# Other-Regarding Preferences in General Equilibrium*

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**Abstract**

We study competitive market outcomes in economies where agents have other-regarding preferences. We identify a separability condition on monotone preferences that is necessary and sufficient for one's own demand to be independent of the allocations and characteristics of other agents in the economy. Given separability, it is impossible to identify other-regarding preferences from market behavior: agents behave as if they had classical preferences that depend only on own consumption in competitive equilibrium. If preferences, in addition, depend only on the final allocation of consumption in society, the Second Welfare Theorem holds as long as an increase in resources can be distributed such that all agents are better off. Nevertheless, the First Welfare Theorem generally does not hold. Allowing agents to care about their own consumption and the distribution of consumption possibilities in the economy, we provide a condition under which agents have no incentive to make direct transfers, and show that this condition implies that competitive equilibria are efficient given prices.

**Keywords**: markets, other-regarding preferences, self-interest, welfare theorems.

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# 1 Introduction

The literature on other-regarding preferences (ORP), which documents and models how decision makers often fail to maximize their narrow self-interest, has recently expanded rapidly.[1] Incorporating these preferences into simple strategic situations enriches the theory. In familiar bargaining and public-goods games, models of other-regarding preferences yield strikingly different, and more accurate, predictions about play than standard theory. In competitive markets, the classical results of general-equilibrium theory tell celebrated stories about existence and efficiency of market equilibria, when agents are selfish. So one may ask whether these results hold in economies with ORP-affected individuals. The paper at hand provides answers.

We present a general model of preferences in which it is possible to compare the outcomes of a general-equilibrium model with ORP to those of a classical model in which each agent maximizes a utility function that depends only on her own consumption. To do this, we look for conditions under which an agent with ORP has well defined preferences over own consumption in market settings. We say that an agent behaves **as if classical** if the agent's demand function depends only on her income and prices. Under standard technical assumptions, we show in Section 2 that an as if classical demand function exists if and only if the preferences of an agent can be represented by a utility function that is separable between her own consumption bundle and the consumption vectors and opportunity sets of others. If this separability condition holds, an agent's preferences induce preferences over the own consumption set that are independent of the consumption bundles of the other agents and of the distribution of budget sets. We refer to these preferences over own consumption as **internal preferences**. The separability assumption is restrictive, but holds for natural generalizations of ORP found in the literature.

Using the results of Section 2, we can relate any economy with separable other-regarding preferences to an economy with classical egoistic agents, whose preferences coincide with the internal preferences of the agents of the original economy. Price-taking agents with other-regarding preferences behave exactly like their classical counterparts. Consequently, as we observe in Section 3, the equilibria of the other-regarding economy coincide with those of the associated classical economy. This result sharply contrasts with re-

[1] Fehr and Gächter [18] and Sobel [41] survey the literature.

sults in strategic settings, where the existence of ORP leads to important qualitative changes in the predictions of models. When the market mechanism governs exchange, classical outcomes are consistent with a broad range of underlying preferences. These results suggest that it is more difficult to express spite and altruism in a market setting than in strategic interactions.

We next investigate the extent to which the Fundamental Welfare Theorems extend to our framework. Walrasian equilibrium is efficient with respect to internal preferences, but it need not be efficient with respect to other-regarding preferences. To investigate the efficiency properties of equilibrium in more detail, we discuss the domain and structure of ORP. The ORP models in the literature were motivated by experimental games that involved the distribution of a single good (money). The natural generalization of classical preferences is to expand the domain of utility functions to the whole payment profile,[2] which leads to basic models of spite and altruism and the richer models proposed by Bolton and Ockenfels [6], Fehr, Kirchsteiger, and A. Riedl [19], Fehr and Schmidt [20], and others. Extending the approach to the multi-good world requires a careful consideration of the domain of other-regarding preferences. We want our model to describe an agent whose welfare depends on the opportunities available to other agents independent of their actual consumption choices. For this reason our formulation of ORP must be general enough to include budget sets and not just consumption profiles. One contribution of the paper is a novel model of preferences that also depend on opportunity sets.[3]

In order to discuss welfare properties of competitive equilibria, we distinguish two classes of ORP. Section 4 introduces and discusses **well-being externalities**, which can be modeled by utilities that depend on the allocations, and the—technically more general—concept of **opportunity-based externalities**, which allow preferences to depend on opportunity sets.

[2]Even in the one-good case, there is evidence that preferences depend not just on the payoff profile, but on other details of the environment including the strategies available to players and the intentions. Cox, Friedman, and Gjerstad [10], Dufwenberg and Kirchsteiger [16], Rabin [36], and Segal and Sobel [40] present models of this kind. In these models the preferences depend on the strategic situation, which is of course not modeled within the general-equilibrium framework. Hence, in this paper we abstract from these phenomena.

[3]Standard references incorporate consumption externalities into general-equilibrium theory (for example, Arrow and Hahn [2] extend standard existence results). But to our knowledge this paper is the first to study preferences that depend on opportunity sets in the context of general-equilibrium theory.

Section 5.1 studies efficient allocations when well–being externalities are present. Efficient allocations need not be equilibria in this case. Indeed, we construct an exchange economy in which efficiency is incompatible with full resource utilization (total consumption equal to total endowment) even when internal preferences are strictly increasing. To rule out this kind of example, we assume that if the resources in the economy increase, then it is possible to make everyone better off. Under this condition, all Pareto-efficient allocations are internally efficient, and hence the Second Welfare Theorem holds.[4] But even in this environment, the First Welfare Theorem does not hold. In fact, equilibria are typically not efficient.[5]

In Section 5.2, we discuss the efficiency of equilibria when agents care only about the consumption opportunities of others,[6] which technically encompasses the case of well-being externalities. We study a condition we call the **Redistributional Loser Property**. The condition requires that a non-trivial redistribution of income in the population must leave someone worse off. The condition therefore places a limit on the importance of distributional concerns. When this condition holds, agents do not wish to make unilateral transfers and, for a given equilibrium price vector, equilibrium allocations are efficient. We show that the condition holds for several natural generalizations of some of the most prominent one-dimensional ORP models found in the literature.

Our analysis assumes price-taking behavior in the context of a market. It makes sense to ask whether agents with ORP would be price takers or instead try to manipulate outcomes either by making exchanges outside of the market or distorting their demand. In Section 6 we examine this question in the way it is traditionally approached in classical economies, by showing the stability of market outcomes and the rationality of price-taking behavior in large economies. Section 6.1 shows that the standard answer also applies

[4] That the Second Welfare Theorem holds has been established by Winter [46] for the case of separable ORP that are increasing in the internal utility of all agents. It has been extended by Borglin [7] and Rader [37] to the class of separable other-regarding preferences that allow for both spitefulness and altruism.

[5] Geanakoplos and Polemarchakis [21] show in a related model that equilibria with consumption externalities are generically inefficient. Noguchi and Zame [34] also observe that equilibria need not be efficient in the presence of consumption externalities.

[6] Models of preferences that depend on opportunity sets are useful in the study of flexibility (Kreps [29] and Dekel, Lipman, Rustichini [13]), diversity (Nehring and Puppe [33]), and freedom (Puppe [35]). Segal and Sobel [40] provide an axiomatic treatment of a related model.

to our framework: Under appropriate continuity conditions, if the economy is large, then price taking is approximately optimal. Focusing on well-being externalities, Section 6.2 provides conditions under which the core (suitably defined) of the economy is contained in the core of the economy defined by internal preferences. When the core is non-empty, this result provides a generalization of the classical core-equivalence theorem.

# 2 Separability

This section introduces a basic model of competitive equilibrium that is classical except that consumers may have ORP. We then identify a separability condition necessary and sufficient to get a well-defined notion of preferences over own consumption.

Consider an economy with $L$ goods indexed by $l = 1, \ldots, L$. Prices are normalized such that the price vector is an element of the unit simplex $P$, i.e. $p_l \geq 0$ for all $l \in L$, and $\sum_{l \in L} p_l = 1$.

There are $J$ profit-maximizing firms. A typical firm $j \in \{1, \ldots, J\}$ is endowed with a production set $Y_j \subseteq \mathbb{R}^L$, with $y_j \in Y_j$ denoting the production plan implemented by firm $j$. As usual, negative components of $y_j$ are inputs, and positive ones are outputs. $Y_j$ is closed and bounded from above for all $j$.[7] The maximum attainable profit of firm $j$ confronted with a price vector $p$ is denoted by $\pi_j(p)$, and since $Y_j$ is closed and bounded from above, $\pi_j(p)$ exists for all $p$. Denote by $y = (y_1, \ldots, y_J)$ the implemented production profile and by $Y = \prod_{j=1}^{J} Y_j$ the set of all feasible production profiles.

There are $I$ agents and the consumption set of a typical agent is assumed to be the nonegative orthant $\mathbb{R}^L_+$. The initial endowment of Agent $i$ is denoted by $e_i$, and the bundle consumed by $i$, consumer $i$'s **own consumption**, is $x_i = (x_{i1}, \ldots, x_{iL}) \in \mathbb{R}^L_+$. $x = (x_1, \ldots, x_I) \in \mathbb{R}^{L \times I}_+$ is the whole consumption profile, i.e. the allocation of goods. Denote by $\bar{e}$ the aggregate initial endowment, $\sum_{i=1}^{I} e_i$. Firms are owned by the consumers and $\theta_{ij}$ denotes $i$'s share of firm $j$. The income of consumer $i$, $w_i$, is the sum of the value of $i$'s initial endowment and the dividends she earns, $w_i = pe_i + \sum_{j \in J} \theta_{ij} \pi_j(p)$.

Let $B = (B_1, \ldots, B_I)$ be a profile of budget sets, where each $B_i$ is a non-empty compact subset of $\mathbb{R}^L_+$, and denote by $\mathcal{B}$ the set of all profiles of

[7]We impose the boundedness assumption for simplicity. Our results go through for unbounded, convex production sets. To this end, one would use the typical compactification argument of general-equilibrium Theory.

budget sets. Endow $\mathcal{B}$ with the Hausdorff topology. Including budget sets in the domain of preferences permit us to describe situations where agents' care for what others could have consumed rather than what others actually consume. Section 4 contains a more detailed discussion of the importance of this type of ORP.

To model general ORPs, we assume that each Agent $i$ has a *preference relation* defined over allocations $x$ and over profiles of budget sets $B$, which we denote by $\succeq_i$. We assume that the agents' preference relations are complete and transitive. To ensure that each agent's preference relation $\succeq_i$ can be represented by a utility function $U_i(x, B)$ defined on the set $\mathbb{R}_+^{L\times I} \times \mathcal{B}$, we assume that $\succeq_i$ is continuous. We also assume that Agent $i$'s preferences are strictly convex over her own consumption—i.e. for all $B$, $x_{-i}$ and $x_i \neq x'_i$, $(x_i, x_{-i}, B) \succeq_i (x'_i, x_{-i}, B)$ implies that $(\alpha x_i + (1-\alpha)x'_i, x_{-i}, B) \succ_i (x'_i, x_{-i}, B)$ for all $\alpha \in (0, 1)$. We do not require strict convexity over allocations, which would be far more stringent. For example, strict convexity over allocations would rule out that an agent is only interested in the consumption bundle she receives, because an appropriate change in the consumption bundle of a fellow agent would have to make her better off. Indeed, if Agent $k$ is better offer with a convex combination of $x_k$ and $x'_k$, then a jealous Agent $i$ could prefer $(x_i, x_{-i}, B)$ and $(x_i, x'_{-i}, B)$ to $(x_i, \alpha x_{-i} + (1-\alpha)x'_{-i}, B)$.

Unless mentioned explicitly otherwise, we also assume strict monotonicity in own consumption, so that for all $B$, $x_{-i}$, $x_i \neq x'_i$ with $x_i$ weakly larger than $x'_i$ in all components we have $(x_i, x_{-i}, B) \succ_i (x'_i, x_{-i}, B)$. With ORP, strict monotonicity rules out, for example, that an agent wants to reduce her consumption because she feels bad whenever she is a lot better off then others.

Finally, an **economy** $\mathcal{E}$ is fully described by a tuple $(I, e, (U_i), Y, \theta)$ of agents, endowments, utility functions, production sets, and ownership shares.

This paper asks whether agents with ORP behave differently from classical agents in perfectly competitive markets. To do so, we study demand behavior. Since agents' preferences can be represented by a continuous utility function and the budget set is compact, the demand correspondence exists. Because we furthermore assume that an agent's preferences over her own consumption bundles are strictly convex, each agent $i$ has a demand function given by

$$d_i(x_{-i}, B) = \arg\max_{x_i \in B_i} U_i(x, B).$$

Most of our results hold when the domain of the preferences includes budget sets profiles that consist of any non-empty, compact subsets of $\mathbb{R}_{+}^{L}$. For some of our results, however, we study budget set profiles that are induced by a system of incomes and prices, i.e. profiles consisting of sets $B_i$ for which there exists a price $p \in P$ and an income $w_i > 0$ such that

$$B_i = \left\{ x_i \in \mathbb{R}_{+}^{L} : p x_i \leq w_i \right\}. \tag{1}$$

For such budget sets, we write the demand function as $d_i\left(p, w_i, x_{-i}, B_{-i}\right)$.

In general, the demand function depends on the consumption choice of other agents $x_{-i}$ and the profile of consumption possibility sets of the others, $B_{-i}$. On the other hand, the utility function of an agent $i$ with classical preferences is independent of $x_{-i}$ and on $B_{-i}$ (but will depend on $B_i$). The same holds of course for the demand function. This consideration leads to the following[8]

**Definition 1.** *Agent $i$ behaves as if classical if $d_i\left(x_{-i}, B\right)$ is independent of $x_{-i}$ and $B_{-i}$.*

To see under when agents behave as if classical, we take a closer look at preferences over own consumption. We say that an agent's preferences are separable if her relative evaluation of own consumption bundles is independent of the consumption of others and the profile of budget sets.

**Definition 2.** *Preferences $\succeq_i$ of Agent $i$ are separable if for all allocations $x = (x_1, \ldots, x_I)$ and $x' = (x'_1, \ldots, x'_I)$ and all profiles of budget sets $B$ and $B'$ we have*

$$(x_i, x_{-i}, B) \succeq_i (x'_i, x_{-i}, B)$$

*if and only if*

$$\left(x_i, x'_{-i}, B'\right) \succeq_i \left(x'_i, x'_{-i}, B'\right).$$

Separable preferences can be represented by a utility function of the form $V_i(m_i(x_i), x_{-i}, B)$. Due to monotonicity in own consumption, $V_i$ is strictly increasing in its first argument. Under our assumptions, $m_i : \mathbb{R}_{+}^{L} \to \mathbb{R}$ can

[8]Our version of behaving as if classical is the strongest possible formulation. If the demand function is independent of $x_{-i}$ and $B_{-i}$ for all possible allocations and budget set configurations, there is no way to distinguish ORP from classical preferences in markets. A weaker version might require $d_i(x_i, B)$ to be independent of $x_{-i}$ and $B_{-i}$ only for equilibrium allocations and budget sets.

be taken to be a continuous, strictly monotone, and strictly quasi-concave function. In this case, $m_i(x_i)$ describes Agent $i$'s preferences when the consumption choices and opportunities of the other agents are fixed. We refer to the function $m_i(x_i)$ as a consumer's **internal utility function**. Loosely speaking, this function is a measure of the consumer's well-being absent any social comparisons.

It is intuitive that if an agent has a utility function that is separable in own consumption, then she would choose the same consumption bundle independent of the consumption and characteristics of others. The following theorem also establishes the converse—that if an agent behaves as if classical, then her preferences can be represented by a separable utility function—under the assumptions of continuously differentiable demand.

**Theorem 1.** *1. If Agent i's preferences can be represented in the form*

$$V_i\left(m_i(x_i), x_{-i}, B\right)$$

*for a strictly quasiconcave, continuous function* $m_i : \mathbb{R}^L_+ \to \mathbb{R}$ *and a function* $V_i : D \subseteq \mathbb{R} \times \mathbb{R}^{(I-1)L} \times \mathcal{B} \to \mathbb{R}$ *that is increasing in its first variable, then Agent i behaves as if classical.*

*2. Consider budget sets profiles induced by a system of incomes and prices. Suppose that Agent i's preferences are smooth enough that the demand function* $d_i(p, w_i, x_{-i}, B_{-i})$ *is continuously differentiable*[9] *in* $(p, w_i)$*. If Agent i behaves as if classical, then her preferences can be represented in the form*

$$V_i\left(m_i(x_i), x_{-i}, B\right)$$

*for a strictly quasiconcave, continuous function* $m_i : \mathbb{R}^L \to \mathbb{R}$ *and a function* $V_i : D \subseteq \mathbb{R} \times \mathbb{R}^{(I-1)L} \times \mathcal{B} \to \mathbb{R}$ *that is increasing in its first variable.*

The proof of Theorem 1 and all subsequent results are in the Appendix.

The separability requirement is quite strong. The class of separable preferences is non-generic in the class of ORP.[10] Nevertheless, these preferences

[9] A sufficient condition for this is that preferences are $\mathcal{C}^2$ in own consumption without critical points, and that the bordered Hessian of $U$ is nonzero at all $x$. See Mas-Colell [31, Chapter 2], or Mas-Colell, Whinston, and Green [32, Chapter 3, Appendix].

[10] For the case of "well-being externalities" that we introduce below, a formal proof is available upon request.

include classical preferences as well as some of the most prominent ORP models as special cases (see Section 4 below).[11] Furthermore, they are the most general class of preferences that induces a consistent measure of individual utility independent of social comparisons.

# 3 Equilibrium Equivalence

In this section we analyze the impact of ORP in a general-equilibrium environment. In order to do so, we have to adjust the equilibrium definition. We also define a hypothetical economy in which all agents have corresponding internal preferences in order to investigate the implications of ORP for behavior and welfare.

A **Walrasian equilibrium** consists of a price vector $p^*$, a feasible allocation $x^*$, a production plan $y^*$, and a profile of budget sets $B^*$ such that every firm maximizes its profits for given price $p^*$, each consumer $i$ chooses her utility maximizing consumption bundle $x_i^*$ for given profile of budget sets $B^*$, and the profile of budget sets $B^*$ is compatible with $p^*$ and $y^*$. That is, for all $i = 1, \ldots, I$, $j = 1, \ldots, J$, we have

$$\begin{aligned} p^* y_j^* &\geq p^* y_j' \text{ for all } y_j' \in Y_j \\ x_i^* &= \arg\max_{x_i \in B_i^*} U_i(x, B^*) \\ B_i^* &= \{x_i : p^* x_i \leq p^* e_i + \sum_{j \in J} \theta_{ij} p^* y_j^*\} \end{aligned}$$

This equilibrium definition implies that consumers are price-takers and producers are profit-maximizers. The price-taking assumption is justified by the analysis in Section 6. When agents have other-regarding preferences, the assumption of profit-maximizing firms is not as straightforward to justify as it is within standard general-equilibrium theory. To illustrate this, consider a firm that is owned by many small shareholders, who together own more than half of the shares, and one big shareholder, who owns the rest. In this case the firm's profits might be important for the big shareholder's wealth, but negligible for the wealth of the other owners. If the small shareholders envy the big shareholder, a coalition of small shareholders might decide that

[11] Maccheroni, Marinacci, and Rustichini [30] and Vostroknutov [45] present models of ORP that do not satisfy our separability assumption.

the firm should not maximize its profits.[12] To exclude such a possibility and to justify profit maximization, we might restrict the analysis to situations where each firm is owned only by one consumer.

To understand the role of other-regarding preferences, we will compare an economy $\mathcal{E} = (I, e, (U_i), Y, \theta)$ to its corresponding **internal economy** $\mathcal{E}^{int} = (I, e, (m_i), Y, \theta)$. In an internal economy each firm has the same production set, and each consumer the same endowment, the same shares, and the same internal preferences as in the original economy $\mathcal{E}$. In the internal economy, however, agents care only about their own direct consumption.

Having defined equilibrium and the internal economy, an immediate consequence of Theorem 1 is that:

**Theorem 2.** *If all agents have separable preferences that are strictly monotone in own consumption, the set of Walrasian equilibria of an economy $\mathcal{E}$ coincides with the set of Walrasian equilibria of its corresponding internal economy $\mathcal{E}^{int}$.*

If all agents' preferences are separable in their own consumption bundles and all agents prefer to spend their entire wealth, concerns such as envy, altruism, or fairness do not influence market outcomes.

Theorem 2 requires not only separability but also that every agent's preferences are monotonic in own consumption. This assumption rules out the possibility that a well off agent may choose not to consume all of her income in order to reduce inequality in society. In this sense, the requirement of monotonicity limits the inequity concerns that agents are allowed to have. The following corollary shows that if monotonicity is violated, the set of Walrasian equilibria with other-regarding preferences might differ from the set of Walrasian equilibria of the internal economy.

**Corollary 1.** *Suppose that all agents have separable preferences. Consider an economy $\mathcal{E}$ and a Walrasian equilibrium $(p^*, x^*, y^*)$ of its corresponding internal economy $\mathcal{E}^{int}$. If for some agent $i$, $\frac{\partial V_i\left(m_i(x_i^*),(x_{-i}^*),B^*\right)}{\partial m_i} < 0$, then $(p^*, x^*, y^*)$ is not part of a Walrasian equilibrium of the economy $\mathcal{E}$.*

The Corollary highlights that Theorem 2 requires, in addition to separability, that $V_i(\cdot)$ be monotonic in its first argument.

[12] On the problem of the firm's objective within an oligopolistic framework, see Dierker and Grodal [15].

# 4 Other-Regarding Preferences in Multi-Good Contexts

The results in Sections 2 and 3 hold for preferences defined on general domains provided that internal utility can be separated from social concerns. For subsequent results, we limit attention to special classes of preferences that we describe in more detail in this section. We wish to emphasize how and why we include opportunities in utility functions, as we believe that this aspect of our model is central to the study of other-regarding preferences but, to our knowledge, has not been discussed in the literature. We begin by discussing special cases of standard models of consumption externalities and then introduce budget sets into preferences.

A traditional way to model other-regarding preferences is to assume that Agent $i$'s utility is a function of the internal utilities of other agents in the economy. Formally, a well-being externality arises if the utility of Agent $i$ depends on $x_i$ and the internal utility levels $m_k(x_k)$ of Agents $k \neq i$. Hence, Agent $i$'s preferences depend non-trivially on the $x_{-i}$, the consumption of other agents in the economy, but not on $B$, the set of opportunities. Well-being externalities are thus a subclass of consumption externalities. However, it is the (internal) well-being of your neighbor, independent of its source, that enters into utility.[13]

When there are well-being externalities, Agent $i$'s preferences can be represented by a function $V_i(m_1, \ldots, m_I)$, where $I$ is the number of agents in the economy. We make the standard assumptions that $m_k(\cdot)$ is strictly increasing for each $k$ and that $V_i(\cdot)$ is strictly increasing in $m_i$.

A leading example of well-being externalities is Edgeworth's [17, page 51] example in which

$$V_i\left(m_1, \ldots, m_I\right) = m_i + \frac{\beta_i}{I-1}\left(\sum_{k=1}^{I} m_k\right). \tag{2}$$

In Equation (2), Agent $i$ cares about his own internal utility and a weighted average of the utilities of the other agents. If $\beta_i > 0$, then Agent $i$ is altruistic or benevolent. If $\beta_i < 0$ (a case that Edgeworth does not consider), then she

[13] If status is measured according to relative consumption of a particular good, then our model of well-being externalities does not include status concerns that could be captured in a general model of consumption externalities.

is envious or spiteful.[14]

Well-being externalities provide a natural way to generalize existing one-dimensional models of other-regarding preferences tailored to allocations of money. Recent literature designed to organize experimental observations in games with monetary outcomes proposes alternative functional forms that can be interpreted as well-being externalities in multi-good settings. For example, the model of Fehr and Schmidt [20] generalizes to

$$V_i(m_1, \ldots, m_I) = \\ m_i - \frac{\alpha_i}{I-1} \sum_k \max\{(m_k - m_i), 0\} - \frac{\beta_i}{I-1} \sum_k \max\{(m_i - m_k), 0\}, \tag{3}$$

where Fehr and Schmidt assume that $\alpha_i \geq \beta_i \geq 0$ and $\beta_i < 1$. The first parameter assumption ensures that agents suffer more from being behind than they gain from being ahead. In the context of a single good, $\beta_i < 1$ ensures that the utility function is monotonically increasing in one's internal utility. Similarly, the simple version of the model of Bolton and Ockenfels [6] can be written

$$V_i(m_1, \ldots, m_I) = m_i - \beta_i \left| m_i - \frac{\sum_k m_k}{I} \right|, \tag{4}$$

where $0 \leq \beta_i < 1$. Finally, the preferences proposed in Charness and Rabin [9][15] are

$$V_i(m_1, \ldots, m_I) = m_i + \frac{\beta_i}{I-1} \left[ \delta_i \min\{m_1, \ldots, m_I\} + (1 - \delta_i) \sum_{k=1}^{I} m_k \right], \tag{5}$$

where $\beta_i, \delta_i \geq 0$ and $\beta_i \delta_i < \frac{1}{I-1}$. Intuitively, one may think of an agent as being maximizing the combination of his own well-being and a given social welfare function. The model of Charness and Rabin can be viewed as extending Edgeworth's example (2) by adding a Rawlsian type concern for the worst off agent to this social welfare function.

Well-being externalities easily capture the preferences of agents who care about the level of (internal) utility of other agents in the economy. They

[14] For other one-good models of envy, see Bolton [5] and Kirchsteiger [27]; additional examples of one-good models of altruism are provided by Andreoni and Miller [1], and by Cox and Sadiraj [11].

[15] Charness and Rabin [9] also include reciprocity concerns in their formulation.

provide a less compelling model of situations in which an agent's welfare depends on interpersonal comparisons. Consider an economy in which there are two agents, Adam and Eve. Imagine that Adam has other-regarding preferences so that he gains or loses utility depending on his position relative to Eve. Adam may, for example, be jealous of Eve whenever he deems her better off than himself but feel sorry for her when she is worse off. One can try to capture this situation as a well-being externality by positing a model in which Adam's total utility decreases when his internal utility is less than (some function of) Eve's internal utility and increases when his internal utility is greater than Eve's. We lack a theory that allows us to make interpersonal comparisons of internal utility, however. Consequently we have no general way in which to identify when Adam should begin to envy Eve's internal utility.

An alternative approach is to assume that Adam envies Eve if he prefers (according to his internal preferences) her consumption to his.[16] This formulation can be described in models in which preferences depend only on (economy-wide) consumption bundles and does not require interpersonal comparisons of utility. On the other hand, what if Eve, due to differences in endowments, could choose bundles that Adam would love to have, but in fact chooses a bundle that Adam is not interested in at all. For example, she may use her budget to buy apples, while Adam, who is allergic to apples, buys pears instead. If Adam *envies* Eve because if he had her budget he would have been able to buy more pears, then we must expand the domain of preferences to include these opportunities.[17]

More generally, individuals who desire equality of opportunity have preferences that depend on more than the final allocation of goods. A thought experiment contrasts well-being externalities from opportunity-based externalities. When there are well-being externalities, it is generally possible to change Agent $i$'s utility by changing Agent $k$'s internal preferences (holding allocations fixed). When there are opportunity-based externalities, Agent $i$'s utility need not depend on the internal utility of other agents.[18] On the

[16] For the case of purely selfish agents, this is defined as envy in Varian [43]. Varian [44] mentions the possibility of envying the possibilities of another agent. Varian, however, does not consider ORP but investigates properties of allocations in a classical environment that are envy-free.

[17] On the other hand, if Adam is jealous because Eve's opportunity set translates into a high level of internal utility for her, this can be captured by well-being externalities.

[18] The functional forms (7), (8), (9), and (10) introduced below are examples.

other hand, when there are opportunity-based externalities, Agent $i$ can be made better off if Agent $k$'s choice set is changed even if the change does not influence Agent $k$'s final allocation. Informally, an individual who prefers that all families can afford child care – whether they choose to use it or not – is consistent with opportunity-based externalities. A childless agent whose preferences exhibit well-being externalities benefits from providing more affordable child care facilities only if by doing so the number of children using the facilities increases.

For our welfare results, we separate social concerns derived from differences in opportunities from those that derive from concern about the well-being of other agents. In order to do this, we study utility functions that depend on own consumption and the economy-wide budget profile, but not directly on the consumption of other agents. To see the generality of this approach, suppose that Agent $i$ evaluates an opportunity set of Agent $k$ as being the value of the best element within this set. If, furthermore, Agent $i$ selects and evaluates this best element according to Agent $k$'s internal utility function, then well-being externalities can be viewed as a special case of the more general opportunity-based externalities.[19] Opportunity-based externalities are clearly far more general than this. For example, Agent $i$ could be envious because of what he would consume given $k$'s opportunity set – that is, Agent $i$ could evaluate the opportunity set of Agent $k$ according to his own and not $k$'s internal utility function.

To contrast the opportunity-based from well-being externalities, we point out alternative generalizations of the one-dimensional models of ORP described above. In order to describe these models, we must introduce some notation. Let $\tilde{m}_i(B_k)$ be the internal utility of Agent $i$ if she could select the item from Agent $k$'s budget set that she prefers most (according to the internal utility function $m_i$). That is, if $B_k$ represents Agent $k$'s budget set,

$$\tilde{m}_i(B_k) = \max m_i(x_k) \text{ subject to } x_k \in B_k. \tag{6}$$

To simplify notation, we write $\tilde{m}_{ki}$ in place of $\tilde{m}_i(B_k)$. If $B$ is the profile of

[19]There is a conceptual difference to the usual interpretation of the well-being externalities. If, for example, an agent would not choose the optimal allocation within his budget set, this would not change the social comparison from an opportunity-based perception while it would do so from a well-being interpretation. Such suboptimal choice could arise if social comparisons lead to non-monotonicity in internal utility.

opportunities in the economy, then

$$V_i\left(m_i(x_i), B\right) = m_i(x_i) + \frac{\beta_i}{I-1}\left(\sum_{k=1}^{I} \tilde{m}_{ki}\right) \tag{7}$$

provides an alternative to (2),

$$\begin{aligned} &V_i\left(m_i(x_i), B\right) = \\ &\quad m_i(x_i) - \frac{\alpha_i}{I-1}\sum_k \max\{(\tilde{m}_{ki} - \tilde{m}_{ii}), 0\} - \frac{\beta_i}{I-1}\sum_k \max\{(\tilde{m}_{ii} - \tilde{m}_{ki}), 0\} \end{aligned} \tag{8}$$

provides an alternative to (3),

$$V_i\left(m_i(x_i), B\right) = m_i(x_i) - \beta_i \left|\tilde{m}_{ii} - \frac{\sum_k \tilde{m}_{ki}}{I}\right|, \tag{9}$$

provides an alternative to (4), and

$$\begin{aligned} &V_i\left(m_i(x_i), B\right) = \\ &\quad m_i(x_i) + \frac{\beta_i}{I-1}\left[\delta_i \min\{\tilde{m}_{1i}, \ldots, \tilde{m}_{Ii}\} + (1-\delta_i)\sum_{k=1}^{I} \tilde{m}_{ki}\right] \end{aligned} \tag{10}$$

provides an alternative to (5). If everyone in the economy shares the same internal utility function ($m_i$ independent of $i$), then for fixed opportunity sets the alternative approaches are the same. In general, the approaches take a different perspective on the appropriate way to make interpersonal comparisons. In one (exemplified by the functional forms (2), (3), (4), and (5)), agents make social comparisons by explicitly comparing interpersonal (internal) utilities. In the other (exemplified by the functional forms (7), (8), (9), and (10)), an agent compares herself to another by evaluating her utility to the utility she would receive in the other person's position. Both points of view are consistent with the general idea of opportunity-based externality.

# 5 Welfare and Stability of Equilibria

We next examine the extent to which the fundamental welfare theorems hold in our setting. Obviously, the first and the second welfare theorem hold with respect to the internal utility functions. In order to refine our understanding of the welfare properties of equilibria with respect to the full ORP, we separate well-being externalities from opportunity-based externalities. Furthermore, we restrict the way in which these externalities enter preferences.

## 5.1 Well-being Externalities and Welfare

This section concentrates on efficiency in the presence of well-being externalities. As already discussed in Section 4 preferences of Agent $i$ are represented by the function $V_i(m_1(x_1), \ldots, m_I(x_I))$, with $m_{k(\cdot)}$ strictly increasing for each $k$ and $V_i(\cdot)$ strictly increasing in its $i$th argument. Since preferences depend only on the allocation, the usual efficiency definition can be used: An allocation $x$ is called **feasible** if there is a production plan $y$ with $y_j \in Y_j$ for all $j$ and $\sum_{i=1}^{I} x_{il} \leq \bar{e}_l + \sum_{j=1}^{J} y_l$ for all commodities $l = 1, \ldots, L$, and a feasible allocation $x$ is **efficient** if there is no other feasible allocation $x'$ that makes every consumer better off in terms of utility, that is, with $U_i(x', B) > U_i(x, B)$ for all $i = 1, \ldots, I$.

Examples (Edgeworth [17, page 51] and Hochman and Rodgers [25]) demonstrate that a Walrasian equilibrium need not be Pareto efficient even when agents have benevolent preferences.[20] Furthermore, Pareto-efficient allocations need not be Walrasian equilibria, as the following example shows.

**Example 1.** ***Hateful Society.*** *Consider an exchange economy with two identical agents each with utility functions $V_i = m_i - 2m_k$, where $m_i(x_i) = (x_{i_1} x_{i_2})^{\frac{1}{2}}$ and $i \neq k$. Let the aggregate endowment be $\overline{e} = (1, 1)$. The allocation $((0,0),(0,0))$, which is obviously not internally efficient as none of the endowment is consumed, is Pareto efficient.*[21] *In this hateful society, it is impossible to make Agent 1 better off without making Agent 2 worse off. Hence, the set of Pareto-efficient allocations is not a subset of the internally efficient allocations and Walrasian equilibria need not be Pareto efficient.*

The preferences in the example exhibit a high degree of spitefulness. We next introduce a condition that rules out such pathological cases. The condition is satisfied by all specific models of ORP discussed above.

[20] We can take preferences to be of the form (2) with $\beta_i \in (0, 1)$ for all $i$.

[21] If the endowment cannot be destroyed, then the edge of the Edgeworth box is the set of Pareto efficient allocations, while the diagonal is the set of internally efficient allocations.

**Social Monotonicity (SM)** For any allocation $x$ and $z \in \mathbb{R}^L_+$, $z \neq 0$ there is a $(z_1, \ldots, z_I)$ such that $z_i \geq 0$ for all $i$, $\sum_{i=1}^{I} z_i = z$, and for all $i$,

$$V_i \left( m_1(x_1 + z_1), \ldots, m_I(x_I + z_I) \right) > V_i \left( m_1(x_1), \ldots, m_I(x_I) \right).$$

The condition states that any increase in the resources available to the economy can be redistributed to make everyone better off. It is clear that SM fails in the above example with hateful agents. Under SM, Pareto-efficient allocations must be internally efficient. SM ensures that if an outcome is not internally efficient, then in the set of allocations in which all agents are internally better off, there exists an element in which the internal gains are divided between all agents in such a way that everyone is better off.[22]

**Theorem 3.** *If SM holds, then the set of Pareto-efficient allocations is a subset of the set of internally-efficient allocations.*

An immediate consequence is the Second Welfare Theorem.

**Corollary 2** (Second Welfare Theorem)**.** *If SM holds, then every Pareto-efficient allocation can be achieved as a Walrasian equilibrium by using suitable lump–sum transfers.*

The following example demonstrates how inequitable outcomes may be inefficient in the presence of ORP.

**Example 2.** ***Inequity Aversion.*** *Fehr and Schmidt [20] introduce ORP of the form given in Equation (3). It is straightforward to check that SM is satisfied. These preferences permit both altruistic and spiteful behavior. In a two-agent exchange economy in which Agent* 1 *has a utility function of the form (3) and Agent* 2 *is classical (*$\alpha_2 = \beta_2 = 0$*), internally efficient allocations in which* $m_1(x_1) \leq m_2(x_2)$ *must be efficient while it is possible for an internally efficient allocation in which* $m_1(x_1) > m_2(x_2)$ *to be inefficient.*

In this example, an inefficiency could arise if the inequity–averse agent's internal payoff is greater than that of an agent who cares only about her own

[22] Rader [37] demonstrates that the conclusion of Theorem 3 holds under a condition on utility-possibility sets implied by SM.

consumption. The reason is that both agents could benefit from transfers from the better off to the worse off agent. Such opportunities for a Pareto improvement do not exist whenever the classical agent consumes more than the inequity–averse one since the classical agent loses utility for sure when she gives to the inequity–averse agent.

In the Hateful Society Example, no agent would want to make transfers but equilibrium is inefficient because in equilibrium agents will consume everything even though efficiency requires collective destruction of resources. But when SM holds, the Inequity Aversion example suggests that inefficiency of competitive equilibria can be traced to the existence of Pareto-improving transfers—raising the question whether voluntary transfers can reestablish efficiency under SM.

SM, however, does not guarantee that an equilibrium with bilateral transfers is efficient. Indeed, the next example highlights that there exist economies in which all equilibria with bilateral transfers are inefficient.[23] In such economies, in order to guarantee efficiency redistribution needs binding coordination among those willing to give.

**Example 3.** ***Inefficiency With Bilateral Transfers.*** *Consider an exchange economy with three agents and one good. Let the initial endowment be* $e = (1, 0, 1)$. *Let the utility of Agent 2 be given by* $x_2$, *i.e. assume that Agent 2 is selfish. Let the utility of Agent 1 be* $x_3 + (2/3)x_2$ *and the utility of Agent 3 be* $x_1 + (2/3)x_2$. *Then, independent of what Agent 3 gives, Agent 1 will never transfer any of the good to Agent 2. Similarly, Agent 3 will not want to transfer any of the good. This allocation, however, is Pareto dominated by the allocation* $(0, 2, 0)$.

## 5.2 Opportunity-Based Externalities and Redistribution

We now consider economies in which agents exhibit opportunity-based externalities. We assume in this section that the preferences of Agent $i$ depend non-trivially on $i$'s direct consumption, $x_i$, and on the budget sets of all agents, but they do not depend directly on the actual consumption of others,

[23] Winter [46] provides an example in which there exists both an efficient and an inefficient equilibrium with bilateral transfers. Goldman [22] provides an example that demonstrates that competitive equilibria need not be efficient when benevolent agents can exchange gifts.

$x_{-i}$. In other words, the utility function can be written as $V_i\left(m_i(x_i), B\right)$ for a strictly quasiconcave, continuous function $m_i : \mathbb{R}^L_+ \to \mathbb{R}$ and a function $V_i : \mathbb{R} \times \mathcal{B} \to \mathbb{R}$ that is increasing in its first variable.

In order to identify some welfare properties of equilibrium, we restrict the analysis to budget set profiles that are induced by a system of incomes and prices and analyze a constrained notion of efficiency.

Due to the normalization of the price vector, each budget set $B_i$ is consistent only with one particular price vector $p$ and only with one particular income level $w$. Denote the price vector and the income level consistent with budget set $B_i$ by $p(B_i)$ and $w(B_i)$, respectively. $w(B) = (w(B_1), \ldots, w(B_I))$ denotes the profile of incomes connected with budget set profile $B$. For the rest of this section we only consider budget set profiles that are consistent with one price vector, i.e. budget set profiles $B$ for which $p(B_i) = p$ for all $i = 1, \ldots, I$.

Since budget set profiles enter the domain of the preferences, we have to define the feasibility of budget sets profiles.

**Definition 3.** *Let $\mathcal{E}$ be an economy with opportunity-based externalities. The triple $(x, y, B) \in X \times Y \times \mathcal{B}$ is feasible for a price $p$, if and only if for all $i \in I$, $j \in J$, and $l \in L$ it holds:*

$$
\begin{aligned}
\textit{i)}\;\; y_j &\in Y_j \\
\textit{ii)}\;\; \sum_{i\in I} x_{il} &\leq \sum_{i\in I} e_{il} + \sum_{j\in J} y_{jl} \\
\textit{iii)}\;\; x_i &\in B_i \\
\textit{iv)}\;\; \sum_{I} w(B_i) &= \sum_{I} pe_i + \sum_{J} py_j
\end{aligned}
$$

In addition to the usual feasibility requirements on the production profile $y$ and the consumption profile $x$, this feasibility notion also requires some consistency between $x, y$, and $B$. In particular, it requires that each individual consumption bundle is in the budget set of the respective consumer, and that the profile of budget sets is feasible for the amount of income available in the economy.

A triple consisting of a production profile, a consumption profile, and a budget set profile is efficient if there is no other triple of a production profile, a consumption profile, and budget set profile that is also feasible for the same price and that makes all consumers weakly and some consumers strictly better off.

**Definition 4.** *In an economy $\mathcal{E}$ with opportunity-based externalities a triple $(x, y, B)$ is efficient with respect to a price vector $p$ if and only if*

1. *$(x, y, B)$ is feasible for $p$.*
2. *there does not exist another triple $(x', y', B')$ which is feasible for $p$, and for which*

$$\begin{aligned} V_i(m_i(x_i'), B') &\geq V_i(m_i(x_i), B) \text{ for all } i, \text{ and} \\ V_i(m_i(x_i'), B') &> V_i(m_i(x_i), B) \text{ for at least one } i. \end{aligned}$$

Since prices determine budget sets and budget sets enter preferences, prices enter the efficiency definition. So this efficiency notion considers the impact of redistributions, but not the impact of redistributions caused only by changes in prices. Clearly, this restriction on the set of feasible changes restricts the welfare implications of any efficiency result. In fact it may be possible to make everyone better off just by changing prices. Take an exchange economy with two equilibria (with different supporting prices) associated with the same initial endowment. These equilibria will not be Pareto ranked for the internal preferences, but could be Pareto ranked when agents have ORP. For example, take a selfish Agent 1 and an altruistic Agent 2. If moving from equilibrium 1 to equilibrium 2 makes Agent 1 better off and decreases Agent 2's internal utility, it could still be that Agent 2's overall utility increases because the positive other-regarding effect dominates the negative effect on 2's internal utility. In this example, the two different equilibrium prices create different budget sets for agents even when initial endowments are fixed. Our definition of efficiency relative to a price does not permit this kind of comparison. Instead it considers efficiency relative to changes in the budget set profiles that leave the implied price vector unchanged.

The following property will turn out to be crucial for the efficiency of the equilibrium allocation with respect to the equilibrium prices.

**Redistributional Loser Property (RLP)** RLP holds at a budget set profile $B$ if for any other profile of budget sets $B' \neq B$ for which there exists a $p$ such that $p(B_i) = p(B'_i) = p$ for all $i$ and $\sum_{i \in I} w(B_i) \geq \sum_{i \in I} w(B'_i)$

$$\begin{aligned} V_k(m_k\left(d_k(B_k)\right), B) \leq V_k(m_k\left(d_k(B'_k)\right), B') \text{ for all } k \implies \\ V_k(m_k\left(d_k(B_k)\right), B) = V_k(m_k\left(d_k(B'_k)\right), B') \text{ for all } k. \end{aligned} \tag{11}$$

RLP holds if implication (11) holds at all $B$.

Notice that (11) holds if there always exists an agent $r$ for whom

$$V_r(m_r\left(d_r(B_r)\right), B) > V_r(m_r\left(d_r(B'_r)\right), B'). \tag{12}$$

If inequality (12) holds, then Agent $r$ loses when budget sets change from $B$ to $B'$.

The Redistributional Loser Property guarantees efficiency of the equilibrium outcome.

**Theorem 4.** *The equilibrium outcome $(x^*, y^*, B^*)$ of an economy with opportunity–based externalities is efficient with respect to the equilibrium price vector $p^*$ if preferences satisfy RLP at $B^*$.*

RLP is thus sufficient for efficiency of competitive markets. Conversely, if the equilibrium outcome $(x, y, B)$ of an economy with opportunity-based externalities is efficient with respect to the equilibrium price vector $p$, then, by the definition of efficiency, (11) holds for all triples $(x', y', B')$ that are feasible with respect to $p$.

While Theorem 4 implies that the internal economy shares some efficiency properties with a family of economies with ORP, one cannot look only at internal preferences to analyze the distributional impact of changes in opportunities even if RLP holds. For example, a change in the wealth profile from $B$ to $B'$ need not have the same impact on consumer $i$ as it would have on her counterpart $i^{int}$ in the corresponding internal economy: A change beneficial for $i^{int}$ (i.e. a change with $B_i \subseteq B'_i$) may hurt $i$, and a change beneficial for $i$ might may hurt $i^{int}$. The theorem only states that all equilibrium outcomes are efficient in the economy with distributional concerns as they are for the corresponding internal economy, provided that RLP holds and that the prices inducing $B$ and $B'$ are the same.

Since RLP implies efficiency, one wonders when RLP does hold. Obviously, RLP holds when an consumer is classical (i.e. selfish) with preferences strictly monotone in his own consumption goods. RLP also holds for prominent specifications of preferences. To illustrate this point, we will analyze opportunity-based externalities that can be represented by the utility functions given by (8), (9), and (10).

Fix $p \in S$. Given $w_i > 0$, let $v_i(w_i) = m_i(d_i(p, w_i))$ be the indirect utility determined by internal preferences. For the remainder of the section, we assume that $v_i(\cdot)$ is differentiable and concave,[24] and we denote the derivative of $v_i(\cdot)$ with respect to wealth by $v_i'(\cdot)$.

Given a profile of budget sets $B$ and associated wealth profile $w_i(B)$ for each $i$. Let $\bar{v}_i(w_i(B)) = \max_k v_i'(w_k(B))$, $\underline{v}_i(w_i(B)) = \min_k v_i'(w_k(B))$, and $b_i = \underline{v}_i/\bar{v}_i$. The next result shows that RLP holds when the utility function takes on one of several functional forms that generalize utility functions used in one-good models of ORP.

**Theorem 5.** *Let $B$ be a profile of budget sets and $w(B)$ the associated wealth profile. RLP holds at $B$ whenever the internal utility function of Agent $i$ is concave and takes one of the following forms:*

*1.*

$$\begin{aligned} &V_i(m_i(x_i), B) \\ &= m_i(x_i) - \frac{\alpha_i}{I-1}\sum_k \max\{\tilde{m}_{ki} - \tilde{m}_{ii}, 0\} - \frac{\beta_i}{I-1}\sum_k \max\{\tilde{m}_{ii} - \tilde{m}_{ki}, 0\} \end{aligned} \tag{13}$$

*with $\alpha_i \geq \beta_i \geq 0$, $\beta_i < 1/\left(1 + \bar{v}_i - \underline{v}_i\right)$ and $I$ large enough.*

*2.*

$$V_i(m_i(x_i), B) = m_i(x_i) - \beta_i \left| \tilde{m}_{ii} - \frac{\sum_k \tilde{m}_{ki}}{I} \right| \tag{14}$$

*with $0 \leq \beta_i < b_i$;*

[24] Since $v_i(\cdot)$ is increasing, it will be differentiable almost everywhere. We assume differentiability for convenience. A sufficient condition for concavity of $v_i(\cdot)$ is concavity of $m_i(\cdot)$.

*3.*

$$V_i\left(m_i(x_i), B\right) \tag{15}$$
$$= \quad m_i(x_i) + \frac{\beta_i}{I-1}\left[\delta_i \min\{\tilde{m}_{1i}, \ldots, \tilde{m}_{Ii}\} + (1-\delta_i)\sum_{k=1}^{I} \tilde{m}_{ki}\right]$$

*with*

$$\frac{b_i}{(1-\delta_i)(b-1)+\delta_i} > \beta_i > -b_i \text{ and } I \text{ large enough};$$

The proof of Theorem 5 demonstrates that for each of the functional forms inequality (12) holds. Furthermore, the Agent $r$ in (12) can always be taken to be one of the agents who loses most from the redistribution of income induced by the changes in opportunity sets (that is, $w(B_r) - w(B_r') = \max_i\{w(B_i) - w(B_i')\}$).

When $\delta_i = 0$, the utility function (15) in Theorem 5 concides with the preferences represented by (7). We conclude that Theorem 5 demonstrates that RLP is satisfied in a wide range of examples, including standard preferences exhibiting the possibility of both altruism and spite.

The extent to which RLP applies depends on three things: the degree of concavity of the indirect utility function defined over own consumption, the inequality of the initial wealth distribution, and the size of the economy.

The parameter restrictions in Theorem 5 depend in an intuitive way on the concavity of the indirect utility function. Consider the special case in which $v_r(\cdot)$ is linear. RLP is most likely to hold in this case since the internal cost of a transfer does not depend on the level of wealth. Indeed, $\bar{v}_r = \underline{v}_r$ and so $b_r = 1$ and the ranges for $\beta_r$ in the theorem agree with those found in the one-dimensional version of the models (designed for risk-neutral agents). On the other hand, the ranges for $\beta_r$ collapse to $\beta_r = 0$ when $\bar{v}_r$ increases to infinity or $\underline{v}_r$ converges to zero. Example 4, below, confirms that the conclusions of Theorem 5 need not hold when the marginal internal utility of income approaches infinity.

$\bar{v}_r - \underline{v}_r$ decreases when the difference between the largest and smallest income in the economy decreases. This observation gives some insight into when market outcomes are likely to be efficient. Consider an economy in which agents have preferences given by one of the functional forms in Theorem 5. If the initial endowments are nearly equal, the range of $\beta_i$ for which RLP holds will be large and therefore it is more likely that the equilibrium outcome will be efficient. When endowments are unequal, however, it will generally be more difficult to satisfy RLP and inefficiency may result.

A common feature of the preferences in Theorem 5 is that the impact of the opportunities of a particular other agent shrinks when the economy grows. This assumption seems appropriate when agents have preferences that take into account the opportunity sets of all other agents symmetrically. As a consequence of the assumption, the parameter restrictions sufficient for RLP are weaker in (13) and (15) as the economy grows larger. For these functional forms, the power of an individual to make a meaningful change to the distribution of income of the economy decreases as the economy grows large.

RLP limits the extent to which an agent can take the opportunities of others into account. It strikes us as incompatible with much real-world charity. A rich agent with low marginal utility of income sacrifices little internal utility when she makes a transfer to a poor agent. RLP assumes that this transfer is unattractive. The following example makes this point concretely. It further demonstrates that when the marginal internal utility of income is unbounded, there may be scope for efficiency enhancing redistribution even if the economy is large.

**Example 4.** ***Inefficiency and no RLP.*** *We consider an exchange economy with two groups of size $n \geq 2$, rich and poor agents. Poor agents are selfish, while rich agents are altruistic. All agents have the same internal utility function $m$. In order to have concrete formulas, we take a Cobb–Douglas utility of the form*

$$m(x_i) = \left( \prod_{l=1}^{L} x_i^l \right)^{\alpha}$$

*for some small $1 > \alpha L > 0$.*

*The preferences of any rich agent $i = 1, \ldots, n$ are*

$$V_i(m_i(x_i), B) = m_i(x_i) + \frac{\beta}{I-1} \sum_{k \neq i} \tilde{m}_{ik}$$

*for some $\beta > 0$ and $\tilde{m}_{ik}$ described in equation (6). Let aggregate endowment be $\bar{e} = n(1+\eta)\mathbf{1}$ and $\eta > 0$ be sufficiently small (specified exactly below). Here, $\mathbf{1}$ is the vector $(1, \ldots, 1)$ of ones in $\mathbb{R}^L$.*

*The initial endowment is given by $e_i = \mathbf{1}$ for the rich agents $i = 1, \ldots, n$ and $e_i = \eta\mathbf{1}$ for the poor agents $i = n+1, \ldots, 2n$. The allocation $e$ is internally* *efficient, and hence, the unique Walrasian equilibrium of an economy where $e$ is the initial endowment. The corresponding equilibrium price $p^*$ is $\frac{1}{L}\mathbf{I}$ , and the corresponding income levels are 1 for $i = 1, \ldots, n$, and $\eta$ for $i = n+1, \ldots, 2n$. The corresponnding profile of budget sets is denoted by $B^*$.*

*We now construct another tuple $(x', B')$ that is feasible for price $p^*$ and dominates $(e, B^*)$. The idea is that every rich agent gives $\epsilon > 0$ of his income to some poor agent, so that income levels are given by $1-\epsilon$ for $i = 1, \ldots, n$, and by $\eta + \epsilon$ for $i = n+1, \ldots, 2n$. Denote the corresponding profile of budget sets by $B'$. The redistribution of incomes lead to optimal consumption bundles of $x'_i = (1-\epsilon)\mathbf{1}$ for $i = 1, \ldots, n$ and $x'_i = (\eta+\epsilon)\mathbf{1}$. The following argument shows that $(x', B')$ dominates $(e, B^*)$ for all $n$. Hence, for all $n$, the Walrasian equilibrium is inefficient.*

*We do the calculation for the rich agent. Let $i \in \{1, \ldots, n\}$. Note that*

$$\frac{n}{2n-1} \geq 1/2 \geq \frac{n-1}{2n-1} \tag{16}$$

$$\begin{aligned}
&V_i(x'_i, B') - V_i(e_i, B^*) \\
&= (1-\epsilon)^{\alpha L} + \frac{\beta}{2n-1}\left((n-1)(1-\epsilon)^{\alpha L} + n(\eta+\epsilon)^{\alpha L}\right) - 1 - \frac{\beta}{2n-1}\left((n-1) + n\eta^{\alpha L}\right) \\
&= (1 + \frac{\beta(n-1)}{2n-1})\left((1-\epsilon)^{\alpha L} - 1\right) + \beta\frac{n}{2n-1}\left((\eta+\epsilon)^{\alpha L} - \eta^{\alpha L}\right) \\
&\geq (1+\beta/2)\left((1-\epsilon)^{\alpha L} - 1\right) + \beta/2\left((\eta+\epsilon)^{\alpha L} - \eta^{\alpha L}\right),
\end{aligned}$$

*where the inequality follows from (16) since $(1-\epsilon)^{\alpha L} - 1 < 0$ and $(\eta+\epsilon)^{\alpha L} - \eta^{\alpha L} > 0$. Hence, it suffices to show that the last expression is strictly positive for $\epsilon$ sufficiently close to 0. As the expression is zero for $\epsilon = 0$, it suffices to show that the right derivative with respect to $\epsilon$ is positive at 0. Taking the derivative and setting $\epsilon = 0$, one has*

$$-(1+\beta/2)\alpha L + \beta/2\alpha L\eta^{\alpha L-1}.$$

*For $\alpha L < 1$, and $\eta < (\beta/(2+\beta))^{1/(1-\alpha L)}$, the above expression is positive and, thus, the altruistic agents are better off after the redistribution. As the poor agents are selfish, they benefit from the redistribution. Thus, we have robust inefficiency even if $n \to \infty$.*

# 6 Large Economies

Throughout the paper, we have assumed that agents take prices as given. But why would agents act as price takers? This question is valid in the standard models, although it perhaps has greater force in our setting with ORP because market outcomes need not be efficient. We consider two possible approaches to this problem. First, we use arguments of Roberts and Postlewaite [38] to show that, as in classical economies, price-taking is approximately optimal in large economies. We limit attention to exchange economies. Second, we examine the classical core-equivalence theorem, which asserts that the set of core allocations shrinks to the set of competitive equilibria as the number of agents grows. Specifically, we consider the Debreu-Scarf thought experiment in which agents' internal preferences are replicated. We show that a generalization of the social monotonicity condition introduced in Section 5.1 implies that the core is contained in the internal core. We focus on well-being externalities because it is unclear how to extend the opportunity-based preferences we have introduced above to cases in which a coalition of agents jointly determine the use of a given set of resources.

## 6.1 Price-Taking Behavior

Roberts and Postlewaite [38] show that the incentives of classical agents to behave strategically converge to zero in well behaved pure-exchange economies when the number of agents increases. Their results extend with essentially no modification to our framework. This section describes the result formally.

To carry out the analysis, we modify our framework to talk about large economies—in which agents have no ability to influence prices—and economies in which the number of agents varies. Also, we allow agents to respond to prices strategically, rather than simply taking prices as given.

When we assume that agents are price takers, Agent $i$ is characterized by an initial endowment $e_i$ and a preference relationship. We maintain the assumptions that preferences are separable and depend on the economy-wide allocation and the distribution of opportunities. Let $\mathbb{B}$ be the set of individual budget sets (that is, the set of compact convex subsets of $\mathbb{R}^L_+$). To permit the possibility for an individual's preferences to be defined for a sequence of economies, assume that there is a measure space $\mathcal{N}$ of all compactly supported probability measures on $\mathbb{R}^L_+ \times \mathbb{B}$. An element of $\mathcal{N}$ describes the consumption vectors and opportunities of the agents in an economy. If $\nu \in \mathcal{N}$

is a counting measure on a finite set, then $\nu$ describes a simple (finite) economy. A measure $\nu^*$ describes the allocations and opportunities of a limiting economy if there is a sequence of simple measures $\nu_n$ that converge to $\nu^*$ in the topology of weak convergence of measures. We assume that preferences of Agent $i$ are continuous in $x_i$ and $\nu$. Continuity of preferences with respect to $x_i$ is a standard assumption. Continuity with respect to $\nu$ holds if, for example,

$$U_i(x_i, \nu) = \lambda m_i(x_i) + \int f(m_i(x_i), m_a(x_a))d\nu_1(x_a), \tag{17}$$

where $\nu_1$ is the marginal distribution of $\nu$ on $\mathbb{R}^L_+$, $\lambda > 0$, and $m_a(\cdot)$ and $f(\cdot)$ are continuous. The functional form in (17) includes inequity aversion and utilitarian preferences as special cases.

To incorporate the idea that agents may respond to prices strategically, let $\mathcal{S}_i$ denote the set of all correspondences $S$ from $P$ to $\mathbb{R}^L$ such that $S(p) + \{e_i\} \in \mathbb{R}^L_+$ and $pz = 0$ for all $p \in S(p)$. That is, $\mathcal{S}_i$ consists of all net-trade correspondences that satisfy the feasibility constraints of Agent $i$. This set contains the price-taking (or competitive) correspondence $\tilde{S}_i$ that consist of the net trades that maximize Agent $i$'s true preferences. The net trades of a strategic agent need only be elements of a "distorted" individual excess demand correspondence in $\mathcal{S}_i$.

An economy $\mathcal{E}$ is a mapping that assigns to each agent $i$ a correspondence $S_i \in \mathcal{S}_i$, an endowment, $e_i$, and a utility function, $U_i(\cdot)$. A price $p$ is market clearing for an economy if there exists $z = (z_1, \ldots, z_I)$ with $z_i \in S_i$ for all $i$ such that $\sum_{i\in I} z_i = 0$ and $x^*_i = z_i(x^*) + e_i$. Let $Q(\mathcal{E})$ be the set of market-clearing prices for $\mathcal{E}$. $Q(\mathcal{E})$ is completely determined by the excess demand correspondences in $\mathcal{E}$.

Given the response correspondences of the other agents, Agent $k$ can manipulate the economy by proposing an alternative response correspondence. A price $\hat{p}$ is **attainable** for $i$ if there is $\hat{S} \in \mathcal{S}_i$ such that $0 \in \sum_{k,k\neq i} S_k(\hat{p}) + \hat{S}_i(\hat{p})$. Let the set of attainable prices be $H_i(\mathcal{E})$. A net trade $z_i$ (consumption $x_i$) is attainable if there exists $p \in H_i(\mathcal{E})$ such that $0 \in \left(\sum_{k,k\neq i} S_k(\hat{p}) + z_i\right)$ $(x_i = e_i + z_i)$. A consumption $x_i$ is **competitive** for agent $i$ if is an attainable consumption given that $i$ uses the competitive net trade correspondence $\tilde{S}_i$.

The competitive response is **individually incentive compatible** for $i$ if, for any consumption vector $x$ attainable by $i$, there exists a competitive consumption $x'$ for $i \in \mathcal{E}$ such that $x'$ is preferred to $x$ for $i$. If $\{\mathcal{E}_n\}$ is a sequence of finite economies, then the competitive mechanism is **limiting individually incentive compatible** if for any $\epsilon > 0$ there exists $\overline{n}$ such that $n > \overline{n}$ implies that for each $x$ attainable by $i$ in $\mathcal{E}_n$ there exists a competitive allocation $x'$ in $\mathcal{E}_n$ such that $U_i(x'; \nu_n) > U_i(x; \nu_n) - \epsilon$.

To formulate the notion of a limit economy, let $\mathcal{S}$ be the set of all correspondences from $P$ to $\mathbb{R}^L$ such that $S \in \mathcal{S}_i$ for some $i$. Give $\mathcal{S}$ a metric topology.[25] Let $\mathcal{M}$ be the measure space of all compactly supported probability measures on $\mathcal{S}$. An element of $\mathcal{M}$ is an abstract economy. If $\mu \in \mathcal{M}$ is a counting measure on a finite set, then $\mu$ describes a simple economy. A measure $\mu^*$ describes a limiting economy if there is a sequence of simple measures $\mu_n$ that converge to $\mu^*$ in the topology of weak convergence of measures. The set of market-clearing prices for an economy $Q(\cdot)$ can be thought of as a mapping from $\mathcal{M}$ into $P$.

In an infinite economy with a non-atomic distribution of agents, no single agent can influence market-clearing prices. Hence, when the economy is truly large, no agent can do no better than use his competitive excess demand correspondence. The competitive response is therefore individually incentive compatible in limit economies. Roberts and Postlewaite formulated conditions under which this result is approximately true in large finite economies. Under the proper continuity assumptions, their result – both statement and proof – continue to hold.

**Theorem 6.** *Let $\{\mathcal{E}_n\}$ be a sequence of finite economies such that $\#\mathcal{E}_n \to \infty$. Suppose that the sequence of simple measures describing $\mathcal{E}_n$ converges to $\mu^*$ and that $Q(\cdot)$ and equilibrium demands are continuous at $\mu^*$. Suppose $\bar{i}$ belongs to $\mathcal{E}_n$ for all $n$ and that an indirect utility function for $\bar{i}$ exists and is continuous in a neighborhood of $Q(\mu^*)$. Then the competitive response is limiting individually incentive compatible for $\bar{i}$ in $\{\mathcal{E}_n\}$.*

Roberts and Postlewaite provide an example to show that the competitive response need not be limiting individually incentive compatible if $Q(\cdot)$ fails to be continuous at the limit point. They also point out that results of Dierker [14] or Hildenbrand [24, pages 168-175] imply if each agent holds a positive amount of each commodity and the set $\mathcal{S}$ are continuously differentiable functions $f$ on the interior of $P$ such that if $p_n \to p$ for $p$ on the

[25] The actual topology does not matter for the statement of Theorem 6. See Hildenbrand [24, pages 169, 170] or the discussion following Theorem 6 for a standard example.

boundary or $P$, then $||f(p_n)|| \to \infty$, then the set of **regular** economies on which the equilibrium correspondence $Q(\cdot)$ is continuous is open and dense in the topology of weak convergence (we metrize this space of demand functions by requiring uniform convergence of the functions and their first derivaives on compact sets).[26] In regular economies, equilibrium allocations also vary continuously in prices. For these results, an economy is characterized as a family of demand functions, without reference to underlying preferences. As such, the results apply without change to our setting.

Since preferences in our model have a larger domain than the preferences in Roberts and Postlewaite [38], the continuity of the indirect utility function requires additional assumptions. The indirect utility function depends on $p$ and $\nu$. By separability, Agent $i$'s optimal choice of $x_i$ exists provided that $i$'s internal utility function is continuous and the budget set is compact. Hence the indirect utility function is defined under these conditions. Since the indirect utility function does not depend on $\nu$, its continuity follows from continuity of $U_i(\cdot)$ with respect to $\nu$ and standard results on the continuity of indirect utility functions (for example, Mas-Colell, Whinston, and Green [32, Theorem 3.D.3., page 56]).

The proof of Theorem 6 requires that the indirect utility viewed as a function of price only (with allocations and opportunities determined by the equilibrium allocation) is continuous. For regular economies, this follows by the continuity of the indirect utility function because equilibrium allocations and the corresponding budget sets vary continuously in $p$ for regular economies.

Theorem 6 extends without problems to economies with production. In this setting, however, less is known about the conditions under which the equilibrium correspondence is continuous.[27]

Roberts and Postlewaite present one possible approach to the study of price-taking behavior in large economies. One criticism of their approach is that, by concentrating on approximate optimization, it does not provide information about the behavior of exact equilibria in large, finite economies. An alternative approach is to study mechanisms that induce agents to report "true" excess demands in finite economies and to provide conditions under

[26] Roberts and Postlewaite identify weaker conditions under with one would expect $Q(\cdot)$ to be continuous for "most" parameters.

[27] Mas-Colell [31, pages 249-252] provides conditions under which large production economies are "regular" in that the equilibrium correspondence varies continuously with prices.

which equilibria of these mechanisms converge to Walrasian outcomes when economies are large. Kovalenkov [28] provides an example of this type of result.

## 6.2 Core Equivalence

In this subsection, we discuss the relationship between the cooperative solution concept core and Walrasian equilibria in large economies. We restrict attention to well-being externalities. We require that any subgroup of agents can find a way to distribute extra endowments among themselves in such a way that every member of the subgroup is better off. Under this **group social monotonicity** assumption, the core of the original economy is a subset of the core of the internal economy. In particular, we get the equal treatment property: agents with the same internal preferences and endowments get the same consumption bundle in every core allocation. The Debreu-Scarf theorem then implies that the core of the limit economy is a subset of the set of Walrasian equilibria. We do not get full core equivalence in general because the core can be empty. We then give a simple sufficient condition for a nonempty core.

The first conceptual problem that we encounter is that of defining the core. An allocation belongs to the core if it can be viewed as the outcome of cooperation among agents. Classically, $x$ is in the core if there is no coalition $C \subseteq I$ that can improve upon or block $x$. $C$ improves upon $x$ if there is a $C$–allocation $x' = (x'_i)_{i \in C}$ such that

- $x'$ is feasible when the coalition $C$ is autarkic,
- and every member of $C$ prefers $x'$ to $x$.

The first verbal statement is easy to define formally: $x'$ is $C$–feasible if

$$\sum_{i \in C} (x'_i - e_i) = 0\,.$$

The second statement is more subtle: given that preferences depend on others' consumption choices, how should we evaluate the actions of agents outside of a coalition once the coalition forms? One possibility is that a coalition $C$ can improve upon $x$ if there is a feasible reallocation within the coalition that ensures a social state preferred by all the agents in $C$ regardless of the strategies the other agents outside the coalition may choose. Formally, we say that $C$ can $\alpha-$improve upon $x$ if there is an $(x'_k)_{k\in C}$ that is feasible for $C$ such that

$$U_i\left((x'_k)_{k\in C}\,,(x_{k'})_{k'\notin C}\right) > U_i(x) \quad \text{for all } i\in C$$

for all $(x'_k)_{k\notin C}$ that is feasible for $\neg C$.[28]

It is difficult for coalitions to improve themselves under the definition of the $\alpha$-core, making it more likely to demonstrate that the core is non-empty. In this section we focus on another notion of core in which improvements are easier to find:

**Definition 5.** *A coalition $C\subseteq\{1,\ldots,I\}$ can improve upon an allocation $x$ if there exists an allocation $x'$ with $\sum_{k\in C}(x'_k-e_k)=0$ and*

$$U_i\left((x'_k)_{k\in C}\,,(x_{k'})_{k'\notin C}\right) > U_i(x) \quad \textit{for all } i\in C.$$

This definition is a generalization of the test for deviations in Nash Equilibrium: holding the allocations of the other agents fixed, a coalition can improve itself if it is able to reallocate its resources in a way that makes all members of the coalition better off. Aumann's [3] strong Nash equilibrium concept, defined for non-cooperative games, makes the same assumption about the behavior of non-coalition members. This definition makes it relatively easy to block a proposed allocation and, thus, creates existence problems. With this notion of stability we can generalize the results in Section 5.1.

In general, we cannot expect to have equality of core and equilibria even in the continuum limit because we know that Walrasian equilibria can be inefficient. On the other hand, we have shown that social monotonicity implies a version of the second welfare theorem. It thus seems plausible that a suitable strengthening of the social monotonicity condition yields that the core of large economies is a subset of the set of Walrasian equilibria.

[28]Aumann and Peleg [4] introduces the $\beta$-core, which consists of those allocations $x$ in which a coalition cannot $\beta$-improve upon $x$. For a coalition to $\beta$-improve upon $x$ it must be that for all $(x_{k'})_{k'\notin C}$ that are feasible for $\neg C$ there is an $(x'_k)_{k\in C}$ that is feasible for $C$ that makes every agent in $C$ better off relative to $x$. Chander and Tulkens [8] introduce the $\gamma$-core. Translated to our framework, outsiders consume their endowments when a coalition is formed.

**Group Social Monotonicity (GSM)** Let $C \subseteq I$ be a coalition. For any allocation $x$ and $z \in \mathbb{R}^L_+ \setminus \{0\}$ there is a redistribution $(z_j)_{j \in C} \geq 0$ with $\sum_{j \in C} z_j = z$ such that the members of $C$ prefer

$$y_j = \begin{cases} x_j + z_j & j \in C \\ x_j & j \notin C \end{cases}$$

over $x$, i.e.

$$U_i(y) > U_i(x) \qquad (i \in C)\,.$$

**Lemma 1.** *Under GSM, the core is a subset of the internal core.*

While intuitive, the above Lemma relies on our definition of the core. For example, the $\alpha$-core need not be a subset of the internal core.[29]

We now perform the Debreu–Scarf thought experiment by replicating an economy many times. Note that replication is not a trivial task with other-regarding preferences. Suppose that Adam is altruistic and benefits from Eve's well-being in a two–person economy. Now replicate them. How does Adam feel about Eve 1 and/or Eve 2 ? There are several more or less natural choices to formalize Adam's preferences in the replicated economy. Somewhat fortunately, our results do not depend on the way the replicated Adams care about the replicated Eves' consumption choices. Let us start with $I$ agents with separable preferences $V_i(m_i(x_i), x_{-i})$. The $n$–replica of the economy $\mathcal{E}_n$ has $NI$ agents. Let us denote by $x_{i,n}$ the consumption choice of the $n$th copy of agent $i$, $n = 1, \ldots, N$. We suppose that preferences of Agent $i, n$ can be represented by $V_{i,n}(m_i(x_{i,n}), x_{-(i,n)})$ that is monotone in $m_i(x_{i,n})$ for all $x_{-i,n}$. Note that all copies of Agent $i$ have the same internal utility function. We leave it completely general, in which way the utility of Agent $i, n$ depends on others' consumption choices. Lemma 1 tells us that the core of the $N$th replica is a subset of the core of the internal $N$th replica economy. As a

[29] Consider a four member economy with two goods. Internally, the first agent cares only about $.7x_{11} + x_{12}$ and the second agent cares about $x_{21} + .7x_{22}$. The other two agents are indifferent between the two goods (all have linear internal preferences). Agent 1 cares about $m_1 + 2m_2$. Agent 2 cares about $m_2 - .5m_4$. Agent 3 cares about $m_3 + m_4$. Agent 4 is selfish. Allocating one unit of Good 1 to Agents 1 and 3 and one unit of Good 2 to Agents 3 and 4 is not in the internal core (Agents 1 and 2 would trade.) On the other hand, this allocation is in the $\alpha$-core because if Agents 1 and 2 form a coalition, Agent 3 can give everything to Agent 4, lowering Agent 2's utility.

consequence of the classical equal-treatment lemma, core allocations treat all agents of type $i$ equal. By the theorem of Debreu and Scarf [12], the internal core shrinks to the set of Walrasian equilibria as $n$ grows large. Let $\mathcal{C}_N$ denote the core of $\mathcal{E}_N$ and let $WE(\mathcal{E})$ be the set of Walrasian equilibria of an economy $\mathcal{E}$. We thus get

**Theorem 7.**

$$\bigcap_{N \in \mathbb{N}} \mathcal{C}_N \subseteq WE(\mathcal{E}) .$$

As we remarked above, one cannot get equality of the limit core and the set of Walrasian equilibria, as these equilibria can be inefficient in general, and in this case the grand coalition could improve. Indeed, while we do have existence of Walrasian equilibria with separable preferences, the core may be empty.

**Example 5.** *Let there be three agents and one consumption good. Let the internal utility functions of all agents be linear. There are 3 units of the consumption good available and individuals each have unit endowment. The utility functions are:*

$$U_1 = m_1 + 2m_2,\ U_2 = m_2 + 2m_3,\ U_3 = m_3 + 2m_1 .$$

*No agent wants to destroy any of the endowment. Thus we can restrict attention to allocations $(a_1, a_2, 1-a_1-a_2)$. The allocation $(1, 1, 1)$ is efficient.*

*To see that the core is empty, observe that any allocation in which $a_1 > 0$ is blocked by the coalition $C = \{1, 2\}$ who prefers to allocate the good to Agent 2. Any allocation in which $a_2 > 0$, is blocked by $C = \{2, 3\}$ and any allocation in which $1 - a_1 - a_2 > 0$ is blocked by $C = \{3, 1\}$. Thus, the core is empty. (The $\alpha$-core is non-empty. For example, Agent 2 would not join the coalition $C = \{1, 2\}$ if Agent 3 can give his entire endowment to Agent 1.)*

In the preceding example, Agent 1 is happy to consume less as long as Agent 2 gets what she gives away. This strong kind of altruism in interplay with a certain circularity in preferences destroys the core. Using a condition to rule out such strong altruism, we obtain a nonempty core.

**Theorem 8.** *Let $x$ be an internal core allocation. Assume that no coalition $C \subseteq \{1, \ldots, I\}$ can find a $C$–feasible allocation in which $m_i(x_i') < m_i(x_i)$ and $U_i(x') > U_i(x)$ for some $i$. Then $x$ belongs to the core of the original economy.*

# 7 Conclusion

We have compared economies with and without other-regarding preferences and proved that under conditions that guarantee that this comparison is meaningful (separability) the equilibria are the same. Hence agents who care directly about the welfare and opportunities of others cannot be distinguished from selfish agents in market settings.[30]

The fact that market behavior may not be affected by other-regarding preferences does not mean that we can ignore the existence of ORP in markets. First, market outcomes need not be efficient. Second, even when market equilibria are efficient – and we have given conditions that imply efficiency – the agents who gain and lose from interventions will depend on the precise nature of preferences.

From a positive perspective, our result can be viewed as a benchmark that highlights when other-regarding preferences are irrelevant for predicting behavior. Of course, other-regarding preferences can be important in other institutional environments than the one we consider. For example, Heidhues and Riedel [23] observe that an altruistic agent has different incentives to contribute to a public good than a classical agent. Hence, in public-good economies other regarding preferences affect behavior. Similarly, in economies with "classical" consumption externalities such as pollution, other regarding preferences will typically also affect behavior. Heidhues and Riedel also point out that our separability assumption is unlikely to hold when there is uncertainty and thus other-regarding preferences may affect behavior in this case also.

[30]Sobel [42] establishes a similar result in a setting where agents have market power.

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# Appendix

**Proof of Theorem 1.** We prove (1) first. As $m_i$ is continuous and strictly quasiconcave, the standard utility maximization problem

$$\max_{x \geq 0, px_i \leq w} m_i(x_i)$$

has a unique solution, which we denote, in a slight abuse of notation, $d_i(p, w)$ for $p \gg 0$ and $w > 0$. This demand function does not depend on $x_{-i}$ or $B_{-i}$. Now take any $x_{-i}$ and $B$. We have for all budget–feasible $x_i$

$$m_i(x_i) < m_i(d_i(p, w))$$

whenever $x_i \neq d_i(p, w)$. As $V_i(m, x_{-i}, B)$ is increasing in $m$, it follows that

$$V_i\left(m_i\left(x_i\right), x_{-i}, B\right) < V_i\left(m_i\left(d_i(p, w)\right), x_{-i}, B\right)$$

whenever $x_i \neq d_i(p, w)$. Thus, $d_i(p, w)$ also uniquely maximizes utility for Agent $i$. In particular, her demand function is independent of $x_{-i}$; in other words, she behaves as if selfish.

Now consider (2). Let $d_i(p, w)$ be the demand function of Agent $i$ which, by assumption, does not depend on $x_{-i}$ and $B_{-i}$. In a first step, we construct an internal utility function on the consumption set $\mathbb{R}^L_+$ of Agent $i$. This is a standard integrability problem. Such a function $m_i(x_i)$ exists if

- $d_i$ is continuously differentiable,
- homogeneous of degree zero,
- $d_i$ has a symmetric and negative semi-definite Slutsky substitution matrix,
- $d_i$ satisfies Walras's law: $pd_i(p, w) = w$ for all $p \gg 0$ and $w > 0$.

By assumption, $d_i$ is continuously differentiable. As demand $d_i$ is derived from utility maximization (albeit with the additional parameters $x_{-i}$ and $B$), homogeneity of degree zero and negative semi-definiteness of the substitution matrix hold true as well. Walras's law follows from monotonicity. We can then apply an integrability theorem of Hurwicz and Uzawa [26] to obtain a utility function $m_i(x_i)$ that rationalizes $x_i$. In particular, we have for all $x_{-i}$ that

$$U_i(x_i, x_{-i}, B) \geq U_i(z_i, x_{-i}, B) \Leftrightarrow m_i(x_i) \geq m_i(z_i) \qquad \left(x_i, z_i \in \mathbb{R}_+^L\right) . \quad (18)$$

We can thus define a function $V_i(\mu, x_{-i}, B)$ on the image of $m$ and $\mathbb{R}_+^{(I-1)L}$ by setting

$$V_i(\mu, x_{-i}, B) = U_i(x_i, x_{-i}, B)$$

for some $x_i$ with $m_i(x_i) = \mu$. This definition does not depend on the particular $x_i$ chosen as we have $U(x_i, x_{-i}, B) = U(z_i, x_{-i}, B)$ for all $x_i, z_i$ with $m_i(x_i) = m_i(z_i)$ by relation (18).

Finally, we have to show that $V_i$ is increasing in $\mu$. Let $\mu > \nu$ for two numbers $\mu, \nu$ in the image of $m$. Choose $x_i, z_i$ with $\mu = m_i(x_i)$ and $\nu = m_i(z_i)$. We then get from $m_i(x_i) > m_i(z_i)$ and (18) that

$$U_i(x_i, x_{-i}, B) > U_i(z_i, x_{-i}, B) .$$

By definition of $V_i$, this is equivalent to

$$V_i(\mu, x_{-i}, B) > V_i(\nu, x_{-i}, B) .$$

Thus, $V_i$ is increasing in its first variable. ■

**Proof of Theorem 3.** Assume that there exists a Pareto-efficient allocation $x$ that is not internally efficient. Hence, there exists a feasible allocation $x'$ such that $m_i(x_i') > m_i(x_i)$ for all $i$. It follows from monotonicity that there exists

$$\tilde{x}' < x' \qquad (19)$$

such that $m_i(\tilde{x}_i') = m_i(x_i)$ for all $i$. The Social Monotonicity Condition guarantees that it is possible to make all agents better off by some distribution of $x' - \tilde{x}'$. ■

**Proof of Corollary 2.** The result follows because every internally efficient allocation can be implemented under SM and the set of Pareto-efficient allocations is a subset of the set of internally efficient payoffs. ■

**Proof of Theorem 4.** Since in equilibrium each agent $i$ chooses a utility maximizing consumption bundle in $B_i^*$, $V_i(d_i(B_i^*), B^*) \geq V_i(x_i', B^*)$, for all $x_i'$. If a change from the equilibrium outcome $(x^*, y^*, B^*)$ to outcome $(x', y^{'}, B^{'})$ constitutes a Pareto improvement, it must therefore be that $B^* \neq B'$. The profile of budget sets $B^*$ induces a profile of incomes $w(B^*)$. Since $p(B_i^*) = p(B_i') = p^*$ for all $i$, $B^* \neq B'$ implies that $w(B^*) \neq w(B')$. Because in equilibrium each firm is profit maximizing, it must hold that

$$\sum_{i=1}^{I} p^* e_i + \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{ij} p^* y_j^* \geq \sum_{i=1}^{I} p^* e_i + \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_{ij} p^* y_j',$$

it follows that, $\sum_{i=1}^{I} w(B_i^*) \geq \sum_{i=1}^{I} w(B_i')$. A change from $(x^*, y^*, B^*)$ to $(x', y', B')$ is a Pareto improvement if and only if $V_i(d_i(B_i^*), B^*) \geq V_i(d_i(B_i'), B')$, for all $i$, with one inequality strict. This is not possible if RLP holds, since if $V_i(d_i(B_i^*), B^*) \geq V_i(d_i(B_i'), B')$, for all $i$, then (11) implies that $V_i(d_i(B_i^*), B^*) = V_i(d_i(B_i'), B')$, for all $i$. ■

**Proof of Theorem 5.** Let $B$ and $B'$ be two profiles of budget sets with $B \neq B'$, $\sum_{i \in I} w(B_i) \geq \sum_{i \in I} w(B_i')$, and $p(B_i) = p(B_k')$ for all $i, k = 1, \ldots, I$. Let $r$ be a consumer who loses most in terms of income by a change from $B$ to $B'$; i.e. for all $i$

$$w(B_r) - w(B_r') \geq w(B_i) - w(B_i'). \tag{20}$$

Let $w_k = w(B_k)$, $w_k' = w(B_k')$, $\bar{w} = \frac{\sum_{i \in I} w(B_i)}{I}$, and $\bar{w}' = \frac{\sum_{i \in I} w(B_i')}{I}$. Note that

$$w_r - w_r' > 0 \tag{21}$$

and

$$\bar{w} \geq \bar{w}'. \tag{22}$$

We let $v(w_r) = m_r(d_r(p, w_r))$ and $v'(\cdot)$ the associated derivative. Let $\bar{v} = \max_k v'(w_k)$ and $\underline{v} = \min_k v'(w_k)$. Note that $\bar{v} = v'(\min_k w_k)$ and $\underline{v} = v'(\max_k w_k)$.

Let

$$V_r = V_r(m_r(d_r(p, w_r)), m_r(d_r(p, w_1)), \ldots, m_r(d_r(p, w_I)))$$

and

$$V_r' = V_r(m_r(d_r(p, w_r')), m_r(d_r(p, w_1')), \ldots, m_r(d_r(p, w_I'))).$$

Finally, let

$$\mu = \frac{\sum_k v_r(w_k)}{I} \text{ and } \mu' = \frac{\sum_k v_r(w'_k)}{I}$$

We need two related preliminary facts.

**Lemma 2.** $\sum_{k \neq r} (v_r(w_k) - v_r(w'_k)) \geq -((\bar{v} - \underline{v})(I-1) + \underline{v})(w_r - w'_r)$.

**Proof.**

$$\begin{aligned}
\sum_{k \neq r} (v_r(w_k) - v_r(w'_k)) &\geq \sum_{k: w_k < w'_k} \bar{v}\,(w_k - w'_k) + \sum_{k: k \neq r, w_k > w'_k} \underline{v}\,(w_k - w'_k) \\
&\geq \sum_{k: w_k < w'_k} (\bar{v} - \underline{v})\,(w_k - w'_k) - \underline{v}\,(w_r - w'_r) \\
&\geq -((\bar{v} - \underline{v})(I-1) + \underline{v})\,(w_r - w'_r),
\end{aligned}$$

Where the second inequality follows from (22) and the third inequality follows from (20) and (22). ■

**Lemma 3.** $\mu - \mu' \geq -(\bar{v} - \underline{v})\,(w_r - w'_r)$.

**Proof.**

$$\begin{aligned}
I\,(\mu - \mu') &= \sum_{k \neq r} (v_r(w_k) - v_r(w'_k)) + v_r(w_r) - v_r(w'_r) \\
&\geq -((\bar{v} - \underline{v})(I-1) + \underline{v})\,(w_r - w'_r) + \underline{v}\,(w_r - w'_r) \\
&\geq -(I-1)\,(\bar{v} - \underline{v})\,(w_r - w'_r),
\end{aligned}$$

where the first inequality uses Lemma 2. ■

1. It suffices to show that

$$\begin{aligned}
& v_r(w_r) - \frac{\alpha_r}{I-1} \sum_{k=1}^{I} \max\{v_r(w_k) - v_r(w_r), 0\} - \frac{\beta_r}{I-1} \sum_{k=1}^{I} \max\{v_r(w_r) - v_r(w_k), 0\} \qquad (23) \\
> \;& v_r(w'_r) - \frac{\alpha_r}{I-1} \sum_{k=1}^{I} \max\{v_r(w'_k) - v_r(w'_r), 0\} - \frac{\beta_r}{I-1} \sum_{k=1}^{I} \max\{v_r(w'_r) - v_r(w'_k), 0\}.
\end{aligned}$$

For any $w_k > w_r$ such that $w_r - w'_r > w_k - w'_k$ decreasing $w'_k$ to $w_k + w'_r - w_r$ increases the right-hand side of (23) without violating (20) or (22). So in order to prove the result it is sufficient to show that inequality (23) when $w_k > w_r$ implies $w_r - w'_r = w_k - w'_k$. Since $w_r > w'_r$, concavity of $v_r(\cdot)$ implies that

$$v_r(w_k) - v_r(w_r) \leq v_r(w'_k) - v_r(w'_r) \tag{24}$$

whenever $w_k > w_r$ Hence (23) will be harder to satisfy if we assume that $w_r - w'_r = w_k - w'_k$ whenever $w_k > w_r$. Hence we take $w_r = \max_{1\leq k \leq I} w_k$. Consequently it suffices to show that

$$\begin{aligned} & v_r(w_r) - \frac{\beta_r}{I-1}\sum_k (v_r(w_r) - v_r(w_k)) \qquad (25) \\ > \; & v_r(w'_r) - \frac{\alpha_r}{I-1}\sum_{w'_k > w'_r, w_k \leq w_r} (v_r(w'_k) - v_r(w'_r)) - \frac{\beta_r}{I-1}\sum_{w'_k < w'_r, w_k \leq w_r} (v_r(w'_r) - v_r(w'_k)). \end{aligned}$$

Since the right-hand side of inequality (25) is no greater than

$$v_r(w'_r) - \frac{\beta_r}{I-1}\sum_{k=1}^{I} (v_r(w'_r) - v_r(w'_k)),$$

inequality (25) is true whenever

$$v_r(w_r) - \frac{\beta_r}{I-1}\sum_k (v_r(w_r) - v_r(w_k)) > v_r(w'_r) - \frac{\beta_r}{I-1}\sum_k (v_r(w'_r) - v_r(w'_k)). \tag{26}$$

To complete the proof it suffices to show that

$$(1-\beta_r)\left(v_r(w_r) - v_r(w'_r)\right) > \frac{\beta_r}{I-1}\sum_{k\neq r}\left(v_r(w'_k) - v_r(w_k)\right). \tag{27}$$

Since $v_r(w_r) - v_r(w'_r) \geq \underline{v}\,(w_r - w'_r)$, it follows from Lemma 2 that RLP holds provided that

$$(1-\beta_r) \geq \beta_r\left((\bar{v} - \underline{v}) + \frac{\underline{v}}{I-1}\right) \text{ or } \beta_r < \frac{1}{1 + \bar{v} - \underline{v} + \frac{\underline{v}}{I-1}}.$$

2. By the triangle inequality,

$$|v_r(w_r) - v_r(w'_r) - (\mu - \mu')| \geq |v_r(w_r) - \mu| - |v_r(w'_r) - \mu'| . \quad (28)$$

If $v_r(w_r) - v_r(w'_r) \geq \mu - \mu' > 0$, then the result follows from (28).
If $v_r(w_r) - v_r(w'_r) \geq 0 > \mu - \mu'$, then let

$$\beta_r \leq \underline{v}/\bar{v}. \quad (29)$$

It follows that

$$\begin{aligned}(1-\beta_r)\left(v_r(w_k) - v_r(w'_k)\right) &\geq (1-\beta_r)\underline{v}\,(w_r - w'_r) \\ &\geq \beta_r\,(\bar{v} - \underline{v})\,(w_r - w'_r)\end{aligned}$$

and therefore

$$v_r(w_r) - v_r(w'_r) \geq \beta_r\left((v_r(w_r) - v_r(w'_r)) + (\bar{v} - \underline{v})\,(w_r - w'_r)\right). \quad (30)$$

Also we have

$$\begin{aligned}v_r(w_r) - v_r(w'_r) + (\bar{v} - \underline{v})\,(w_r - w'_r) &\geq v_r(w_r) - v_r(w'_r) - (\mu - \mu') \\ &> |(v_r(w_r) - \mu)| - |(v_r(w'_r) - \mu')| ,\end{aligned}$$

where the first inequality follows from Lemma 3 and the second inequality follows from (28). It follows that if (29) holds, then RLP holds.

Finally we must consider the case in which $v_r(w_r) - v_r(w'_r) < \mu - \mu'$. In this case, it follows from (28) that it suffices to show that

$$v_r(w_r) - v_r(w'_r) > \beta_r\left(\mu - \mu' + v_r(w_r) - v_r(w'_r)\right). \quad (31)$$

Inequality (31) holds if

$$\beta_r < \frac{\underline{v}}{\bar{v} - \underline{v}},$$

since $v_r(w_r) - v_r(w_r') \geq \underline{v}(w_r - w_r')$ and $(w_r - w_r')\bar{v} \geq \mu - \mu'$.[31] The result follows because

$$\frac{\underline{v}}{\bar{v} - \underline{v}} > \frac{\underline{v}}{\bar{v}}.$$

3. Note that if $v_r(w_i) = \min_k v_r(w_k)$, then

$$\begin{aligned} \min\{v_r(w_1), \ldots, v_r(w_I)\} - \min\{v_r(w_1'), \ldots, v_r(w_I')\} &\geq v_r(w_i) - v_r(w_i') \\ &\geq \bar{v}\left(w_i - w_i'\right) \qquad (32) \\ &\geq -(I-1)\left(w_r - w_r'\right). \end{aligned}$$

The third inequality follows from (20).

When $\beta_r > 0$,

$$V_r - V_r' \geq (w_r - w_r')\left(\underline{v} - \beta_r(1-\delta_r)\left(\bar{v} - \underline{v} + \frac{\underline{v}}{I-1}\right) - \beta_r\delta_r\bar{v}\right), \quad (33)$$

where the inequality follows from concavity of $v_r(\cdot)$, (32), and Lemma 2. Consequently, RLP holds whenever

$$\frac{\underline{v}}{(1-\delta_r)\left(\bar{v} - \underline{v} + \frac{\underline{v}}{I-1}\right) + \delta_r\bar{v}} > \beta_r > 0.$$

When $\beta_r < 0$,

$$\begin{aligned} V_r - V_r' &\geq v_r(w_r) - v_r(w_r') + \frac{\beta_r(1-\delta_r)}{I-1}\left(\sum_{k\neq r}\left(v_r(w_k) - v_r(w_k')\right)\right) + \beta_r\delta_r\left(v_r(w_i) - v_r(w_i')\right) \\ &\geq \left(v_r(w_r) - v_r(w_r')\right)\left(1 + \frac{\beta_r\delta_r}{I-1}\right) + \frac{\beta_r(1-\delta_r)}{I-1}\left(\sum_{k:w_k<w_k'}\bar{v}\left(w_k - w_k'\right)\right) \\ &\geq \underline{v}\left(1 + \frac{\beta_r\delta_r}{I-1}\right)\left(w_r - w_r'\right) + \beta_r\bar{v}\left(w_r - w_r'\right). \end{aligned}$$

[31] We assumed that $\underline{w}' \equiv \min_k w_k' \geq \min_k w_k$ to obtain the bound $(w_r - w_r')\bar{v} \geq \mu - \mu'$. This assumption is without loss of generality, for if it failed, then RLP would become harder to satisfy. To see this, assume that $\underline{w} < \min_k w_k$. If $\underline{w}' < w_r'$, then you can raise the minimum and either lower another $w_k'$ keeping $\mu'$ fixed provided some can be lowered. Otherwise all $w_k'$ are either lowered to the minimum or lowered by the maximum amount. All $w_k'$ can be increased in such a way that $\mu' - v_r(w_r')$ remains constant. If $\underline{w}' = w_r'$, then raising all of the lowest $w_k'$ in such a way that leaves $\mu' - v_r(w_r')$ constant is feasible and makes RLP harder to satisfy.

The first inequality hold when $v_r(w'_i) = \min_k v_r(w'_k)$. One obtains the second inequality by discarding positive terms and using the definition of $\bar{v}$. Provided that

$$1 + \frac{\beta_r \delta_r}{I-1} > 0$$

(which holds for sufficiently large $I$), the third inequality follows from (20) and (22). Hence RLP holds provided that

$$\beta_r > -\frac{\underline{v}}{\bar{v} + \frac{\beta_r \delta_r \underline{v}}{I-1}}$$

■

**Proof of Theorem 6.** Let $\mathcal{E}_n$ be described by the simple measure $\mu_n$, suppose $\bar{i}$ belongs to each $\mathcal{E}_n$, and suppose $S_{n,\bar{i}}$ is the response used by $\bar{i}$ in $\mathcal{E}_n$. The choice of a response $S'_{n,\bar{i}}$, different from $S_{n,\bar{i}}$, by $\bar{i}$ in $\mathcal{E}_n$ defines a new "apparent economy" described by a simple measure $\rho^n$, where

$$\rho_n(F) = \frac{\#\{F \cup [(\text{support } \mu_n) \cap \{S'_{n,\bar{i}}\} \setminus \{S_{n,\bar{i}}\}\}}{\#\{\text{support } \mu_n\}}$$

for all Borel sets $F$ of $\mathcal{S}$.

As $\{\mu_n\}$ converges to $\mu^*$, so too will the sequence $\{\rho_n\}$, since the corresponding measures differ on a point whose measure goes to zero. By the continuity of $Q(\cdot)$, given any $\epsilon > 0$, there exist $n^*$ such that $n \geq n^*$ implies $d(Q(\mu^*), Q(\mu_n)) < \epsilon$ and $d(Q(\mu^*), Q(\rho_n)) < \epsilon$, where $D(\cdot)$ is the Hausdorff metric on subsets of $P$ (the price simplex). It follows from the triangle inequality that $d(Q(\mu_n), Q(\rho_n)) < 2\epsilon$. Thus, for large enough $n$, any equilibrium price of the $n$th "apparent economy" is arbitrarily close to an equilibrium price of $\mathcal{E}_n$, while both lie within the neighborhood of $Q(\mu^*)$ on which the indirect utility function of agent $\bar{i}$ is continuous. To complete the proof, we take $S'_{n,\bar{i}}(\cdot) = \tilde{S}_{\bar{i}}(\cdot)$ to be the excess demand correspondence derived from the true preferences of agent $\bar{i}$. ■

**Proof of Lemma 1.** If $x$ is not in the internal core, then there is a coalition $C$ and a $C$–feasible allocation $x' = (x'_k)_{k \in C}$ such that $m_i(x'_i) > m_i(x_i)$ for

$i \in C$. From monotonicity and continuity of $m_i(\cdot)$, we can find $\eta_i \leq x_i', i \in C$, $\eta_i \neq x_i'$, such that $m_i(\eta_i) = m_i(x_i)$ for $i \in C$. Let

$$z = \sum_{i \in C} (x_i' - \eta_i) \geq 0, z \neq 0\,.$$

GSM implies that the coalition $C$ can improve upon $x$. Hence, $x$ is not in the core. ■

**Proof of Theorem 8.** The core of the internal economy is not empty because internal preferences are convex (Scarf [39]). We want to show that $x$ belongs to the core of the original economy. If not, there is a coalition $C$ and a $C$–feasible allocation $x'$ such that all members in $C$ prefer $x'$ to $x$. As $x$ belongs to the internal core, some members of $C$ must have a lower internal utility. This contradicts the assumption. ■