

# RISK AND RETURN IN ENVIRONMENTAL ECONOMICS\*

by

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**Abstract:** I examine the risk/return tradeoff for environmental investments, and its implications for policy choice. Consider a policy to reduce carbon emissions, and thereby limit global warming and the damages that might result. To what extent does the value of such a policy depend on the expected future damages from warming versus uncertainty over those damages, i.e., on the expected benefits from the policy versus their riskiness? And to what extent should the policy objective be a reduction in the expected temperature increase versus a reduction in uncertainty over that increase? Using a simple model of a stock externality (e.g., temperature) that evolves stochastically, I examine the “willingness to pay” (WTP) for alternative policies that would reduce the expected damages under “business as usual” (BAU) versus the variance of those damages. I also show how one can compute “iso-WTP” curves (social indifference curves) for combinations of risk and expected returns as policy objectives. Given cost estimates for reducing risk and increasing expected returns, one can then compute the optimal risk-return mix for a policy, and the policy’s social surplus. I illustrate these results by calibrating the model to data for global warming.

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# 1 Introduction.

The adoption of an environmental policy usually imposes costs on society, but is expected to yield a social return in the form of a stream of benefits, e.g., health benefits from less pollution. Those benefits, however, usually occur in the future and may be highly uncertain, so that (as with other private or public investments) the actual return is uncertain. This is especially true for environmental policies involving stock externalities, such as increases in greenhouse gas (GHG) concentrations, the acidification of lakes and oceans, and the accumulation of toxic waste. All of these problems involve long time horizons and uncertain impacts. This paper examines the risk/return tradeoff for environmental investments and the implications of that tradeoff for policy choice. I utilize a very simple and stylized model which focuses on climate change, because of the long time horizon and considerable uncertainty involved. However, the framework developed here could easily be applied to other environmental policy problems, such as the management of toxic (including nuclear) waste.<sup>1</sup>

Consider a policy designed to limit the accumulation of atmospheric greenhouse gases. An example would be a simple abatement policy, implemented via the imposition of a carbon tax. That policy would have some cost to society, but by reducing GHG emissions it would reduce the extent of warming by some uncertain amount, and thus it would yield uncertain future benefits. The uncertainty arises because (a) we have limited knowledge of the relationship between GHG concentrations and temperature; and (b) we know even less about how higher temperatures would affect GDP growth or other indicia of welfare. Thus we could ask two questions. First, how large are the expected social benefits from a given policy relative to the risk involved? Second, to what extent is the social value of the policy (measured, e.g., by society's willingness to pay for it) driven by expected outcomes without the policy versus uncertainty over those outcomes.

Now consider the design of an environmental policy, such as one to limit global warming. Alternative policies might have different impacts on the *expected* change in temperature

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<sup>1</sup>For a general discussion of the nature of the uncertainties involved in environmental policies, whether or not stock externalities are involved, see Pindyck (2007).

versus the *variance* of that change. For example, rather than (or in addition to) GHG emissions abatement, money might be spent to improve our knowledge of how GHG emissions affect temperature, and how changes in temperature affect economic output. Therefore a purely research-focused policy might do little or nothing to reduce the expected change in temperature (and hence the expected reduction in GDP growth), but might reduce the variance of our estimates of future temperature and GDP growth. As a general matter, there is value in reducing the expected damages from GHG emissions *and* in reducing the variance of those damages. At issue is the tradeoff between the two. To what extent should policy be aimed at reducing expected damages versus the variance of those damages?

The framework I use to address these questions is “willingness to pay” (WTP). Consider a policy that would substantially reduce expected future temperature increases, and possibly reduce the uncertainty over those temperature increases. The willingness to pay for such a policy is the *maximum* percentage reduction in current and future consumption that society should be willing to give up in order to achieve those reductions in the expectation and/or variance of future temperature increases.<sup>2</sup> This does not mean it will be possible to achieve these reductions given the revenues from this WTP; doing so may be more costly, or less costly. WTP relates only to the *demand side* of policy, i.e., society’s reservation price for the achievement of a particular policy objective.

In Pindyck (2011a, 2012) I calculated WTP for GHG abatement policies that would limit future increases in temperature to some specified amount (e.g., 3°C), based on a probability distribution for the rate of warming absent the policy, i.e., under “business as usual” (BAU). Of course WTP depends on the probability distribution, and in particular, depends on both the *expected* increase in temperature under BAU and the *standard deviation* of the increase in temperature. I calculated the trade-off between the expected value of a temperature increase versus its standard deviation under BAU as determinants of WTP. That trade-off is the marginal rate of substitution along an “iso-WTP curve,” i.e., the locus of combinations of the expected value versus the standard deviation of the change in temperature such that

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<sup>2</sup>I used a related measure of WTP to examine climate change policies under uncertainty; see Pindyck (2011a, 2012). WTP has also been used in the context of climate change by Heal and Kriström (2002).

the WTP (e.g., to prevent any temperature increase) is constant. The results showed that uncertainty over temperature change can be a stronger driver of WTP than expectations, and thus should be a major focus of climate change policy.

That earlier work focused on the *starting conditions* for policy, i.e., the relative contributions to WTP of the expected value versus the variance of the change in temperature under BAU. Here I turn to the *target* of policy, and examine the trade-off between reducing the expected rate of the increase in temperature versus reducing the variance of future increases in temperature. In particular, what combinations of drift reduction and variance reduction yield the same WTP? Those combinations trace out a social indifference curve — combinations of changes in drift and variance that are welfare-equivalent.

Given estimates of the costs of drift reduction and variance reduction, we can take this a step further. We can determine whether the cost of a particular combination of drift and variance reduction is less or greater than the WTP for that combination. If the cost is less than the WTP, the policy yields a positive social surplus. If the cost exceeds the NPV, the policy is not economically feasible. Finally, assuming the costs are convex (as we would expect), we can determine the optimal risk-return tradeoff for policy design.

In the next section I lay out the simple model that I will use to address these questions. Despite its simplicity, the model captures the key features of environmental policy risk and expected returns that are of interest. In Section 3, I use the model to analyze risk and return for a policy that reduces the expected rate of temperature increase by some incremental amount. Section 4 addresses policies to change the expected rate of temperature increases and/or the variance of future temperature increases, and shows how we can calculate the willingness to pay for such policies. In Section 5 I calibrate this model to information on global warming and its impact, and then use it to address risk-return tradeoffs.

## 2 A Simple Model.

I use a simple model in which an environmental stock variable, in this case temperature change, follows an arithmetic Brownian motion (ABM) with positive drift and some volatil-

ity.<sup>3</sup> The impact on consumption is through the real growth rate of GDP, which is linearly related to the change in temperature. Welfare is measured using CRRA utility. Various expectations and variances of utility can be found analytically, making the model very tractable.

## 2.1 Temperature and GDP Growth.

Let  $X_t$  denote the anthropomorphic increase in temperature from its current level. I assume that under “business as usual,”  $X_t$  follows an arithmetic Brownian motion (ABM) with drift and volatility  $\alpha_X$  and  $\sigma_X$  respectively. I also assume that the impact of an increase in  $X_t$  is to reduce the real rate of growth of consumption,  $g_t$ . This differs from the damage function specifications used in most integrated assessment models of climate change, where  $X_t$  is assumed to reduce consumption directly.<sup>4</sup> Theoretical arguments and empirical evidence support the view that higher temperatures (and environmental damage in general) should affect the growth rate of consumption rather than its level, as in the model I use here.<sup>5</sup>

The process for  $X_t$  is thus:

$$dX_t = \alpha_X dt + \sigma_X dz , \tag{1}$$

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<sup>3</sup>In the context of global warming, the actual stock variable is the atmospheric GHG concentration, which in turn drives temperature change, but with a lag. I ignore the lag, and thus can treat temperature change itself as the stock variable. In the context of toxic waste, the stock variable could be the quantity or concentration of waste material over some geographic area.

<sup>4</sup>Most economic studies of climate change relate the temperature increase  $T$  to GDP through a “loss function”  $L(T)$ , with  $L(0) = 1$  and  $L' < 0$ , so GDP at a horizon  $H$  is  $L(T_H)GDP_H$ , where  $GDP_H$  is but-for GDP with no warming. These studies — see, e.g., Nordhaus (2008) — typically use an inverse-quadratic or exponential-quadratic function. The loss function  $L(T)$  implies that if temperatures rise but later fall, GDP could return to its but-for path with no permanent loss.

<sup>5</sup>There are several reasons to expect higher temperatures to affect the growth rate of GDP as opposed to the level. First, some effects of warming will be permanent; e.g., destruction of ecosystems, extinction of species, and deaths from weather extremes. A growth rate effect allows warming to have a permanent impact. Second, there is theoretical support for a growth rate effect. The resources needed to counter the impact of warming will reduce those available for R&D and capital investment, reducing growth. Adaptation to rising temperatures is equivalent to the cost of increasingly strict emission standards, which, as Stokey (1998) has shown with an endogenous growth model, reduces the rate of return on capital and lowers the growth rate. (Also see the related study by Brock and Taylor (2010).) Finally, there is empirical support for a growth rate effect. Using historical data on temperatures and precipitation over 50 years for a panel of 136 countries, Dell, Jones, and Olken (2008, 2009) have shown that higher temperatures reduce GDP growth rates but not levels. Also, using economic and financial market data for a panel of 147 countries over the period 1950 to 2007, Bansal and Ochoa (2011a, b) show that increases in temperature have a strong negative impact on economic growth. For an analysis of the policy implications of direct versus growth rate impact, see Pindyck (2011a).

where  $dz$  is the increment of a Weiner process, and  $X_0 = 0$ . The growth rate of consumption is given by the following linear relationship:

$$g_t = g_0 - \gamma X_t , \quad (2)$$

where  $g_0$  is the growth rate absent any change in  $X_t$ . Thus  $g_t$  also follows an ABM:

$$dg = -\gamma\alpha_X dt - \gamma\sigma_X dz \equiv -\alpha dt - \sigma dz . \quad (3)$$

Eqn. (2) implies that at any time  $s$ ,

$$g(s) = g_0 - \alpha s - \sigma \int_0^s dz = g_0 - \alpha s - \sigma z(s) ,$$

so consumption at a future time  $t$  can be written as:

$$C_t = C_0 e^{\int_0^t g(s) ds} = C_0 e^{g_0 t - \frac{1}{2} \alpha t^2 - \sigma \int_0^t z(s) ds} , \quad (4)$$

To measure welfare, I use a constant relative risk aversion (CRRA) social utility function:

$$U(C_t) = C_t^{1-\eta} / (1-\eta) , \quad (5)$$

where  $\eta$  is the index of relative risk aversion (and  $1/\eta$  is the elasticity of intertemporal substitution). At time 0, welfare (under business as usual) is then given by:

$$W_0 = \frac{1}{1-\eta} \mathcal{E}_0 \int_0^\infty C_t^{1-\eta} e^{-\delta(t-s)} dt \quad (6)$$

where  $\delta$  is the rate at which utility is discounted.

## 2.2 Willingness to Pay.

Suppose that with some expenditure over time, society could reduce  $\alpha_X$  and/or  $\sigma_X$ , i.e., reduce the expected rate of temperature change and/or the volatility of temperature change (and hence the variance of the change in temperature at any arbitrary date in the future). Let  $\alpha_X$  and  $\sigma_X$  be the drift and volatility of  $X_t$  under BAU, and  $\alpha'_X$  and  $\sigma'_X$  be the corresponding drift and volatility under a policy that has a permanent cost to society of  $w$  percent of

consumption. Here  $\alpha'_X \leq \alpha_X$  and  $\sigma'_X \leq \sigma_X$ . Willingness to pay is the maximum value of  $w$  that society would accept to reduce  $\alpha_X$  to  $\alpha'_X$  and reduce  $\sigma_X$  to  $\sigma'_X$ .

Note that WTP is society's reservation price for achieving the policy objective  $(\alpha_X, \sigma_X) \rightarrow (\alpha'_X, \sigma'_X)$ . However, it is a reservation price for a particular type of policy and a particular form of pricing: At time  $t = 0$  the policy is adopted (with no option to wait for more information), and the drift and volatility of  $X_t$  are immediately changed to their new values. The payment flow, however, starts at  $t = 0$  and continues forever, i.e., takes the form of a permanent reduction of consumption of  $w$  percent (yielding a permanent flow of revenue to pay for the policy objective). One might naturally argue that this is unrealistic. First, we would expect that changing  $\alpha_X$  and/or  $\sigma_X$  should take time, and perhaps considerable time. Second, there is little reason to expect that the flow cost of achieving this policy objective will be a fixed percentage of GNP or consumption. Thus WTP might instead be defined as the willingness to give up a particular time-varying percentage of consumption in return for a shift in  $\alpha_X$  and/or  $\sigma_X$  that occurs over some period of time. However, the simplification that I use ( $\alpha_X$  and/or  $\sigma_X$  change instantly and the flow cost is a constant percentage of consumption) adds clarity to the basic results.

Welfare under the policy is given by:

$$W_1(\alpha'_X, \sigma'_X) = \frac{(1-w)^{1-\eta}}{1-\eta} \mathcal{E}'_0 \int_0^\infty C_t^{1-\eta} e^{-\delta t} dt, \quad (7)$$

where  $\mathcal{E}'_0$  denotes the expectation at  $t = 0$  when the drift and volatility of  $X_t$  are  $\alpha'_X$  and  $\sigma'_X$ . Under BAU (i.e., no policy), welfare is:

$$W_2 = \frac{1}{1-\eta} \mathcal{E}_0 \int_0^\infty C_t^{1-\eta} e^{-\delta t} dt, \quad (8)$$

where  $\mathcal{E}_0$  is the expectation under the original drift and volatility,  $\alpha_X$  and  $\sigma_X$ . Then WTP is the value  $w^*$  that equates  $W_1$  and  $W_2$ . Note that  $w^*$  is a percentage of consumption. I treat GDP and consumption as interchangeable, and assume that all losses from higher temperatures, including health and ecosystem damage, can be monetized and included in GDP.

From eqns. (2) and (3), the drift and volatility of temperature correspond to a drift and

volatility of the real growth rate  $g_t$ , i.e.,  $\alpha = \gamma\alpha_X$ ,  $\sigma = \gamma\sigma_X$ ,  $\alpha' = \gamma\alpha'_X$ , and  $\sigma' = \gamma\sigma'_X$ . In what follows I will refer to changes in  $\alpha$  and  $\sigma$  rather than  $\alpha_X$  and  $\sigma_X$ .

Suppose that under BAU, the drift and volatility of  $g_t$  are  $\alpha_0$  and  $\sigma_0$  respectively. If we specify a target drift and volatility,  $\alpha'$  and  $\sigma'$ , we can calculate the WTP for a policy that would achieve this target. Suppose that WTP is  $w_1^*$ . We are interested, however, in the tradeoff between drift reduction and variance reduction as targets of policy. That is, we want to know what combinations of drift reduction and variance reduction will yield the same WTP,  $w_1^*$ , assuming we start at  $\alpha_0$  and  $\sigma_0$ . We can compute the locus of such combinations and thereby obtain an “iso-WTP curve,” i.e., combinations of  $\alpha$  and  $\sigma$  for which, starting at  $\alpha_0$  and  $\sigma_0$ , all have the same WTP, and thus are welfare-equivalent.

An iso-WTP curve is illustrated in Figure 1. Point A is the starting drift and volatility, and points B and C are two targets, both of which have the same WTP of  $w_1^* = .03$ . Moving from A to *any* point on the curve will have the same WTP of .03. Note that  $\alpha$  increases as we move *down* the vertical axis, so the curve represents a social indifference curve between expected return (a lower value of  $\alpha$ ) and risk (a higher value of  $\sigma$ ). It is thus analogous to the return-risk indifference curve of an investor choosing a portfolio of equities and a risk-free asset. The figure also shows another iso-WTP curve, labelled  $w_2^* = .05$ ; moving from  $\alpha_0$  and  $\sigma_0$  to any point on this curve would have a WTP of .05.

If we knew the cost of reducing  $\alpha$  and the cost of reducing  $\sigma$ , we could plot an iso-cost line. If those costs were linear in the change in  $\alpha$  and the change in  $\sigma$ , the iso-cost line would be a straight line, as drawn in Figure 1. Its tangency with the iso-WTP curve  $w_1^* = .03$  at point B is the cost-minimizing target combination of  $\alpha$  and  $\sigma$ . There are other target combinations of  $\alpha$  and  $\sigma$  that have a WTP of .03, but they would be more costly to achieve.

In Figure 1, moving from point A to point B has the same WTP = .03 as moving from point A to point C. We can also calculate the combination of *starting values* for  $\alpha_0$  and  $\sigma_0$  that yield the same WTP = .03 when moving to point B. Doing so is useful because it allows us to determine combinations of temperature drift and volatility under BAU that have the same welfare implications. In other words, how important is the expected change in temperature (under BAU) relative to its variance as determinants of the demand for policy?



This is illustrated in Figure 2, which shows two iso-WTP curves. Curve 1 gives combinations of starting values  $\alpha_0$  and  $\sigma_0$  such that the movement to point B has a WTP = .03. Thus  $w^*(A \rightarrow B) = w^*(D \rightarrow B) = w^*(A \rightarrow C) = .03$ . But  $w^*(D \rightarrow A) = w^*(B \rightarrow C) = 0$ , so  $w^*(D \rightarrow C) = .03$ . Thus moving from any point on Curve 1 to any point on Curve 2 will have the same WTP.

The iso-WTP and iso-cost curves in Figures 1 and 2 are just illustrative examples. Later I will calibrate this simple model to a set of estimates of temperature change and confidence intervals compiled by the IPCC (2007) and others, and then use the calibrated model to obtain iso-WTP and iso-cost curves along the lines of those shown in Figures 1 and 2.

### 3 Risk and Return for Policy Outcomes.

Consider an emissions abatement policy that reduces the expected growth rate of temperature and thereby increases the expected rate of growth of consumption. Putting aside the cost of the policy, we want to compare the expected benefit to the uncertainty over that benefit, i.e., compare the expected return to the riskiness of that return. To do this, I begin with a simple two-period example in which consumption today is known but consumption at a future time  $T$  is uncertain. I examine the benefit that results from a small increase in the drift of the growth rate (which would result from a small decrease in the expected rate of increase of temperature). I then turn to a continuous-time model, and again consider a policy that results in a small increase in the drift of the growth rate. Of interest is the expectation and variance of the cumulative return from the policy over the entire horizon.

#### 3.1 Two Periods.

In this simple case,  $C_0 = 1$  and  $C_T$  is uncertain, the outcome of the stochastic growth process, as given by eqn. (4). Given  $\alpha$  and  $\sigma$ , welfare is stochastic and is given by:

$$\tilde{W}_T = \frac{1}{1 - \eta} \left[ 1 + (C_T^{1-\eta} e^{-\delta T}) \right] \quad (9)$$

Now consider a policy that results (at  $t = 0$ ) in an incremental decrease in  $\alpha_X$ , the expected change in temperature, and hence a decrease  $\alpha = \gamma\alpha_X$ , the expected change in the growth

rate of consumption. The (stochastic) return, measured in terms of increased welfare from that policy is

$$\tilde{r}_{\alpha T} = -\frac{\partial \tilde{W}_T}{\partial \alpha} = -\frac{1}{1-\eta} \frac{\partial}{\partial \alpha} C_T^{1-\eta} e^{-\delta T} . \quad (10)$$

Note that

$$C_T^{1-\eta} = e^{(1-\eta) \int_0^T g(s) ds} ,$$

and from eqn. (3), the growth rate  $g(s)$  at time  $s > 0$  is given by

$$g(s) = g_0 - \alpha s - \sigma \int_0^s dz = g_0 - \alpha s - \sigma z(s) .$$

Thus

$$C_T^{1-\eta} = e^{(1-\eta)g_0 T - \frac{1}{2}(1-\eta)\alpha T^2 - \sigma(1-\eta) \int_0^T z(s) ds} , \quad (11)$$

and

$$-\frac{\partial}{\partial \alpha} C_T^{1-\eta} = \frac{1}{2}(1-\eta)T^2 C_T^{1-\eta} . \quad (12)$$

The (stochastic) return from this policy is thus

$$\tilde{r}_{\alpha T} = \frac{1}{2}T^2 C_T^{1-\eta} e^{-\delta T} . \quad (13)$$

We want the expectation and variance of this return. The Appendix derives a formula for the expectation of  $C_T^{1-\eta}$ . Using that formula, the expected return is:

$$\bar{r}_{\alpha T} = \frac{1}{2}T^2 \mathcal{E}_0(C_T^{1-\eta}) e^{-\delta T} = \frac{1}{2}T^2 e^{-\rho_0 T - \frac{1}{2}\alpha(1-\eta)T^2 + \frac{1}{6}\sigma^2(1-\eta)^2 T^3} \quad (14)$$

where

$$\rho_0 \equiv \delta + (\eta - 1)g_0 .$$

Note that  $\partial \bar{r} / \partial \alpha > 0$  and  $\partial \bar{r} / \partial \sigma^2 > 0$ . The first inequality simply results from the fact that the higher is the starting value of  $\alpha$ , the greater is the value of reducing  $\alpha$ . The second inequality follows from the fact that  $\mathcal{E}_0(C_T^{1-\eta})$  is a convex function of the rate of increase in temperature, so that on average stochastic fluctuations in temperature increase the value of

reducing its rate of growth. Finally,  $\partial\bar{r}/\partial\eta$  can be positive or negative, depending on the relative size of the starting growth rate  $g_0$ .<sup>6</sup>

To get the standard deviation of this return, note from eqn. (12) that

$$\mathcal{E}_0\left(\frac{\partial}{\partial\alpha}C_T^{1-\eta}\right)^2 = \frac{1}{4}(1-\eta)^2T^4\mathcal{E}_0(C_T^{2-2\eta}). \quad (15)$$

Now use results in the Appendix to get  $\mathcal{E}_0(C_T^{2-2\eta})$ , so that

$$\mathcal{E}_0\left(\frac{\partial}{\partial\alpha}C_T^{1-\eta}\right)^2 = \frac{1}{4}(1-\eta)^2T^4e^{-2\rho_0T-\alpha(1-\eta)T^2+\frac{2}{3}\sigma^2(1-\eta)^2T^3}.$$

Thus the variance of the return is given by

$$\begin{aligned} \mathcal{V}(r_{\alpha T}) &= \mathcal{E}_0(\tilde{r}_{\alpha T}^2) - \bar{r}_{\alpha T}^2 \\ &= \frac{1}{4}T^4\mathcal{E}_0(C_T^{2-2\eta})e^{-2\delta T} - \bar{r}_{\alpha T}^2 \\ &= \frac{1}{4}T^4e^{-2\rho_0T-\alpha(1-\eta)T^2+\frac{2}{3}\sigma^2(1-\eta)^2T^3} - \frac{1}{4}T^4e^{-2\rho_0T-\alpha(1-\eta)T^2+\frac{1}{3}\sigma^2(1-\eta)^2T^3} \\ &= \frac{1}{4}T^4e^{-2\rho_0T-\alpha(1-\eta)T^2+\frac{1}{3}\sigma^2(1-\eta)^2T^3}[e^{\frac{1}{3}\sigma^2(1-\eta)^2T^3} - 1] \\ &= \frac{1}{4}T^4[\mathcal{E}_0(C_T^{1-\eta})e^{-\delta T}]^2[e^{\frac{1}{3}\sigma^2(1-\eta)^2T^3} - 1] \end{aligned} \quad (16)$$

The standard deviation of the return is therefore

$$\text{SD}(r_{\alpha T}) = \frac{1}{2}T^2\mathcal{E}_0(C_T^{1-\eta})e^{-\delta T}[e^{\frac{1}{3}\sigma^2(1-\eta)^2T^3} - 1]^{1/2} = \bar{r}_{\alpha T}[e^{\frac{1}{3}\sigma^2(1-\eta)^2T^3} - 1]^{1/2}. \quad (17)$$

$\text{SD}(r_{\alpha T})$  is proportional to  $\bar{r}_{\alpha T}$ , so a convenient way to characterize the relative riskiness of this return is by its Sharpe ratio, i.e., the ratio of the expected return to its standard deviation. From eqns. (14) and (17), the Sharpe ratio is

$$S_{\alpha T} = \frac{\bar{r}_{\alpha T}}{\text{SD}(r_{\alpha T})} = [e^{\frac{1}{3}(1-\eta)^2\sigma^2T^3} - 1]^{-1/2} \quad (18)$$

The Sharpe ratio is useful because it summarizes the effects of changes in various parameters, such as volatility,  $\sigma$ . As can be seen from eqns. (14) and (17), and increase in  $\sigma$  causes an

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<sup>6</sup> $\partial\bar{r}/\partial\eta = [\frac{1}{2}\alpha T^2 - \frac{1}{3}\sigma^2(1-\eta)T^3 - g_0T]\bar{r}_{\alpha T}$ . The first two terms in the brackets are positive and the last is negative. If  $g_0T$  is large relative to the first two terms,  $\partial\bar{r}/\partial\eta$  will be negative. Higher  $g_0T$  implies higher  $C_T$  and thus greater reduction in marginal utility at  $T$  from an increase in  $\eta$ , and hence a lower return from reducing  $\alpha$ . A higher value of  $\alpha T$  implies the opposite — lower  $C_T$ . A higher value of  $\sigma^2T^2$  implies a higher value of marginal utility at  $T$  (via Jensen's inequality).

increase in *both* the expected return and the standard deviation of the return. But the effect on the standard deviation is larger, so that an increase in  $\sigma$  causes a decrease in  $S_{\alpha T}$ .

Note that the Sharpe ratio is independent of the drift,  $\alpha$ . The reason is that the exponential growth rates for both the expected return and the standard deviation include the factor  $\frac{1}{2}\alpha(1-\eta)T^2$ , which cancels out of the ratio. Also note that if  $\eta \neq 1$ ,  $S_{\alpha T} \rightarrow 0$  as  $T \rightarrow \infty$ . The reason is that the standard deviation of the return grows faster with  $T$  than the expected return. Finally, an increase in the index of relative risk aversion,  $\eta$ , leads to a decrease in the Sharpe ratio. The reason is that the return itself is risk-adjusted, i.e., it is proportional not to the change in future consumption but to the change in the *utility* from future consumption. Thus an increase in  $\eta$  has the same effect as an increase in  $\sigma$ .

### 3.2 Continuous-Time, Finite Horizon.

Now suppose we have a horizon  $T$  so that welfare is given by the integral of discounted utility over  $t = 0$  to  $T$ . Taking future consumption as unknown, welfare is:

$$\tilde{W} = \frac{1}{1-\eta} \int_0^T C_t^{1-\eta} e^{-\delta t} dt \equiv \int_0^T \tilde{W}_t dt . \quad (19)$$

Thus  $\tilde{W}_t$  is (uncertain) discounted utility at time  $t > 0$ .

Once again we consider a policy that increases  $\alpha$  incrementally. As before, it is easily seen that the (uncertain) return from this policy is

$$\tilde{r}_\alpha = \int_0^T \frac{1}{2} t^2 C_t^{1-\eta} e^{-\delta t} dt = \int_0^T \frac{1}{2} (1-\eta) t^2 \tilde{W}_t dt . \quad (20)$$

We can now examine returns over the full time horizon.

We want the expectation, standard deviation, and Sharpe ratio for the cumulative return over the horizon 0 to  $T$ ,  $\bar{r}_\alpha$ . The expectation is straightforward; it is simply

$$\bar{r}_\alpha = \int_0^T \frac{1}{2} t^2 \mathcal{E}_0(C_t^{1-\eta}) e^{-\delta t} dt = \int_0^T \frac{1}{2} t^2 e^{-\rho_0 t - \frac{1}{2}\alpha(1-\eta)t^2 + \frac{1}{6}\sigma^2(1-\eta)^2 t^3} dt . \quad (21)$$

Note that  $\partial \bar{r}_\alpha / \partial \eta$  can be positive or negative depending on the relative magnitudes of  $g_0$ ,  $\alpha$  and  $\sigma$ , and can change with the time horizon  $T$ .

The variance of the return is  $\mathcal{V}(\tilde{r}_\alpha) = \mathcal{E}_0(\tilde{r}_\alpha^2) - \bar{r}_\alpha^2$ , so we need to find

$$\mathcal{E}_0(r_\alpha^2) = \mathcal{E}_0 \left( \int_0^T \frac{1}{2} t^2 C_t^{1-\eta} e^{-\delta t} dt \right)^2 \quad (22)$$

We can write the integral as:

$$\begin{aligned} r_\alpha^2 &\approx \left[ \sum_{j=0}^N \frac{1}{2} j^2 C_j^{1-\eta} e^{-j\delta} \right]^2 \\ &\approx \sum_{i=0}^N \sum_{j=0}^N \frac{1}{4} j^2 j^2 C_i^{1-\eta} C_j^{1-\eta} e^{-\delta(i+j)} \end{aligned} \quad (23)$$

where  $N = T/\Delta t$ .

As shown in the Appendix,

$$\mathcal{E}_0(C_i^{1-\eta} C_j^{1-\eta}) = C_0^{2(1-\eta)} e^{(1-\eta)g_0(i+j)+b(i,j)}, \quad (24)$$

where

$$b(i, j) = \frac{1}{12} \sigma^2 (1 - \eta)^2 (i + j)^3 - \frac{1}{2} \alpha (1 - \eta) (i^2 + j^2). \quad (25)$$

Setting  $C_0 = 1$  and letting  $\Delta t \rightarrow 0$ ,

$$\mathcal{E}_0(r_\alpha^2) = \int_0^T \int_0^T \frac{1}{4} t^2 s^2 e^{-\rho_0(t+s) - \frac{1}{2} \alpha (1-\eta)(t^2+s^2) + \frac{1}{12} \sigma^2 (1-\eta)^2 (t+s)^3} dt ds. \quad (26)$$

Using eqns. (21) and (26), we can find (numerically) the expectation, standard deviation, and Sharpe ratio for the cumulative return  $\tilde{r}_\alpha$ . Figure 3 shows the Sharpe ratio as a function of  $\eta$  for two values of  $\sigma$ , a “base case” value of .00012, as well as .00024, with a time horizon  $T = 300$  years. (The values of  $\alpha$  and  $\sigma$  come from a calibration to climate change information that is discussed later.) Figure 4 shows  $E(r)$  and  $\text{SD}(r)$  as functions of  $\eta$  for  $\sigma = .00012$ . Note that the standard deviation is first increasing and then decreasing in  $\eta$ .

## 4 WTP and Policy Targets.

In the previous section I considered marginal policies, i.e., policies that would result in a marginal reduction in  $\alpha$ . This allowed for a simple characterization of such policies in terms of a Sharpe ratio. I turn now to policies that would result in a substantial change in the

drift  $\alpha$  and/or the volatility  $\sigma$ , and evaluate those policies in the context of willingness to pay. I begin with policies for which changes are made at one or more discrete points in time. This is simpler than the continuous-time case, which I turn to next.

#### 4.1 WTP: Consumption Now and at Future $T$ .

Again,  $C_0 = 1$ , and  $C_T$  is uncertain. Under BAU (no abatement policy),  $\alpha = \alpha_0$  and  $\sigma = \sigma_0$ . We want to find the fraction of current and future consumption society would give up to decrease  $\alpha$  and/or decrease  $\sigma$ . Define  $a(\alpha_1, \sigma_1, T) = -(1/2)\alpha_1(1-\eta)T^2 + (1/6)\sigma_1^2(1-\eta)^2T^3$ . Then to obtain the WTP to move from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ , write the social welfare for this change as:

$$W_{1T} = \frac{[1 - w(\alpha_1, \sigma_1, T)]^{1-\eta}}{1-\eta} [1 + e^{-\rho_0 T + a(\alpha_1, \sigma_1, T)}] . \quad (27)$$

Under BAU, welfare is

$$W_{2T} = \frac{1}{1-\eta} [1 + e^{-\rho_0 T + a(\alpha_0, \sigma_0, T)}] \quad (28)$$

Equating  $W_{1T}$  and  $W_{2T}$  yields the WTP for this change:<sup>7</sup>

$$w^*(\alpha_1, \sigma_1, T) = 1 - \left[ \frac{1 + e^{-\rho_0 T + a(\alpha_0, \sigma_0, T)}}{1 + e^{-\rho_0 T + a(\alpha_1, \sigma_1, T)}} \right]^{\frac{1}{1-\eta}} , \quad (29)$$

where  $\rho_0 \equiv \delta + (\eta - 1)g_0$  as before.

We would like to generate iso-WTP curves, i.e., different combinations of  $(\alpha', \sigma')$  such that  $w^*(\alpha', \sigma', T) = w^*(\alpha_1, \sigma_1, T)$ . This would describe a “risk-return” trade-off, i.e., a trade-off between decreasing  $\alpha$  and decreasing  $\sigma$  that yields the same WTP as for moving from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ . For this simple two-period case, this is easily done analytically. Let  $a(T)$  denote the function  $a$  with  $\alpha$  and  $\sigma$  set to their starting (calibrated for BAU) values of  $\alpha_0$  and  $\sigma_0$ . Then using eqn. (29),  $w^*(\alpha', \sigma', T) = w^*(\alpha_1, \sigma_1, T)$  implies:

$$a(\alpha', \sigma', T) = a(\alpha_1, \sigma_1, T) , \quad (30)$$

which in turn implies that

$$\sigma' = \sqrt{\sigma_1^2 + \frac{3(\alpha' - \alpha_1)}{(1-\eta)T}} , \quad (31)$$

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<sup>7</sup>Of course it may or may not be possible to shift from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ . This WTP is simply the most society would sacrifice for this change in  $\alpha$  and  $\sigma$ , were it feasible to do so with the resulting revenue.

or equivalently,

$$\alpha' = \alpha_1 + \frac{1}{3}(1 - \eta)T(\sigma'^2 - \sigma_1^2) . \quad (32)$$

Thus moving from  $(\alpha_0, \sigma_0)$  to any combination  $(\alpha', \sigma')$  that satisfies eqn. (32) will have the same WTP =  $w^*(\alpha_1, \sigma_1, T)$ .

The slope of the iso-WTP curve is the marginal rate of substitution (MRS) between the two policy targets,  $\alpha'$  and  $\sigma'$ . From (32), the MRS is:

$$\text{MRS} = \frac{d\alpha'}{d\sigma'} = \frac{2}{3}\sigma'(1 - \eta)T . \quad (33)$$

Note that as  $T$  increases, the MRS becomes larger in magnitude, i.e., it takes larger and larger increases in  $\alpha'$  to compensate for any given reduction in  $\sigma'$ . (This is because of the  $\sigma^2 T^3$  term versus  $\alpha T^2$  in the function  $a$ .)

It is important to keep in mind that the  $(\alpha', \sigma')$  combinations on the iso-WTP curve represent different *ending* positions that result from a policy. In other words, we are always starting from  $(\alpha_0, \sigma_0)$ . All policies that move from this starting position to an ending position on the curve have the same WTP.

## 4.2 Multiple Time Periods; Action at $t = 0$ .

Suppose there are  $n$  50-year time intervals:  $T_1 = 50$ ,  $T_2 = 100$ , ...,  $T_n = 50n$ , and suppose we can decrease  $\alpha$  and/or reduce  $\sigma$  starting immediately by reducing consumption now (i.e., at  $t = 0$ ) and at every future  $T_i$  by some fraction. In this case, to find the WTP to move from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ , write the welfare measures  $W_1$  and  $W_2$  as:

$$W_1 = \frac{[1 - w(\alpha_1, \sigma_1)]^{1-\eta}}{1 - \eta} \left[ 1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)} \right] \quad (34)$$

$$W_2 = \frac{1}{1 - \eta} \left[ 1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)} \right] \quad (35)$$

Thus to obtain  $w^*(\alpha_1, \sigma_1)$ , just replace eqn. (29) with:

$$w^*(\alpha_1, \sigma_1) = 1 - \left[ \frac{1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)}}{1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)}} \right]^{\frac{1}{1-\eta}} . \quad (36)$$

As before, we can generate “iso-WTP” curves, i.e., different combinations of  $(\alpha', \sigma')$  such that  $w^*(\alpha', \sigma') = w^*(\alpha_1, \sigma_1)$ . These “iso-WTP” curves will depend on  $n$  because as we saw above, the  $d\sigma^2/d\alpha$  that keeps  $W_T$  fixed varies with  $T$ . Also, we cannot generate these curves analytically. From eqn. (36), the  $(\alpha', \sigma')$  that gives the same WTP as  $(\alpha_1, \sigma_1)$  is the solution of:

$$\sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha', \sigma', T_i)} = \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)} \quad (37)$$

Eqn. (37) can be solved numerically for combinations of  $\alpha'$  and  $\sigma'$ .

Eqn. (37) applies to a specific WTP, namely the WTP that applies to moving from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ . Starting from  $(\alpha_0, \sigma_0)$ , however, we can find combinations of  $\alpha'$  and  $\sigma'$  that yield the same *arbitrary* WTP. Denoting the arbitrary WTP by  $w^*$ , we find the combinations of  $\alpha'$  and  $\sigma'$  by solving:

$$\sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha', \sigma', T_i)} = (1 - w^*)^{\eta-1} \left[ 1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)} \right] - 1 . \quad (38)$$

### 4.3 WTP: Continuous Time with Immediate Action.

Welfare over some future time horizon, with no policy intervention, is given by

$$W_2 = \frac{1}{1 - \eta} \int_0^\infty e^{-\rho_0 t + a(\alpha_0, \sigma_0, t)} dt . \quad (39)$$

Now once again consider a policy that would move from  $(\alpha_0, \sigma_0)$  to  $(\alpha_1, \sigma_1)$ . Denote the WTP for this policy by  $w_1$ , and note that with the intervention, welfare is:

$$W_1 = \frac{1}{1 - \eta} \int_0^\infty (1 - w_1)^{1-\eta} e^{-\rho_0 t + a(\alpha_1, \sigma_1, t)} dt , \quad (40)$$

where once again,  $a(\alpha_1, \sigma_1, t) = -(1/2)\alpha_1(1 - \eta)t^2 + (1/6)\sigma_1^2(1 - \eta)^2 t^3$ . Equating  $W_1$  and  $W_2$ , we get:

$$w_1 = 1 - \left[ \frac{G(\alpha_1, \sigma_1)}{G(\alpha_0, \sigma_0)} \right]^{\frac{1}{\eta-1}} , \quad (41)$$

where

$$G(\alpha_0, \sigma_0) = \int_0^\infty e^{-\rho_0 t + a(\alpha_0, \sigma_0, t)} dt , \quad (42)$$



and likewise for  $G(\alpha_1, \sigma_1)$ . Thus, given starting values of  $\alpha$  and  $\sigma$  (based, say, on a calibration of a future  $T$  against the IPCC or other distributions), we can calculate the WTP to increase  $\alpha$  and/or decrease  $\sigma$ .

As before, we would like to obtain iso-WTP curves, i.e., combinations of  $\alpha'$  and  $\sigma'$  for which the WTP is again  $w_1$ . This can be done numerically by finding combinations of  $\alpha'$  and  $\sigma'$  that satisfy

$$G(\alpha', \sigma') = G(\alpha_1, \sigma_1) . \quad (43)$$

We can also obtain combinations of  $\alpha'$  and  $\sigma'$  for which the WTP is equal to some arbitrary number,  $w$ . From eqn. (41), we find those combinations that satisfy

$$G(\alpha', \sigma') = (1 - w)^{\eta-1} G(\alpha_0, \sigma_0) . \quad (44)$$

Finally, note that the integration over time cannot be carried out over an arbitrarily large horizon. It is necessary to limit the time horizon to some number of years, e.g., 200 or 300.

#### 4.4 Costs of Changing $\alpha$ and $\sigma$ .

We have seen how we can determine the WTP for moving from a starting drift and volatility,  $(\alpha_0, \sigma_0)$ , to a new  $(\alpha', \sigma')$ , and how we can trace out combinations of  $\alpha$  and  $\sigma$  targets that have the same WTP. However, we also want to know the cost of moving from  $(\alpha_0, \sigma_0)$  to  $(\alpha', \sigma')$ . If that cost exceeds the WTP (which is society's reservation price), the policy is economically infeasible. If the cost is less than the WTP, the policy yields a positive surplus.

If we knew the costs of changing  $\alpha$  and  $\sigma$  we could plot an iso-cost line. If those costs were linear in the change in  $\alpha$  and the change in  $\sigma$ , the iso-cost line would be a straight line, as drawn in Figure 1, and its tangency with an iso-WTP curve would be the cost-minimizing target combination of  $\alpha$  and  $\sigma$ . There are other target combinations of  $\alpha$  and  $\sigma$  with the same WTP, but they would be more costly to achieve. The minimum cost, however, might be greater or less than the particular WTP. Given a starting point  $(\alpha_0, \sigma_0)$ , the WTP to move to a target  $(\alpha', \sigma')$  depends on the distance to the target and also on the parameters  $\eta$ ,  $\delta$ , and  $g_0$ . For any given shift in  $\alpha$  and  $\sigma$ , for example, a reduction in  $\eta$  will increase the

WTP, which implies that for any fixed WTP, the required change in  $\alpha$  and  $\sigma$  will be smaller, so that the corresponding cost will be smaller.

## 5 Calibration and Results.

Although it is exceedingly simple, the model specified in Section 2 can be informative about risk and return in the context of climate change policy. In this section, I calibrate the model by determining values of the parameters  $\alpha_X$ ,  $\sigma_X$ , and  $\gamma$  that are consistent with recent studies of global warming and its impact, along with estimates of the cost of reducing  $\alpha_X$  and  $\sigma_X$ . I then calculate iso-WTP curves and discuss their implications.

### 5.1 Calibration.

To obtain numerical results we need to find values for  $\alpha$  and  $\sigma$  in eqn. (2), the parameters  $\eta$ ,  $\delta$ , and the initial growth rate  $g_0$ , and rough estimates of the costs of reducing  $\alpha$  and  $\sigma$ .

**Calibration of  $\alpha$  and  $\sigma$  under BAU.** The most recent IPCC report (2007) states that growing GHG emissions (i.e., under BAU) would likely lead to a doubling of the atmospheric CO<sub>2</sub>e concentration relative to the pre-industrial level by mid-century, which would “most likely” cause an increase in global mean temperature between 2.0°C to 4.5°C by 2100, with an expected value of 2.5°C to 3.0°C. The IPCC report indicates that this range, derived from the results of 22 scientific studies the IPCC surveyed, represents a roughly 66- to 90-percent confidence interval, i.e., there is a 5 to 17-percent probability of a temperature increase above 4.5°C. The 22 studies surveyed by the IPCC also provide rough estimates of the outer tail of the temperature distribution. In summarizing them, the IPCC translated the implied outcome distributions into a standardized form that allows comparability across the studies, and created graphs showing multiple distributions implied by groups of studies. Those distributions suggest that there is a 5% probability that a doubling of the CO<sub>2</sub>e concentration relative to the pre-industrial level would lead to a global mean temperature increase,  $T$ , of 7°C or more.

I will use, for a horizon of 100 years,  $\mathcal{E}(X) = 3^\circ\text{C}$  and a 5% probability of a temperature

increase  $\geq 7^\circ\text{C}$ . The 5% point is 1.65 standard deviations above the mean (because eqn. (1) implies that  $T_t$  is normally distributed), so one standard deviation is  $4/1.65 = 2.42$ . Thus the drift and volatility of the process for temperature are  $\alpha_X = 3/100 = .03$ , and  $\sigma_X = 2.42/\sqrt{100} = 0.242$ . More recent studies, e.g., Sokolov et al. (2009), have larger numbers for the expected temperature change and for the 5% point, but I will use these earlier (perhaps overly optimistic) estimates.

We need the process for the growth rate  $g_t$ , and thus we need an estimate of  $\gamma$ , which relates  $g_t$  to  $X_t$ . One way to estimate  $\gamma$  is to use the equation for expected future consumption derived in the Appendix, combined with estimates of the expected loss of GDP at a specific temperature change. The Appendix shows that expected future consumption is given by:

$$\mathcal{E}_0(C_t) = e^{g_0t - (1/2)\alpha t^2 + (1/6)\sigma^2 t^3} . \quad (45)$$

In earlier work (2011a, 2012), I used the IPCC (2007) estimates that the loss of GDP resulting from a temperature change of  $4^\circ\text{C}$  is “most likely” in the range of 1% to 5%. These IPCC estimates are based on several integrated assessment models (IAMs). Taking the upper end of the range, i.e., 5%, implies that with  $\alpha'_X = .04$ , i.e.,  $\mathcal{E}_0(X_{100}|\alpha'_X = .04) = 4^\circ\text{C}$ , and using eqn. (45), for  $t = 100$ ,

$$\mathcal{E}_0(C_t) = e^{g_0t - (1/2)\alpha'_X \gamma t^2 + (1/6)\sigma_X^2 \gamma^2 t^3} = .95e^{g_0t} ,$$

so that

$$-\frac{1}{2}\alpha'_X \gamma t^2 + \frac{1}{6}\sigma_X^2 \gamma^2 t^3 = \ln(.95) . \quad (46)$$

Substituting  $t = 100$ ,  $\alpha'_X = .04$  and  $\sigma_X = .242$  yields the following quadratic equation for  $\gamma$ :

$$\gamma^2 - .0205\gamma + .00000526 = 0$$

If  $\sigma_X = 0$ ,  $\gamma = .000152$ , so we want the smaller root of this equation, which yields  $\gamma = .00026$ .

This, of course, is an indirect estimate of  $\gamma$ , and relies on the integrated assessment models surveyed by the IPCC that largely posit, rather than estimate, a relationship between temperature and GDP. An alternative way to obtain  $\gamma$  is to utilize direct econometric estimates. Unfortunately, few such estimates are available; the only ones I am aware of are

those of Dell, Jones, and Olken (2008, 2009) and Bansal and Ochoa (2011a, b). The latter set of estimates are based on a more extensive data set (147 countries over 1950–2007) and allow for dynamic adjustment of GDP to changes in temperature. Bansal and Ochoa find that a 0.2°C increase in temperature leads to about a .2 percentage point reduction in the growth of real GDP (a number close to that in Dell, Jones and Olken (2008)). However, the Bansal-Ochoa impact number is largely transitory; GDP growth largely recovers after 10 years. In my model, the impact of temperature on GDP growth is permanent, so the Bansal-Ochoa number would imply a value of  $\gamma$  in eqn. (2) of around .001. This is four times as large as the value obtained from the IPCC’s survey of IAMs, so I will take as “consensus” number something in the middle, namely  $\gamma = .0005$ .

Thus we have  $\alpha_X = .03$ ,  $\sigma_X = .242$ , and  $\gamma = .0005$ , so that  $\alpha = \alpha_X\gamma = .000015$  and  $\sigma = \sigma_X\gamma = .00012$ . These are the values for  $\alpha$  and  $\sigma$  that correspond to the starting values  $(\alpha_0, \sigma_0)$  when calculating WTP and iso-WTP curves.

**Other Parameter Values.** WTP also depends on the index of relative risk aversion  $\eta$ , the rate of time preference  $\delta$ , and the base level growth rate  $g_0$ . Thus we must select values for these parameters.

The finance and macroeconomics literatures have estimates of  $\eta$  ranging from 1.5 to 6, and estimates of  $\delta$  ranging from .01 to .04. The per capita real growth rate  $g_0$ , measured from historical data, is about .02. It has been argued that for intergenerational comparisons  $\delta$  should be close to zero, on the grounds that society should not value the well-being of our great-grandchildren less than our own. Without taking sides in that (essentially ethical) argument, I set  $\delta = 0$  for the calculations that follow. (For values of  $\delta$  of .01 or higher, WTP is close to zero, and it is difficult to justify *any* abatement policy.)

Likewise, while values of  $\eta$  above 4 may be consistent with the (relatively short-horizon) behavior of investors, we might apply lower values to welfare comparisons involving future generations. In the (deterministic) Ramsey growth model, the optimal savings rate is  $s^* = (R - \delta)/\eta R$ , where  $R$  is the consumption discount rate and the real return on investment. If  $R = .04$  and  $\delta = .02$ ,  $s^* = 1/2\eta$ , suggesting  $\eta$  should be in the range of 2 to 4. Stern (2007) used a value of 1 for  $\eta$ , which is below the consensus range. Lower values of  $\eta$  result

in higher values of WTP; I will generally set  $\eta = 2$ , but I will also show how various results depend on  $\eta$ .

**Costs of Reducing  $\alpha$  and  $\sigma$ .** I assume a linear cost function, i.e., the cost of moving from  $(\alpha_0, \sigma_0)$  to  $(\alpha', \sigma')$  is given by:

$$\text{Cost} = c_1(\alpha_0 - \alpha') + c_2(\sigma_0 - \sigma') \quad (47)$$

Note that the total cost and its two components are expressed in terms of percentages of consumption that must be sacrificed annually. Thus  $c_1(\alpha_0 - \alpha')$  is the percentage of consumption that must be given up each year in order to reduce  $\alpha$  from  $\alpha_0$  to  $\alpha' < \alpha_0$ .

There are no direct estimates of  $c_1$  or  $c_2$  that I am aware of, so coming up with numbers requires extrapolation of the few cost estimates that exist, along with some guesswork. I start with the Kyoto Protocol, which if enacted and adhered to is expected to limit any increase in global mean temperature by the end of the century to roughly  $3^\circ\text{C}$  (which is the *expected* temperature increase under BAU). The U.S. Energy Information Administration (1998) estimated that compliance with the Protocol would cost some 1 to 3% of GDP annually. Estimates from country cost studies in IPCC (2007c) also put the cost of limiting the increase in temperature to  $3^\circ\text{C}$  at around 1 to 3% of GDP. I will take the mid-range of 2% of GDP and translate “limiting the increase in temperature to  $3^\circ\text{C}$ ” to mean reducing the expected temperature increase from  $3^\circ\text{C}$  to  $1.5^\circ\text{C}$  with a standard error of about  $1.5^\circ\text{C}$ . This means cutting the drift  $\alpha$  in half, i.e., from  $.000015$  to  $.0000075$ , which in turn implies that  $c_1 = (.02)/(7.5 \times 10^{-4}) = 2667$ .

Obtaining an estimate for  $c_2$ , the cost of reducing the volatility and hence the standard deviation of future temperature change, is even more difficult. According to numbers compiled by Palmer (2011), public funding for climate change research in the U.S. during 2009 was about \$5 billion. These expenditures could arguably improve our understanding of global warming and reduce our uncertainty over future temperatures; they exclude public R&D spending on renewable energy, new technologies, electricity and fossil fuel production, and related technologies.<sup>8</sup> I will make the bold assumption that a doubling of these expen-

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<sup>8</sup>For the six focus areas of The US Global Change Research Program, this figure includes only “Improving

ditures to \$10 billion per year could result in a halving of the volatility  $\sigma_0$ . In other words, reducing  $\sigma$  from .00012 to .00006 would require a sacrifice of about 0.067% of GDP annually (based on a \$15 trillion GDP), which in turn implies that  $c_2 = .00067/.00006 = 11.17$ .

## 5.2 Numerical Results.

I begin by calculating iso-WTP curves using the parameter values discussed above. Figure 5 shows three iso-WTP curves, generated using the starting values  $\alpha_0 = .000015$  and  $\sigma_0 = .00012$  based on the calibration described above. Other parameters are  $g_0 = .02$ ,  $\delta = 0$ ,  $\eta = 2$ , and the time horizon is  $T = 300$  years. The starting values  $(\alpha_0, \sigma_0)$  are shown as the small circle near the bottom of the diagram. The solid line in the middle is the iso-WTP curve for  $WTP = .0365$ , which is the WTP to reduce  $\alpha$  to zero but leave  $\sigma$  unchanged. Thus moving from  $(\alpha_0, \sigma_0)$  to any point on that curve has a WTP of .0365. The other two curves in Figure 5 show combinations of  $\alpha$  and  $\sigma$  that, starting from  $\alpha_0$  and  $\sigma_0$ , have WTPs of .02 and .06. (Note that all of the values of  $\alpha$  on the top curve are negative.)

Figure 6 shows an iso-WTP curve (labeled B) for the same starting and ending values of the drift and volatility,  $(\alpha_0, \sigma_0) = (.000015, .00012)$  and  $(\alpha', \sigma') = (0, .00012)$ , and the same values for the other parameters, so that the WTP is again .0365. The curve labeled A shows alternative combinations of starting values for  $\alpha_0$  and  $\sigma_0$  that give the same  $WTP = .0365$  for any target on curve B. Thus shifts from any points on curve A to any points on curve B have a WTP of .0365 and thus are welfare-equivalent.

The iso-WTP curves in Figure 5 show the most that society would be willing to pay (in terms of percentage of consumption sacrificed) in order to move from a starting point  $(\alpha_0, \sigma_0)$  to an ending point on the curve. They do not, however, tell us anything about the cost of moving from  $(\alpha_0, \sigma_0)$  to an ending point. If the cost exceeds the WTP, the policy is economically infeasible; if the cost is less than the WTP, the policy has a positive social surplus. In addition, since all points on an iso-WTP curve are welfare-equivalent, we might

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Knowledge,” “Improving Understanding,” and “Modelling and Prediction.” It also includes about \$1 billion of science expenditures under the 2009 Recovery Act that are climate related. The U.S. GAO (2003) puts the 2003 expenditure at about \$4 billion.

like to know which point has the least cost. To address these questions we utilize the iso-cost lines implied by eqn. (47), with the calibrated values for  $c_1$  and  $c_2$ , i.e., the cost equation

$$\text{Cost} = 2667(\alpha_0 - \alpha') + 11.17(\sigma_0 - \sigma') . \quad (48)$$

Figure 7 shows an iso-WTP curve that plots combinations of target values of  $\alpha$  and  $\sigma$  that have a WTP of .02, given the starting point  $(\alpha_0, \sigma_0) = (.000015, .00012)$ , which is shown as a small circle near the bottom of the figure. Also shown is an iso-cost line that is tangent to the iso-WTP curve, i.e., at the point where the slope of the iso-WTP curve is  $-c_2/c_1$ . (Remember that  $\alpha$  is *decreasing* as we move up the vertical axis.) Thus at this point the cost of moving from  $(\alpha_0, \sigma_0)$  to the iso-WTP curve is minimized. In this example, that cost turns out to be about .020, i.e., the same as the WTP. Thus the policy illustrated by Figure 7 is just feasible, i.e., society would be paying its reservation price for the policy, so the social surplus would be zero. On the other hand, if we were to double both  $c_1$  and  $c_2$  (so that the slope and tangency point remained the same), the cost would also double, making the policy economically infeasible. And if we were to instead halve both  $c_1$  and  $c_2$ , the cost would drop to .010, so that the policy would have a positive social surplus of .01, i.e., 1% of GDP.

Given eqn. (48), the cost of any policy is a simple function of the changes in  $\alpha$  and  $\sigma$ . But the WTP of the changes in  $\alpha$  and  $\sigma$  depends not only on the magnitudes of the changes, but also on the parameters  $\eta$ ,  $\delta$ , and  $g_0$ . For any given shift in  $\alpha$  and  $\sigma$ , for example, a reduction in  $\eta$  will increase the WTP.<sup>9</sup> Equivalently, a lower value of  $\eta$  implies that for any fixed WTP, the required change in  $\alpha$  and  $\sigma$  will be smaller. This is illustrated by Figure 8, which is the same as Figure 7 except that  $\eta = 1.5$  instead of 2. Compared to  $\eta = 2$ , if  $\eta = 1.5$  it takes a much smaller reduction in  $\alpha$  and  $\sigma$  to have a WTP of .02, and thus the cost of the policy is much smaller (.008 versus .020). In this case the policy yields a positive social surplus.

Table 1 shows the target  $(\alpha', \sigma')$ , the cost, and the net social surplus for other parameter values. The first row shows the base case corresponding to Figure 7, and the second row corresponds to Figure 8 (in which  $\eta$  is reduced to 1.5 but the WTP is fixed at .02, so that the

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<sup>9</sup>Reducing  $\eta$  reduces the effective consumption discount rate, which, ignoring uncertainty, is given by  $R_t = \delta + \eta g_t$  (the deterministic Ramsey rule). Thus reducing  $\eta$  increases the present value of future losses of consumption resulting from higher temperatures. For further discussion of this point, see Pindyck (2012).

Table 1: WTP AND COST

Note: Entries show changes from base case, for which $c_1 = 2667$ , $c_2 = 11.17$ , $\eta = 2$ , $g_0 = .02$ , and $\delta = 0$ . Throughout, starting point is $\alpha_0 = .000015$ and $\sigma_0 = .00012$ .					
Change from Base Case	WTP	$\alpha'$	$\sigma'$	Cost	Social Surplus
BASE CASE	.02	.0000077	.000046	.020	0
$\eta = 1.5$	.02	.000012	.000068	.008	.012
$\eta = 3$	.02	-.0000187	.000046	.091	-.071
$\eta = 3$	.00032	.0000152	.000041	.00032	0
$g_0 = .01$	.02	.0000131	.000033	.0060	.014
$\delta = .01$	.02	-.0000031	.000067	.049	-.029
$c_2 = 22.34$	.02	.0000074	.000090	.021	-.001

cost falls and the social surplus is positive). Increasing  $\eta$  to 3 implies that for any fixed target  $(\alpha', \sigma')$ , the WTP will fall, so that obtaining a WTP of .02 requires a much larger shift in  $(\alpha', \sigma')$ , and thus a much higher cost (in this case .091 so that the social surplus is negative). As the fourth row shows, with  $\eta = 3$  we can still obtain a positive social surplus, but only when the WTP is very small. As the fifth row of the table shows, reducing  $g_0$  has the same effect as reducing  $\eta$  (the initial deterministic consumption discount rate is  $R_0 = \delta + \eta g_0$ ), so that the target  $(\alpha', \sigma')$  is closer to the initial  $(\alpha_0, \sigma_0)$  and the cost falls. Increasing  $\delta$  has the same directional effect as increasing  $\eta$ , so that with WTP fixed at .02, the cost increases. Finally, in the last row of the table,  $c_2$ , the cost of reducing  $\sigma$ , is doubled. As we would expect, this increases the overall cost (but not by much), and causes the target  $(\alpha', \sigma')$  to shift to a larger reduction in  $\alpha$  and smaller reduction in  $\sigma$ .

## 6 Concluding Remarks.

Using a simple model of a stock externality (e.g., temperature) that evolves stochastically, I examined the extent to which the value of a costly policy (e.g., emissions abatement) depends on the policy's expected future benefits versus their riskiness. I also presented a framework by which one can estimate the extent to which the objective of policy should be a reduction



in the expected future damages versus a reduction in the uncertainty over those damages. The framework is based on the computation of “iso-WTP” curves (social indifference curves) for combinations of risk and expected returns as policy objectives. Given cost estimates for reducing risk and increasing expected returns, one can then compute the optimal risk-return mix for policy, and evaluate the policy’s social surplus.

I have illustrated this framework using a simple Brownian motion process for the stock variable, in this case temperature, and a simple CRRA utility function. This has the advantage of yielding simple expressions for expected future consumption and utility. However, the framework can also be applied to alternative stochastic processes for the stock variable and alternative social preferences, although numerical methods may be needed to obtain results.

In the case of climate change, there is a broad consensus on the expected rate of temperature increase under BAU — about 3°C. There is considerable disagreement, however, over the nature and extent of uncertainty around that expected value. That disagreement relates to the choice of probability distribution that best applies to future temperatures (e.g., whether the distribution should be fat- or thin-tailed), and to the parameterization of any particular distribution.<sup>10</sup> I have used a simple (thin-tailed) normal distribution to describe temperature change, but one can incorporate alternative distributions (resulting from alternative stochastic processes for  $X_t$ ). Thus the framework presented here can be used to evaluate the implications of alternative characterizations of uncertainty for determining the targets of climate change policy.

It is an understatement to say that some caveats are needed. First, costs of reducing  $\alpha$  and  $\sigma$  are probably convex, not linear as in eqn. (47). But a linear cost function may be a reasonable first approximation, and in any case our ability to estimate these costs is limited. More troublesome is the assumption that whatever the cost, we can change  $\alpha$  and  $\sigma$  instantly. One would expect that shifts in these parameters would occur gradually in response to ongoing expenditures on abatement and R&D. Also, I have calculated WTP

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<sup>10</sup>Pindyck (2011b) addresses the question of fat- versus thin-tailed distributions in the context of climate change.

under the restriction that the flows of costs are constant percentages of GDP. In fact, those costs might fall as a result of technological change making it cheaper to limit GHG emissions. And even if the costs do not fall, WTP might be calculated under the alternative assumption that costs increase or decrease over time as the policy becomes more or less stringent. The model can be modified to allow for gradual changes in  $\alpha$  and  $\sigma$  and/or costs that are time-varying (although both changes would be at the cost of complicating the numerical solution of the model).

Even maintaining the simple assumptions used here (in which the change in  $\alpha$  and  $\sigma$  occur instantly and the flow cost is a constant percentage of GDP), the framework can be extended in a number of directions. For example, the timing of policy implementation can be made endogenous. Suppose a policy is imposed only when temperature reaches a critical threshold (e.g., 3°C). Given a cost for the policy, what is the temperature threshold that maximizes net welfare?

## Appendix: Some Expectations

There are some key expected values that are used throughout this paper. They are derived below, beginning with  $\mathcal{E}_s(C_t^{1-\eta})$ .

**Expected Utility.** Denote  $F(C, g, s) = \mathcal{E}_s(C_t^{1-\eta})$ , for  $t > s$ . Then  $F$  must satisfy the following partial differential equation (i.e., Kolmogorov Forward Equation):

$$dF = \frac{1}{2}\sigma^2 F_{gg} - \alpha F_g + gCF_C + F_s = 0 , \quad (49)$$

with boundary conditions  $F(C, g, t) = C_t^{1-\eta}$  and  $F(0, g, s) = 0$ . Try, and then verify by substitution, a function of the form:

$$F(C, g, s) = C_s^{1-\eta} e^{(1-\eta)g_s(t-s)+a(s)} . \quad (50)$$

The partial derivatives are:

$$\begin{aligned} F_g &= (1-\eta)(t-s)F \\ F_{gg} &= (1-\eta)^2(t-s)^2F \\ F_C &= (1-\eta)F/C \\ F_s &= [-(1-\eta)g_s + a'(s)]F \end{aligned}$$

After substituting into eqn. (49) and dividing through by  $F$ , we have:

$$a'(s) = -\frac{1}{2}\sigma^2(1-\eta)^2(t-s)^2 + \alpha(1-\eta)(t-s) \quad (51)$$

so that the function  $a(s)$  is:

$$a(s) = \frac{1}{6}\sigma^2(1-\eta)^2(t-s)^3 - \frac{1}{2}\alpha(1-\eta)(t-s)^2 . \quad (52)$$

Note that if  $\eta > 1$  and  $\alpha > 0$ ,  $a(s) > 0$ .

Thus starting at time  $s = 0$  with  $C_0 = 1$ ,

$$\mathcal{E}_0(C_t^{1-\eta})e^{-\delta t} = e^{-\delta t + (1-\eta)g_0 t - (1/2)\alpha(1-\eta)t^2 + (1/6)\sigma^2(1-\eta)^2 t^3} . \quad (53)$$

Note that if  $\eta > 1$ , as  $t$  increases,  $\mathcal{E}_0(C_t^{1-\eta})$  first decreases and then eventually increases without bound. Thus the integration over time to compute welfare must be done for a finite horizon.

**Expected Consumption.** We will also need an expression for  $\mathcal{E}_0(C_t)$ . Following the same steps as above it is easy to show that if  $C_0 = 1$ ,

$$\mathcal{E}_0(C_t) = e^{g_0 t - (1/2)\alpha t^2 + (1/6)\sigma^2 t^3} . \quad (54)$$

Note that if  $\alpha^2 > 2g_0\sigma^2$ ,  $\mathcal{E}_0(C_t)$  is first increasing in  $t$  up to a local maximum, then will decrease to a local minimum, and then increase without bound. But if  $\alpha^2 < 2g_0\sigma^2$ , then  $\mathcal{E}_0(C_t)$  will always be increasing in  $t$ .

**Expectations of Products.** We will want to calculate the variance of the return from an abatement policy. To do this we need the expectation of  $C_i^{1-\eta}C_j^{1-\eta}$  for two different future times  $i$  and  $j$ . Denote  $G(C, g, s) = \mathcal{E}_s(C_i^{1-\eta}C_j^{1-\eta})$  for  $i, j > s$ . Then  $G$  satisfies the following Kolmogorov Forward Equation:

$$dG = \frac{1}{2}\sigma^2 G_{gg} + \alpha G_g + gCG_C + G_s = 0 , \quad (55)$$

Try and confirm that the solution has the form:

$$G(C, g, s) = C_s^{2(1-\eta)} e^{(1-\eta)g_s[(i-s)+(j-s)]+b(s)} . \quad (56)$$

Substituting the partial derivatives  $G_g, G_{gg}$ , etc. into eqn. (55) yields

$$b'(s) = -\frac{1}{2}\sigma^2(1-\eta)^2[(i-s)+(j-s)]^2 + \alpha(1-\eta)[(i-s)+(j-s)] \quad (57)$$

so that the function  $b(s)$  is:

$$b(s) = \frac{1}{12}\sigma^2(1-\eta)^2[(i-s)+(j-s)]^3 - \frac{1}{2}\alpha(1-\eta)[(i-s)^2+(j-s)^2] . \quad (58)$$

I use this result when calculating the variance of incremental returns from an abatement policy.

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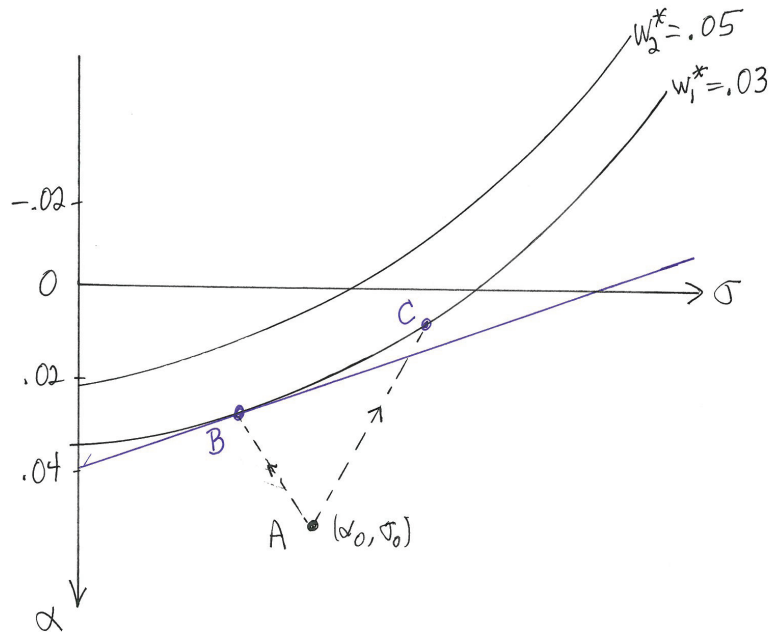


Figure 1: Iso-WTP Curves and Iso-Cost Line

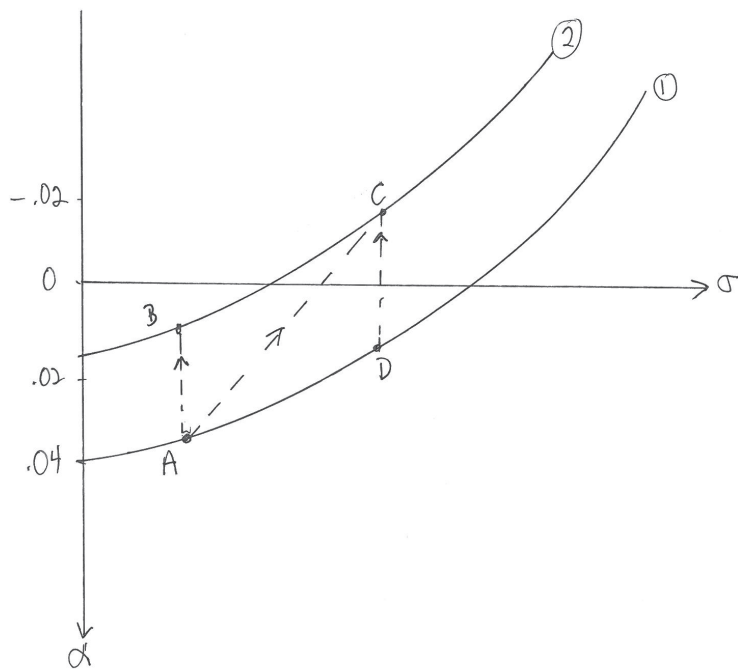


Figure 2: Iso-WTP Starting Curve and Target Curve

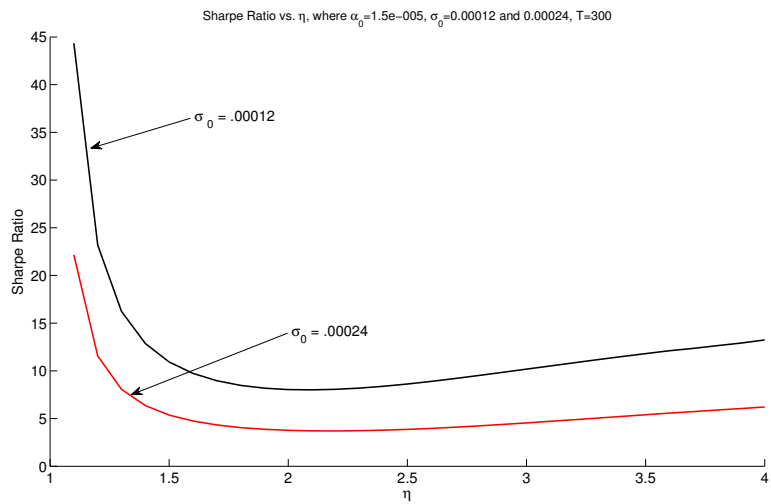


Figure 3: Sharpe Ratio vs.  $\eta$  ( $g_0 = .020$ ,  $\delta = 0$ ,  $T = 300$ ,  $\sigma = .00012$  and  $.00024$ )

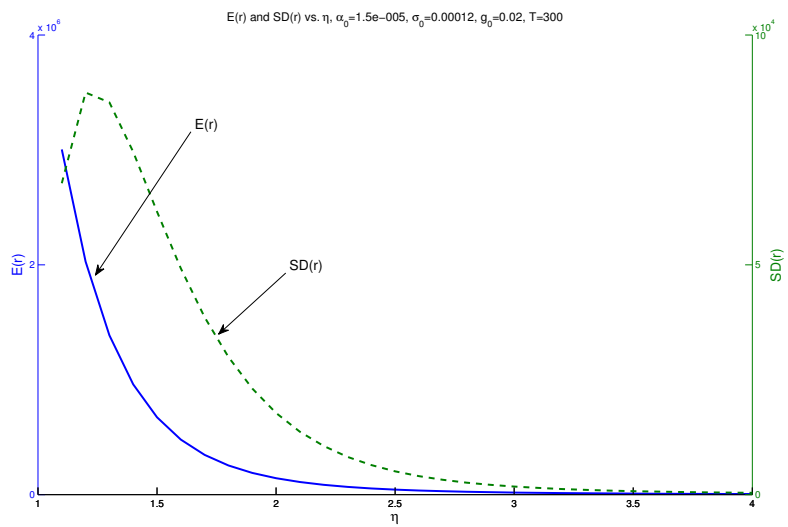


Figure 4: Mean and Standard Deviation of Return vs.  $\eta$  ( $g_0 = .020$ ,  $\delta = 0$ ,  $T = 300$ ,  $\sigma = .00012$ )



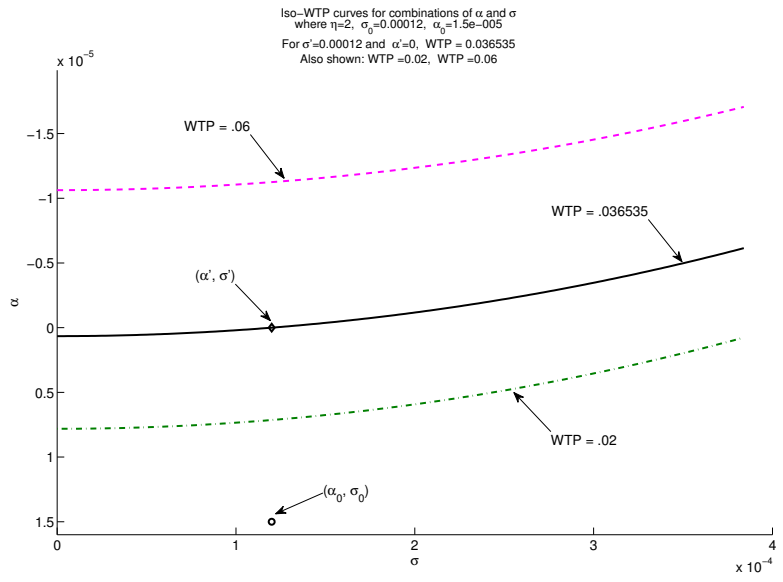


Figure 5: Iso-WTP Curves ( $g_0 = .020$ ,  $\delta = 0$ ,  $\eta = 2$ ,  $T = 300$ )

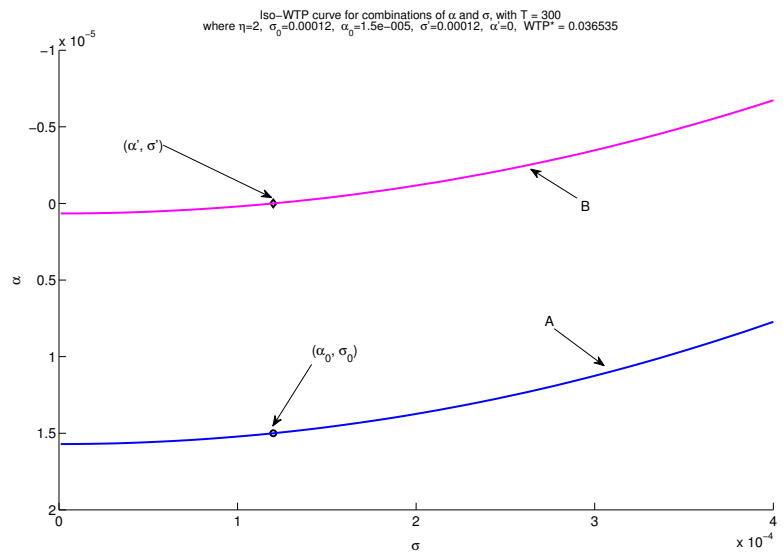


Figure 6: Iso-WTP Starting and Target Curves ( $g_0 = .020$ ,  $\delta = 0$ ,  $\eta = 2$ ,  $T = 300$ )

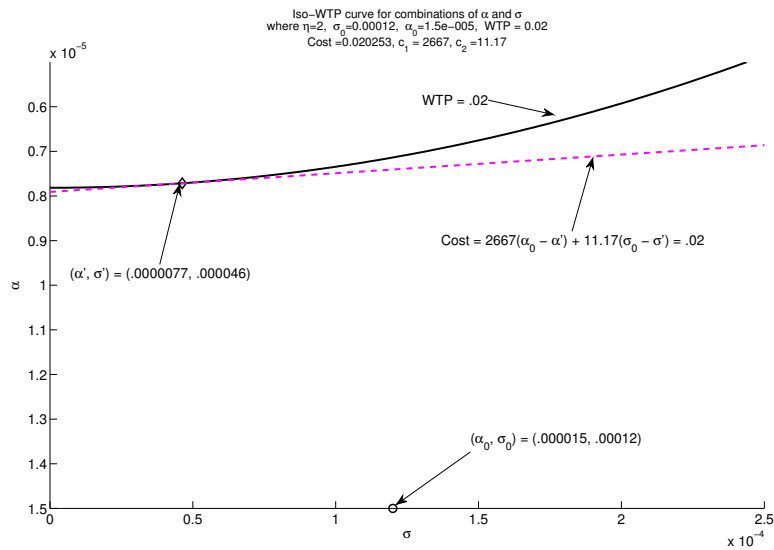


Figure 7: Iso-WTP and Iso-Cost Curves ( $g_0 = .020$ ,  $\delta = 0$ ,  $\eta = 2$ ,  $T = 300$ )

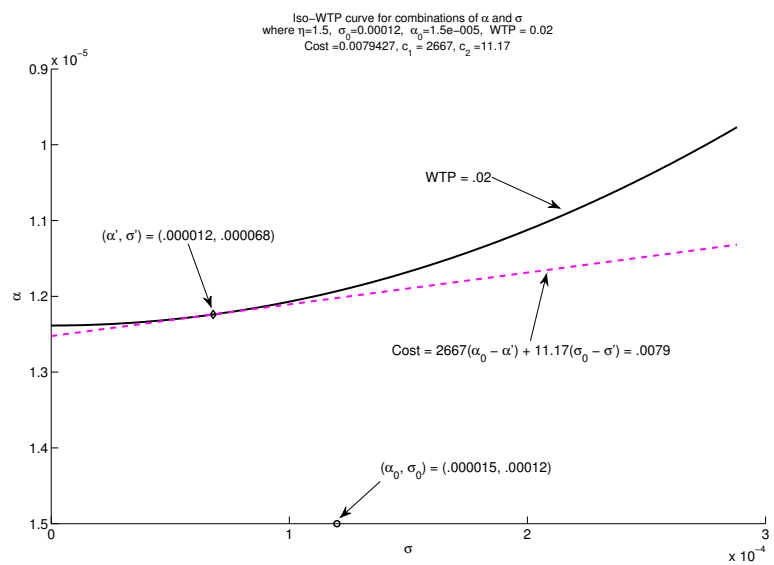


Figure 8: Iso-WTP and Iso-Cost Curves ( $g_0 = .020$ ,  $\delta = 0$ ,  $\eta = 1.5$ ,  $T = 300$ )