Demand Shocks that Look Like Productivity Shocks*

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Abstract

The use of competitive search protocols in final goods allows for the specification of models where demand shocks generate propagations that look like productivity shocks. In such environments there is a role for expansion of public expenditures during recessions. We have shown how these shocks can replicate the movements of the Solow residual that the RBC literature typically identifies with technology shocks. We find that correlated shocks to the demand of consumption and investment can replicate the propoerties of the standard real business cycle models in terms of the comovements of macroeconomic variables. If anything, our model performs marginally better. When allowing in our model economy the coexistence of demand shocks and technology shocks a full Bayesian estimation imputes essentially no role to technology shocks even in shaping the properties of the Solow residual. In addition, our shopping economies have implications for other macroeconomic variables (relative price of consumption and investment, the stock market, capacity utilization) over which standard models are silent. Such implications are remarkably consistent with the data.

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1 Introduction

In this paper we construct a model where measured productivity, output, and the labor input are all strongly correlated. While real business cycle models deliver these features via persistent shocks to TFP, we generate such movements with shocks to demand. In our case these are preference shocks, but the exact nature is not so important. Moreover, our model economy, henceforth the shopping model, is built on top of a growth model with capital and features a balanced growth path.

So how can we have simultaneously an increase in labor, decreasing returns to scale, no productivity shocks or externalities in production and procylical measured productivity? By explicitly posing the role of spenders in generating output. Restaurants need diners, hospitals need patients, in sum, firms need consumers and investors. For households and firms, the acquisition of goods is an active process that involves costs that are not measured in NIPA. When shocks directly lower these costs households and firms change their spending, firms see increases in demand that allow them to deliver goods that would otherwise rot. In a sense, ours is a model of a modern economy, filled with services and perishable goods that if not sold, cannot be stored and do not show up in measured GDP. Consequently, our model displays notions of potential output, or full capacity.

The shopping economy looks like a real business cycle economy with productivity shocks except that there is no such a thing. In fact, we estimate processes for preference shocks to match the measured process for TFP. It has been almost 30 years since the seminal real business cycle paper Kydland and Prescott (1982) that brought to center stage productivity shocks, yet we have learned very little about such shocks, and in fact productivity movements remain pretty much like a black box. Moreover, casual discussions with firm managers do not seem to highlight the issue: business is good when customers show up and not when productivity has just increased.

Technically, the paper poses a search shopping friction that affects both consumption and investment and that is resolved via competitive search. The model poses shocks to the willingness to spend by consumers and investors which we model in this paper as preference shocks and shopping technology shocks. However, we think of them as stand ins for other things such as financial shocks, shocks to real exchange rate, wealth shocks, monetary policy and so on, and it should be easy and the stuff of future work to pose such alternatives explicitly. We estimate the processes that drive those shocks from the Solow residual explicitly.

In our model the friction is intrinsic to the process of production and is not arbitrarily imposed:

markets clear, and no agent has incentives to do things different. While some other models (of the new keynesian variety) have been able to generate demand induced shocks, they do that by making agents trade at prices that are not the equilibrium ones. In this paper there is no such a thing. We see our paper somehow pursuing one of the keynesian traditions. Not the fixed price tradition followed by the new keynesian literature in otherwise standard RBC models. Not the coordination problem tradition that sees a recession as a bad outcome within environments susceptible to multiple equilibria, but a tradition where failures of demand generate recessions via infrautilization of productive capacity. In our environment "animal spirits" modeled as shocks to agents' forecasts would generate recessions. Technically however, we see our environment as impeccably neoclassical.

Quantitative Findings A variety of demand shocks can generate a process for the Solow residual in exactly the same way that an exogenous process would. Demand shocks that affect both consumption and investment are capable of generating the same comovements among the main macroeconomic variables as the data and the standard RBC model. When giving the chance to differentiate between productivity shocks and demand shocks as the source of movements in the Solow residual, Bayesian estimation imputes to demand shocks 90% of the variance while technology shocks are responsible for less than 10%. The well known inability of the RBC model to generate large movements in hours worked (for instance a structural estiation of output and productivity within the RBC model imputes less than 5% of the volatility of hours to technogly shocks) also holds for the shopping model.

To summarize our findings, we think that our model compares favorably with the standard RBC model as a model of business cycles. Moreover, we think that with very little additional work most macroeconomic models of aggregate fluctuations triggered by productivity shocks and where labor market fluctuations get amplified via a variety of frictions, can use demand demand shocks as the main mechanism that triggers fluctuations.

Additional contributions Besides the proposal of an alternative structure to the standard RBC model as a base to study aggregate fluctuations, our work has other contributions. First, it shows how in models with production and competitive search, to achieve optimality it is necessary to index markets, not only by price and market tightness, but also by the quantity of the good traded. Second, we provide a theory of the cyclical changes of the relative price between investment and consumption goods that is not based on exogenous technology shocks[1]. We provide a theory of

[1] As was done by Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Fisher (2006) cite just a couple of papers.

capacity utilization. We also provide a theory of stock market movements that are associated not to adjustment costs but to how firms can match with customers and this feature depends on economic conditions.

Literature . We relate to various literatures. V: Still to be done

1. Capacity utilization papers, Basu and Fernald (1997), Greenwood, Hercowitz, and Huffman (1988) and others that we do not know yet.
2. Labor Hoarding Burnside, Eichenbaum, and Rebelo (1990).

We start discussion a simple version of the model with Lucas trees in Section 2 and we go to a full version with the neoclassical growth model underlying the economy in Section 3. We then map the model to data in Section 4. In Section 5 we estimate processes for demand shocks that replicate the properties of the Solow residual while in Section 6 we explore the business cycle properties of economies with demand shocks and we asses the findings in detail. In Section 7 we study the performance of the Economy with various shocks simultaneously. We explore the robustness of our findings in Section 8. Section 9 concludes while an Appendix provides some proofs and additional tables and computational and data details.

2 The simple Lucas-trees version of the model

We start by illustrating the workings of the model in a simple search model where output is produced by trees instead of capital and labor. We show that the sarch process impacts aggregate output in a way that appears as a level effect on the Solow residual. In an example we show how shocks to preferences are partly accommodated by increase in prices and interest rates and partly accommodated by increases in quantities, and demonstate that absent the search friction, the same shocks translate only to price increases.

2.1 Technology and preferences

There is a continuum of trees (i.e., suppliers), with measure $T = 1$. Each tree yields one piece of fruit every period. A standard search friction makes it difficult for consumers to find trees. To overcome this friction the consumer sends out a number of shoppers searching for fruit. The aggregate number of fruits found, Y, is given by the Cobb-Douglas matching function:

$$Y = A\ D^{\alpha}\ T^{1-\alpha}, \tag{1}$$

where D is the aggregate measure of shoppers searching for fruit (we sometimes call it aggregate demand) and A and α are parameters of the matching technology.

Following Moen (1997), we assume a competitive search protocol where agents can choose to search in specific locations indexed by both the price and market tightness, defined as the ratio of trees per shopper, $Q = T/D$. The probability that a tree is found (i.e., matched with a shopper) is $\Psi_T(Q) = A\ Q^{-\alpha} = A\ D^{\alpha}/T^{\alpha}$. Once a match is formed, then the fruit is traded at the posted price p. By the end of the period, all fruit which is not found is lost. The trees pay out sales revenues as dividends and the expected dividend is $\varsigma = p\Psi_T(Q)$.

The economy has a continuum of identical, infinitely lived households of measure one. Their preferences are given by

$$E\left\{\sum_t \beta^t\ u\left(c_t, d_t, \theta_t\right)\right\}, \tag{2}$$

where c_t is consumption, d_t is the measure of shopping units (search effort) by the household, and θ is a preference shock that follows a stochastic Markov process. The probability that an individual shopping unit is successful is given by the ratio of matches to the aggregate number of shoppers. Given the matching technology (1), this can be expressed in terms of market tightness as $\Psi_d(Q) = A\ Q^{1-\alpha}$, so c_t is given by

$$c = d\ \Psi_d(Q) = d\ A\ Q^{1-\alpha}. \tag{3}$$

Trees are owned by households and are traded every period. We normalize the price of the tree to unity and use it as the numeraire good. Let s denote the number of shares owned by the household. The aggregate number of shares is unity. Consequently, with identical households the aggregate state of the economy is just θ, while the individual state includes also individual wealth

s. The representative household problem cen then be expressed recursively as

$$v(\theta, s) \;=\; \max_{c,d,s'} u(c, d, \theta) + \beta E\{v(\theta', s')|\theta\} \tag{4}$$

$$\text{s.t.} \quad c = d\ \Psi_d[Q(\theta)] \tag{5}$$

$$P(\theta)\ c + s' = s\ [1 + \varsigma(\theta)] \tag{6}$$

2.1.1 Equilibrium

A competitive search equilibrium is defined by a set of individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$, the aggregate allocations $D(\theta)$, $Q(\theta)$, and $C(\theta)$, good prices $P(\theta)$, and the rate of return on trees $\varsigma(\theta)$ so that

1. The individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$ solve the household problem (4).
2. The individual decision rules are consistent with the aggregate functions

$$C(\theta) = c(\theta, 1) \qquad D(\theta) = d(\theta, 1) \qquad s'(\theta, 1) = 1$$

3. The aggregate consumption is consistent with aggregate search

$$C(\theta) = A\ D(\theta)^{\alpha} \tag{7}$$

4. Firms' dividends equal firm sales

$$\varsigma(\theta) = p(\theta)\ \ A\ D(\theta)^{\alpha}$$

5. The pair $(p(\theta), D(\theta))$ clears the goods market and aggregate tightness is $Q(\theta) = 1/D(\theta)$.

2.1.2 Competitive Search in the Market for Goods

We start by analyzing the equilibrium in the market for goods. There are differentiated markets indexed by the price, each of them having its own market tightness of number of trees per shopper. Let ς denote the outside value for firms of going to the most attractive market, yet to be determined. Clearly a market can attract trees only if it offers trees at least ς. This puts a constraint on the feasible combinations of prices and market tightness that shoppers can offer:

$$\varsigma \leq \; p \, \Psi_T(Q) \tag{8}$$

The expected contribution to household utility of a shopper that chooses the best price–tightness pair is[2]

$$\Phi = \max_{Q,p} \left\{ u_d(\theta, s) \; + \; \Psi_d(Q) \; \left(u_c(\theta, s) \; - \; p \; m\right) \right\} \qquad \text{s.t. } \varsigma \leq \; p \, \Psi_T(Q), \tag{9}$$

where $u_d(\theta, s)$ and $u_c(\theta, s)$ are the marginal utility of increases in d and c, respectively. Moreover, $m = m(\theta, s'(\theta, s))$ is the expected discounted marginal utility of an additional unit of savings:

$$m(\theta, s') = \beta \; E \left\{ \frac{P(\theta) \; [1 + \varsigma(\theta')]}{P(\theta')} \; \frac{\partial v(\theta', s')}{\partial s'} | \theta \right\}$$

Solve (9) by substituting (8) at equality and take the first-order condition w.r.t. Q. This yields the unique equilibrium price p and value of the tree ς as functions of market tightness Q;

$$p \;\; = \;\; (1 - \alpha) \; \frac{u_c(\theta, s)}{m}, \tag{10}$$

$$\varsigma \;\; = \;\; pA \; Q^{-\alpha}. \tag{11}$$

2.1.3 Characterizing the equilibrium

The competitive search equilibrium and its efficiency properties can then be characterized by the following proposition.

Proposition 1. *1. Aggregate search D is determined by the functional equation*

$$0 = \alpha \; A \; D(\theta)^{\alpha - 1} \;\; u_c\left(AD(\theta)^{\alpha}, D(\theta), \theta\right) + u_d\left(AD(\theta)^{\alpha}, D(\theta), \theta\right) \tag{12}$$

The functions C, Q and ς then follow directly from the equilibrium conditions $C(\theta) = A \; D(\theta)^{\alpha}$, $Q(\theta) = 1/D(\theta)$, and $\varsigma(\theta) = p(\theta) \;\, A \; D(\theta)^{\alpha}$.

[2] To derive this, take the first-order condition of (4) with respect to d and consider the price-tightness posting problem when the cost $\bar{u}_d$ of sending an additional shopper has been borne. The idea is that each shopper is equipped with a credit card and if no fruit is found, then utility is not affected over and above the sunk search cost.

2. *The equilibrium price is defined by the functional equation*

$$(1-\alpha)\ \theta_c\ u_c\left(C\left(\theta\right),D(\theta),\theta\right) = \\ p\left(\theta\right)\ \beta\ E\left\{\frac{P(\theta)\left[1+\varsigma(\theta')\right]}{P(\theta')}\ \left[u_c\left(C\left(\theta'\right),D(\theta'),\theta'\right)+\frac{u_d\left(C\left(\theta\right),D(\theta),\theta\right)}{\Psi_d[Q(\theta')]}\right]|\theta\right\} \quad (13)$$

3. *The competitive equilibrium is efficient.*

Proof. Substitute $d = c/\Psi_d\left(\hat{D}\right)$ into the period utility $U = \theta_c u(c) - \theta_d\ \overline{u}(c/\Psi_d(\hat{D}))$ and differentiate U w.r.t. c to get

$$\frac{dU}{dc} = \theta u_c + \frac{\partial U}{\partial d}\frac{\partial d}{\partial c} = \theta u_c - \frac{1}{\Psi_d(\hat{D})}\ \overline{u}_d\left(\frac{c}{\Psi_d(\hat{D})}\right)$$

The Euler equation can then be expressed as

$$\theta u_c - \frac{1}{\Psi_d(\hat{D})}\ \overline{u}_d\left(\frac{c}{\Psi_d(\hat{D}\left(\theta\right))}\right) = P(\theta)m.$$

Impose the representative-agent conditions $c = C = AD^\alpha$ and $d = D = \hat{D}$, and use (10) to substitute out the term $P(\theta)m$. This gives one functional equation:

$$\theta u_c\left(C(\theta)\right) - \frac{1}{\Psi_d(D\left(\theta\right))}\ \overline{u}_d\left(\frac{C(\theta)}{\Psi_d(D\left(\theta\right))}\right) = (1-\alpha)\ \theta\ u_c(C(\theta)).$$

Rearranging this equation and using (7) yields the functional equation (12). The functional equation (13) is derived from the equilibrium price equation (10) and the definition of m, where we exploit the envelope theorem and (5) to express $\partial v/\partial s$ as

$$\frac{\partial v\left(\theta,s\right)}{\partial s} = \theta u_c(c) - \bar{u}_d(d)\frac{c}{\Psi_d[D(\theta)]}$$

At the equilibrium, the agents' budget constraint (6) is satisfied. Given (12)-(13), the first-order conditions of (4) hold, which guarantees individual optimization.

Consider the planner problem $\max_{C,D}\left\{\theta u(C) - \bar{u}(D)\right\}$ subject to the aggregate resource constraint $C = A\ D^\alpha$. It is straightforward to verify that the solution to equation (12) solves this planner problem, which establishes efficiency. ∎

2.2 An example

Consider a version of this economy where

$$u(c, d, \theta) = \theta_c \ \log c \ - \ \theta_d \ d, \tag{14}$$

where the stochastic preference weights θ_c and θ_d are i.i.d. with $E\{(\theta_c') \ |\theta\} = E\{(\theta_d') \ |\theta\} = 1$ and $\theta_c \ > \ 0$ and $\theta_d \ > \ 0$. For notational convenience we define $\theta \equiv (\theta_c, \theta_d)$. Given these preferences, the equilibrium condition in equation (12) becomes

$$0 = \alpha \ A \ D(\theta)^{\alpha-1} \ \frac{\theta_c}{AD(\theta)^{\alpha}} - \theta_d.$$

Simplifying this expression and using (7), yields the equilibrium allocation:

$$\begin{aligned} D(\theta) &= \alpha \frac{\theta_c}{\theta_d} \\ C(\theta) &= A \ (D(\theta))^{\alpha} = A\left(\alpha \frac{\theta_c}{\theta_d}\right)^{\alpha} \end{aligned}$$

As is clear from this allocation, preference shocks generate aggregate fluctuations.

A central point of our paper is that in the presence of search frictions, the measured total factor productivity (TFP) is a function of the search effort and, hence, fluctuates in response to preference shocks. The only other production factor in this economy is the fixed stock of capital, $T = 1$. Therefore, if search effort is ignored, the contribution from search will show up as a Solow residual Z, defined as $C = Z \ T$;

$$Z(\theta) = A \ (D(\theta))^{\alpha} = A\left(\alpha \frac{\theta_c}{\theta_d}\right)^{\alpha}$$

Given the allocation, it is straightforward to verifcd cdy that the equilibrium price satisfying (13) is given by

$$P(\theta) = \left(\frac{1}{\beta} - 1\right) \frac{1}{A\alpha^{\alpha}} (\theta_d)^{\alpha} \ (\theta_c)^{1-\alpha}.$$

The dividends (which is also the rate of return on the tree), is then

$$\varsigma(\theta) = p(\theta)\ A\left(\alpha\,\frac{\theta_c}{\theta_d}\right)^{\alpha} = \left(\frac{1}{\beta}-1\right)\ \theta_c.$$

Thus, the rate of return is the discount rate times the preference shock. [3]

To get some intuition for the price expression, consider the price of the tree in terms of consumption units:

$$\frac{1}{p(\theta)} = \frac{\beta}{1-\beta}A\alpha^{\alpha}\,(\theta_d)^{-\alpha}\,(\theta_c)^{\alpha-1} = \frac{\beta}{1-\beta}\,\frac{C(\theta)}{\theta_c} = \frac{\beta}{1-\beta}\,\frac{1}{\theta_c\ u_c}.$$

Thus, the price of the tree in terms of consumption units is the discounted value of the current marginal utility of consumption for ever. Finally, note that the equilibrium interest rate in terms of the consumption good is given by

$$1 + r(\theta) = \frac{(\theta_d)^{\alpha}\,(\theta_c)^{1-\alpha}}{\beta\ E\left\{(\theta_d')^{\alpha}\,(\theta_c')^{1-\alpha}\right\}}$$

Note that this implies that in steady state, with $\theta' = \theta$, the interest rate would be $r(\theta) = \varsigma(\theta) = 1/\beta - 1$. In conclusion, the interest rate is increasing in θ_c and decreasing in θ_d. Note also that the elasticity of aggregate consumption (to changes in θ) is α.

It is instructive to compare the model to a version of the standard Lucas tree model without the search friction. As $\alpha \to 0$, the search economies converge to the standard Lucas tree model. In this case $Y = C = A$, $D = 0$, and

$$\begin{aligned} P(\theta) &= \left(\frac{1}{\beta}-1\right)\frac{1}{A}\,\theta_c \\ 1 + r(\theta) &= \frac{\theta_c}{\beta\ E\left\{\theta_c'\right\}} = \frac{\theta_c}{\beta}. \end{aligned}$$

Clearly, in this case the aggregate consumption is invariant to the demand shock (so the elasticity

[3] It follows that when expressed in terms of consumption units, the dividends are given by

$$\frac{R(\theta)}{p(\theta)} = \left(\alpha\,\frac{\theta_c}{\theta_d}\right)^{\alpha} = C(\theta).$$

is zero), and all the adjustment to the shock takes place through the prices. Specifically, the elasticity of r and p to θ_c is unity, compared to $1-\alpha$ in the search case.

2.3 Fiscal policy in the Lucas-tree model

We now analyze a verison of the model where the government provides a public good financed with lump-sum taxation in terms of the consumption good. For simplicity, we assume that the government does not run deficits, so government expenditures G_t equals the lump-sum tax each period. Due to Ricardian equivalence, this assumption is without loss of generality. Since government consumption is in terms of the consumption good, the aggregate resource constraint is

$$Y_t = C_t + G_t = A\ D_t^{\alpha}\ T^{1-\alpha}.$$

Agents now search for fruit for both own consumption and tax purposes, so their budget constraint becomes

$$p_t\ (c_t + G_t) + s_{t+1} = s_t\ [1 + \varsigma_t],$$

and the search constraint becomes

$$c_t + G_t = d_t\ \Psi_d[Q_t].$$

Government expenditures G_t finances investment in government capital which is long-lived. We model this in a stark way by assuming that the marginal value of additional public-good spending is constant, so individual preferences are

$$E\left\{\sum_t\ \beta^t\ \left[\theta_{c,t}\log(c_t) - \theta_{d,t}\ d_t + G_t\right]\right\}.$$

2.3.1 Exogenous fiscal policy

Suppose first that the sequence of G_t is exogenous. As above, the competitive equilibrium allocations are constrained efficient, given the exogenous sequence of G_t. The equilibrium allocation is then a solution to the following static problem:

$$\max_{C,D} \{\theta_c \log(C) - \theta_d\, D + G\} \quad (15)$$
$$\text{s.t.} \quad C + G = A\, D^{\alpha}\, T^{1-\alpha},$$

and the solution is the unique positive solution $D(\theta, G)$ to the following equation[4]

$$\frac{\theta_c}{\theta_d} \alpha A D^{\alpha-1} + G = A\, D^{\alpha}, \quad (16)$$

and equilibrium consumption is given by

$$C(\theta) = \frac{\theta_c}{\theta_d} \alpha A\, D(\theta)^{\alpha-1}. \quad (17)$$

The solution $D(\theta, G)$ is increasing in G and in the ratio θ_c/θ_d.[5] Intuitively, the aggregate search effort incrases if current consumption is more valuable ($\theta_c \uparrow$) or if search effort is less costly ($\theta_d \downarrow$). Increased government consumption will crowd out private consumption.[6] Therefore, a larger G will incrase the marginal utility of consumption and, hence, induce a larger search effort D. A larger D in turn increases aggregate output. Note, however, that the "fiscal multiplier" $\partial Y/\partial G$ is smaller than unity – an increase in G generates a less than one-for-one increase in output. Note also that an increase in the ratio θ_c/θ_d and an exogenous in G would generate lower aggegate output and, hence a fall in the Solow residual.

2.3.2 Endogenous fiscal policy

Consider now the Ramsey problem of public good provision in this economy. Since the search equilibrium is constrained efficient, the Ramsey planner problem amounts to maximize (15) over G, C, and D:

$$\max_{C,D,G} \Lambda = \{\theta_c \log(C) - \theta_d\, D + G - \lambda (A\, D^{\alpha} - C - G)\}$$

[4] To see that D is unique, note that the right-hand side is zero for $D = 0$ and monotone increasing. The left-hand side is monotone decreasing for $D > 0$ and converges to ∞ for $D \to 0$.

[5] To see this, note that both a larger G and a larger ratio θ_c/θ_d will increase the left-hand side of (16). Since the right-hand side of (16) is increasing in D, the equilibrium D must increase.

[6] To see this, use (17) to substitute D for C in equation (16) to obatin an expression of G as a decreasing function of C. Therefore, a larger G is associated with a lower C, *ceteris paribus*.

The first-order conditions to that problem are

$$\begin{aligned}\frac{\partial \Lambda}{\partial G} &= 1-\lambda = 0 \\ \frac{\partial \Lambda}{\partial D} &= \theta_d - \lambda\alpha A D^{\alpha-1} = 0 \\ \frac{\partial \Lambda}{\partial C} &= \frac{\theta_c}{C} - \lambda = 0\end{aligned}$$

The implied optimal allocation is given by

$$\begin{aligned}D &= \left(\frac{\alpha A}{\theta_d}\right)^{\frac{1}{1-\alpha}} \\ C &= \theta_c \\ G &= \left(\frac{A}{\theta_d}\right)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{\alpha}{1-\alpha}} - \theta_c\end{aligned}$$

It follows that optimal search is decreasing in θ_d, that consumption is increasing in θ_c, and that optimal fiscal policy reacts to these shocks. In particular, a fall in θ_c would, with exognous fiscal policy, generate a recession. The planner offsets this shock completely by increasing government expenditures one for one, so aggregate output is constant (countercyclical fiscal policy). However, when θ_d falls, fiscal policy exacerbates the aggregate fluctuations relative to an economy with exogenous fiscal policy. In the exogenous-fiscal-policy model above a fall in θ_d increases D and C (since G is constant by assumption in that model). This reduces the marginal utility of consumption which, in turn, mitigates the increase in search and, hence, mitigates the increase in aggregate output. However, when G is endogenous, it is optimal to let the whole increase in output go toward increasing G. The reason is that the marginal utility of G does not fall when G increases. Thus, fiscal policy stablizes private consumption but increases the volatility of aggregate output (procyclical fiscal policy).

3 Growth Model

We now turn to embody our ideas on a growth model suitable for business cycle analysis. There is capital, and increasing the stock requires both investment goods and professionals to shop for those goods. In addition, work generates a disutility. These two features generate substantial complications that we now analyize. We start with the problems faced by households and firms, we then look at the operation of competitive search for consumption and investment goods and we define equilibrium. Along the way we prove a few results that guarantee that all firms make the same choices of labor and investment. We also establish the Pareto Optimality of Equilibrium. We finish the section discussion some subtle issues of National Accounting that are involved in the determination of labor share and in the calculation of the Solow residual.

3.1 Households

There is a measure one of households that have preferences over consumption, shopping, and working and is affected by shocks that are perfectly correlated across households. All this is summarized by utility function $u(c, d, n, \theta)$.

We will proceed by assuming (which we will later verify) that the state of the economy is given by the pair (Θ, K), this is the aggregate preference shock and aggregate capital. The additional state variables for the household are the values of its own shock and the shares of the stock market that it owns or the vector (Θ, K, θ, s). We will denote with superindex c or i to indicate whether we refer to consumption or investment goods (which are identical from the point of view of production but not from the point of view of distribution).

The aggregate variables that are determined in equilibrium and that the households take as given include market tightness, price and quantity in the consumption good market which we denote by $\{Q^c, P^c, F^c\}$ and that are functions of (Θ, K); they also include the rate of return of the economy in terms of the numeraire (shares of the stock market), the wage, and the law of motion of capital denoted $G(\theta, K)$.

Household problem The representative household solves

$$\begin{aligned} v(\Theta, K, \theta, s) \quad &= \quad \max_{c,d,n,s'} \quad u(c, d, n, \theta) + \beta \; E\left\{v(\Theta', K', \theta', s')|\Theta, \theta\right\} && (18)\\ \text{s.t.} \qquad & \qquad c = d \; \Psi_d[Q^c(\Theta, K)] \; F^c(\Theta, K), && (19)\\ & \qquad P^c(\Theta, K) \; c + s' = s \; [1 + R(\Theta, K)] + n \; w(\Theta, K), && (20)\\ & \qquad K' = G(\Theta, K). && (21) \end{aligned}$$

Equation (19) shows that consumption requires the household's shopping effort, but it also depends on market tightness and the amount produced by consumption produced firms. Equation (20) is the budget constraint in terms of shares.

The solution to this problem is a set of functions $d(\Theta, K, \theta, s)$, $c(\Theta, K, \theta, s)$, $n(\Theta, K, \theta, s)$, and $s'(\Theta, K, \theta, s)$. Anticipating equilibrium conditions, we can bring the aggregate counterparts of these functions, yielding,

$$C(\Theta, K) = c(\Theta, K, \Theta, 1) \tag{22}$$

$$D^c(\Theta, K) = d(\Theta, K, \Theta, 1), \tag{23}$$

$$N(\Theta, K) = n(\Theta, K, \Theta, 1) \tag{24}$$

$$s'(\Theta, K, \Theta, 1) = 1. \tag{25}$$

Notice that the use of stock market shares in lieu of assets as the state variable of the household accounts for the last condition.

Using these aggregate conditions, the intertemporal Euler is

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = \beta \; E\left\{\frac{P^c(\Theta, K)\,[1 + R(\Theta', K')]}{P^c(\Theta', K')}\; u_c\,[C(\Theta', K'), D(\Theta', K'), N(\Theta', K'), \Theta']\,|\Theta\right\}, \tag{26}$$

while the intratemporal is

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = u_n\,[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta]\;\frac{P^c(\Theta, K)}{w(\Theta, K)}. \tag{27}$$

Let $m(\Theta, K, \theta, s)$, denote the value in terms of marginal utility of an additional unit of savings and let $M(\Theta, K)$ be its aggregate counterpart

$$M(\Theta, K) = \beta \; E\left\{\frac{[1 + R(\Theta', K')]}{P^c(\Theta', K')}\; u_c\,[C(\Theta', K'), D(\Theta', K'), N(\Theta', K'), \Theta']\,|\Theta,\right\}. \tag{28}$$

The dynamic Euler equation becomes

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = P(\Theta, K)\; M(\Theta, K). \tag{29}$$

3.2 Firms

There is a measure one of firms each one of them constituted by a plant or location or tree. The firm has certain amount of capital installed in that location. There is a technology that transforms capital services and labor into goods that is described by production function $f(k, n)$. To install new capital for the following period, the firm like the households has to shop. The firm has a technology than transforms one unit of labor for shopping (that cannot be used for production) into ζ shopping units.

Each firm has to choose three sets of things: whether to produce for investment or consumption, the specific submarket to which to go, and how much to invest.

Firm's problems The choices of the firm can be separated. First it chooses the type of good, then the submarket and then the investment choice. Firms choose to produce whichever good that gives higher value, i.e.

$$\Omega(\Theta, K, k) = \max\{\Omega^c(\Theta, K, k), \Omega^i(\Theta, K, k)\}, \tag{30}$$

where $\Omega^c(\Theta, K, k)$ is the best value for producing consumption goods. Firms obtain this value by choosing among the best available triplet of (Q^c, P^c, F^c) in the consumption submarket,

$$\Omega^c(\Theta, K, k) = \max \ \widetilde{\Omega}^c(\Theta, K, k, Q^c, P^c, F^c) \qquad \text{for all available } (Q^c, P^c, F^c).$$

imilarly, for investment. We denote by $T(\Theta, K)$ the share of firms that produce consumption goods.

A firm in a (Q^c, P^c, F^c) consumption goods submarket chooses labor for shopping n^k, investment i and capital to install tomorrow k' to solve the following problem,

$$\begin{aligned}
\widetilde{\Omega}^c(\Theta, K, k, Q^c, P^c, F^c) \;\; &= \;\; \max_{n^k, k', i} \; \frac{\Psi_d(Q^c)}{Q^c} \, P^c F^c - w(\Theta, K) \, [\widetilde{n}(k, F^c) + n^k] \\
&\qquad - P^i(\Theta, K) \; i + E\left\{ \frac{\Omega(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \, \Theta \right\} && (31) \\
\text{s.t.} \qquad & i = n^k \;\; \zeta \;\; \Psi_d[Q^i(\Theta, K)] \;\; F[K, N^i(\Theta, K)] && (32) \\
& k' = i + (1 - \delta) k && (33) \\
& K' = G(\Theta, K) && (34)
\end{aligned}$$

where $\widetilde{n}(k, y)$ is the inverse function of the production function $y = f(k, n)$ for a given k; $\zeta 1$ is the technological requirement for firms of how many shopping workers it needs to provide a unit of shopping service. Finally $N^i(\Theta, K)$ is the equilibrium amount of production workers in investment goods producing forms so $f(K, N^i(\Theta, K))$ is the amount of investment good produced by investment producing firms.

Similarly, a firm in the investment good market (Q^i, P^i, F^i) chooses labor for shopping n^k, investment i, and future capital stock k' to solve the following problem

$$\widetilde{\Omega}^i(\Theta, K, k, Q^i, P^i, F^i) = \max_{n^k, k', i} \frac{\Psi_d(Q^i)}{Q^i} P^i F^i - w(\Theta, K)\,[\widetilde{n}(k, F^i) + n^k)] - P^i(\Theta, K)\, i + E\left\{ \frac{\Omega(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \Theta \right\} \quad (35)$$

subject to (32), (33), and (34).

The FOC of this problem is given by

$$E\left\{ \frac{\Omega_k(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \Theta \right\} = \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^i(\Theta, K)]} + P^i(\Theta, K). \quad (36)$$

Let $\varsigma(\Theta, K, k)$ denote the outside revenue for firms of going to the best market in units of shares. It is the best revenue obtained either by serving the consumption submarket or by serving the investment submarket, i.e.

$$\varsigma(\Theta, K, k) = \max\{\varsigma^c(\Theta, K, k), \varsigma^i(\Theta, K, k)\},$$

where $\varsigma^c(\Theta, K, k)$ is the maximum over the available submarkets of $\varsigma^c(\Theta, K, k, Q^c, P^c, F^c)$,

$$\varsigma^c(\Theta, K, k, Q^c, P^c, F^c) = P^c\, \frac{\Psi_d(Q^c)}{Q^c}\, F^c - w(\Theta, K)\ \widetilde{n}(k, F^c). \quad (37)$$

Similarly for investment goods.

We will proceed by establishing a series of lemmas that will allow us to verify our conjecture. We start by establishing that firms in all submarkets have the same expected revenue, so we write

Lemma 2. $\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ *implies* $\varsigma^c(\Theta, K, k) = \varsigma^i(\Theta, K, k)$.

This lemma simplifies later the competitive search analysis. It allows firms to only consider the current revenue (and so the current labor demand) instead of lifetime revenue (and so the lifetime labor demands) in deciding which markets to enter.

3.3 Competitive Search in the Market for Consumption Good

In addition to the decisions made by the households with respect to the main elements of the allocation (how much to consume, shop, work and save) its shoppers choose how to implement the shopping, this is which market to go to. In our environment with competitive search this means choosing which market tightness, price, and output size, (Q^c, P^c, F^c). These choices will give us two conditions to be satisfied by these 3 variables.

To simplify notation let $\eta = u_d \ \frac{1}{\Psi_d(Q^c)F^c}$ denote the sunk utility cost in terms of current consumption of having an extra shopper and $u_c(\Theta, K)\ u_c[C(\Theta, K)]$. Recall that $m(\Theta, K, \theta, s')$ is the value in terms of marginal utility of an additional unit of savings. We can write the contribution to the utility of a household of a shopper that chooses the best price–tightness pair as

$$\Phi = \max_{Q^c, P^c, F^c} \quad -\eta + \Psi_d(Q^c)F^c\ (\theta\ u_c(\Theta, K) - P^c m)$$

subject to the constraint

$$\varsigma^c \leq P^c \frac{\Psi_d(Q^c)}{Q^c} F^c - w(\Theta, K)\widetilde{n}(k, F^c). \tag{38}$$

This constraint reflects the fact that the only relevant markets are those that guarantee certain expected revenue for the firms. Substituting (38) with $k = K$, and $\Psi_d(Q)$, we rewrite the problem as

$$\Phi = \max_{Q^c, F^c} \ -\eta + A\ (Q^c)^{1-\alpha}\ F^c \left(\theta u_c(\Theta, K) - m\ \frac{\varsigma^{*c} + w(\Theta, K)\ \widetilde{n}(k, F^c)}{A(Q^c)^{-\alpha} F^c}\right). \tag{39}$$

The FOC over Q^c is given by

$$\begin{aligned} 0 &= (1-\alpha)\frac{A\theta u_c(\Theta, K)}{(Q^c)^\alpha} F^c - m\,[\varsigma^{*c} + w(\Theta, K)\widetilde{n}(k, F^c)] \\ &= (1-\alpha)\,\frac{A\theta u_c(\Theta, K)}{(Q^c)^\alpha} F^c - m\frac{AP^c}{(Q^c)^\alpha} F^c \end{aligned}$$

which yields an equation for the unique price equilibrium P^c

$$P^c(\Theta, K) = (1-\alpha)\frac{\Theta\ u_c(\Theta, K)}{M(\Theta, K)}. \tag{40}$$

The FOC over F^c is given by

$$0 = A(Q^c)^{1-\alpha}\theta u_c(\Theta, K) - Q^c m\, w(\Theta, K)\frac{\partial \widetilde{n}}{\partial F^c}.$$

Substituting equilibrium price from equation (40) and the relation $\partial\widetilde{n}/\partial F^c = 1/f_n$ for the given $k = K$, we have

$$\frac{w(\Theta, K)}{P^{c*}(\Theta, K)} = \frac{1}{1-\alpha}A(Q^c)^{-\alpha} f_n\left[K, n^c(\Theta, K)\right],$$

where $N^c(\Theta, K) = \widetilde{n}(K, F^c(\Theta, K))$ is the labor associated with $k = K$ and proposed F^c. The market tightness Q^c is a function of the equilibrium value for producing consumption goods $\varsigma^c(\Theta, K)$,

$$Q^c(\Theta, K) = \left[\frac{A\, P^c(\Theta, K)\, f(K, N^c(\Theta, K))}{\varsigma^c(\Theta, K) + w(\Theta, K)\, N^c(\Theta, K)}\right]^{\frac{1}{\alpha}}. \tag{41}$$

3.4 Competitive search in the market for investment goods

In the same way than household shoppers, firm shoppers choose how to implement the shopping, this is which market tightness, price, and output size, (Q^i, P^i, F^i), and these choices yield two conditions to be satisfied by these 3 variables.

A shopper for the firm chooses the best price-tightness pair $\{Q^i, P^i, F^i\}$ to solve

$$\begin{aligned} \Phi_F &= \max_{Q^i, P^i, F^i} -w(\Theta, K) + \zeta\, \Psi_d(Q^i)F^i\left[E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - P^i\right] \\ \text{s.t.}\quad \varsigma^i &\leq P^i\frac{\Psi_d(Q^i)}{Q^i}F^i - w(\Theta, K)\widetilde{n}(k, F^i). \end{aligned}$$

Substituting P^i, we can rewrite the problem as

$$\max_{Q^i, F^i} -w(\Theta, K) + \zeta\, A(Q^i)^{1-\alpha}F^i\left[E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - \frac{\varsigma^i + w(\Theta, K)\widetilde{n}(k, F^i)}{A(Q^i)^{-\alpha}F^i}\right]$$

The first order condition over Q^i is given by

$$0 = \frac{(1-\alpha)\zeta A}{(Q^i)^\alpha}E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\}F^i - \zeta(\varsigma^i + w(\Theta, K)\widetilde{n}(k, F^i)),$$

which implies that the equilibrium price is

$$P^i(\Theta, K) = (1-\alpha)\; E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\}. \tag{42}$$

The first order condition over F^i is given by

$$0 = \zeta\; A\; (Q^i)^{1-\alpha}\; E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - \zeta\; Q^i w(\Theta, K)\frac{\partial \widetilde{n}}{\partial F^i},$$

Substituting equation (42) for price and $\partial\widetilde{n}/\partial F^i = 1/f_n$ for given $k = K$, we have

$$\frac{w(\Theta, K)}{P^{i*}(\Theta, K)} = \frac{1}{1-\alpha} A(Q^i)^{-\alpha} f_n(K, N^i(K, \Theta))$$

where $n^i(\Theta, K) = \widetilde{n}(K, F^i(\Theta, K))$ is the labor associated with $k = K$ and the amount produced by investment firms proposed F^i. The equilibrium market tightness Q^i as a function of ς^i is given by

$$Q^i(\Theta, K) = \left[\frac{A\; P^i(\Theta, K)\; F[K, N^i(\Theta, K)]}{\varsigma^i(\Theta, K) + w(\Theta, K) n^i(\Theta, K)}\right]^{\frac{1}{\alpha}}. \tag{43}$$

3.5 Equilibrium

We have now established the necessary conditions for equilibrium that arise from the households' and the firm's problems and from the shoppers' problems under competitive search. Before formally defining equilibrium we provide a series of lemmas that show that our conjecture that a sufficient characterization of the state is the pair (Θ, K) is correct. They amount to show that if all firms are equal then the choice of sector does yield a different choice of either labor or output or capital for the following period and hence all firms the following period will have identical capital K'.

Lemma 3. *All firms with $k = K$ choose markets with the same output $F = F^c = F^i$ and also the same labor input for production.*

Lemma 4. *The expected revenue per unit of output is the same in both sectors:*

$$P^c(\Theta, K)\frac{\Psi_d[Q^c(\Theta, K)]}{Q^c(\Theta, K)} = P^i(\Theta, K)\frac{\Psi_d[Q^i(\Theta, K)]}{Q^i(\Theta, K)}. \tag{44}$$

Lemma 5. *Firms with the same k chooses the same k' as future capital stock.*

The following lemma characterize firms' optimal choices over capital accumulation. When making decisions for future capital, firms face an explicit cost of investment (the price paid) and an implicit cost (the wages of shoppers). Interestingly, the Euler equation in equilibrium looks almost exactly like the one in a standard RBC model, this is, the implicit wage cost disappears. The reason is that competitive search links the implicit wage cost and explicit cost together. In equilibrium, the following equation holds

Lemma 6. *The Euler equation of a firm equals the price of investment to the value of capital tomorrow.*

$$E\left\{\frac{P^i(\Theta', K')}{1+R(\Theta', K')}\left[\frac{\Psi_d(Q^i)}{Q^i} f_k\left(K', N^y(K', \Theta')\right) + (1-\delta)\right]\right\} = P^i(Ta, K). \tag{45}$$

where we use $N^y(\Theta, K)$ to denote the labor used in production.

We are now ready to define equilibrium in this economy (it extends that of an environment in Stokey and Lucas (1989)). For simplicity and to reduce notation, we take advantage of some of the above lemmas.

Definition 7. *Equilibrium is a set of decision rules and values for the household* $\{c, d, n, s', v\}$ *as functions of its state* (Θ, K, θ, s), *firms decisions and values* $\{n^y, n^i, i, k', \Omega\}$ *as functions of its state* (Θ, K, k), *and aggregate functions for shopping for investment* D^i, *shopping for consumption* D^c *consumption* C, *labor* N, *labor for production* N^y, *labor for shopping* N^k, *investment* I, *aggregate capital* G, *the measure of consumption producing firms* T, *wages* w, *consumption good prices* P^c, *consumption market tightness* Q^c, *production of firms* F *expected revenues* ς, *investment good prices* P^i, *investment market tightness* Q^i, *and the rate of return of the economy* R *as functions of the aggregate state* (Θ, K) *that satisfy*

1. Households choices and values $d(\Theta, K, \theta, s)$, $c(\Theta, K, \theta, s)$, $n(\Theta, K, \theta, s)$, $s'(\Theta, K, \theta, s)$, and $v(\Theta, K, \theta, s)$ satisfy (18-20) and (26-27).

2. Firms choose $n^k(\Theta, K, k)$, $i(\Theta, K, k)$, $k'(\Theta, K, k)$ and $\Omega(\Theta, K, k)$ to solve their problem (30). They satisfy conditions (32-33) and (36).

3. Competitive Search Conditions. Shoppers and sellers go to the appropriate submarkets, i.e., equations (40– 41) and (42– 43).

4. Representative Agent and Equilibrium Conditions:

$$\begin{aligned}
C(\Theta, K) &= c(\Theta, K, \Theta, 1) && (46)\\
D^c(\Theta, K) &= d(\Theta, K, \Theta, 1) && (47)\\
D^i(\Theta, K) &= \zeta N^k(\Theta, K) && (48)\\
N(\Theta, K) &= n(\Theta, K, \Theta, 1) && (49)\\
N^y(\Theta, K) &= n^y(\Theta, K, K) && (50)\\
N^k(\Theta, K) &= n^k(\Theta, K, K) && (51)\\
I(\Theta, K) &= i(\Theta, K, K) && (52)\\
K'(\Theta, K) &= G(\Theta, K) = k'(\Theta, K, K) && (53)
\end{aligned}$$

5. Market Clearing Conditions:

$$\begin{aligned}
s'(\Theta, K, \Theta, 1) &= 1. && (54)\\
I(\Theta, K) &= D^i(\Theta, K)\, \Psi_d(Q^i(\Theta, K))\, f(K, N^y(\Theta, K)) && (55)\\
C(\Theta, K) &= D^c(\Theta, K)\, \Psi_d(Q^c(\Theta, K))\, f(K, N^y(\Theta, K)) && (56)\\
N(\Theta, K) &= N^y(\Theta, K) + N^k(\Theta, K) && (57)\\
Q^c(\Theta, K) &= \frac{T(\Theta, K)}{D^c(\Theta, K)} && (58)\\
Q^i(\Theta, K) &= \frac{1 - T(\Theta, K)}{D^i(\Theta, K)} && (59)\\
& && (60)
\end{aligned}$$

Comments Note that $\Omega(\Theta, K, K) = 1 + R(\Theta, K)$, a direct implication from the fact that the numeraire is the stock market pre-dividends. The financial wealth of the household is $s(1+R)$ and is equal to the stock market today including the dividends. Note also that the share of consumption expenditure equals the share of firms T, i.e. $\frac{P^c C}{P^c C + P^i I} = T$.

3.6 Equilibrium are Optimal

We now turn to establish optimality of equilibrium. V: (Kjetil, Can you write this?)

3.7 Understanding Solow Residual and Labor Share

To analyze movements in the measured Solow residual in this economy, we need to define real GDP and labor share. In the appendix we provide a detailed analysis of various definitions of output in the data and we show that the historical measure of GDP based on base year prices is very similar to the current measure based on a Tornqvist index. This is important because of the changing relative prices of consumption and investment. We measure GDP using base year prices, which is given by

$$Y = P_0^c C + P_0^i I \tag{61}$$

where P_0^c is the base year consumption price and P_0^i is the base year investment price. Replacing C and I with the aggregate production function the matching terms, we can write GDP as

$$Y = \left[P_0^c\, A(D^c)^{\alpha}(T^c)^{1-\alpha} + AP_0^i\,(D^i)^{\alpha}(T^i)^{1-\alpha}\right] zK^{\gamma_k}(N^y)^{\gamma_n}.$$

Denote labor labor share by γ, the Solow residual $\overline{Z}$ is defined as

$$\overline{Z} = \frac{Y}{K^{1-\gamma}N^{\gamma}}.$$

Substituting Y in the Solow residual, we have

$$\overline{Z} = z\left[P_0^c\, A(D^c)^{\alpha}(T^c)^{1-\alpha} + AP_0^i(D^i)^{\alpha}(T^i)^{1-\alpha}\right] K^{\gamma_k+\gamma-1}(N^c)^{\gamma_n}N^{-\gamma}, \qquad \text{or}$$

$$\overline{Z} = A\, z\, \underbrace{\left[P_0^c\,(D^c)^{\alpha}(T^c)^{1-\alpha} + P_0^i(D^i)^{\alpha}(T^i)^{1-\alpha}\right]}_{\text{Demand Effect } 1.04} \overbrace{\left(\frac{N^y}{N}\right)^{\gamma_n}}^{\text{Effective Work } -.00} \underbrace{K^{\gamma_k-(1-\overline{\gamma})}N^{\gamma_n-\overline{\gamma}}}_{\text{Share's Error } -.00}. \tag{62}$$

The Solow residual depends on the demand effects, increases in demand without changes in technology increase the Solow residual. The two additional terms are due to the mismeasure of productive labor and to the imputation of CRS with a measure of labor share.

Labor share is defined as the ratio of total labor compensation over GDP, $\gamma = wN/Y$. Some algebra yields the following awful expression for labor share.

$$\gamma = \frac{\frac{1}{1-\alpha}\gamma_n P_0^c A(D^c)^\alpha (T^c)^{-\alpha} K^{\gamma_k} (N^y)^{\gamma_n - 1} N}{P_0^c A(D^c)^\alpha (T^c)^{1-\alpha} K^{\gamma_k} (N^y)^{\gamma_n} + P_0^i \gamma_n A(D^i)^\alpha (T^i)^{1-\alpha} K^{\gamma_k} (N^y)^{\gamma_n}}.$$

Labor share will be time varying but we can use steady states to both define base year prices and to compute labor share which yields a much more palatable expression.

$$\gamma = \frac{1}{1-\alpha} \frac{\gamma_n (N^y)^{-1} N}{T^c + \frac{P_0^i}{P_0^c} \frac{A(D^i)^\alpha (T^i)^{-\alpha}}{A(D^c)^\alpha (T^c)^{-\alpha}} T^i} = \frac{1}{1-\alpha} \frac{\gamma_n (N^y)^{-1} N}{T^c + T^i} = \frac{1}{1-\alpha} \gamma_n \frac{N}{N^y}$$

Further manipulations allow us to get the ratio of N and N^y from the parameters of the economy yielding an expression for steady state labor share that depends only on the parameters of the economy and that is given by

$$\gamma = \frac{1}{1-\alpha}\gamma_n + \frac{\alpha\delta}{(1-\alpha)(1/\beta - 1 + \delta)}\gamma_k, \tag{63}$$

which reflects the fact that there are workers in production in shopping. When $\alpha = 0$, we get the same labor share of γ_n as the standard model.

In empirical applications $N - N^y$ is small and γ_n is close to γ. This implies that the last two terms in equation (62) are almost constant over the business cycle and that all movements in the Solow residual are due to movements in demand.

4 Mapping the model to data

We now turn to map the model to data describing which properties of the steady state of the model we are seeking. We also make the case for a few functional forms that we find appropriate. We need to determine 11 parameters.

Preferences We pose separable preferences to be able to clearly discuss the role of the Frisch labor elasticity and also isolate the role of shopping. The per period utility function or felicity is given by (64)

$$u(c, n, d) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - d \tag{64}$$

where we omit the shocks to preferences for simplicity. This choice of utility function requires that we specify four parameters: the discount rate β, the coefficient of risk aversion, σ, Frisch elasticity of labor, ν, and the parameter that determines average hours worked, χ. That the disutility of shopping is linear is not important as the shocks that affect it will shape its properties.

Production Technology [5] Firms have a Cobb-Douglas production function with decreasing returns.

$$f(k,n) = z \;\; k^{\gamma_k} \, (n^y)^{1-\gamma_n} \tag{65}$$

There is no need to impose constant returns to scale given the fact that there are a limited number of locations that are valuable.[7] Note also that z is a parameter to determine units (in later sections we consider it as shock). In addition, a worker devoted to shop for investment goods produces ζ units of shopping services, allowing for the possibility that firms could be a lot more efficient shoppers than people. Capital depreciates at rate δ.

Matching Technology [2] As we have explicitly imposed throughout the paper matching in goods markets occurs via competitive search where they are indexed by tightness, prices, and quantities via a Cobb-Douglas matching technology

$$A \, D^{\alpha} \; T^{1-\alpha}. \tag{66}$$

4.1 Calibration

We now turn to describe the calibration process. We first describe the steady state targets for the model economy and then we estimate the process for the shocks. Some of the targets are standard macroeconomic variables, while others are specific to this economy. Some of those are not exactly steady state targets as much as values of parameters themselves (the intertemporal elasticity of substitution and the Frisch elasticity of labor). Table 1 reports the targets and, following tradition, reports also the parameter values most associated with those targets. The targets are defined in yearly terms even though the model period is a quarter.

The first group of parameters are those that can be set independently of the equilibrium allocation. In this regard, the value of 2 for the intertemporal elasticity of substitution is quite

[7] The model could be easily extended to accommodate the costly creation of such locations.

Table 1: Calibration Targets, Implied Aggregates and (quarterly) Parameter values

Targets	Value	Parameter	Value
First Group: Parameters are set autonomously			
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\psi}$	0.72
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	16.81
Physical Capital to Output Ratio	2.75	δ	0.07
Consumption Share of Output	0.80	γ_k	0.23
Labor Share of income	0.67	γ_n	0.59
Units of output	1.	z	2.03
Third Group: Targets specific to this economy			
Share of production workers	97.%	ζ	3.16
Capacity Utilization of Consumption Sector	0.81	A	0.97
Capacity Utilization of Investment Sector	0.81	α	0.09
Fourth Group: Aggregate Variables Implied			
Percentage of GDP payble to Shoppers	2.%		
Percentage of Cost of New Capital that is internal	9%		
Wealth to output ratio	3.33	From hhold budget constraint	
Relative price of investment in terms of consumption	1.	Equal capacity utilizations	

standard and it does not merit discussion, and neither does the value of the rate of return of 4.%. The Frisch elasticity is much argued among economists, we choose a value .72 based on the measurements of Heathcote, Storesletten, and Violante (2008) who take into account the response of hours worked by he whole household (and not only its head). We study in Section 8.3 an economy with higher Frisch elasticity.

The targets in the second group are what we label standard in the sense that they are targets commonly used in the literature. Perhaps, the only one that deserves some discussion is the consumption to output ratio that we set at 0.8 rather than at 0.75 percent which is more usual. We do this because we take a stand that investment is business investment and not household investment given that there are separate shopping technologies.

The third group of targets is not standard because they relate to variables that are not present in standard real business cycle models. We set the share of workers that are not productive workers but are in charge of purchasing to 3%. While there is no obvious statistic to compare this variable with, we think that it deems a low value. Note that, as the fourth group of variables reports, 2 percent of GDP is used to pay for these workers and the the cost of installing new capital involves internal costs (a form of adjustment costs) as well as the cost of the investment itself and the internal costs are 9% of the total cost of new capital. In the same fashion we target capacity utilization to be 81 percent in both sectors. While our notion of capacity utilization is not the same as that collected by the Federal Reserve System, it is clearly related.[8] Our choices for steady state capacity in both sectors are close to the postwar average of 0.81 (see Corrado and Mattey (1997)) of the official data series.

The fourth group of variables in Table 1 report the implied variables of some interesting aggre-

[8]The Federal Reserve Statistical Release in its oficial documentation of Industrial Production and Capacity Utilization has some Capacity Utilization Explanatory notes that states *"Federal Reserve Board constructs estimates of capacity and capacity utilization for industries in manufacturing, mining, and electric and gas utilities. For a given industry, the capacity utilization rate is equal to an output index (seasonally adjusted) divided by a capacity index. The Federal Reserve Board's capacity indexes attempt to capture the concept of sustainable maximum output-the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place.*

Coverage. Capacity indexes are constructed for 89 detailed industries (71 in manufacturing, 16 in mining, and 2 in utilities), which mostly correspond to industries at the three- and four-digit NAICS level. Estimates of capacity and utilization are available for a variety of groups, including durable and nondurable manufacturing, total manufacturing, mining, utilities, and total industry. Manufacturing consists of those industries included in the North American Industry Classification System, or NAICS, definition of manufacturing plus those industries-logging and newspaper, periodical, book and directory publishing-that have traditionally been considered to be manufacturing and included in the industrial sector. Also, special aggregates are available, such as high-technology industries and manufacturing excluding high-technology industries."

gates. The one that we have not talked about yet is the wealth to output ratio.[9]

5 Estimating Demand Shocks off the Solow Residuals

In this Section we estimate processes for shocks to all elements of preferences, to the shopping technology of firms and to technology (as a mulitiplicative shock to the production function) so that the shopping model generates a Solow residual with the same properties as the data. Note that all these shocks induce changes in the behavior of economic agents which in turn implies that the capacity at which sectors operate is changing. So we are looking for the properties of univariate shocks to induce movements in capacity (and in the case of the technology shock joint movements in capacity and technology) so that the measured Solow residual looks like the data.

The specific shocks that we use are preference shocks (θ_n, θ_d) in the utility function, $\frac{c^{1-\sigma}}{1-\sigma} - \theta_n \frac{n^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \theta_d\, d$, the firm's shopping technology ζ, and the technology shock z.

Table 2: Estimates of the Processes for the Shocks look like TFP shocks

	Our Model with Shopping					Standard RBC
Shocks to	Shop. Disut θ_d	Cons Ut θ_c	Labor Disut θ_n	Firm's Shop. ζ	Tech z	Tech z
ρ	0.942	0.788	0.720	0.990	0.954	0.940
st. dev.	0.022	0.072	0.060	0.007	0.018	0.023
t-stat	43.80	10.98	12.06	135.99	52.08	40.97
σ	0.086	0.164	0.170	0.333	0.006	0.006
st. dev.	0.004	0.008	0.009	0.017	0.0003	0.0003
t-stat	20.00	19.96	20.00	19.99	20.03	20.04
Likelihood	738.75	739.90	739.96	737.21	738.54	738.78
Var of $\bar{Z}$	3.08	3.05	3.01	2.54	3.02	3.06
Autocorr of $\bar{Z}$	0.94	0.94	0.94	0.93	0.94	0.94

The estimates are in Table 2. The last column of the table reports the estimates and its properties of the Solow residual as an exogenous process as is done in the RBC literature. As we can see all these shocks can generate in the context of our model a process for the Solow residual like that in the data without any changes in technology. The estimates are quite precise and they

[9]This statistic is implied by the steady state budget constraint of the household given the rate of return, consumption share, and labor share.

are as good (in terms of the likelyhood) as those of a univariate process for the Solow residual itself, the process that is identified with the productivity shock. The standard deviations of these shocks are, however, quite larger than those of the productivity shock. Interestingly the process of the technology shock in the shopping economy does not have a smaller variance than that of the shock in a standard RBC model, if anything (due to the larger autocorrelation) technology shocks as the only shocks of the shopping economy have a larger variance. This property indicates that the shopping economy does not amplify TFP shocks.

6 Business cycle properties of economies with demand shocks

Whether demand shocks provide a good rationale or not for business cycles does not depend only on their ability to generate movements in the Solow residual. Indeed, what has made the model with technology shocks so popular over the years is its ability to generate the right comovements: the Solow residual, output, the components of output and hours worked are all strongly correlated and investment is much more volatile than output which in turn is more volatile than consumption. We then turn now to explore the business cycle properties of the shopping economy when it is subject to each one of the shocks that we have described. We start revisiting the business cycle properties of both the U.S. data and a standard RBC model with the same steady-state targets and Frisch elasticity that the shopping economy in Table 3.[10] Notice that we report variances and not standard deviations and that all variables move together. Note also that the variance of hours is quite small. This is due to the relatively low Frisch elasticity of substitution that we use in our calibration (that we are using variances gives an additional optical illusion of being small). Despite the low volatility of hours worked, the standard RBC model economy displays the same type of comovements than the U.S. economy.

The business cycle statistics of the shopping economies are in Table 4. A fast look is sufficient to note that the business cycle properties are all over the place. These economies are very different from each other, and also, from the RBC economy and the U.S. data. The only feature they really have in common is that output and the Solow residual move together, something that should be implied almost automatically by the way that the Solow residual is constructed. Let's discuss each one of the shopping economies.

1. *The Shopping disutility shock (Panel (a))*. Labor is negatively correlated with output. This is clearly counterfactual. Shocks of this type by themselves cannot be the trigger of fluctuations.

[10] Table A-1 shows the calibration targets that we used for the standard RBC.

Table 3: Main Business Cycle Moments: U.S. Data and Standard RBC Model

	U.S. Data			Standard RBC		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.19	0.43	0.94	3.06	0.99	0.94
Y	**2.38**	1.00	0.86	**0.96**	1.00	0.71
N	**2.50**	0.87	0.91	**0.10**	0.97	0.71
C	1.55	0.87	0.87	0.05	0.93	0.76
I	34.15	0.92	0.80	17.07	0.97	0.71
cor(C, I)	**0.74**			**0.91**		

All variables except the Solow residual are HP-filtered.

Table 4: Main Business Cycle Moments: Various Economies with Demand Shocks

	(a) Shop Disut θ_d			(b) Labor Disut θ_n		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.08	1.00	0.94	3.01	**-0.85**	0.94
Y	**0.38**	1.00	0.71	**35.84**	1.00	0.58
N	**0.06**	**-1.00**	0.72	**98.13**	0.99	0.55
C	0.50	1.00	0.71	2.89	0.76	0.84
I	0.06	0.99	0.69	634.89	0.98	0.55
cor(C, I)	**0.99**			**0.63**		
	(c) Firms' Shopping Tech ζ			(d) Technology Shock z		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	2.53	0.78	0.93	3.02	0.98	0.94
Y	**1.68**	1.00	0.75	**0.62**	1.00	0.73
N	**0.50**	0.78	0.69	**0.01**	**-0.38**	0.96
C	0.42	**-0.52**	0.75	0.22	0.98	0.78
I	66.43	0.96	0.70	66.43	0.96	0.70
cor(C, I)	**-0.73**			**0.91**		

All variables except the Solow residual are HP-filtered.

These shocks generate a positive wealth effect: consumption goes up and work goes down. Consumer shoppers are more effective and consumption producing firms operate at higher capacity allowing for lower work effort.

2. *The Working disutility shock (Panel (b))*. This shock has also a very large variance and it generates a tremendous volatility of output, six times that of the data. The volatility of hours is even larger, about 15 times that of the data. Clearly, to induce a movement in the Solow residual as large as that in the data the movement in hours has to be tremendous. So much, in fact, as to make the Solow reisudual countercyclical which is due to the induced reduction in the search effort. This economy generates comovements that are dramatically different from those associated to business cycles in the data.

3. *The shock to the firm's shopping technology (Panel (c))*. Investment is a small part of output, therefore this shock has to be stretched to be solely responsible for the movements in measured GDP. Clearly, the shock increases investment quite dramatically, it does so with all the mechanism at the disposal of the economy, more labor, more search and less consumption. The latter behavior is clearly countercyclical.

4. *The technology shock (Panel (d))*. First, note that because this shock induces incentives to change the searching patterns, this shock does not coincide with the Solow residual. Indeed, while consumption and investment goes the right way, labor goes down slightly. In this sense, the shopping economy behaves differently from the standard RBC model in terms of how it accommodates productivity shocks.

   In the shopping model, a positive productivity shock induces enough of a wealth effect and decreases consumer's shopping effort. It, however, makes firms allocate more labor into searching due to a lower investment goods price arising from the positive productivity shock. According to the Solow residual decomposition shown in Section 3.7, both lower consumer shopping and fewer production workers decrease the Solow residual, while higher firm shopping increases the Solow residual. In sum, these effects cancel out somehow and generate almost no amplification of the productivity shock. This also explains why the estimated volatility of innovation in our model is similar as that in the standard RBC model.

To summarize, the shopping economies with only one shock demand shocks do not display the business cycle properties of the data in terms of the comovements of the major variables. In these economies either consumption or labor is countercyclical, which is dramatically counterfactual. The

RBC economies, display the right comovements of variables while inducing a very low variability of labor.

7 Multiple Shocks to the Economy

We now turn to explore whether multiple shocks in the shopping economy may generate the observed type of comovements of the macro variables. The standard RBC model gives a very incomplete picture of business cycles without an additional mechanism that moves hours and a way to generate business cycles like those in the data is to add a shock to the marginal rate of substitution (MRS) between consumption and leisure[11] (see Chari, Kehoe, and McGrattan (2007)). Since the shopping model generates very counterfactual comovements for the main macro variables with only one shock, we compare in Section 7.1 the implications of a two shocks version of the shopping model with two those of a two shock version of the standard RBC model. We complement the main engine of fluctuations, productivity shocks in the standard RBC and pure demand shocks in the shopping model (this is shocks to the cost of shopping), with shocks to the MRS.

We proceed by fully estimating using Bayesian methods[12] those two economies. In Section 7.2 we estimate all the shocks jointly what allows us to do a variance decomposition. We find that demand shocks account for most of the variability of output and the Solow residual and that, like in the RBC model, the volatility of hours is mostly due to shocks to the MRS. In this sense, we find our shopping economy as a substitute for the RBC model. We cannot however, use this model to understand based solely on demand shocks the fluctuations in labor.

7.1 The Shopping Model confronts the standard RBC Model

We want to have demand shocks that involve both consumption and investment in a parsimonious way. One way to do this is to have a common shock that affects both the demands for consumption and for investment. Furthermore, we want to get accomodate for the possibility of other mechanisms that move hours, which we know is a challenge for models were the household optimally chooses hours worked. Specifically, we estimate a process for three shocks, θ_d, ζ, and θ_n, which is a shock to the marginal rate of substitution between consumption and leisure (MRS). We assume that the shocks on shopping incentives θ_d and ζ are perfectly correlated and that they have the

[11]We are ignoring other type of business cycle models with frictions in labor markets that prevent households from optimally choosing hours worked.

[12]See An and Schorfheide (2007) or Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2009) for details.

same persistence but different variability. Thus we estimate the autocorrelations of θ_n and θ_d, the standard deviation of the two innovations, and the relative volatility of ζ and θ_d. We can think of this structure as having one demand shock that feeds both consumption and investment, and we estimate the strength with which it does so. For comparison, we estimate the standard RBC model with two shocks, z and θ_n. We assume that these two shocks are not correlated.

The estimations for both models use Bayesian methods over Solow residual and output data. We assume that autocorrelations follow Beta distributions and that innovations of shocks follow Inverse-Gamma distributions. The priors, posteriors and the estimated parameters are reported in Table 5. The log likelihood for the shopping model is 1492, significantly higher than that in the standard RBC model of 1484.[13] The 90% intervals are tight, indicating significant estimated parameters. All shock are persistent. The volatility of the demand shocks is much larger than that of the technology shock of the RBC model, but the volatility and persistence of the shock to the MRS is a bit lower.

Table 5 also reports the variance decomposition over the major business cycle variables. The shopping model and the standard RBC model have similar implications over the variance decomposition. In the shopping model, demand shocks account for most of the variance of output, the Solow residual an investement, while the shock to the MRS accounts for half of the volatility of consumption and 96% of that of hours. Things are strikingly similar in the RBC model, if anything the technology shock accounts for less of the variance of output and consumption. Consequently, the MRS shock plays a more important role in the RBC. In any case most models pose a very minor role to non MRS shocks in moving hours.

The last panel of Table 5 presents the business cycle statistics for the major variables, for both the shopping model and the standard RBC model. With a combination of demand shocks, the shopping model generates the right comovements of major variables with output. With the MRS shock, both models improve over their prediction in the hour variability. The two models have similar implications over the business cycle statistics.

We conclude that the shopping model has the same properties as the RBC model the main shock accounts for most of the movements of output and TFP, but both models require the shock to the MRS to move hours beyond a pittance.

[13] The specification that we chose involves one extra parameter than the estimation of the RBC model. Still, the likelyhood of the shopping model is higher for a large range of relative volatilities of ζ and θ_d.V: Yan could you verify that this is correct?

Table 5: Comparison of Shopping Model and Standard RBC Model

Priors and Posteriors for the Shock Parameters

Shopping model (likelihood = 1492)

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	**0.921**	[0.894, 0.947]
σ_d	Inverse Gamma	0.20	0.20	**0.064**	[0.056, 0.072]
σ_ζ/σ_d	Normal	0.50	0.50	**1.51**	[1.06, 1.92]
ρ_n	Beta	0.95	0.05	**0.982**	[0.968, 0.999]
σ_n	Inverse Gamma	0.06	0.20	**0.019**	[0.017, 0.021]

RBC model (likelihood = 1484)

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_z	Beta	0.93	0.05	**0.952**	[0.925, 0.981]
σ_z	Inverse Gamma	0.006	0.20	**0.006**	[0.0055, 0.0065]
ρ_n	Beta	0.94	0.05	**0.993**	[0.981,1.000]
σ_n	Inverse Gamma	0.02	0.20	**0.021**	[0.018, 0.023]

Variance Decomposition (%)

	Shopping model		RBC model	
	θ_d, θ_ζ	θ_n	z	θ_n
Y	72.86	27.14	63.13	36.87
Solow	98.44	1.56	100.00	0.00
N	4.27	95.73	3.08	96.92
C	47.38	52.62	19.71	80.29
I	86.84	13.16	86.02	13.98

Business Cycle Statistics

	Shopping model		RBC model	
	Variance	Cor w Y	Variance	Cor w Y
Y	1.17	1.00	1.27	1.00
Solow	2.44	0.96	4.19	0.79
N	0.94	0.56	1.09	0.73
C	0.28	0.76	0.31	0.88
I	13.74	0.97	14.58	0.96

7.2 Full Estimation

We now turn to estimating a model with all shocks, what we have described as demand shocks, a shock to the MRS and a productivity shock within the shopping model. The estimation allows us to do a variance decomposition, this is to impute the contribution of each disturbance to aggregate fluctutations. We use again Bayesian methods. We assume that the two demand shocks (shocks to θ_d and ζ) are correlated and we estimate such correlation together with other shock parameters. The data that we use are the Solow residual, output, hours, and consumption, all of them linearly detrended. We assume that the autocorrelations follow a Beta distribution, that the standard deviation of the innovations follow an inverse-Gamma distribution, and that the correlation between demand shocks follows a normal distribution.

Table 6 shows the priors and posteriors for all shock parameters. The 90% intervals are tight except for the correlation between the demand shocks, implying that the estimated shock parameters are significant. The standard deviations of the demand shocks are high, and the volatility of the MRS is also on the high side as well as its persistence. The volatility of the productivity shock, is however, about half of what is obtained in a standard RBC model with a little bit less persistence, so its overall variability is much smaller than in the RBC model.

Table 6 also reports the variance decompositions of the major aggregate variables. The first thing to see is that demand shocks are much more important than the productivity shock, not only in terms of its contribution to output (60% relative to 4%) but also in terms to its contribution to the Solow residual itself (90% relative to 8%). Hours are still dependent mostly on shocks to the MRS, but the demand shocks contribute 14% while productivity shocks have no effects on hours. Demand shocks also account for more than half of consumption and 90% of investment while the contribution of TFP shocks is less than 5%.

The 90% interval of the estimated demand-shock correlation is not tight. To see how sensitive are the variance decompositions to this parameter, we reestimated the shocks while fixing the demand-shock correlation at different levels. When we vary the demand-shock correlation from -0.2 to 0.2, the log-likelihood ratios are flat and not significant from each other. As the correlation increases, the contribution of demand shocks to the Solow residual goes from 93% to 49%, while that of output goes from 68% to 34%. (See Table A-2 in the Appendix for details).

Lastly, we report the major business cycle statistics for the estimations conducted in this Section in the last panel of Table 6.The shopping economy gets the right comovements between the main

Table 6: Full Estimation of the Shopping Model

Priors and Posteriors for the Shock Parameters (Lilkelihood = 2244.23)					
Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	**0.957**	[0.934, 0.982]
σ_d	Inverse Gamma	0.06	0.20	**0.067**	[0.053, 0.080]
ρ_ζ	Beta	0.95	0.05	**0.957**	[0.934, 0.982]
σ_ζ	Inverse Gamma	0.15	0.20	**0.126**	[0.107, 0.144]
ρ_z	Beta	0.95	0.05	**0.933**	[0.866, 0.999]
σ_z	Inverse Gamma	0.004	0.20	**0.003**	[0.001, 0.005]
ρ_n	Beta	0.95	0.05	**0.996**	[0.992,1.000]
σ_n	Inverse Gamma	0.02	0.20	**0.022**	[0.020, 0.024]
Cor(θ_d, ζ)	Normal	0.00	0.20	**0.037**	[-0.172, 0.262]

	Variance Decomposition (%)				Business Cycle Statistics	
	θ_d	ζ	z	θ_n	Variances	Cor w Y
Y	29.09	30.62	4.17	36.12	1.07	1.00
Solow	75.54	14.34	8.15	1.98	5.96	0.63
N	2.97	10.83	0.11	86.09	1.28	0.61
C	50.00	15.46	3.19	31.36	0.69	0.62
I	1.16	87.58	1.37	9.89	16.45	0.76

variables. With respect to the variances, it is too high for the Solow residual and too low for output and labor.

8 Robustness of Findings

In this section, we explore additional interesting properties of the shopping economies. In section 8.1 we compare the performance of the shopping economy with that of the standard RBC economy ignoring the role of the shock to the MRS that is used to generate more volatility of hours. This is a standard way of assessing a RBC model. In Section 8.2 we study the business cycle behavior of aggregate variables that are constant in the standard RBC model. Finally, in Section 8.3 we compare the shopping economy and the RBC economy with an alternative (higher) Frisch elasticity.

8.1 Business cycle Statistics without the MRS Shock

The performance of the standard RBC model is usually assessed when facing only the productivity shock, without posing a shock to the MRS to pump up the volatility of hours. Accordingly, we now look at the shopping model with perfectly correlated demand shocks (to the utility of shopping of households and to the cost of shopping for firms) that are estimated off the Solow residual alone and we compare the business cycle statistics with those of the RBC economy in Table 7. The table clearly indicates that both models are similar: they predict a pitifylly small variance of hours, and they have the right comovements of variables. Unlike the one shock only versions of the shopping model, a shopping economy with perfectly correlated demand shocks for consumption and investment, can match the performance of the RBC model (and the data) in terms of comovements.

8.2 Other Business Cycle Statistics

The shopping model also has implications for other important macroeconomics variables, the relative price of investment, stock market price, and capacity utilization. The standard RBC is silent about these variables. Table 8 reports the properties of the data and the two main versions of the shopping model that attempt to match output volatility.

1. *The relative price of investment.* The role of the relative price of investment in shaping economic performance has been studied in various contexts, from its role in shaping the skill premia (Krusell, Ohanian, Rios-Rull, and Violante (2000)) to its role as a direct source of

Table 7: Business Cycle Statistics without MRS Shock

Priors and Posteriors for the Shock Parameters

Shopping model with (θ_d, ζ), likelihood = 734

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.96	0.05	0.970	[0.948, 0.995]
σ_d	Inv. Gamma	0.05	0.20	0.052	[0.043, 0.059]
σ_ζ/σ_d	Normal	2.60	0.50	2.63	[1.769, 3.476]

RBC model with z, likelihood = 735

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_z	Beta	0.95	0.05	0.949	[0.918, 0.985]
σ_z	Inv. Gamma	0.006	0.20	0.006	[0.0055, 0.0065]

Main Business Cycle Statistics

	Variance			Correlation with Y		
	Data	(θ_d, ζ)	RBC	Data	(θ_d, ζ)	RBC
Solow	3.19	3.48	3.59	0.43	0.96	1.00
Y	2.38	0.84	0.92	1.00	1.00	1.00
N	2.50	0.05	0.08	0.87	0.56	0.96
C	1.55	0.10	0.05	0.87	0.76	0.95
I	34.15	13.68	15.40	0.92	0.97	1.00

Table 8: Other Business Cycle Statistics for the Full Estimation Shopping Model

	Variance			Correlation with Y		
	Data	$\theta_d, \zeta, \theta_n$	θ_d, ζ z, θ_n	Data	$\theta_d, \zeta, \theta_n$	θ_d, ζ z, θ_n
p_i/p_c	0.47	0.15	2.68	-0.23	-0.96	-0.31
Stock Market (S&P 500)	42.64	0.58	1.80	0.41	0.79	0.33
Capacity Utilization	4.21	0.72	0.59	0.89	0.85	0.72

business cycles (Fisher (2006)). In most of these cases such relative price is taking to be an exogenous object that depends purely on technological considerations (an exception is Valles (1997) that uses a non-linear production possibility frontier). In our economy the relative price of consumption and investment is purely an economic object since consumption and investment are perfect substitutes in production. In the shopping environment a reduction in the distutility (or cost) of shopping translates into a willingness to shop longer while facing a cheaper price for consumption (or the investement) good. In the data, the relative price of investment is countercyclical and less volatile than output. The model economies also pose a countercylical relative price of investment. The two versions of the model differ drastically in their performance with respect to the variance.

2. *The stock market price.* Given our normalization, the stock market is just the inverse of the price of consumption. In the shopping economy the value of firms changes not only because of changes in the cost of shopping for new capital, but also because the value of locations capable of matching with shoppers changes. As is well known, the stock market price in the data is extremely volatile with a volatility about 20 times that of output. It is also procyclical. Our model does indeed generate a procyclical stock price. Not surprisingly, the variance of the price is much smaller thant that of the data, but still sizeable especially given the low risk aversion that we have posed.

3. *Capacity utilization.* In the shopping economies the ratio of output to potential output is constantly changing, with higher utilization resulting in higher measured productivity. In the U.S., the Federal Reserve Board has constructed a series of capacity utilization that while does not coincide with our notion, is clearly related. In the data, the capacity utilization is about twice as volatile as output, and very procyclical. Our model without frictions in production also has procyclical utilization although with lower volatility than the data.

8.3 Alternative calibration targets

Finally, we want to explore the extent to which our findings are tied to the specific value of the Frisch elasticity of output that we have chosen. For this reason we posed model economies with a much larger Frisch elasticity, a value of 1.3.[14] We reestimted the shopping economies with perfectly correlated demand shocks and MRS shock, the 4 shock version and the standard RBC and we have compared the implications. Table A-3 in the Appendix displays the vlaues of the estimates, while Table 9 shows the variance decomposition and the business cycle statistics. Our findings are clearly the same with the higher Frisch elasticity: The shopping model can generate all of the variance in the Solow residual even when given technology shocks a change to contribute. The shopping models account for a small part of the volatility of hours but a much higher one than the RBC model. The comevements are similar. If anything, the higher Frisch elasticity improves the estimates for all the models.

9 Conclusions and Extensions

In this we have developed a model where demand shocks generate procyclical productvity via a cyclical use of production capacity that implies inconveniences for shoppers. We have developed the theory using search frictions under competitive search protocoles that imply optimality (and hence existence and uniqueness) of equilibrium and that provide a rationale for stimulus packages. We have shown how these shocks can replicate the movements of the Solow residual that the RBC literature typically identifies with technology shocks. We shock that correlated shocks to the demand of consumption and investment can replicate the propoerties of the standard real business cycle models in terms of the comovements of macroeconomic variables. If anything, our model performs marginally better. When allowing in our model economy the coexistence of demand shocks and technologies a full Bayesian estimation imputes essentially no role to technology shocks even in shaping the properties of the Solow residual. In addition, our shopping economies have implications for other macroeconomic variables (relative price of consumption and investment, the stock market, capacity utilization) over which standard models are silent. Such implications are remarkably consistent with the data.

Our assessment of the findings is that our modeling structure provides an alternative to technology shocks as a source of fluctuations. The failure of both models in generating enough volatility of hours is addressed in the literature in a variety of ways (using non market clearing labor markets by

[14] A recent defense for this value is in Kaplan and Ríos-Rull (2011).

Table 9: Business Cycle Statistics under Frisch Elasticity 1.3

Variance Decomposition

	$\theta_d, \zeta, \theta_n$		RBC		$\theta_d, \zeta, \theta_n, z$			
	θ_d, ζ	θ_n	z	θ_n	θ_d	ζ	z	θ_n
Y	76.18	23.82	60.69	39.31	20.48	33.65	7.03	38.85
Solow	98.30	1.70	100.00	0.00	71.70	11.99	14.07	2.24
N	14.70	85.30	2.64	97.36	4.70	15.59	0.20	79.50
C	45.73	54.27	39.12	60.88	48.79	12.95	5.26	33.00
I	88.87	11.13	71.32	28.68	0.19	83.76	2.80	13.25

Main Business Cycle Statistics

	Variance			Correlation with Y		
	$\theta_d, \zeta, \theta_n$	RBC	θ_d, ζ	$\theta_d, \zeta, \theta_n$	RBC	θ_d, ζ
		z, θ_n	z, θ_n		z, θ_n	z, θ_n
Solow	2.93	7.14	5.66	0.79	0.78	0.58
Y	1.33	1.29	1.15	1.00	1.00	1.00
N	1.03	1.18	1.60	0.68	0.73	0.64
C	0.21	0.24	0.65	0.72	0.97	0.60
I	21.51	14.89	18.33	0.96	0.99	0.80
Other Statistics						
p_i/p_c	1.19	–	2.57	-0.91	–	-0.36
S&P 500	0.53	–	1.75	0.39	–	0.37
Cap Ut	0.68	–	0.54	0.87	–	0.67

the new Keynesians, writing explicity frictions in the labor market a la Morternsen and Pissarides) on models based on technology shocks. We see no reason why these approaches cannot be used on top of our shopping models. The research agenda we think is clear and has two directions: to pose deep models of demand shocks: financial shocks, shocks to real exchange rate, wealth shocks, monetary policy and so on; and to accomodate mechanisms that substitute the frictionless labor market.

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APPENDIX

A Proof of some results

Lemma 2. *$\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ implies $\varsigma^c(\Theta, K, k) = \varsigma^i(\Theta, K, k)$.*

Proof. From the firms' first order condition over k' (36) it is clear that both marginal return and marginal cost of capital are independent of firm's current choice over which goods to produce and choice of labor for production. This implies that firms simply search for the markets that give them the best current revenue. Thus $\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ implies $\varsigma^{c*}(\Theta, K, k) = \varsigma^{i*}(\Theta, K, k)$. ∎

Lemma 3. *All firms with $k = K$ choose markets with the same output $F = F^c = F^i$ and also the same labor input for production.*

Proof. Let's define $n^c(K, \Theta)$ as the necessary labor for a consumption-producing firm with capital $k = K$ to produce output $y^c(\Theta, K)$, namely $n^c(K, \Theta) = \widetilde{n}(K, y^c(\Theta, K))$. Similarly, we define $n^i(K, \Theta) = \widetilde{n}(K, y^i(\Theta, K))$ for investment-producing labor. In equilibrium, firms are indifferent of producing consumption goods or investment goods, i.e. $\varsigma^{c*} = \varsigma^{i*}$. By definition, $\varsigma^{c*} = P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha} y^c(\Theta, K) - w(\Theta, K)n^c(\Theta, K)$. We can further rewrite ς^{c*} using equilibrium conditions from the competitive search,

$$\begin{aligned}
\varsigma^{c*} &= P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha} y^c(\Theta, K) - w(\Theta, K)n^c(\Theta, K) \\
&= w(\Theta, K)\left[\frac{P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha} y^c(\Theta, K)}{w(\Theta, K)} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{A[Q^c(\Theta, K)]^{-\alpha} y^c(\Theta, K)}{A[Q^c(\Theta, K)]^{-\alpha} f_n[K, n^c(\Theta, K)]} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{y^c(\Theta, K)}{f_n[K, n^c(\Theta, K)]} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{zK^{\gamma_k}(n^c(\Theta, K))^{\gamma_n}}{\gamma_n zK^{\gamma_k}(n^c(\Theta, K))^{\gamma_n - 1}} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{n^c(\Theta, K)}{\gamma_n} - n^c(\Theta, K)\right] \\
&= \frac{1-\alpha-\gamma_n}{\gamma_n} w(\Theta, K)n^c(\Theta, K).
\end{aligned}$$

The third equality uses the following equilibrium condition from the competitive search problem of consumption goods

$$\frac{w(\Theta, K)}{P^c(\Theta, K)} = \frac{1}{1-\alpha} A(Q^c)^{-\alpha} f_n[K, n^c(\Theta, K)].$$

The fifth equality uses the definition of the production function $f(k, n) = zk^{\gamma_k} n^{\gamma_n}$. Similarly, we have

$$\varsigma^{i*} = \frac{1-\alpha-\gamma_n}{\gamma_n} w(\Theta, K) n^i(\Theta, K).$$

Equalizing ς^{i*} and ς^{c*} implies that $n^c(\Theta, K) = n^i(\Theta, K)$, the labor inputs for production are the same for the firms with the same capital $k = K$. Their outputs must be the same too. ∎

Lemma 4. *In equilibrium, the following holds*

$$P^c(\Theta, K) \frac{\Psi_d[Q^c(\Theta, K)]}{Q^c(\Theta, K)} = P^i(\Theta, K) \frac{\Psi_d[Q^i(\Theta, K)]}{Q^i(\Theta, K)}. \quad \text{(A-1)}$$

Proof. Under Lemma 3, firms with the same $k = K$ have the same labor input for production, the same output, and the same ς^*. This implies equation (44). ∎

Lemma 5. *Firms with the same k chooses the same k' as future capital stock.*

Proof. According to Lemma 3, firms with the same k choose the same labor input for both consumption producing and investment producing. For a firm that considers to produce consumption tomorrow, the first order condition over k' is given by

$$E\left\{ \frac{-w(\Theta', K')\tilde{n}_k(k', y(\Theta', K')) + (1-\delta)\left(\frac{w(\Theta', K')}{\zeta\, \Psi_d[Q^i(\Theta', K')]\, f[(\Theta', K')]} + P^i(\Theta', K')\right)}{1 + R(\Theta', K')} \middle| \Theta \right\}$$
$$= \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^y(\Theta, K)]} + P^i(\Theta, K). \quad \text{(A-2)}$$

For a firm that considers to produce investment tomorrow, the first order condition over k' is

$$E\left\{ \frac{-w(\Theta', K')\widetilde{n}_k(k', y(\Theta', K')) + (1-\delta)\left(\frac{w(\Theta',K')}{\zeta\, \Psi_d[Q^i(\Theta',K')]\, f[K'^i(\Theta',K')]} + P^i(\Theta', K')\right)}{1+R(\Theta', K')} \middle| \Theta \right\}$$
$$= \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^y(\Theta, K)]} + P^i(\Theta, K),$$

With Lemma 3, it is easy to see that the first order conditions for k' of future consumption producing firms and investment producing firms are identical. Thus, all the firms with the same current capital choose the same future capital. ∎

Lemma ??. *In equilibrium, the following holds*

$$\frac{w(\Theta, K)}{\zeta\Psi_d[Q^i(\Theta, K)]f(K, N^y(\Theta, K))} = \frac{\alpha}{1-\alpha}P^i(\Theta, K) \tag{A-3}$$

Proof. From investment-producing firms' first order condition over k', we have

$$E\left\{ \frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')} \middle| \Theta \right\} = \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^i(\Theta, K)]} + P^i(\Theta, K) \tag{A-4}$$

The equilibrium search in the investment goods market implies

$$P^i(\Theta, K) = (1-\alpha)\, E\left\{ \frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')} \right\}. \tag{A-5}$$

Putting together the above two equations proves the Lemma. ∎

Lemma 6. *In equilibrium, investment-producing firm has the Euler equation with the usual form*

$$E\left\{ \frac{P^i(\Theta', K')}{1+R(\Theta', K')} \left[\frac{\Psi_d(Q^i)}{Q^i} f_k\left(K', N^y(K', \Theta')\right) + (1-\delta) \right] \right\} = P^i. \tag{A-6}$$

Proof. The FOC for future capital stock evaluated at K' for an investment-producing firm is given by

$$E\left\{ \frac{-w(\Theta',K')\widetilde{n}_k(K',y(\Theta',K')) + (1-\delta)\left(\frac{w(\Theta',K')}{\zeta\,\Psi_d[Q^i(\Theta',K')]\,f[K',N^y(\Theta',K')]} + P^i(\Theta',K')\right)}{1+R(\Theta',K')} \middle| \Theta \right\}$$
$$= \frac{w(\Theta,K)}{\zeta\,\Psi_d[Q^i(\Theta,K)]\,f[K,N^y(\Theta,K)]} + P^i(\Theta,K),$$

According to Lemma **??**, we can replace $\frac{w(\Theta,K)}{\zeta\Psi_d[Q^i(\Theta,K)]f(K,N^y(\Theta,K))}$ with $\frac{\alpha}{1-\alpha}P^i(\Theta,K)$. Similarly for the future variables. The Euler becomes

$$E\left\{ \frac{-w(\Theta',K')\widetilde{n}_k(K',y(\Theta',K')) + (1-\delta)\frac{P^i(\Theta',K')}{1-\alpha}}{1+R(\Theta',K')} \middle| \Theta \right\} = \frac{P^i(\Theta,K)}{1-\alpha}$$

Multiplying $1-\alpha$ on both hand side and reorganizing the equation, we have

$$E\left\{ \frac{P^i(\Theta',K')}{1+R(\Theta',K')}\left[-(1-\alpha)\frac{w(\Theta',K')\widetilde{n}_k(K',y(\Theta',K'))}{P^i(\Theta',K')} + (1-\delta)\right] \middle| \Theta \right\} = P^i(\Theta,K).$$

Substituting $w(\Theta',K')/P^i(\Theta',K')$ with $\frac{1}{1-\alpha}\frac{\Psi_d(Q^i)}{Q^i}f_n(K,N^y(\Theta',K'))$ from the competitive search problem, we can rewrite the Euler as

$$E\left\{ \frac{P^i(\Theta',K')}{1+R(\Theta',K')}\left[-\frac{\Psi_d(Q^i)}{Q^i}f_n(K,N^y(\Theta',K'))\widetilde{n}_k(K',y(\Theta',K')) + (1-\delta)\right] \middle| \Theta \right\} = P^i(\Theta,K).$$

Under the production function $f(k,n) = zk^{\gamma_k}n^{\gamma_n}$, $\widetilde{n}_k(k,y) = -\gamma_k n/(\gamma_n k)$ and $f_n = \gamma_n z k^{\gamma_k} n^{\gamma_n - 1}$. Thus $-f_n\widetilde{n}_k = \gamma_k z k^{\gamma_k - 1} n^{\gamma_n} = f_k$. Substituting $-f_n\widetilde{n}$ with f_k in the Euler equation, we have

$$E\left\{ \frac{P^i(\Theta',K')}{1+R(\Theta',K')}\left[\frac{\Psi_d(Q^i)}{Q^i}f_k(K,N^y(\Theta',K')) + (1-\delta)\right] \middle| \Theta \right\} = P^i(\Theta,K).$$

∎

Table A-1: Calibration for the Standard RBC model

Targets	Value	Parameter	Value
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\psi}$	0.72
Fraction of time spent working	30%	χ	18.49
Consumption Share of Output	0.80	δ	0.06
Labor Share of income	0.67	γ_n	0.67
Units of output	1.	z	0.94

Table A-2: Sensitivity Analysis over Demand-Shock Correlation

Corrrelation	−0.4	−0.3	−0.2	−0.1	0	0.1	0.2	0.3	0.4
Likelihood	2234.46	2240.48	2242.90	2244.46	2245.58	2244.30	2243.93	2244.48	2244.63
ρ_d	0.959	0.958	0.934	0.957	0.956	0.957	0.958	0.957	0.963
σ_d	0.079	0.077	0.067	0.073	0.071	0.066	0.061	0.057	0.054
ρ_n	0.998	0.997	0.993	0.997	0.996	0.996	0.996	0.996	0.998
σ_n	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022
ρ_ζ	0.959	0.958	0.934	0.957	0.956	0.957	0.958	0.957	0.963
σ_ζ	0.142	0.136	0.134	0.130	0.128	0.123	0.119	0.118	0.114
ρ_z	0.968	0.929	0.945	0.940	0.924	0.918	0.911	0.919	0.905
σ_z	0.002	0.002	0.002	0.002	0.002	0.003	0.004	0.005	0.005
					Variance decomposition for output				
θ_d	46.38	40.81	35.82	29.92	26.56	14.00	8.36	4.97	4.01
ζ	23.86	26.43	28.63	30.39	32.12	27.40	25.22	22.14	19.41
z	1.34	1.50	3.88	1.75	3.37	22.38	30.31	37.17	41.31
θ_n	28.43	31.26	35.81	33.81	37.96	36.22	36.12	35.72	35.27
					Variance decomposition for the Solow residual				
θ_d	84.94	83.03	81.25	76.18	76.43	48.67	36.65	29.28	28.62
ζ	10.91	12.26	13.35	14.23	14.91	13.17	12.25	10.92	8.98
z	2.68	3.06	3.57	7.63	6.57	36.17	49.12	57.83	60.51
θ_n	1.47	1.66	1.83	1.96	2.09	1.99	1.98	1.96	1.89

Table A-3: Bayesian Estimation under Frisch Elasticity 1.3

Priors and Posteriors for the Shock Parameters

Shopping model with $(\theta_d, \zeta, \theta_n)$, likelihood = 1505

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	0.961	[0.939, 0.982]
σ_d	Inverse Gamma	0.07	0.20	0.050	[0.043, 0.055]
σ_ζ/σ_d	Normal	2.60	0.50	2.91	[2.241, 3.576]
ρ_n	Beta	0.95	0.05	0.971	[0.959, 0.986]
σ_n	Inverse Gamma	0.02	0.20	0.013	[0.012, 0.015]

RBC model with (z, θ_n), likelihood = 1483

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_z	Beta	0.95	0.05	0.971	[0.956, 0.988]
σ_z	Inverse Gamma	0.006	0.20	0.006	[0.0055, 0.0065]
ρ_n	Beta	0.95	0.05	0.989	[0.979, 1.000]
σ_n	Inverse Gamma	0.02	0.20	0.015	[0.013, 0.017]

Shopping model with $(\theta_d, \zeta, \theta_n, z)$, likelihood = 2229

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.95	0.05	0.965	[0.941, 0.987]
σ_d	Inverse Gamma	0.07	0.20	0.062	[0.047, 0.078]
σ_ζ	Inverse Gamma	0.12	0.20	0.117	[0.095, 0.136]
ρ_z	Beta	0.95	0.05	0.927	[0.861, 0.999]
σ_z	Inverse Gamma	0.005	0.20	0.004	[0.002, 0.006]
ρ_n	Beta	0.95	0.05	0.991	[0.986, 0.997]
σ_n	Inverse Gamma	0.02	0.20	0.017	[0.016, 0.019]
Cor(σ_d, ζ)	Normal	0.0	0.30	0.185	[-0.172,0.522]