Illiquidity Component of Credit Risk*

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Abstract

We describe and contrast three different measures of an institution's credit risk. "Insolvency risk" is the conditional probability of default due to deterioration of asset quality if there is no run by short term creditors. "Total credit risk" is the unconditional probability of default, either because of a (short term) creditor run or (long run) asset insolvency. "Illiquidity risk" is the difference between the two, i.e., the probability of a default due to a run when the institution would otherwise have been solvent. We discuss how the three kinds of risk vary with balance sheet composition. We provide a formula for illiquidity risk and show that it is (i) decreasing in the "liquidity ratio" - the ratio of realizable cash on the balance sheet to short term liabilities; (ii) increasing in the "outside option ratio" - a measure of the opportunity cost of the funds used to roll over short term liabilities; and (iii) increasing in the "fundamental risk ratio" - a measure of ex post variance of the asset portfolio.

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1 Introduction

Credit risk refers to the risk of default by borrowers. In the simplest case, where the term of the loan is identical to the term of the borrower's cash flow, credit risk arises from the uncertainty over the cash flow from the borrower's project. However the turmoil in credit markets in the financial crisis that erupted in 2007 has highlighted the limitations of focusing just on the value of the asset side of banks' balance sheets. The problem can be posed most starkly for institutions such as Bear Stearns or Lehman Brothers that financed themselves through a combination of short-term and long-term debt, but where the heavy use of short-term debt made the institution vulnerable to a run by the short term creditors.[1]

The issue is highlighted in an open letter written by Christopher Cox, the (then) chairman of the US Securities and Exchange Commission explaining the background and circumstances of the run on Bear Stearns in March 2008.[2]

> "[T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.
>
> Specifically, even at the time of its sale on Sunday, Bear Stearns' capital, and its broker-dealers' capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity - not inadequate capital - caused Bear's demise."

Thus, in spite of Bear Stearns meeting the letter of its regulatory capital requirements, it got into trouble because its lenders stopped lending. The implication is that the run was liquidity based rather than solvency based. However, even on this score, the issues are more complex than meets the eye. Bear Stearns was regulated by the SEC under its Consolidated Supervised Entity (CSE) Program which stipulated a liquidity requirement as well as a

[1] See Morris and Shin (2008) and Brunnermeier et al. (2009) for a reappraisal of financial regulation in a system context.

[2] Letter to the Chairman of the Basel Committee on Banking Supervision, dated March 20th 2008, posted on the SEC website on: http://www.sec.gov/news/press/2008/2008-48.htm

Basel II-style capital requirement. The fact that Bear Stearns suffered its crippling run suggests that the liquidity requirement in place was inadequate. We will return to examine this issue in more detail below, and interpret our theoretical framework in the light of the events surrounding Bear Stearns's collapse.

The idea that self-fulfilling bank runs are possible is well established in the banking literature (see Bryant (1980) and Diamond and Dybvig (1983)). But the sharp distinction between solvency and liquidity in the SEC Chairman's letter may not be so easy to draw in practice, even ex post. Our current understanding of the relation between insolvency risk and illiquidity risk is not well developed. Existing models tend to focus on one or the other and not on the interaction between the two. We regard this division of attention as untenable. Runs don't happen out of the blue; they tend to occur when there are also concerns about the quality of the assets, as in the case of Bear Stearns in 2008 and as documented by Gorton (1988) for U.S. bank runs during the 1863-1914 National Banking Era. It is sometimes difficult to tell (even ex post) whether the run merely hastened the failure of a fundamentally insolvent bank, or whether the run scuppered an otherwise sound institution. Nevertheless, the distinction between insolvency and illiquidity is meaningful as a counterfactual proposition asking what *would* have happened in unrealized states of the world. The distinction is also important for understanding the policy alternatives. However, in order to address counterfactual questions we need a theoretical framework, and this is the task we take up here.

For the ex ante pricing of total credit risk, it is important to take account of the probability of a run. This is both because the occurrence of a run will undermine the debt value, and also that a run will tend to destroy recovery values through disorderly liquidation under distressed circumstances. Merely focusing on the credit risk associated with the fundamentals of the assets will underestimate the total credit risk faced by a long term creditor.

In what follows, we provide a framework can be used to address these questions. A leveraged financial institution funds its assets using short- and long-term debt, as well as its own equity. Short-term debt earns a lower return, but short-term creditors have the choice not to renew funding at an interim date. We use global game methods (introduced by Carlsson and van Damme (1993) and used in Morris and Shin (1998, 2003)) to solve for the unique equilibrium in the roll-over game among short-term creditors. In particular, we provide an accounting framework to decompose total credit risk into two components. First, the eventual asset value realization may be too low to pay off all debt. We dub this "insolvency risk". Second, a run by the short-term creditors may precipitate the failure of the institution even though, in the absence of the run, the asset realization would have been high enough to pay all creditors. We refer to this second part as "illiquidity risk". We demonstrate how total credit risk can be decomposed into insolvency risk and illiquidity risk, and how the two are jointly determined as a function of the underlying parameters of the problem.

Earlier papers such as Morris and Shin (1998, 2004), Rochet and Vives (2004) and Goldstein and Pauzner (2005) used global game methods to address coordination failure in roll-over games. However, the earlier literature has focused on how coordination failure depends on current fundamentals rather than on how *future* fundamental uncertainty interacts with strategic uncertainty today. For this reason, the insights from the earlier literature are not well-suited to answer the main question we pose in this paper - namely, how illiquidity risk depends on future insolvency risk. In order to pose the questions in the most stark way, our framework has the feature that illiquidity risk would disappear if there were no future insolvency risk.

Two elements determine the size of the illiquidity risk. The "outside option ratio" measures the opportunity cost to short-term creditors of using their funds elsewhere. The "liquidity ratio" is the ratio of the cash that can be realized relative to the maturing short-term obligations. The cash that can be realized includes liquid assets on the balance sheet but also considers the cash that can be raised by selling or borrowing against risky assets with a haircut and fire sale discount. Since we also have a (standard) expression for the insolvency risk that the bank faces, we can calculate the impact on total credit risk of shifting assets to safe, liquid, low return assets from risky, illiquid, higher expected return assets. In particular, we can characterize the terms of the tradeoff when risky assets are reduced in favor of cash. We show a switch to cash will reduce total credit risk most when the bank is more highly leveraged, there is greater fundamental uncertainty, and where the repo haircut and fire sale discounts are large.

In contrast to the basic philosophy underpinning the Basel approach to capital regulation which emphasizes the size of the capital cushion relative to risk-weighted assets, our analysis points to the importance of examining the composition of the asset side of the balance sheet, and the ratio of cash to short-term debt. We have argued elsewhere (Morris and Shin (2008)) that for regulatory purposes, the single-minded focus on capital requirements needs to give way to a broader range of balance sheet indicators, including the ratio of liquid assets to total assets and short-term liabilities to total liabilities. Our results provide further theoretical backing to our earlier arguments.

Our analysis highlights that cash holdings and fundamental asset insolvency risk interact strongly so that total credit risk is affected through two channels. First, the asset insolvency risk enters directly in credit in the conventional way, where the realized value of assets at the terminal date of the project falls short of the notional obligations. However, there is also a second effect that works through the risk of runs. As the fundamentals of the bank weaken, the probability of the failure of the bank through a run by its short-term creditors also increases. In this way, there is an interaction between the asset fundamentals and the risk of a run. Thus, the SEC chairman's distinction between a run and fundamental solvency is not easily drawn.

The outline of our paper is as follows. We begin in section 2 with a framework where a bank holds cash and illiquid risky assets financed through three sources – equity, short-term debt and long-term debt. Short-term debt holders face the choice of rolling over their claims at an intermediate date. We solve a global game model where the outcome of the coordination problem faced by the short-term creditors determines the threshold value of the asset realization below which the run outcome takes place. In section 3, we use the model to define our decomposition of total credit risk into insolvency risk and illiquidity risk. The core of our paper is the comparative statics analysis of section 5 showing how the balance sheet composition impacts total credit risk.

Our benchmark model makes a number of stark modeling assumptions in order to bring out what we believe to be the key mechanisms in decomposing and analyzing credit risk. In section 6, we show how our results can be generalized to incorporate arbitrary collections of assets, general distributions of returns, fire sale discounts and haircuts that reflect current market conditions, "partial" liquidation of the bank, alternative assumptions about the resolution of the coordination problem, small ex ante uncertainty about conditions when the short-run creditors make their withdrawal decisions and "partial" payouts to creditors.

2 Benchmark Model

2.1 The Balance Sheet and the Funding Game

We will analyze the balance sheet of a leveraged financial institution, called a "bank" for convenience.

There are three dates, ex ante (0), interim (1) and ex post (2). The bank holds a risky asset, such as loans or risky securities. Each unit of the risky asset pays a gross amount θ_2 in the final period (period 2). We write θ_0 and θ_1 for the expected value of θ_2 in periods 0 and 1 respectively. We assume that $\theta_1 = \theta_0 + \sigma_1\varepsilon_1$ and $\theta_2 = \theta_1 + \sigma_2\varepsilon_2$, where ε_1 and ε_2 are independently distributed with means 0. In the benchmark model, we normalize the units of the risky asset so that $\theta_0 = 1$. We also start by assuming that both ε_1 and ε_2 are uniformly distributed on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. We will relax this assumption shortly. The parameters σ_1 and σ_2 measure the size of interim and final period uncertainty respectively. We will refer to the ratio

$$\rho = \frac{\sigma_2}{\sigma_1}$$

as the "fundamental risk ratio." It measures the size of the standard deviation of the final period innovation, normalized by the standard deviation of the initial innovation.

The bank's balance sheet in the benchmark model takes a simple form.[3] On the asset side, the bank holds two assets: cash M and Y units of the risky asset. The bank finances these assets with three sources of funding - short term debt, long term debt and equity. We denote by S_2 the face value of short term debt (the amount promised to short-term debt holders) at date 2, and denote by L_2 the face value of long term debt at date 2. Thus, the (ex post) balance sheet of the bank at date 2 can be written as follows.

Assets	Liabilities
Cash, M Risky Assets $\theta_2 Y$	Equity E_2 Short Debt S_2 Long Debt L_2

The residual payoff of the bank's owners is given by the ex post equity E_2. The bank is solvent if the ex post equity is positive, i.e.,

$$M + Y\theta_2 \geq S_2 + L_2$$

[3] We later (in section 6.1) extend the analysis to a more general asset portfolio.

or, equivalently,

$$\theta_2 \geq \frac{S_2 + L_2 - M}{Y} \equiv \theta^*. \tag{1}$$

This "solvency point" θ^* will play a crucial role in our analysis. We assume that if the bank is insolvent in period 2 - i.e., when $\theta_2 < \theta^*$ - then the bank goes into liquidation. In the benchmark model, we assume that if the bank goes into liquidation then neither short nor long term debtors receive any payoff. In a later section (in section 6.6), we relax this assumption to see how our analysis is affected by positive recovery rates. Although the zero recovery assumption of the benchmark model is admittedly stark, we have seen in the crisis of 2008 that when a securities firm goes bankrupt, the recovery values tend to be very low. The recovery rate for debt issued by Lehman Brothers has been around 8 cents to the dollar, reflecting the disruptive nature of the failure of a highly leveraged institution with a large derivatives book. Thus, even the benchmark model with zero recovery values may be of some relevance in thinking about credit risk in the context of financial intermediaries operating in the capital markets.

At the intermediate date 1, the short-term creditors face a decision on whether to rollover their lending. If the positions of short run debt holders are not rolled over, then additional assets must be pledged to raise new funding, or sold into the market to raise cash. A key quantity in our model is how much cash can be raised from the risky asset portfolio. We assume that the cash that can be raised from a unit of the risky asset is $\psi\theta_0$, where ψ represents the amount that can be borrowed by pledging one unit of the risky asset of the risky asset as collateral. Given our normalizing assumption that $\theta_0 = 1$, the total cash that is available to the bank at the interim date is

$$A^* = M + \psi Y \tag{2}$$

The parameter ψ plays an important role in our analysis. The larger is ψ, the larger is the cash pool that the bank can draw on at the interim period. Our interpretation of ψ is in terms of the cash that can be borrowed when one unit of the risky asset is pledged to the lender as collateral. The parameter ψ reflects the size of the "haircut" demanded by the lender in the collateralized transaction. The runs on Bear Stearns and Lehman Brothers in 2008 have highlighted the crucial role played by haircuts in the financial crisis. Gorton and Metrick (2009) provides striking evidence of fluctuations in haircuts and the way in which haircuts on collateralized borrowing transactions soared in the financial crisis. For lower rated asset-based securities (ABSs), the haircut rose to 100% in the aftermath of the run on Lehman Brothers, implying that such securities could not be pledged at all for secured borrowing.

We mentioned at the outset that Bear Stearns and other security broker dealers were regulated by the SEC and that they were subject to a liquidity requirement, as well as a Basel-style capital requirement. In September 2008, the SEC's Office of Inspector General published the results of an audit into the run on Bear Stearns (SEC (2008)).[4] The first of its ten official Findings states that "Bear Stearns was compliant with the CSE program's capital ratio and liquidity requirements, but the collapse of Bear Stearns raises questions about the adequacy of these requirements."[5] The liquidity requirement in place at the time governed the amount of cash set aside to meet the non-renewal of unsecured funding such as commercial paper, but did not require a liquidity buffer against the ballooning of haircuts on secured borrowing. In the event, it was the inability of Bear Stearns to roll over its secured funding that drove it to failure.

In the benchmark model, we assume that if the run is unsuccessful (i.e. the run does not drive the bank into failure), then the fundamentals of the risky asset remains unaffected and the eventual payoff of the risky assets are unaffected by the extent of the run in the interim period. This assumption is in contrast to the usual assumption in models of bank runs where the bank has to liquidate long-term assets ("dig up potatoes planted in the field") to pay the early withdrawers. The possibility of such "partial liquidations" complicates the analysis of bank run models, but we will side-step this complication in the benchmark version of the model. We address partial liquidations in section 6.3.

Let us denote by S (without any subscript) as the face value of the short-term debt at date 1, the interim date. Thus the bank fails from a run if the proportion of short term debt holders not rolling over is more than

$$\lambda = \frac{A^*}{S},$$

which we dub the *liquidity ratio.* It is the value of the cash that can be realized in the short run relative to short run liabilities. We will focus on the

[4]We thank Pete Kyle for pointing us to this reference and for helping us to understand some of the implications.

[5]SEC (2008, p.10)

case where

$$\lambda < 1.$$

If the liquidity ratio were to exceed 1, runs would be impossible and there would be no illiquidity risk. Note that in the benchmark model, the amount of cash available to the bank is assumed to depend only on the ex ante face value of the risky asset. We later (in section 6.2) discuss how the analysis changes if we allow fire sale prices to reflect θ_1, the innovation in the risky asset value.

Finally, we assume that short run debt holders have an alternative investment opportunity in which they can earn gross return r^*. Let r_S be the notional return to short-term debt from date 1 to date 2. In other words, r_S is the amount promised at the ex post date for each dollar that the short-term debt holder claims at the interim date.

$$r_S = \frac{S_2}{S}$$

A crucial parameter will be

$$\mu = \frac{r^*}{r_S},$$

which we will refer to as the *outside option ratio.* It measures the outside option value to short run creditors of their funds at the roll-over date, relative to the amount promised by the bank.

We make the assumption that if the run is successful, then the short-term creditors receive a payoff of zero. For secured lending arrangements where the creditors hold collateral, such an assumption may seem to be extreme. However, the legal underpinnings surrounding the bankruptcy of securities firms is crucial. The failure of Lehman Brothers illustrates the issues well. US bankruptcy rules ring-fence the collateral assets in a repurchase agreement so that the creditor has claim on the collateral security in case the borrower defaults. However, this is not true in other jurisdictions. In the UK, the creditors to the UK arm of Lehman Brothers have the collateral assets tied up the general bankruptcy procedure. At the time of writing (September 2009), the creditors have yet to be repaid. Our assumption that the short-term creditors receive zero in the case of a successful run is very much motivated with this case in mind.

The liquidity ratio λ, the outside option ratio μ and the fundamental risk ratio ρ will be key parameters in our analysis. We will eventually be

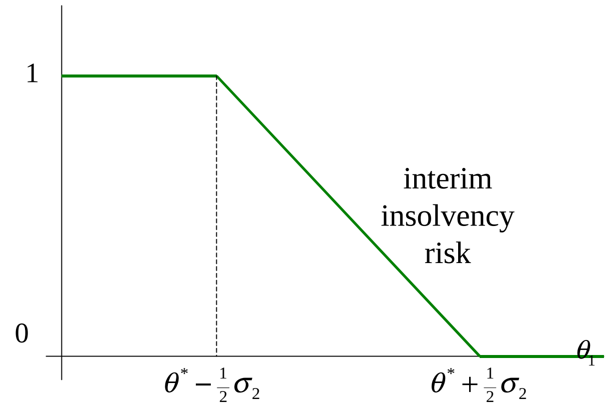

Figure 1: Interim solvency risk

interested in analyzing how ex ante credit risk depends on these parameters. But we must solve by backward induction by first analyzing what happens at the intermediate stage.

2.2 The Rollover Decision of Short Term Creditors and Interim Credit Risk

We now turn to the solution of our model. We first describe how we will deal with the coordination problem among short run debt holders in the interim period. The interim insolvency risk - the probability that the bank will fail *if there is no run* - is given by

$$\mathcal{N}_1(\theta_1) = \Pr(\theta_2 \leq \theta^* | \theta_1) \tag{3}$$

$$= \begin{cases} 1, \text{ if } \theta_1 \leq \theta^* - \frac{1}{2}\sigma_2 \\ \frac{1}{2} + \frac{\theta^* - \theta_1}{\sigma_2}, \text{ if } \theta^* - \frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^* + \frac{1}{2}\sigma_2 \\ 0, \text{ if } \theta^* + \frac{1}{2}\sigma_2 \leq \theta_1 \end{cases}, \tag{4}$$

and is depicted in figure 1. The notation $\mathcal{N}_1(\theta_1)$ indicates the <u>in</u>solvency risk at date 1, which is a function of θ_1, and hence can be dubbed the "interim insolvency risk". Thus the total expected return to rolling over, conditional on there not being a run, is

$$r_S\left(1-\mathcal{N}_1\left(\theta_1\right)\right)=\begin{cases} 0, \text{ if } \theta_1 \leq \theta^*-\frac{1}{2}\sigma_2 \\ \left(\frac{1}{2}+\frac{\theta_1-\theta^*}{\sigma_2}\right) r_S, \text{ if } \theta^*-\frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^*+\frac{1}{2}\sigma_2 \\ r_S, \text{ if } \theta^*+\frac{1}{2}\sigma_2 \leq \theta_1 \end{cases},$$

while the return to not rolling over is r^*.

We introduce our global game element here. Suppose that instead of perfectly observing θ_1, each short run creditor observed it with a small amount of noise. Let π be the proportion of short-term creditors who do not roll over their debt. We appeal to the standard result for global games that, in the limiting case as noise goes to zero, and conditional on being at the switching point between rolling over and not rolling over short-term debt, the density over π is the uniform density over $[0,1]$.[6] We later (in section 6.4) describe how the results would change under different models pinning down equilibrium strategic uncertainty. The indifference condition where payoffs to rolling over and not rolling are set equal each other determines the *run point* θ^{**} of θ. The run point θ^{**} is the unique value of θ_1 solving

$$r^*=\begin{cases} 0, \text{ if } \theta_1 \leq \theta^*-\frac{1}{2}\sigma_2 \\ \left(\frac{1}{2}+\frac{\theta_1-\theta^*}{\sigma_2}\right) \frac{A^*}{S} r_S, \text{ if } \theta^*-\frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^*+\frac{1}{2}\sigma_2 \\ \frac{A^*}{S} r_S, \text{ if } \theta^*+\frac{1}{2}\sigma_2 \leq \theta_1 \end{cases}. \tag{5}$$

Recalling that $\lambda=\frac{A^*}{S}$ and $\mu=\frac{r^*}{r_S}$, this gives

$$\theta^{**}=\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right).$$

The interim *illiquidity risk* is the probability that the bank will fail because of a run, when it would not have been insolvent in the absence of a run. It is given by

$$\mathcal{L}_1\left(\theta_1\right)=\begin{cases} 0, \text{ if } \theta \leq \theta^*-\frac{1}{2}\sigma_2 \\ \frac{1}{2}-\frac{1}{\sigma_2}\left(\theta^*-\theta\right), \text{ if } \theta^*-\frac{1}{2}\sigma_2 \leq \theta \leq \theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right) \\ 0, \text{ if } \theta>\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right) \end{cases}. \tag{6}$$

[6] The argument is reviewed in the appendix for a special parameterized case. For the general argument, the reader is referred to Morris and Shin (2003), section 2.

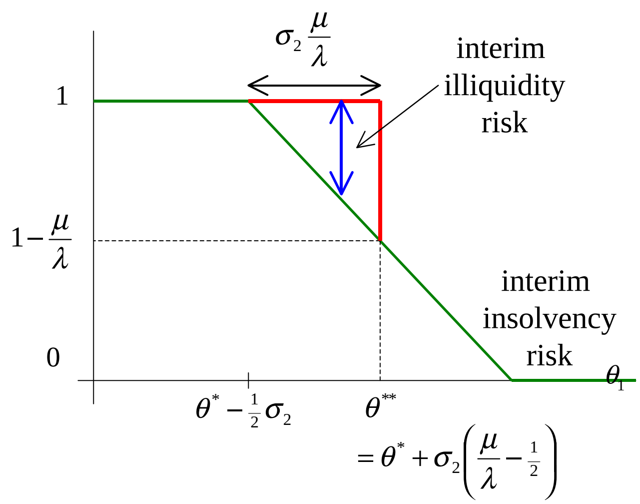

Figure 2: Total interim credit risk

The notation $\mathcal{L}_1(\theta_1)$ indicates the illiquidity risk at date 1, which is a function of θ_1. Summing the insolvency risk and illiquidity risk gives the interim (total) credit risk, which is

$$\mathcal{C}_1(\theta_1) = \begin{cases} 1, \text{ if } \theta \leq \theta^* + \sigma_2\left(\frac{\mu}{\lambda} - \frac{1}{2}\right) \\ \frac{1}{2} + \frac{1}{\sigma_2}(\theta^* - \theta), \text{ if } \theta^* + \sigma_2\left(\frac{\mu}{\lambda} - \frac{1}{2}\right) \leq \theta \leq \theta^* + \frac{1}{2}\sigma_2 \\ 0, \text{ if } \theta^* + \frac{1}{2}\sigma_2 \leq \theta \end{cases}.$$

Figure 2 illustrates the interim total credit risk $\mathcal{C}_1(\theta_1)$ consisting of the insolvency risk and the illiquidity risk.

3 Ex Ante Credit Risk

We now want to characterize ex ante credit risk, i.e., probability of default, for a holder of long run debt in the initial period 0. We will do this analysis under the assumption that the fundamental risk ratio is sufficiently large, i.e.,

$$\rho = \frac{\sigma_2}{\sigma_1} > 1.$$

The reason for examining this case first is that it gives rise to a particular simple accounting decomposition of illiquidity risk and insolvency risk. As we will see below, this case leads to an additively separable decomposition of illiquidity and insolvency. Without this assumption, additive decompositions are no longer possible, but we will describe in section 6.5 the case where σ_1 is small relative to σ_2. In the benchmark model, we will also make the assumption that the prior mean of returns is not so far from solvency point that the analysis becomes trivial. In particular, we assume:

$$\theta_0 \in \left[\theta^* - \frac{1}{2}(\sigma_1 - \sigma_2), \theta^* + \frac{1}{2}(\sigma_1 - \sigma_2)\right].$$

The ex ante insolvency risk is now

$$\begin{aligned} \mathcal{N}_0(\theta_0) &= \int_{\varepsilon=-\frac{1}{2}}^{\frac{1}{2}} \mathcal{N}_1(\theta_0 + \sigma_1\varepsilon) \\ &= \frac{1}{\sigma_1}\left[\theta^* - \left(\theta_0 - \frac{1}{2}\sigma_1\right)\right] \\ &= \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1}. \end{aligned} \tag{7}$$

The ex ante illiquidity risk is

$$\begin{aligned} \mathcal{L}_0(\theta_0) &= \int_{\varepsilon=-\frac{1}{2}}^{\frac{1}{2}} \mathcal{L}_1(\theta_0 + \sigma_1\varepsilon) \\ &= \frac{1}{2\sigma_1}\left[\left[\theta^* + \sigma_2\left(\frac{\mu}{\lambda} - \frac{1}{2}\right)\right] - \left[\theta^* - \frac{1}{2}\sigma_2\right]\right]\frac{\mu}{\lambda} \\ &= \frac{\rho}{2}\left(\frac{\mu}{\lambda}\right)^2. \end{aligned} \tag{8}$$

The ex ante credit risk is now the sum of the ex ante insolvency risk and the ex ante illiquidity risk.

$$\begin{aligned} \mathcal{C}_0(\theta_0) &= \mathcal{N}_0(\theta_0) + \mathcal{L}_0(\theta_0) \\ &= \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1} + \frac{\rho}{2}\left(\frac{\mu}{\lambda}\right)^2 \end{aligned} \tag{9}$$

where

$$
\begin{aligned}
\text{Ex Ante Insolvency Risk} &= \text{Prob. (returns below solvency point)} \\
&= \text{Prob. } (\theta_2 \leq \theta^*) \\
&= \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1}.
\end{aligned}
$$

$$
\text{Ex Ante Illiquidity Risk} \quad = \quad \frac{\rho}{2}\left(\frac{\mu}{\lambda}\right)^2,
$$

$$
\text{where} \quad \begin{array}{l} 1.\ \rho \text{ is the fundamental risk ratio} \\ 2.\ \mu \text{ is the outside option ratio} \\ 3.\ \lambda \text{ is the liquidity ratio} \end{array}
$$

We note from (9) that our benchmark case allows the additive separability of insolvency risk and illiquidity risk. The reason for this additive separability comes from our assumption that the initial shock is uniformly distributed and that σ_1 is large. The illiquidity risk is given by the expectation of the triangle indicated in Figure 2, where the expectation is taken with respect to the realization of the initial shock $\sigma_1 \varepsilon_1$. Since ε_1 is uniform and σ_1 is large, the expectation of the illiquidity triangle does not depend on the solvency point θ^* or the run point θ^{**}. Indeed, we see that the illiquidity risk only depends on the three paramters ρ, μ and λ.

The reason why ex ante illiquidity risk depends on future fundamental uncertainty σ_2 can also be seen from Figure 2. The illiquidity risk is the expectated value of the triangle indicated there. However, the size of the triangle depends on σ_2. When σ_2 is large, the triangle becomes elongated, while maintaining the same height. For this reason, the illiquidity risk in our model is a function of future fundamental risk. To put it more succinctly, illiquidity risk is parasitic on fundamental uncertainty.

The additive decomposition of insolvency and illiquidity highlights the role of the key parameters in determining credit risk. Illiquidity risk is increasing in the fundamental risk ratio, increasing in the outside option ratio and decreasing in the liquidity ratio. We note the following properties of insolvency risk and illiquidity risk in the benchmark model.

- $\mathcal{N}_0(\theta_0) = \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1}$. The ex ante insolvency risk is independent of μ and λ once the solvency point θ^* is given.

- $\sigma_1 \frac{d\mathcal{N}_0}{d\theta^*} = 1$. In other words, the insolvency risk increases at a constant rate to changes in the solvency point θ^*.

- $\mathcal{L}_0(\theta_0) = \frac{\sigma_2}{2\sigma_1}\left(\frac{\mu}{\lambda}\right)^2$. The illiquidity risk is independent of the solvency point θ^*. Illiquidity risk disappears when $\sigma_2 = 0$. That is, the illiquidity risk disappears when there is no future fundamental uncertainty.

- $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda} = -\frac{2\sigma_2\mu^2}{\sigma_1\lambda^3}$. The ex ante illiquidity risk is increasing linearly in the future fundamental uncertainty σ_2. It is increasing in the outside option ratio μ and is decreasing in the liquidity ratio λ.

These nice properties and the simple expressions for insolvency and illiquidity risks arise from the stark assumptions used in the benchmark case. We now examine how far these features survive in a setting with more general densities governing the fundamental uncertainty.

4 General Distributions

We now relax the assumption of uniform densities, and examine the case where the ex ante shocks ε_1 and ε_2 are distributed with smooth densities f_1 and f_2 with corresponding c.d.f.s F_1 and F_2. We will assume that the densities have full support on the real line. We will retrace the argument used above for this more general case, while leaving the other assumptions of the benchmark case unchanged. The interim solvency risk becomes

$$\mathcal{N}_1(\theta_1) = F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right). \tag{10}$$

The total expected return to rolling over, conditional on there not being a run, is

$$r_S\left(1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\right);$$

while return to not rolling over is r^*.

Appealing to the strategic uncertainty in the limiting case of the global game where noise becomes small, the probability that the run is successful conditional on being at the switching point is A^*/S. Hence, the indifference condition characterizing the run point is

$$\frac{A^*}{S}\left(1 - F_2\left(\frac{\theta^* - \theta^{**}}{\sigma_2}\right)\right) r_S = r^*;$$

or

$$\theta^{**} = \theta^* - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right). \tag{11}$$

Now the interim illiquidity risk is

$$\mathcal{L}_1(\theta_1) = \begin{cases} 1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right), \text{ if } \theta \leq \theta^{**} \\ 0, \text{ if } \theta > \theta^{**} \end{cases}. \tag{12}$$

Total interim credit risk is $\mathcal{C}_1(\theta_1) = \mathcal{N}_1(\theta_1) + \mathcal{L}_1(\theta_1)$, so that

$$\mathcal{C}_1(\theta_1) = \begin{cases} 1, \text{ if } \theta \leq \theta^{**} \\ F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right), \text{ if } \theta > \theta^{**} \end{cases}. \tag{13}$$

Ex ante insolvency risk is

$$\mathcal{N}_0(\theta_0) = \int_{\theta_1=-\infty}^{\infty} F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1. \tag{14}$$

Ex ante illiquidity risk is

$$\mathcal{L}_0(\theta_0) = \int_{\theta_1=-\infty}^{\theta^* - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right)} \left(1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1. \tag{15}$$

Total ex ante credit risk is the sum of the two

$$\begin{aligned} \mathcal{C}_0(\theta_0) &= \mathcal{N}_0(\theta_0) + \mathcal{L}_0(\theta_0) \\ &= F_1\left(\frac{\theta^* - \theta_0 - \sigma_2 F_2^{-1}(1-q)}{\sigma_1}\right) \\ &\quad + \int_{\theta_1=\theta^* - \sigma_2 F_2^{-1}\left(1 - \frac{\mu}{\lambda}\right)}^{\infty} F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right) \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1. \end{aligned} \tag{16}$$

Recall that in the benchmark case with uniform distributions and sufficiently large σ_1, we had:

1. $\mathcal{N}_0(\theta_0) = \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1}$, and thus independent of μ and λ.

2. $\sigma_1 \frac{d\mathcal{INS}_0}{d\theta^*} = 1$.

3. $\mathcal{L}_0(\theta_0) = \frac{\sigma_2}{2\sigma_1}\left(\frac{\mu}{\lambda}\right)^2$, and thus independent of θ^*.

4. $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda} = -\frac{2\sigma_2\mu^2}{\sigma_1\lambda^3}$.

Here, we claim that qualitatively similar results hold with general distributions. In particular, we have:

1. $\mathcal{N}_0(\theta_0)$ is independent of μ and λ.

2. as $\sigma_1 \to \infty$, $\sigma_1 \frac{d\mathcal{N}_0}{d\theta^*}$ is constant (i.e., independent of θ^*).

3. as $\sigma_1 \to \infty$, $\sigma_1 \mathcal{L}_0(\theta_0)$ is independent of θ^*.

4. as $\sigma_1 \to \infty$, $\sigma_1 \frac{d\mathcal{L}_0}{d\lambda}$..
   (a) is negative;
   (b) is linear in σ_2;
   (c) has absolute value decreasing in λ.

To show these claims, recall that insolvency risk is given by (14). Thus it does not depend on μ and λ (claim 1). As $\sigma_1 \to \infty$

$$\begin{aligned}\sigma_1 \frac{d\mathcal{N}_0}{d\theta^*} &\to \int_{\theta_1=-\infty}^{\infty} \frac{1}{\sigma_2} f_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right) f_1(0)\, d\theta_1 \\ &= f_1(0),\end{aligned}$$

which is constant (claim 2). Illiquidity risk is given by (15) which can be re-written, with the change of variables

$$r = 1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right),$$

as

$$\sigma_1 \mathcal{L}_0(\theta_0) = \int_{r=0}^{\frac{\mu}{\lambda}} r f_1\left(\frac{\theta^* - \theta_0 - F_2^{-1}(1-r)}{\sigma_1}\right) \frac{\sigma_2}{f_2\left(F_2^{-1}(1-r)\right)} dr.$$

As $\sigma_1 \to \infty$,

$$\sigma_1 \mathcal{L}_0(\theta_0) = \int_{r=0}^{\frac{\mu}{\lambda}} f_1(0) \frac{\sigma_2 r}{f_2\left(F_2^{-1}(1-r)\right)} dr.$$

This is independent of θ^* (claim 3). Now as $\sigma_1 \to \infty$,

$$\sigma_1 \frac{d\mathcal{L}_0(\theta_0)}{d\lambda} \to -\frac{\mu^2 \sigma_2 f_1(0)}{\lambda^3 f_2\left(F_2^{-1}\left(1-\frac{\mu}{\lambda}\right)\right)}.$$

This expression in negative (claim 4a) and linear in σ_2 (claim 4b). It depends on λ through $\frac{1}{\lambda^3 f_2\left(F_2^{-1}\left(1-\frac{\mu}{\lambda}\right)\right)}$. Setting $x = F_2^{-1}\left(1-\frac{\mu}{\lambda}\right)$, this equals

$$\frac{(1-F_2(x))^3}{\mu^3 f_2(x)} = \left(\frac{(1-F_2(x))^2}{\mu^3}\right)\left(\frac{1-F_2(x)}{f_2(x)}\right).$$

If f_2 further satisfies the non-decreasing monotone hazard ratio condition that $\frac{f_2(z)}{1-F_2(z)}$ is non-decreasing in z, then we can conclude that that $\frac{1-F_2(x)}{f_2(x)}$ is non-increasing in x; the full support assumption implies that $\frac{(1-F_2(x))^2}{\mu^3}$ is decreasing in x, and so $\frac{(1-F_2(x))^3}{\mu^3 f_2(x)}$ is decreasing in x. Now since $x = F_2^{-1}\left(1-\frac{\mu}{\lambda}\right)$ is increasing in λ, we have that $\frac{\mu^2 \sigma_2 f_1(0)}{\lambda^3 f_2\left(F_2^{-1}\left(1-\frac{\mu}{\lambda}\right)\right)}$ is decreasing in λ (claim 4c).

We can conclude as follows. Many of the features of the uniform density benchmark case results survive in a more general framework with general densities. For the result that illiquidity risk is decreasing in the liquidity ratio λ (feature 4(c)), we have used the non-decreasing monotone hazard ratio condition that $\frac{f_2(z)}{1-F_2(z)}$ is non-decreasing in z. Otherwise, the spirit of our earlier results follow through with very few modifications, giving some confidence that the results of the benchmark case has more general applicability.

5 Balance Sheet Impact on Credit Risk

At the outset, we stated one of our goals as investigating the effect of changed asset composition, where risky assets are replaced by cash. We will pose the question by asking what is the impact on total credit risk of converting risky assets into cash at the ex ante stage. We will first analyze the impact on insolvency risk, then the impact on illiquidity risk, and finally examine the total impact.

Recall that the solvency point, θ^*, is a sufficient statistic for the impact of the balance sheet on insolvency risk and that

$$\theta^* = \frac{S_2 + L_2 - M}{Y}.$$

where S_2 is the notional value of short term debt at date 2 and L_2 is the notional value of long-term debt at date 2. Thus the impact of cash on the solvency point is

$$\frac{d\theta^*}{dM} = -\frac{1}{Y},$$

while the impact of the risky asset is

$$\frac{d\theta^*}{dY} = -\frac{S_2 + L_2 - M}{Y^2}.$$

To interpret this expression, it is useful to define an equity measure E given by

$$E = M + Y - S_2 - L_2,$$

The equity E is the difference between asset values and liabilities when the risky asset holding is valued at its ex ante value Y (equivalently, when the ex post asset realization $\theta_2 = 1$). Using this notation, observe that

$$\frac{d\theta^*}{dY} = \frac{1}{Y}\left(\frac{E}{Y} - 1\right).$$

Now from (7)

$$\frac{d\mathcal{N}_0}{d\theta^*} = \frac{1}{\sigma_1},$$

so

$$\frac{d\mathcal{N}_0}{dM} = \frac{d\mathcal{N}_0}{d\theta^*}\frac{d\theta^*}{dM} = -\frac{1}{\sigma_1 Y},$$

while

$$\frac{d\mathcal{N}_0}{dY} = \frac{d\mathcal{N}_0}{d\theta^*}\frac{d\theta^*}{dY} = \frac{1}{\sigma_1 Y}\left(\frac{E}{Y} - 1\right).$$

Thus the net effect of shifting from risky assets to cash is to increase insolvency risk as long as equity is positive:

$$-\frac{d\mathcal{N}_0}{dY} + \frac{d\mathcal{N}_0}{dM} = \frac{E}{\sigma_1 Y}.$$

Recall that

$$\lambda = \frac{A^*}{S} = \frac{M + \psi Y}{S}.$$

Thus the impact of cash on the liquidity ratio is

$$\frac{d\lambda}{dM} = \frac{1}{S},$$

while the impact of the risky asset is

$$\frac{d\lambda}{dY} = \frac{\psi}{S}.$$

Now from (8)

$$\frac{d\mathcal{L}_0}{d\lambda} = -\frac{2\sigma_2\mu^2}{\sigma_1\lambda^3},$$

so

$$\frac{d\mathcal{L}_0}{dM} = \frac{d\mathcal{L}_0}{d\lambda}\frac{d\lambda}{dM} = -\frac{2\sigma_2\mu^2}{\sigma_1\lambda^3 S}$$

while

$$\frac{d\mathcal{L}_0}{dY} = \frac{d\mathcal{L}_0}{d\lambda}\frac{d\lambda}{dY} = -\frac{2\sigma_2\mu^2\psi}{\sigma_1\lambda^3 S}.$$

Thus the net effect of shifting from risky assets to cash is to decrease illiquidity risk as long as the product of the expected return and fire sale discount of the risk asset is less than one (i.e., $\psi\theta_0 < 1$):

$$-\frac{d\mathcal{L}_0}{dY} + \frac{d\mathcal{L}_0}{dM} = -\frac{2\sigma_2\mu^2(1-\psi)}{\sigma_1\lambda^3 S}.$$

The assumption $\psi\theta_0 < 1$ says that the return to holding cash to period 1 is higher than the return to holding the risky asset and selling it in the intermediate period at its fire sale discount price.

Now the net effect on total credit risk of shifting assets to cash is:

$$\begin{aligned} -\frac{d\mathcal{C}_0}{dY} + \frac{d\mathcal{C}_0}{dM} &= -\frac{d\mathcal{N}_0}{dY} + \frac{d\mathcal{N}_0}{dM} - \frac{d\mathcal{L}_0}{dY} + \frac{d\mathcal{L}_0}{dM} \\ &= \frac{1}{\sigma_1}\left(\frac{E}{Y} - \frac{2\sigma_2\mu^2(1-\psi)}{\lambda^3 S}\right). \end{aligned} \tag{17}$$

For a highly leveraged institution with a small holding of cash, the ratio E/Y will be close to zero. Morris and Shin (2008) discuss how market-based intermediaries such as Bear Stearns and Lehman Brothers, equity is around 3% of total assets, while the holding of cash is also similarly low. For such institutions, the ratio E/Y will be small, so that the negative term in (17) will dominate. For such institutions, a change in the composition of the portfolio toward a greater holding of cash would lower overall credit risk. Moreover, the lowering of the credit risk will be more pronounced when:

- equity (E) is low relative to risky asset holdings Y
- ex post uncertainty (σ_2) is high
- the fire sale discount (ψ) is high
- outside option ratio (μ) is high
- the liquidity ratio (λ) is low

Arguably, all these features figured prominently for the case of highly leveraged financial intermediaries such as Bear Stearn and Lehman Brothers.

6 Generalizations

We made a number of modeling choices in our earlier analysis to highlight the importance of key variables in determining illiquidity risk. In this section, we analyze what happens under a sequence of alternative formulations. This analysis serves three purposes. First, it demonstrates the robustness of the qualitative conclusions from our earlier analysis. Second, it assists in comparisons with the related literature (in section 6.6). Third, it illustrates the flexibility of our approach to incorporate many alternative institutional scenarios.

6.1 General Balance Sheet

We first establish that our results extend straightforwardly to more general asset portfolios. There were only two assets in our benchmark model: a riskless, liquid, zero return asset called "money" and a risky, illiquid and positive expected return asset. But "riskiness" is not the same as "illiquidity," and so it is desirable to have a framework that allows the two concepts to be analyzed separately.

Let the bank hold assets in $N+1$ categories indexed by $i \in \{0, 1, ..., N\}$. We denote by A_i the face value of assets in asset class i. Assume that the total return of asset i at final date 2 be $\alpha_i + \beta_i \theta_2$. Thus we assume - for simplicity - that the returns to all asset categories are thus perfectly correlated. Let ψ_i be the pledgeable value of one unit of asset i.

The model analyzed in the previous sections corresponds to the case with two assets, where asset 0 is money, so $A_0 = M$, $\alpha_0 = 1$, $\beta_0 = 0$ and $\psi_0 = 1$; and asset 1 is the risky asset, so $A_1 = Y$, $\alpha_0 = 0$, $\beta_0 = 1$ and $\psi_0 = \psi$. Now the relevant solvency point is

$$\theta^* = \frac{S_2 + L_2 - \sum_{i=0}^{N} \alpha_i A_i}{\sum_{i=0}^{N} \beta_i A_i},$$

while the cash that can be raised from the bank's assets becomes

$$A^* = \sum_{i=0}^{n} \lambda_i \left(\alpha_i + \beta_i \theta_0 \right) A_i.$$

With these alternative formulas for A^* and θ^*, the expressions for ex ante credit risk and its decomposition are unchanged.

6.2 Firesale Prices and Current Market Conditions

We made the simplifying assumption that the cash that can raised by pledging the risky asset portfolio depends on the face value. A more realistic assumption would be that the cash that can be raised depends on the realization of θ_1. In this more realistic formulation, the cash that is available to meet withdrawals by short-term creditors is a function of θ_1, and given by

$$A^* \left(\theta_1 \right) = M + \psi \theta_1 Y.$$

We show (in appendix 8.1.1) that a first order approximation (for small ψ) for illiquidity risk is then

$$\frac{\sigma_2}{2\sigma_1} \left[\left(\frac{\mu}{\lambda} \right)^2 - \frac{\psi \mu Y}{\lambda^2 S} \left(\theta^* + \sigma_2 \left(\frac{\mu}{\lambda} - \frac{1}{2} \right) \right) \right],$$

where $\lambda = \frac{M}{S} = \left. \frac{A^*(\theta_1)}{S} \right|_{\theta_1 = 0}$.

6.3 Long Run Implications of Partial Liquidation

Our benchmark model assumed that if the bank was able to meet period 1 withdrawals, there was no long run impact on the bank's solvency. We argued that such an assumption makes sense in the context of collateralized borrowing arrangements where the bank covers claims by borrowing against its illiquid assets. This assumption allowed us to conduct the analysis without worrying about the effect of partial liquidaitons of the assets, where the incidence of the run impacts on the solvency point θ^*.

If we wish to introduce partial liquidations, we need to modify our analysis by explicitly modeling the dependence of θ^* on the incidence of the run. In our benchmark model, if the bank survived until the last period, it was solvent if

$$\theta_2 \geq \theta^* \equiv \frac{S_2 + L_2 - M}{Y}.$$

Now suppose the bank was forced to meet non-renewal of funding by liquidating illiquid assets at fire sale prices and that proportion π of short term creditors withdrew their money. If the claims of the short run creditors could be met with cash, so that

$$\pi S \leq M,$$

then there will be notional claims $(1-\pi) S_2 + L_2$ outstanding in the last period and cash $M - \pi S$, so the bank will be solvent if

$$\theta_2 \geq \theta^* (\pi) \equiv \frac{(1-\pi) S_2 + L_2 - M + \pi S}{Y}.$$

But if

$$M \leq \pi S \leq A^*,$$

the bank can pay claims, but must sell $\frac{\pi S - M}{\psi}$ units of the risky asset to meets its claims. In this case, the bank will be solvent if

$$\theta_2 \geq \theta^* (\pi) \equiv \frac{(1-\pi) S_2 + L_2}{Y - \frac{\pi S - M}{\psi}}.$$

Because of this dependence of the solvency point on the proportion of withdrawals, we can show (in appendix 8.1.2) that the run point can differ from the solvency point even as ex post uncertainty disappears. Specifically, simplifying algebra by setting $S_2 = S$, if we write $\theta^{**} (\sigma)$ for the critical value of

θ_1 below which there is a run, we have

$$\theta^{**}(\sigma_2) \to \begin{cases} \theta^* \equiv \frac{S+L_2-M}{Y}, \text{ if } \mu \leq \frac{M}{S} \\ \frac{(1-\mu)S+L_2}{Y-\frac{\mu S-M}{\psi}}, \text{ if } \mu \geq \frac{M}{S} \end{cases} .$$

This in turn implies that the insolvency risk is as before as $\sigma_2 \to 0$ but the illiquidity risk (as $\sigma_2 \to 0$) is

$$\frac{1}{\sigma_1}\left(\frac{\mu S - M}{\psi}\right)(S + L_2 - M - \psi Y)$$

and thus, in particular, strictly positive, if

$$M < \mu S \leq A^*.$$

As we discuss in section 6.6, this illiquidity risk, that arises even with no ex post uncertainty from partial liquidation, is the focus of Rochet and Vives (2004).

6.4 Public Signals, Optimism and Strategic Uncertainty

In the benchmark model, we assumed that the marginal short term creditor deciding whether to roll over his debt faced the standard global game "Laplacian" uncertainty about how other short run traders would behave and described assumptions from the global games literature that would endogenously lead to this conclusion in a rational model without common knowledge.

However, one might believe that short term creditors were more optimistic (or pessimistic) about other short term creditors withdrawal decisions; this will occur in a rational common prior model if the realized return is low (high) relative to ex ante public information (e.g., as in Morris and Shin (2004)). It can also occur because "behavioral" agents anticipate that others will be more optimistic than them (Izmalkov and Yildiz (2009), Morris and Shin (2007)). There is experimental evidence that subjects choose an action closer to the efficient outcome than the Laplacian prediction (Heinemann, Nagel and Ockenfels (2004)).

Such optimism (or pessimism) can be factored into the risk decomposition straightforwardly. Suppose that, whatever a creditor's expected value of θ_1 in the interim period, the probability that he attaches to less than proportion

z of his creditors having a lower expectation is $f(z)$, where $f : [0,1] \to [0,1]$. Then the adjusted run point will be

$$\theta^{**} = \theta^* + \sigma_2 \left(\frac{\mu}{f(\lambda)} - \frac{1}{2} \right)$$

and the adjusted ex ante illiquidity risk will be

$$\frac{1}{2}\rho \left(\frac{\mu}{f(\lambda)} \right)^2 .$$

The Laplacian assumption implied that f is the identity map, but qualitative comparative statics will remain the same for any increasing f.

6.5 Small Ex Ante Uncertainty

In our benchmark model, we calculated ex ante credit risk and its decomposition into insolvency and illiquidity risk under the assumption that there was a significant amount of ex ante uncertainty (relative to ex post uncertainty). If ex ante uncertainty is small, then the decomposition of credit risk will of course depend on how the ex ante expected return relates to the "run point" for θ_1 and the "solvency point" for θ_2. In particular, we show in appendix 8.1.3 how illiquidity risk, and the impact of portfolio changes on illiquidity risk, become small if the ex ante probability of being close to the run point is low, but explode if the ex ante probability of being close to the run point is high.

We will see in the conclusion section and in Figure 4 that one feature of the recent period in the run-up to the financial crisis was the rapid increase in overnight funding of U.S. primary dealers. A short horizon for lenders between making a loan and their withdrawal decision means that little information will be realized and thus - in our model - σ_1 is small. The case of small σ_1 is therefore more realistic when viewed from an asset pricing perspective. The analysis in appendix 8.1.3 reveals how the formulas change from the benchmark case. Further development of the asset pricing consequences of our model would yield additional insights.

6.6 Partial Payouts and Related Literature

We conclude this section by discussing in more detail the relation to other papers using global game methods to address liquidity risk.

Morris and Shin (2004) analyzed the impact of illiquidity risk on ex ante pricing of long run debt. It emphasized the distinctive impact of *public* information (via its impact on strategic uncertainty) on the pricing of debt. This issue - briefly discussed in section 6.4 above - is absent from the benchmark model in the current paper. Because there was no ex post uncertainty in the sense of the current paper, the relationship between insolvency risk and illiquidity risk was not captured in the earlier paper.

The stylized portfolio we study is essentially that of Rochet and Vives (2004) [RV]: if we simultaneously allowed general distributions (as discussed above in section **??**), firesale prices reflecting current information (as in section 6.2), and partial liquidation (as in section 6.3), our model would be close to theirs. For example, our observation in section 6.3 that the run point will equal the solvency point for small ex post uncertainty if $\mu \leq \frac{M}{S}$, but will be higher if $\mu > \frac{M}{S}$ is a re-statement of their proposition 2.

The crucial difference between our model and RV is that we allow for ex post uncertainty. In our benchmark model, this is the only source of illiquidity risk (i.e., illiquidity risk goes to zero as ex post uncertainty disappears). We think that ex post uncertainty is key to understanding the link between illiquidity risk and insolvency risk. We believe that focussing only on the partial liquidation effect that underlies the difference between insolvency risk and illiquidity risk in RV may be missing the primary channel.

Both RV and this paper have short term creditors withdraw only if the probability that the bank is both liquid (so creditors can be paid in period 1) and solvent (so creditors can be paid in period 2) is above some critical threshold. RV justify this assumption by a reduced form agency problem: the withdrawal decision is not made by the creditors themselves but by agents whose rewards are sensitive only to the probability of failure. But in this paper, this behavioral rule was rational for the creditors themselves, under our assumption of exogenous returns to rolling over short term debt and (lower) returns for investing the money elsewhere. We were able to use this alternative modelling because we assumed away partial payouts in our benchmark model.

Goldstein and Pauzner [GP] (2005) consider the classic Diamond and Dybvig (1983) model of demand-deposit banking under the assumption that there is uncertainty about whether long term investments will pay out, with creditors observing noisy signals of the true probability. In this setting, some investors have "liquidity needs" in the intermediate period but others may withdraw only because they expect others to withdraw. They solve for the critical signal where runs occur (analogous to the "run point" in our model). It would be natural to define a "solvency point" corresponding to when the bank would fail if it was possible to constrain creditors without liquidity needs from withdrawing in the interim period. In this way, credit risk could be decomposed into "insolvency risk" and "illiquidity risk" and it is driven by ex post uncertainty. Although they do not carry out this comparative static, one could presumably show that greater ex post uncertainty (in their model, this would be reflected in probabilities of failure a long way from 0 or 1) leads to greater illiquidity risk. Because GP, like Diamond and Dybvig (1983), model the bank as a cooperative acting in the interest of depositors, the bank portfolio and creditor payoffs are different from this paper, although we would expect analogous comparative static effects to exist in their model.

Creditors' payoffs in GP are complex, both because they are residual claimants in the final period, and because, in the event all claims cannot be paid in the intermediate period, only a random subset of creditors are paid. In order to solve their model, they had to extend global game arguments to deal with withdrawal decisions that were not strategic complements. In our benchmark model, we make the simplifying - but, we think, plausible assumption - that creditors either end up getting the face value of their claim or do not get paid at all. If we wanted to allow for more complicated partial payments to creditors, we would have to rely on the arguments developed by GP to solve the model. This would require stronger assumptions on the information structure but would not change the qualitative conclusions.

7 Concluding Remarks

Our benchmark model provided a tractable way of decomposing credit risk into insolvency risk and illiquidity risk. As a positive model, it highlights factors that will lead to increased illiquidity risk. In the current financial crisis, commercial banks and some hedge funds may have faced similar negative shocks to their asset portfolios, but hedge fund "gates" (restricting early withdrawals) reduced early withdrawals and thus exposure of hedge fund creditors to losses. Our framework is highly flexible and - within the two period framework - can incorporate a wide range of assumptions about balance sheets and institutional rules. The extension to a richer timing structure is an important open question, although it could be done appealing to tools for modelling dynamic bank runs developed in Guimaraes (2006) and He and

Xiong (2009a,b).

Our results have particular significance in the light of two recent trends that played a role in the 2008 credit crisis. The first is the secular decline in the cash holdings by banks over the last thirty or so years, until the outbreak of the recent financial crisis. Figure 3 shows the proportion of cash assets in the total assets of US commercial banks, drawn from the Federal Reserve's H8 series. Even as recently as the early 1980s, cash assets were around 10% of total assets, but that ratio fell below 3% by the eve of the crisis. However, the cash ratio has risen sharply since September 2008 following the failure of Lehman Brothers.

Although there are several special circumstances that surround the events since September 2008 (such as actions of the Federal Reserve to expand its balance sheet), our theory points to the financial stability consequences both of the long, secular decline in cash holdings until 2008, as well as the portfolio choice motives of banks when faced with credit market turmoil. In particular, our theory highlights the vulnerability of leveraged institutions to the combination of low cash ratios and the prevalence of short term debt. To that extent, the secular decline in the cash ratio in recent years suggests vulnerabilities were increasing in the US banking sector.

A second trend in recent years has been the shortening maturity of bank liabilities, especially for the broker dealer sector of the financial system. The broker-dealer sector is the sector that included the major Wall Street investment banks.

Figure 4 plots the trend on the composition of liabilities of the banking system comparing the total amount of overnight repurchase agreements with the amount of longer maturity term repo agreements. We can see that the period preceding the current financial crisis saw a dramatic increase in the use of overnight repos, compared to longer maturity term repo agreements (see also Adrian and Shin (2007) and Brunnermeier (2009)).

Our analysis reveals the importance of the maturity of debt through the tradeoffs faced by the short-term creditors. We discuss in section 6.5 how when the short-term debt becomes very short term, the illiquidity component of credit risk increases due (paradoxically) to the reduced uncertainty over the short-term as compared to long-term outcomes. The very feature of short-term debt that makes it safe for the creditors (its safety over the short-term) makes the bank vulnerable to a run. Thus the fact that short-term debt becomes *ultra*-short term can be seen as an important consideration when thinking of the fragility of the financial system.

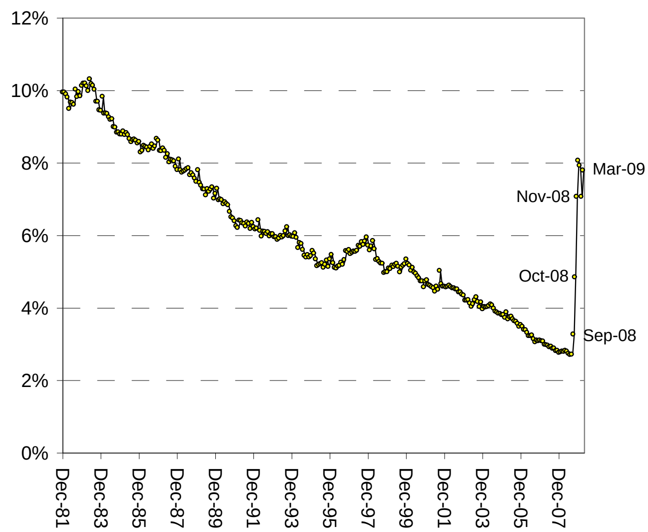

Figure 3: US Commercial Bank Cash Ratio (Source: Federal Reserve H8)

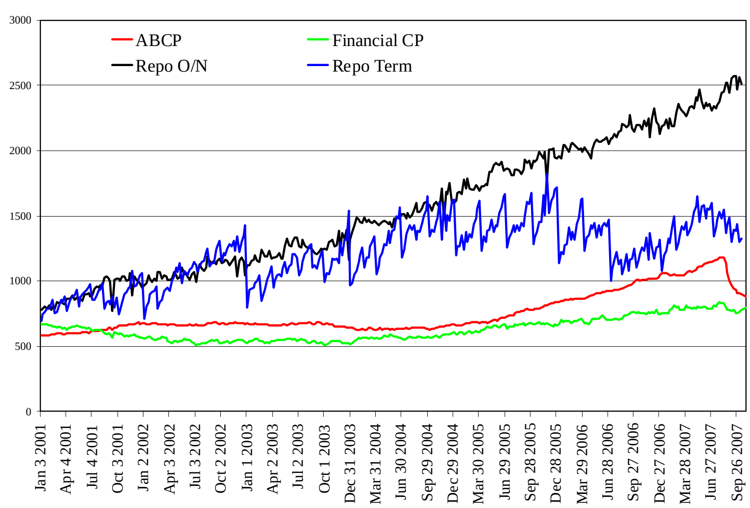

Figure 4: Outstanding Repurchase Agreements of US Primary Dealers ($ Billion): Source, Adrian and Shin (2008)

Our results have implications for financial regulation. We discuss these issues in more detail in our Brookings Papers piece on financial regulation (Morris and Shin (2008)). When the spillover effects across financial institutions are taken into account, liquidity requirements take on greater significance as a policy tool. If the debtor bank held more cash in place of illiquid assets, it could meet the withdrawals more easily, thereby lowering the threshold in the coordination game among the creditors to the bank. The cost of miscoordination for the creditor banks could also be reduced if they held more cash, since they would be less vulnerable to a run themselves. A more liquid creditor bank would be less jittery. Our theory suggests that understanding credit risk depend on fully grasping the interactions of illiquidity risk and fundamental insolvency risk. More research beckons in exploring further the themes outlined in this paper.

8 Appendix

8.1 Analysis for the Variations Section

8.1.1 Firesale prices and current market conditions

We now assume that assets available to short run creditors are

$$A^* (\theta_1) = M + \psi \theta_1 Y,$$

instead of

$$A^* = M + \psi \theta_0 Y.$$

Recall that in our benchmark analysis, we had set $\theta_0 = 1$. The run point θ^{**} is now (for small ψ) the value of $\theta_1 \in \left[\theta^* - \frac{1}{2}\sigma_2, \theta^* + \frac{1}{2}\sigma_2\right]$ solving the equation

$$\frac{M + \psi \theta_1 Y}{S} \left(\frac{1}{2} + \frac{\theta_1 - \theta^*}{\sigma_2} \right) = \mu. \tag{18}$$

This implies a quadratic expression in θ_1:

$$\psi Y \theta_1^2 + \left[M - \psi Y \left(\theta^* - \frac{1}{2}\sigma_2 \right) \right] \theta_1 - \left[\left(\theta^* - \frac{1}{2}\sigma_2 \right) M + \sigma_2 \mu S \right] = 0;$$

so that

$$\theta^{**} = \frac{\psi Y \left(\theta^* - \frac{1}{2}\sigma_2\right) - M + \sqrt{\left[\psi Y \left(\theta^* - \frac{1}{2}\sigma_2\right) - M\right]^2 + 4\psi Y \left[\left(\theta^* - \frac{1}{2}\sigma_2\right) M + \sigma_2 \mu S\right]}}{2\psi Y}$$

(only the positive root will give a solution in the relevant range).

To provide some intuition, we hear solve for the case where ψ is close to zero. Writing $\lambda = \frac{M}{S} = \left.\frac{A^*(\theta_1)}{S}\right|_{\theta_1 = 0}$, we know that the solution when $\psi = 0$ is

$$\theta^* + \sigma_2 \left(\frac{\mu}{\lambda} - \frac{1}{2} \right).$$

Let us assume that

$$\theta^{**} = \theta^* + \sigma_2 \left(\frac{\mu}{\lambda} - \frac{1}{2} \right) + \delta,$$

and solve for δ when as a function of ψ in the following equation, when ψ is small:

$$\left(\lambda + \frac{\psi Y \left(\theta^* + \sigma_2 \left(\frac{\mu}{\lambda} - \frac{1}{2}\right) + \delta\right)}{S} \right) \left(\frac{\mu}{\lambda} + \frac{\delta}{\sigma_2} \right) = \mu. \tag{19}$$

Totally differentiating with respect to ψ, we have

$$\left\{\begin{array}{l}\frac{Y\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)+\delta\right)}{S}\left(\frac{\mu}{\lambda}+\frac{\delta}{\sigma_2}\right)\\ +\frac{\psi Y}{S}\left(\frac{\mu}{\lambda}+\frac{\delta}{\sigma_2}\right)\frac{d\delta}{d\psi}\\ +\frac{1}{\sigma_2}\left(\lambda+\frac{\psi Y\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)+\delta\right)}{S}\right)\frac{d\delta}{d\psi}\end{array}\right\}=0. \tag{20}$$

Evaluating at $\delta = \psi = 0$, we have

$$\frac{\mu Y\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)\right)}{\lambda S}+\frac{\lambda}{\sigma_2}\left.\frac{d\delta}{d\psi}\right|_{\delta=\psi=0}=0 \tag{21}$$

or

$$\left.\frac{d\delta}{d\psi}\right|_{\delta=\psi=0}=-\frac{\sigma_2\mu Y\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)\right)}{\lambda^2 S}.$$

Thus for small ψ,

$$\theta^{**}\approx\left(1-\frac{\sigma_2\psi\mu Y}{\lambda^2 S}\right)\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)\right)$$

This implies that

$$\begin{aligned}ILL_0\left(\theta_0\right) &= \frac{\sigma_2}{2\sigma_1}\left[\left(1-\frac{\sigma_2\psi\mu Y}{\lambda^2 S}\right)\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)\right)-\left(\theta^*-\frac{1}{2}\sigma_2\right)\right]\frac{\mu}{\lambda}\\ &= \frac{\sigma_2}{2\sigma_1}\left[\left(\frac{\mu}{\lambda}\right)^2-\frac{\psi\mu Y}{\lambda^2 S}\left(\theta^*+\sigma_2\left(\frac{\mu}{\lambda}-\frac{1}{2}\right)\right)\right].\end{aligned}$$

8.1.2 Long Run Implications of Partial Liquidation

We showed in the main body of the paper that with irreversible partial liquidation, the solvency point becomes a function of the proportion of short run creditors, π, who choose not to rollover. In particular, if $\pi S \leq M$, the solvency point will be

$$\theta^*\left(\pi\right)\equiv\frac{\left(1-\pi\right)S_2+L_2-M+\pi S}{Y}$$

and if $M \leq \pi S \leq A$, the solvency point will be

$$\theta^* (\pi) \equiv \frac{(1-\pi) S_2 + L_2}{Y - \frac{\pi S - M}{\theta_0 \psi}}.$$

Now writing F_2 for the c.d.f. of the uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$,

$$F_2 (z) = \begin{cases} 0, \text{ if } z \leq -\frac{1}{2} \\ z + \frac{1}{2}, \text{ if } -\frac{1}{2} \leq z \leq \frac{1}{2} \\ 1, \text{ if } \frac{1}{2} \leq z \end{cases},$$

the run point θ^{**} must now solve

$$\int_{\pi=0}^{1} \left(1 - F_2 \left(\frac{\theta^* (\pi) - \theta_1}{\sigma_2}\right)\right) d\pi = \mu.$$

Rather than provide a general closed form solution, we will identify the run point as $\sigma_2 \to 0$.

If $\left|\frac{\theta_1 - \theta^*}{\sigma_2}\right| < \frac{1}{2}$, then $1 - F_2 \left(\frac{\theta^* (\pi) - \theta_1}{\sigma_2}\right) \in (0, 1)$ for all $\pi \leq \frac{M}{S}$; but if $\pi > \frac{M}{S}$, we have $\theta^* (\pi) > \theta^*$ and thus, if $\left|\frac{\theta_1 - \theta^*}{\sigma_2}\right| < \frac{1}{2}$, $\frac{\theta^* (\pi) - \theta_1}{\sigma_2} > \frac{1}{2}$, so $F_2 \left(\frac{\theta^* (\pi) - \theta_1}{\sigma_2}\right) = 1$ and $1 - F_2 \left(\frac{\theta^* (\pi) - \theta_1}{\sigma_2}\right) = 0$. Thus if $\mu \leq \frac{M}{S}$, the solution tends to

$$\frac{S + L_2 - M}{Y}$$

as $\sigma_2 = 0$.

Now suppose that $\theta_1 = \frac{(1-\mu) S + L_2}{Y - \frac{\mu S - M}{\theta_0 \psi}}$ and that $\frac{M}{S} \leq \mu \leq \lambda$. Now, for sufficiently large σ_2,

$$\frac{\theta^* (\pi) - \theta_1}{\sigma_2} \leq -\frac{1}{2}$$

for all $\pi < \mu$ and

$$\frac{\theta^* (\pi) - \theta_1}{\sigma_2} \geq \frac{1}{2}$$

Thus if $\frac{M}{S} \leq \mu \leq \lambda$, the solution tends to

$$\frac{(1-\mu) S + L_2}{Y - \frac{\mu S - M}{\theta_0 \psi}}.$$

8.1.3 Small Ex Ante Uncertainty

In the benchmark model, we calculated ex ante credit risk and its decomposition into insolvency and illiquidity risk under the assumption that there was a significant amount of ex ante uncertainty (relative to interim uncertainty). In this section, we examine how results would change if there was only a small amount of ex ante uncertainty.

In particular, we see the impact of only a small amount of information being released between times 0 and 1. We will use the notation

$$q \equiv \frac{\mu}{\lambda}$$

In order to examine the case of small ex ante uncertainty, we look at the case where

$$\sigma_1 < \sigma_2 (1 - q) .$$

In this case, we derive simple expressions for ex ante credit risk as follows. Define the following cut-off values of θ_0.

$$\begin{aligned} a_1 &\equiv \theta^* + \sigma_2 q - \frac{1}{2}\sigma_2 - \frac{1}{2}\sigma_1; \\ a_2 &\equiv \theta^* + \sigma_2 q - \frac{1}{2}\sigma_2 + \frac{1}{2}\sigma_1; \\ a_3 &\equiv \theta^* + \frac{1}{2}\sigma_2 - \frac{1}{2}\sigma_1; \\ a_4 &\equiv \theta^* + \frac{1}{2}\sigma_2 + \frac{1}{2}\sigma_1. \end{aligned}$$

Then we have: $\mathcal{C}_0 (\theta_0) =$

$$\left\{ \begin{array}{l} 1, \text{ if } \theta_0 \leq a_1 \\ \left(\begin{array}{c} 1 - \frac{1}{\sigma_1} \left(\theta_0 - \theta^* - \sigma_2 \left(q - \frac{1}{2} \right) + \frac{1}{2}\sigma_1 \right) q \\ - \frac{1}{2\sigma_1\sigma_2} \left(\theta_0 - \theta^* - \sigma_2 \left(q - \frac{1}{2} \right) + \frac{1}{2}\sigma_1 \right)^2 \end{array} \right), \text{ if } a_1 \leq \theta_0 \leq a_2 \\ \frac{1}{2} + \frac{1}{\sigma_2} (\theta^* - \theta_0), \text{ if } a_2 \leq \theta_0 \leq a_3 \\ \frac{1}{2\sigma_1\sigma_2} \left(\theta^* - \theta_0 + \frac{1}{2}\sigma_2 + \frac{1}{2}\sigma_1 \right)^2, \text{ if } a_3 \leq \theta_0 \leq a_4 \\ 0, \text{ if } a_4 \leq \theta_0 \end{array} \right. .$$

Figure 5 plots $\mathcal{C}_0 (\theta_0)$ for the case when there is small interim uncertainty

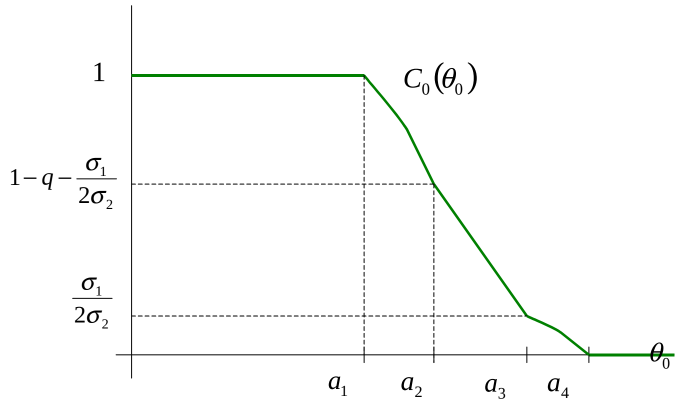

Figure 5: Total credit risk with small interim uncertainty

(writing q for $\frac{\mu}{\lambda}$). Thus

$$\sigma_1 \frac{d\mathcal{C}_0}{d\theta^*}(\theta_0) = \begin{cases} 0, \text{ if } \theta_0 \le a_1 \\ q + \frac{1}{\sigma_2}\left(\theta_0 - \theta^* - \sigma_2\left(q - \frac{1}{2}\right) + \frac{1}{2}\sigma_1\right), \text{ if } a_1 \le \theta_0 \le a_2 \\ \frac{\sigma_1}{\sigma_2}, \text{ if } a_2 \le \theta_0 \le a_3 \\ \frac{1}{\sigma_2}\left(\theta^* - \theta_0 + \frac{1}{2}\sigma_2 + \frac{1}{2}\sigma_1\right), \text{ if } a_3 \le \theta_0 \le a_4 \\ 0, \text{ if } a_4 \le \theta_0 \end{cases}$$

and

$$\sigma_1 \frac{d\mathcal{C}_0}{dq}(\theta_0) = \begin{cases} 0, \text{ if } \theta_0 \le a_1 \\ \sigma_2 q, \text{ if } a_1 \le \theta_0 \le a_2 \\ 0, \text{ if } a_2 \le \theta_0 \end{cases}.$$

These expressions are illustrated in figure 6. Now the impact of the solvency point and illiquidity index on ex ante credit risk is extremely sensitivity to the initial value of θ_0. In fact, we see that the illiquidity index will have no impact except in the range $\theta_0 \in \left[\theta^* + \sigma_2 q - \frac{1}{2}\sigma_2 - \frac{1}{2}\sigma_1, \theta^* + \sigma_2 q - \frac{1}{2}\sigma_2 + \frac{1}{2}\sigma_1\right]$ and, in the limit, the solvency will have most of its impact in that range. In that range, it remains the case that the illiquidity index has a relatively large impact if σ_2 is large. In particular,

$$\sigma_1 \frac{d\mathcal{C}}{d\theta^*} \to \begin{cases} 0, \text{ if } \theta_0 \le a_1 \\ q, \text{ if } a_1 \le \theta_0 \le a_2 \\ 0, \text{ if } a_2 \le \theta_0 \end{cases}.$$

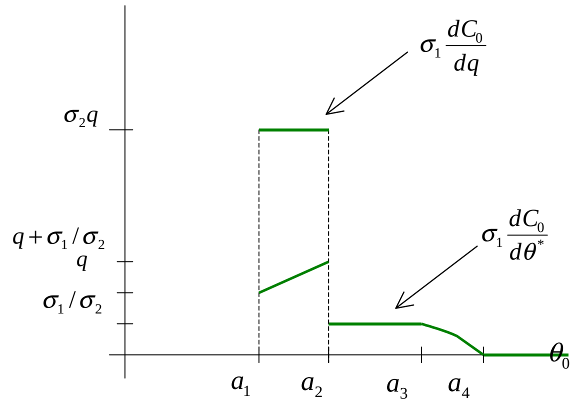

Figure 6: $\sigma_1 \frac{d\mathcal{C}_0}{dq}$ and $\sigma_1 \frac{d\mathcal{C}}{d\theta^*}$

8.2 Global Game Foundations for the Laplacian Rule

We first present the argument for Laplacian beliefs for the Gaussian case. Let player i's private signal be given by

$$x_i = \theta + \varepsilon_i$$

where θ is a Gaussian random variable with mean y and variance $1/\alpha$, and ε_i is Gaussian with mean zero and variance $1/\beta$. The random variables $\{\varepsilon_i\}$ are mutually independent, and independent of θ. Denote by $\pi(x)$ the proportion of players whose signal is x or less. Then, $\pi(x)$ is a random variable with realizations in the unit interval, and which is a function of the random variables $\{\theta, \varepsilon_i\}_{i\in[0,1]}$ and the threshold x. We derive the density function of $\pi(x_i)$ conditional on x_i. Let

$$G(z|x_i) \tag{22}$$

be the cumulative distribution function of $\pi(x_i)$ conditional on x_i, evaluated at z. In other words,

$$G(z|x_i) = \Pr(\pi(x_i) \leq z|x_i) \tag{23}$$

so that, $G(z|x_i)$ is the probability that the proportion of players with signal lower than x_i is z or less, conditional on x_i. Figure 7 illustrates the derivation of $G(z|x_i)$.

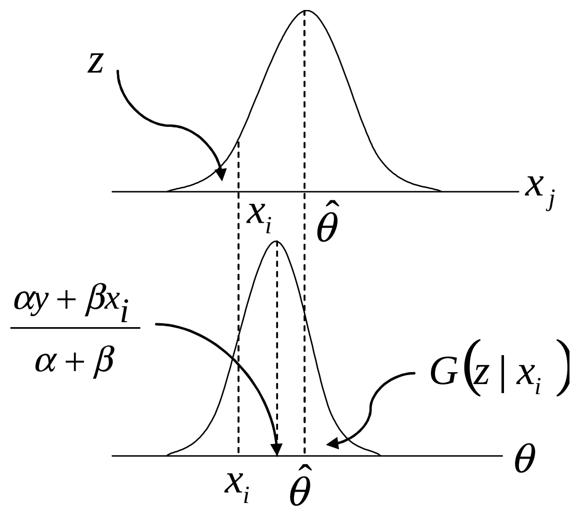

Figure 7: Deriving $G(z|x_i)$

Given θ, the proportion of players who have signal below x_i is

$$\Phi\left(\sqrt{\beta}\,(x_i - \theta)\right) \tag{24}$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal. Let $\hat{\theta}$ be the realization of θ at which this proportion is exactly z. In other words,

$$\hat{\theta} = x_i - \frac{\Phi^{-1}(z)}{\sqrt{\beta}} \tag{25}$$

The top panel of figure 7 illustrates $\hat{\theta}$. When $\theta \geq \hat{\theta}$, the proportion of players that have signal below x_i is z or less. In other words, $\pi(x_i) \leq z$ whenever $\theta \geq \hat{\theta}$. Hence, $G(z|x_i)$ is the probability of $\left\{\theta|\theta \geq \hat{\theta}\right\}$ conditional on x_i. The bottom panel of figure 7 illustrates the argument. Conditional on x_i, the density over θ is normal with mean

$$\frac{\alpha y + \beta x_i}{\alpha + \beta} \tag{26}$$

and precision $\alpha+\beta$. The probability that $\theta \geq \hat{\theta}$ is the area under this density to the right of $\hat{\theta}$, namely

$$1 - \Phi\left(\sqrt{\alpha+\beta}\left(\hat{\theta} - \tfrac{\alpha y + \beta x_i}{\alpha+\beta}\right)\right) \tag{27}$$

This expression gives $G(z|x)$. Substituting out $\hat{\theta}$ by using (25) and re-arranging, we can re-write (27) to give:

$$G(z|x_i) = \Phi\left(\frac{\alpha}{\sqrt{\alpha+\beta}}(y - x_i) + \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(z)\right) \tag{28}$$

In the special case when $\beta \to \infty$, the private signal becomes infinitely precise. In this limit,

$$G(z|x) \to \Phi\left(\Phi^{-1}(z)\right) = z$$

so that G is the identity function. In other words, the c.d.f. of $\pi(x_i)$ is the 45 degree line, and hence the density over $\pi(x_i)$ is uniform. Thus, in this limit, player i believes that he is "typical" in quite a strong sense, in that he puts equal weight on every realization of $\pi(x_i)$.

Morris and Shin (2003) show that the above argument is valid for general densities provided that some mild regularity conditions on the smoothness of densities are preserved. We refer the reader to that discussion for the full argument.

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