  $\theta$ , such that two equilibria exist if and only if Condition I holds*

$$\text{Condition I:} \quad \left[1 - e^{-\lambda_n \hat{t}^1(0)}\right] \left[e^{-\rho \hat{t}^1(0)} - e^{-\rho T}\right] > \left[1 - e^{-\lambda_n \hat{t}^0(0)}\right] \left[e^{-\rho \hat{t}^0(0)} - e^{-\rho T}\right].$$

*In one equilibrium, all inmates with non-trivial rehabilitation choices choose effort  $e = 1$ , and in the other, they all choose effort  $e = 0$ .*

**Corollary 1** *Condition I is satisfied if and only if the incapacitation dominates the sorting effect.*

For these two equilibria to co-exist, optimistic beliefs must be consistent with high effort and pessimistic beliefs with low effort. The sorting effect, where time

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<sup>17</sup>See, e.g., Sah (1991), Murphy, Shleifer and Vishny (1993) or Burdett, Lagos and Wright (2003).

allows for inmates to be more carefully separated, always operates in the direction of encouraging more effort. The low effort equilibrium leads to greater time for sorting. Therefore, if the sorting effect dominates, then when  $\hat{t}(0)$  is greater and there is more time for sorting, inmates should have stronger, rather than weaker, incentives to undertake effort in that equilibrium. As a consequence, for these two equilibria to co-exist it must be that the incapacitation effect dominates. Inmates gain from being released earlier and this encourages effort in the optimistic equilibrium. The longer delay in the pessimistic equilibrium diminishes this benefit and it is not offset by the longer time for sorting.

## 4 Parole Eligibility

The objective that we attribute to a parole board is simple: Parole boards care about releasing rehabilitated inmates. This objective completely abstracts from the deterrence value of longer incarceration periods, as well as any justice issues associated with early release. This objective can easily conflict with the objectives of judges who may also care about deterrence. By releasing inmates early parole boards may, in fact, end up undermining the efforts of judges to deter crime. Despite the fact that it is *ex post* beneficial to release a rehabilitated inmate early, it may increase crime *ex ante*.<sup>18</sup> To mitigate this problem, authorities may want to restrict the discretion of parole boards by prescribing that a minimum time be served before granting parole. For example, both the US and Canada require that at least one-third of a sentence be completed before an inmate becomes eligible for full parole.<sup>19</sup> Taking this restriction as given, we explore its impact on both parole board release decisions and, more importantly, on inmates' rehabilitation effort choices. Using the model previously laid out with  $\lambda_r = 0$  and  $a + c \rightarrow 1$ , we consider a policy where inmates only become eligible for parole after serving a minimum proportion of  $\gamma$  of their total sentences.

Prior to  $\gamma T$ , no one can be released, even if the benefit of parole exceeds the value of continued incarceration. After such a date, the parole board will release an inmate without any incidents only if  $V_T^{\gamma T}(0) \geq W_T^{\gamma T}(0)$ . If such a condition is not satisfied, the constraint is simply not binding and incarceration will continue until  $\hat{t}(0)$ . This leads us to focus on a setting where the sentence  $T$  is long enough that  $\hat{t}(0) < \gamma T$ . Inmates'

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<sup>18</sup>Boadway et al. (1996) discuss a closely related time consistency issue.

<sup>19</sup>Inmates can be eligible for "day parole" before one-third of their sentence is completed.

decisions of whether or not to exert rehabilitative effort follow the same pattern as before: inmates pursue rehabilitation if and only if  $\Omega(0) - \Omega(1) \geq 0$ . When inmates only become eligible for parole after serving a minimum  $\gamma T$  of incarceration, we now have

$$\Omega(0) - \Omega(1) = \mu(1 - e^{-\lambda_n \gamma T}) \left[ \frac{e^{-\rho \gamma T} - e^{-\rho T}}{\rho} \right] D - \theta. \quad (13)$$

An increase in the eligibility requirement  $\gamma$  influences inmates' decisions of whether to invest in rehabilitation effort. Rehabilitated inmates are more likely to be in a position where this eligibility requirement binds, so longer requirements tend to reduce rehabilitation incentives. However, the sorting effect works in the opposite direction: Additional scrutiny due to delayed early releases provides more incentive to undertake effort. Increasing  $\gamma$  acts in the same way as increasing  $c$ : both push  $\hat{t}(0)$  further away.

**Proposition 7** *An increase in the eligibility requirement  $\gamma$  from  $\gamma^1$  to  $\gamma^2$  increases incentives to undertake rehabilitation effort if and only if Condition II holds:*

$$\text{Condition II: } \left[ 1 - e^{-\lambda_n \gamma^1 T} \right] \left[ e^{-\rho \gamma^1 T} - e^{-\rho T} \right] > \left[ 1 - e^{-\lambda_n \gamma^2 T} \right] \left[ e^{-\rho \gamma^2 T} - e^{-\rho T} \right].$$

**Corollary 2** *Condition II is satisfied if and only if the sorting effect dominates the incapacitation effect.*

By restricting discretion and delaying early release, this result mirrors that of the previous section. Delaying release can only serve to encourage more rehabilitation if the sorting effect dominates. By delaying release, the parole board has more time for to learn about an inmate's rehabilitation status. This effect encourages rehabilitative effort. However, by delaying release, the benefit of early release is reduced (the incapacitation effect), which diminishes effort incentives. Effort increases only if the first force is stronger than the second. While it is not obvious which effect should or will dominate, more discounting always makes effort less appealing.

## 5 Concluding Comments

This paper demonstrates that the longer is a sentence, the greater is the option value of keeping an inmate in prison. Simply put, the value of information gleaned from

observing an inmate's behavior is greater the longer the period of time affected by decisions based on it. This leads the state to release an inmate with a shorter sentence before one with a longer sentence, even though the state has identical beliefs about their prospects for rehabilitation. In turn, the increase in expected time served can exceed the sentence increase. To the extent that inmates whom the state believes are less likely to be rehabilitated receive longer sentences, this effect is reinforced.

We identify conditions under which maximal sentences encourage rehabilitation, but then show how when these conditions do not hold, increased sentences can dissuade inmates from attempting to rehabilitate themselves. When information is easy to disentangle, i.e. when non-rehabilitated individual are vastly more often involved in prison incidents, maximal sentence are likely to be desirable. However, if prison incidents are a poor signal of rehabilitation, maximal sentences can reduce rehabilitative effort. This offers another reason for the non-optimality of maximal sentences.

Because parole boards may fail to internalize the deterrence benefits of punishment severity, there may be rationales for discretion restrictions such as mandatory minimum sentences, or other forms of parole limitation. However, such reductions in discretion not only have direct costs, but they may also result in less rehabilitation, an effect that may not occur were the state simply to increase the sentence length.

Finally, we observe that our modeling abstracts from any incentives that inmates may have to behave in prison and appear rehabilitated, as in in Garoupa (1996) or Miceli (1994). In the context of our model, inmates with longer sentences would have stronger incentives to appear rehabilitated, as they would get more time off for "good behavior". Since this would make it harder for the state to distinguish rehabilitated inmates from recidivists, this, too, would lead the state to make inmates with longer sentences serve more time. More generally, this moral hazard has ambiguous consequences. Because those with longer sentences must behave for longer, it may not pay for recidivists to do so, so that it may be easier for the state to distinguish those who are reformed. One can only conclude that, *ceteris paribus*, those who receive longer sentences must serve more time before release.

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## 7 Appendix

*Proof of Lemma 1:* Inmate Upperbar will be released only if  $\bar{V}_T^t - \bar{W}_T^t \geq 0$ . We can rewrite  $\bar{V}_T^t - \bar{W}_T^t = \bar{V}_T^t - E[\bar{Z}_T^{t+\delta}]$ . Give that  $\bar{Z}_T^{t+\delta} = \max\{\bar{V}_T^{t+\delta}, \bar{W}_T^{t+\delta}\}$ , it implies that  $\bar{V}_T^t - \bar{W}_T^t \geq \bar{V}_T^t - E[\bar{V}_T^{t+\delta}]$ . Because  $\bar{\pi} > \underline{\pi}$ , it follows that  $\bar{V}_T^t - \bar{W}_T^t \geq \bar{V}_T^t - E[\bar{V}_T^{t+\delta}] \geq \underline{V}_T^t - E[\underline{V}_T^{t+\delta}]$ . Since Underbar is released at  $t$ , it must be that  $\underline{V}_T^t \geq E[\underline{Z}_T^{t+\delta}]$ . Consequently,  $\bar{V}_T^t - \bar{W}_T^t \geq E[\underline{Z}_T^{t+\delta}] - E[\underline{V}_T^{t+\delta}] \geq 0$ , so Upperbar is also released. ■

*Proof of Lemma 2:* If an inmate is not released at  $t$ , then  $V_T^t(j) < W_T^t(j)$ . If the inmate is released at  $t + \delta$  after an additional incident, then Lemma 1 implies that he would be released even if he did not have an incident. But then

$$0 < W_T^t(j) = \begin{aligned} & [\pi_t(j)e^{-\lambda_r\delta} + (1 - \pi_t(j))e^{-\lambda_n\delta}] V_T^{t+\delta}(j), \\ & + [\pi_t(t)\lambda_r e^{-\lambda_r\delta} + (1 - \pi_t(t))\lambda_n e^{-\lambda_n\delta}] V_T^{t+\delta}(j+1), \end{aligned}$$

$$\begin{aligned} \frac{W_T^t(j)}{(T-t-\delta)} &= [\pi_t(1-\lambda_r) + (1-\pi_t)(1-\lambda_n)] \left[ \frac{(1-\lambda_r)\pi_t}{(1-\lambda_r)\pi_t + (1-\pi_t)(1-\lambda_n)} (B+C) - C \right] \\ &+ [\pi_t\lambda_r + (1-\pi_t)\lambda_n] \left[ \frac{\lambda_r\pi_t}{\lambda_r\pi_t + (1-\pi_t)\lambda_n} (B+C) - C \right], \end{aligned}$$

$$W_T^t(j) = (T-t-\delta)[\pi_t(B+C) - C] < V_T^t(j) = (T-t)[\pi_t(B+C) - C].$$

Connecting the inequalities, we obtain  $V_T^t(j) > W_T^t(j)$ , so that it would have been optimal to release the inmate at  $t$ , a contradiction. ■

*Proof of Proposition 1:* Obviously  $\pi_{t+\Delta T}(j) > \pi_t(j)$ , so  $V_{T+\Delta T}^{t+\Delta T} > V_T^t(j)$ . The result then follows from Lemma 1. ■

*Proof of Proposition 2:* Define  $\hat{W}_{T+k}^t(j)$  to be the value at time  $t$  of continuing to incarcerate an inmate with a sentence of length  $T+k$  when subsequent (potentially sub-optimal) decisions to release are made for a sentence of length  $T$  instead (in particular, all are released at  $T$ ). An inmate with a sentence of length  $T$  and  $j$  incidents is released at  $t$  if and only if  $V_T^t(j) \geq W_T^t(j)$ . An inmate with a sentence of length  $(T+k)$  and  $j$  incidents is released at  $t$  if and only if  $V_{T+k}^t(j) \geq W_{T+k}^t(j)$ . So, suppose that the proposition is false, i.e. that

$$V_{T+k}^t(j) \geq W_{T+k}^t(j),$$

but

$$V_T^t(j) < W_T^t(j).$$

Subtracting yields

$$V_{T+k}^t(j) - V_T^t(j) = k[\pi_t(j)(B + C) - C] > W_{T+k}^t(j) - W_T^t(j).$$

Since with  $\hat{W}_{T+k}^t(j)$ , future release decisions are not necessarily optimal, it implies that  $W_{T+k}^t(j) \geq \hat{W}_{T+k}^t(j)$ , and so

$$W_{T+k}^t(j) - W_T^t(j) \geq \hat{W}_{T+k}^t(j) - W_T^t(j) = k[\pi_t(j)(B + C) - C],$$

a contradiction. ■

*Proof of Lemma 3:* At time  $t$ , it is preferable to keep an individual with  $j$  incidents in prison longer if  $V_T^t(j) - W_T^t(j) < 0$ , and it is preferable to release the individual when  $V_T^t(j) - W_T^t(j) > 0$ . From Proposition 2, we know that  $V_T^t(j) - W_T^t(j)$  is strictly increasing in  $t$ . Consequently, we can solve for a unique optimal release date  $\hat{t}(j) \in [0, T]$  for an inmate with  $j$  incidents, and  $\hat{t}(j)$  is given by  $V_T^{\hat{t}(j)}(j) - W_T^{\hat{t}(j)}(j) = 0$ . Note that if  $V_T^0 - W_T^0 > 0$  the inmate will be released at  $t = 0$ , and if  $V_T^T(j) - W_T^T(j) < 0$  the individual will only be released at  $T$ . Given Lemma 2,  $\hat{t}(j)$  is increasing in  $j$ . ■

*Proof of Lemma 4:*  $\Omega(0) - \Omega(1)$  is decreasing in  $\rho$  as long as

$$-\rho \left[ \hat{t}(0)e^{-\rho\hat{t}(0)} - Te^{-\rho T} \right] - \left[ e^{-\rho\hat{t}(0)} - e^{-\rho T} \right] < 0,$$

which is equivalent to

$$\rho\hat{t}(0)e^{\rho T} - \rho Te^{\rho\hat{t}(0)} > - \left[ e^{\rho T} - e^{\rho\hat{t}(0)} \right].$$

Since the right-hand side of the condition above is negative, a sufficient condition only requires that  $\rho\hat{t}(0)e^{\rho T} \geq \rho Te^{\rho\hat{t}(0)}$ . This condition is equivalent to

$$\rho T - \rho\hat{t}(0) \geq \ln(\rho T) - \ln(\rho\hat{t}(0)),$$

which holds as the log function is concave. ■

*Proof of Proposition 3:* Since  $\hat{t}(0) \leq T$ , this implies that the  $\lim_{T \rightarrow 0} \left[ e^{-\rho\hat{t}(0)} - e^{-\rho T} \right] \rightarrow 0$ . And note that the following is a continuous function of  $T$

$$\mu(1 - e^{-\lambda_n \hat{t}(0)}) \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho\hat{t}(0)}}{\rho} \right] D.$$

Therefore, for  $T$  sufficiently close to 0, the following holds

$$\theta > \mu(1 - e^{-\lambda_n \hat{t}(0)}) \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}(0)}}{\rho} \right] D.$$

*Proof of Proposition 4:* Longer sentences increase the incentive to choose  $e = 1$ , as long as  $\Omega(0) - \Omega(1)$  is increasing with sentence  $T$ . Note that were  $\pi_0 = 0$ , then no inmates ever receive early release, i.e.,  $\hat{t}(0) = T$ , so no effort is ever undertaken. However,  $a > 0$  eliminates the possibility that  $\pi_0 = 0$  can arise in equilibrium.

Suppose that inmates with non-trivial decisions set  $e = 1$  and define the corresponding beliefs and  $\hat{t}(0)$ . Now consider whether, as  $T$  increases, inmates have an incentive to set  $e = 1$  when  $\rho = 0$ :

$$\frac{\partial [\Omega(0) - \Omega(1)]}{\partial T} = \mu \left\{ \lambda_n e^{-\lambda_n \hat{t}(0)} \frac{\partial \hat{t}(0)}{\partial T} [T - \hat{t}(0)] + \left(1 - e^{-\lambda_n \hat{t}(0)}\right) \left[1 - \frac{\partial \hat{t}(0)}{\partial T}\right] \right\} D.$$

The first term on the right-hand side represents the difference in sorting effects, and is always positive. The relative strength of the incapacitation effects is given by the second term, and is also positive whenever  $\rho = 0$ . ■

*Proof of Proposition 5:* Given early release date  $\hat{t}^1(0)$ , the  $b$  inmates choose  $e = 1$  if and only if:

$$\theta \leq \mu \left[1 - e^{-\lambda_n \hat{t}^1(0)}\right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}^1(0)}}{\rho} \right] D.$$

Similarly, for a given early release date  $\hat{t}^0(0)$ ,  $b$  inmates choose  $e = 0$  if and only if:

$$\theta > \mu \left[1 - e^{-\lambda_n \hat{t}^0(0)}\right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}^0(0)}}{\rho} \right] D.$$

Consequently, there exists a range of  $\theta$  that satisfy these two equations simultaneously if and only if Condition I is satisfied. ■

*Proof of Corollary 1:* Since  $\hat{t}^0(0) > \hat{t}^1(0)$ , Condition 1 implies that  $\bar{\theta}$  must be decreasing in  $\hat{t}$ , where:

$$\bar{\theta} = \mu \left[1 - e^{-\lambda_n \hat{t}}\right] \left[ \frac{1 - e^{-\rho T}}{\rho} - \frac{1 - e^{-\rho \hat{t}}}{\rho} \right] D.$$

Since the incapacitation effect is negative and the sorting effect is positive,  $\bar{\theta}$  is increasing in  $\hat{t}$  if and only if the incapacitation effect is stronger. ■

*Proof of Proposition 6:* Similar to Proposition 5. ■

*Proof of Corollary 2:* Similar to Corollary 1. ■