Coagglomeration

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Abstract

Traditional models of systems of cities focus almost entirely on the polar cases of completely diverse and completely specialized cities. The former are shown to be the product of urbanization economies, and the latter of localization economies. This paper specifies models that, in addition to the polar cases, can also generate the intermediate case of cities that feature the coagglomeration of some but not all industries.

The paper's analysis shows that coagglomeration is likely to be inefficient. In some situations, there will be excessive coagglomeration, and cities will be inefficiently diverse. In others, there will be unrealized coagglomeration that could have enhanced efficiency, a failure of matching. These results are important as extensions to the theory of systems of cities and as bases for urban policy analysis. They are also important for shedding light on observed patterns of coagglomeration and what they imply about the nature of agglomeration economies.

I. Introduction

No city is really a one industry town, not even Hollywood or the Silicon Valley. Neither is any city simply a share of the diverse national population. New York's diversity is different than that of Los Angeles. The idea of industry clusters, as popularized by Porter (1990), is that certain industries are related to each other, and these industry groups tend to agglomerate. Porter argues persuasively that this clustering is important for firm location decisions and for industrial policy. Ellison and Glaeser (1997) coined the term "coagglomeration" to refer to the more general tendency of various industries to locate together.

There is a growing empirical literature on coagglomeration. Ellison and Glaeser (1997) document coagglomeration patterns, and Ellison et al (2010) consider how the tendency of industries to coagglomerate relates to industry characteristics. This work establishes clearly that there is both great variety in the forms that cities take and also important regularities in the equilibrium pattern of coagglomeration. Ellison et al (2010) takes the additional step of using evidence of coagglomeration to provide evidence on the "relative importance" across industries of potential sources of agglomeration economies. This is some of the most powerful evidence in the literature on the microfoundations of agglomeration.

There is only limited theoretical work on coagglomeration in a system of cities. This literature begins with Henderson's (1974) classic general equilibrium model of urbanization. The heart of the paper is its characterization of the tradeoff between the benefits and costs of agglomeration. This tradeoff is used to characterize optimal cities and relate their size to the nature of agglomeration economies. With urbanization economies, cities will be diverse. With localization economies, cities will be specialized. Regarding the intermediate case of coagglomeration, Henderson (1974, p. 641) writes that:

> [C]ities will probably specialize in bundles of goods, where, within each bundle, the goods are closely linked in production. They may use a common specialized labor force or a common intermediate input.

While Henderson (1974) has a careful formal analysis of the polar cases of specialization and diversity, there is no formal analysis of coagglomeration. Abdel-Rahman and Fujita (1993) extends the literature in an important way by considering the polar cases of completely diverse and completely specialized cities in a model with two industries. For our purposes here, the most important result is the characterization of the conditions under which an efficient system of cities is diverse. The efficient system is identical to the equilibrium because the model assumes that there exist city developers who arbitrage away any inefficiencies. This paper does not consider the sort of inefficient allocations that could arise without developers. While there are a few papers that deal with aspects of coagglomeration (discussed in Section II below), it seems clear that coagglomeration has not been a central focus of the agglomeration literature.[1]

This paper will provide a system-of-cities model of coagglomeration. The model captures a range coagglomeration patterns that are intermediate between the polar cases of complete specialization and complete diversity. The model will be specified to parallel the standard model as closely as possible. The key result is that equilibrium coagglomeration is likely to be inefficient. In some situations, industries will coagglomerate inefficiently, and there will be excessive diversity. In others, there will be unrealized coagglomeration that could have enhanced efficiency.

The source of the key result is that individual migration is a weak instrument for promoting efficiency. To understand this heuristically, imagine a situation where a worker chooses a location taking other location decisions as given. The worker may be willing to choose a city that because of coagglomeration is larger than the agglomeration economies versus congestion costs tradeoff would dictate if the alternative is forming a new city (autarky). This is a coagglomeration corollary to the familiar migration pathology in the systems-of-cities literature (see the many references in the comprehensive survey by Abdel-Rahman and Anas (2004)). In fact, the result is quite different in the coagglomeration case, with the Pareto efficient allocation no longer being consistent with equilibrium. As will be seen, this is a consequence of externalities

[1]It is worth noting that the word "coagglomeration" does not appear in the indices of either the most recent *Handbook of Regional and Urban Economics* (Henderson and Thisse, eds., 2004) or the comprehensive textbook on agglomeration by Fujita and Thisse (2002).

between industries that arise in the coagglomeration case, but not in the traditional analysis.

There is an additional difference in the coagglomeration case that creates a new type of inefficiency related to the industry mix. Imagine a situation where the tradeoffs suggest that a worker would benefit both from agglomeration with his or her own type and also from coagglomeration with workers in another given industry. The worker, however, may not be offered the does ideal choice. This may lead the worker to choose a city with many own-type workers even if it does not have the ideal partner industry as well. And workers in the partner industry may make the same choices. The individual migration choice is, in this situation, not enough to discipline the inefficient cities. As in Bewley's (1981) analysis of competition between local governments, it is possible that this inefficiency could be remedied through the actions of entrepreneurial governments or profit-maximizing city developers. However, the fact remains that cities are formed primarily by individual migration with at most modest intervention by entrepreneurial agents. City governments have the wrong incentives and insufficient power to exclude. City developers are the exception and not the rule in urban development.[2]

This inefficient coagglomeration result is potentially very important. The system-of-cities literature continues to embrace the idea that localization economies create industry towns, while urbanization economies create completely diverse cities. This claim is based on a strong informal welfare theorem that agglomeration should be efficient. By considering coagglomeration explicitly, we show otherwise. It is, unfortunately, quite possible to have equilibrium cities that fail to involve efficient coagglomeration.

The result is also important because of its empirical implications. Audretsch and Feldman (1996) and Rosenthal and Strange (2001) estimate relationships between industry agglomeration and industry characteristics. The idea is that if agglomeration is based on, for instance, knowledge spillovers, then one should observe agglomeration of knowledge-intensive industries. In an important recent paper, Ellison et al (2010) employ the creative identification strategy of looking at the flows of ideas, people, and goods to

[2] See also the survey by Scotchmer (2002). She writes (p. 2002) that "…the idea of a price-taking equilibrium is rather from afield from the way local public economies operate, and this is why I consider club theory to be a motivator for the subject of local public economics, but not the subject itself."

better understand microfoundations. The analysis in this paper shows that coagglomeration need not be efficient. Potentially beneficial coagglomeration may not necessarily occur, and the coagglomeration that does occur may not be that which creates the greatest benefits. This means that there might be unobserved patterns of industrial location that would be beneficial were they to arise. This argues for caution in dismissing the relative importance of possible microfoundations based on either cross sectional patterns of industrial agglomeration or coagglomeration.

The remainder of the paper is organized as follows. Section II provides a more detailed review of the current state-of-knowledge regarding coagglomeration. Section III describes the basics of our model and applies the model to the case of the coagglomeration of two industries. Section IV extends the model to an arbitrary number of industries, which allows the consideration of a wider range of coagglomeration patterns. Section V considers the empirical implications of our results on coagglomeration. Section VI concludes.

II. Literature

The classics of the agglomeration literature all touch on coagglomeration only lightly. Marshall (1890) is, of course, primarily concerned with localization, the specialization of an area in one industry. There are only a few places where he touches explicitly on coagglomeration. One is his analysis of the possibility that "supplementary" industries may co-locate (Marshall, 1890, Book IV.X.10):

> On the other hand a localized industry has some disadvantages as a market for labour if the work done in it is chiefly of one kind, such for instance as can be done only by strong men. In those iron districts in which there are no textile or other factories to give employment to women and children, wages are high and the cost of labour dear to the employer, while the average money earnings of each family are low. But the remedy for this evil is obvious, and is found in the growth in the same neighbourhood of industries of a supplementary character. Thus textile industries are constantly found congregated in the neighbourhood of mining and engineering industries, in some cases having been attracted by almost imperceptible steps; in others, as for instance at Barrow, having been started deliberately on a large scale in order to give variety of employment in a place

> where previously there had been but little demand for the work of women and children.

Marshall (1890, Book IV.X.12) also identifies a statistical basis for efficient coagglomeration:

> A district which is dependent chiefly on one industry is liable to extreme depression, in case of a falling-off in the demand for its produce, or of a failure in the supply of the raw material which it uses. This evil again is in a great measure avoided by those large towns or large industrial districts in which several distinct industries are strongly developed. If one of them fails for a time, the others are likely to support it indirectly; and they enable local shopkeepers to continue their assistance to workpeople in it.

These passages, while without formal theory, are impressively precise in describing microfoundations for coagglomeration. They do not, however, have much to say about how a location equilibrium is reached or about coagglomeration as a general phenomenon.

Vernon (1960) and Jacobs (1969) are more interested in diverse cities, so their analysis has more to say about coagglomeration. In Vernon (1960), much attention is given to the nature of "increasing returns industries," those that can benefit from the size and diversity of New York's markets. In this analysis, co-location arises because of similarities between the industries (they need similar environments) rather than their differences (drawing on male vs. female workers, as in Marshall). In Jacobs (1969), "new work" arises from prior activities. For example, leather working arises from agriculture in early settlements, and sliding door manufacturing arises from airplane manufacturing in the modern era. In this case, there is little discussion of entrepreneurs choosing locations. Instead, co-location arises from the process of technological advance. Again, there is no general treatment of coagglomeration or of location equilibrium.

The theoretical literature on microfoundations has formalized these ideas, and other forces that might encourage coagglomeration as well. See Duranton and Puga (2004) for a survey. For the most part, this literature does not consider the forces that might lead to coagglomeration, and when it does most of the analysis deals with the polar cases of specialization and diversity. Krugman's (1991a) analysis of labor market pooling is an interesting exception. It formalizes the Marshallian treatment of risk sharing. See also the extension in the Duranton-Puga survey. Another interesting exception is the Duranton-Puga analysis of "nursery cities," which provides a formal model of innovation in the spirit of Jacobs and Vernon. Its key implications for coagglomeration are that young firms and industries benefit more from diversity (coagglomeration with a lot of other industries) than do their more mature counterparts.

The theoretical literature that our paper draws from deals with systems-of-cities. See Abdel-Rahman and Anas (2004) for a survey. As noted above, Henderson (1974) is seminal. It presents analysis that can be taken as a model of the formation of a system of specialized cities or a system of diverse cities. In a series of important papers, including Abdel-Rahman and Fujita (1993) and Abdel-Rahman (1996), Abdel-Rahman and co-authors have considered the circumstances under which a specialized or a diverse system of cities will arise. These models have explicit microfoundations. In Abdel-Rahman and Fujita (1993), it is explicitly assumed that cities are formed by developers. In Abdel-Rahman (1996, p. 10), the analysis focuses on efficient allocations, motivating this by noting that "...any city size larger than [the efficient size] will not be sustained, since it will pay to form smaller sized cities that provide higher utility", which would be true if there were developers. The papers, thus, are more concerned with the issue of when equilibrium will be specialized or diverse than about its efficiency. They are also specified as two good models, which means that they cannot deal with intermediate coagglomeration, where a city produces more than one good but not all goods in the economy.[3]

Finally, it is worth saying a little more about what coagglomeration means. We have thus far employed standard usage by referring to the coagglomeration of industries. And we will continue to employ this usage below. However, the joint location of firms in the same NAICS industry is certainly not the only pattern of coagglomeration that might arise. Duranton and Puga (2005) present a model showing that changes in technology can lead to an urban system based on functional specialization rather than sectoral or industry-based specialization. Various papers on urban labor markets have considered the possibilities of complementarities between workers with various levels of education

[3] Because of the monopolistic competition specification employed, models in the New Economic Geography tradition (i.e., Krugman (1991b)) typically consider cities that produce the entire range of goods.

(Ciccone and Peri (2006) or horizontally differentiated types of skill (Bacolod et al, 2008 and 2009). All of this means that our paper's results can be interpreted as reflecting on various sorts of coagglomeration.

In sum, the literature has focused more on the microfoundations of coagglomeration than on the implications of individual migration for the efficiency of coagglomeration. The folk wisdom is that despite the various increasing return issues associated with city formation, coagglomeration is likely to be efficient. The rest of the paper will employ a system-of-cities model to consider this folk wisdom.

III. Migration

A. Basics

This section develops a basic model of a system of cities with complementarities in production between workers in different industries. There are two types of workers and two industries, indexed by $i \in \{1,2\}$. Only type-i workers are employed by firms in industry i. The number of type-i workers in city $j \in \{1,2,\ldots,J\}$ is $n_{ij} \geq 0$. The population of city j is $N^j = n_{1j} + n_{2j}$. The aggregate number of type-i workers in the economy is $N_i = \sum_j n_{ij}$. A generalization to many types of workers, and thus many industries, is presented in the next section.

Firms and industries are perfectly competitive. Output for a firm in industry i in city j is

$$q_{ij} = g_i(n_{1j},n_{2j})m_{ij}, \; i \in \{1,2\}, \tag{III.1}$$

where m_{ij} is employment of an individual firm in industry i. Output per worker $g_i(\cdot)$ satisfies $\partial g_i/\partial n_{1j} > 0$, $\partial g_i/\partial n_{2j} > 0$, $\partial^2 g_i/\partial n_{1j}{}^2 < 0$, $\partial^2 g_i/\partial n_{2j}{}^2 < 0$, and $\partial^2 g_i/\partial n_{1j}\partial n_{2j} = \partial^2 g_i/\partial n_{2j}\partial n_{1j} > 0$. These assumptions imply that $g_i(\cdot)$ is strictly quasi-concave. Thus, we assume that there are external economies both within and between the worker types. More specifically, we assume that output per worker is an increasing and concave function of the number of workers of either type, holding the number of workers of the other type fixed, and that the marginal product of a type 1 (type 2) worker is increasing in the number of type 2 (type 1) workers in the city. These complementarities are the driving force behind coagglomeration in our model. Finally, we assume that, for $n_{1j} = n_{2j}$, $\partial g_i/\partial n_{ij} > \partial g_i/\partial n_{kj}$, $k \neq i$. This condition ensures that a city that maximizes the utility of a type-i worker contains more type-i workers than type-k workers.

The profit of a firm in industry i in city j is

$$\pi_{ij} = p_i g_i(n_{1j},n_{2j})m_{ij} - w_i m_{ij}, \tag{III.2}$$

where p_i is the exogenous price of industry i's output and w_i is the wage paid to a type-i worker. Profit maximization implies

$$w_i(n_{1j},n_{2j}) = p_i g_i(n_{1j},n_{2j}). \tag{III.3}$$

This wage function inherits the properties of the production function shifter, so $\partial w_i/\partial n_{1j} > 0$, $\partial w_i/\partial n_{2j} > 0$, $\partial^2 w_i/\partial n_{1j}^{\,2} < 0$, $\partial^2 w_i/\partial n_{2j}^{\,2} < 0$, $\partial^2 w_i/\partial n_{1j}\partial n_{2j} = \partial^2 w_i/\partial n_{2j}\partial n_{1j} > 0$, implying strict quasi-concavity, and, for $n_{1j} = n_{2j}$, $\partial w_i/\partial n_{ij} > \partial w_i/\partial n_{kj}$, $k \neq i$. Since the wage function captures the key elements of the technology, we will not need to refer explicitly to the behavior of firms in the remainder of the paper.

The utility of a type-i worker in city j is the excess of the wage over the cost of living in the city,

$$V_i(n_{1j},n_{2j}) = w_i(n_{1j},n_{2j}) - c(n_{1j} + n_{2j}), \tag{III.4}$$

where the cost function $c(\cdot)$ is increasing and convex. Note that $c(\cdot)$ treats worker types and cities symmetrically; we are assuming that crowding or congestion is anonymous and that all city sites are identical. This model closely parallels standard competitive models of city systems with single industry cities (see Abdel-Rahman and Anas, (2004)).

B. Type optimal cities

Under the assumptions that we have made about $w_i(\cdot)$ and $c(\cdot)$, $V_i(\cdot)$ has the familiar inverted-U shape as a function of either of the worker types, holding the number of workers of the other type fixed. For example, for fixed n_{2j}, utility $V_i(n_{1j},n_{2j})$ is increasing in n_{1j} for $n_{1j} < n^{*i}_{1j}(n_{2j})$, and decreasing for $n_{1j} > n^{*i}_{1j}(n_{2j})$, where $n^{*i}_{1j}(n_{2j}) = \text{argmax}_{n1j}V_i(n_{1j},n_{2j})$ satisfies

$$\partial V_i(n_{1j},n_{2j})/\partial n_{1j} \equiv \partial w_i(n_{1j},n_{2j})/\partial n_{1j} - c'(n_{1j} + n_{2j}) = 0. \qquad \text{(III.5)}$$

The second-order condition, $\partial^2 w_i/\partial n_{1j}{}^2 - c''(n_{1j} + n_{2j}) < 0$, is satisfied globally under our assumptions. Applying the implicit function theorem to (III.5) yields

$$dn^{*i}_{1j}/dn_{2j} = - [\partial^2 w_i/\partial n_{1j}\partial n_{2j} - c''(n_{1j} + n_{2j})]/[\partial^2 w_i/\partial n_{1j}{}^2 - c''(n_{1j} + n_{2j})]. \qquad \text{(III.6)}$$

(III.6) implies that $dn^{*i}_{1j}/dn_{2j} > (<) 0$ as $\partial^2 w_i/\partial n_{1j}\partial n_{2j} > (<) c''(n_{1j} + n_{2j})$. Intuitively, if complementarity is sufficiently strong relative to the cost of accommodating population in a city, then an increase in the number of type 2 workers will lead to an increase in the utility maximizing number of type 1 workers, and hence to an increase in city size. We assume that $dn^{*i}_{1j}/dn_{2j} > 0$ throughout the paper. Obviously, the size and composition of a city are co-determined in this setting.

We use the term *type-i optimal city* to refer to the $(n^{*i}_{1j},n^{*i}_{2j})$ pair that maximizes $V_i(n_{1j},n_{2j})$. The first-order conditions that characterize a type-i optimal city are (III.5) and

$$\partial V_i(n_{1j},n_{2j})/\partial n_{2j} \equiv \partial w_i(n_{1j},n_{2j})/\partial n_{2j} - c'(n_{1j} + n_{2j}) = 0. \qquad \text{(III.7)}$$

Thus, a type-i optimal city contains the two types of workers in magnitudes such that the marginal external product of a worker of either type equals the marginal cost of accommodating population in the city. Since congestion cost is anonymous, this implies that, at the type-optimum, adding another worker of either type has the same, positive impact on productivity of workers in industry i:

$$\partial w_i(n_{1j},n_{2j})/\partial n_{1j} = \partial w_i(n_{1j},n_{2j})/\partial n_{2j}. \qquad \text{(III.8)}$$

Since $w_i(\cdot)$ is strictly quasi-concave, (III.8) and the assumption that, for $n_{1j} = n_{2j}$, $\partial w_i/\partial n_{ij} > \partial w_i/\partial n_{kj}$, $k \neq i$, imply that $n_{ij} > n_{kj}$ in a type-i optimal city. The second-order conditions require concavity in the numbers worker of either type, as discussed above, and

$$[\partial^2 w_i/\partial n_{1j}{}^2 - c''(n_{1j} + n_{2j})][\partial^2 w_i/\partial n_{2j}{}^2 - c''(n_{1j} + n_{2j})] - [\partial^2 w_i/\partial n_{1j}\partial n_{2j} - c''(n_{1j} + n_{2j})]^2 > 0. \quad \text{(III.9)}$$

Type optimal cities for both types (labeled 1 and 2, respectively) are illustrated in Figure 1, together with the elliptical level sets of each of the utility functions $V_i(n_{1j},n_{2j})$ (labeled V_1 and V_2, respectively). Since this section focuses on allocations in which all cities are identical, we can simplify the notation by dropping the j subscript. For the remainder of this section, $V_1(n_1,n_2)$ will denote the utility of a type-1 worker, and $V_2(n_1,n_2)$ will denote the utility of a type-2 worker.

C. Constrained Pareto efficient coagglomeration

An unconstrained efficient coagglomerated city maximizes the utility of workers of one type subject to the constraint that all workers of the other type achieve a fixed utility level. Without loss of generality we can characterize a Pareto efficient allocation by choosing n_1 and n_2 to maximize

$$V_1(n_1,n_2) \equiv w_1(n_1,n_2) - c(n_1 + n_2) \;\; \text{s.t} \;\; V_2(n_1,n_2) \equiv w_2(n_1,n_2) - c(n_1 + n_2) = U. \quad \text{(III.10)}$$

Denoting the multiplier by λ, the first order conditions for this problem require that the constraint be satisfied, and that,

$$\partial w_1/\partial n_1 - c'(n_1 + n_2) + \lambda[\partial w_2/\partial n_1 - c'(n_1 + n_2)] = 0, \quad \text{(III.11a)}$$

$$\partial w_1/\partial n_2 - c'(n_1 + n_2) + \lambda[\partial w_2/\partial n_2 - c'(n_1 + n_2)] = 0. \quad \text{(III.11b)}$$

(III.11a) and (III.11b) in turn imply,

$$(\partial w_1/\partial n_1 - c'(n_1 + n_2))/(\partial w_1/\partial n_2 - c'(n_1 + n_2)) = (\partial w_2/\partial n_1 - c'(n_1 + n_2))/(\partial w_2/\partial n_2 - c'(n_1 + n_2)). \quad \text{(III.12)}$$

(III.11) and the constraint characterize the contract curve – the set of Pareto efficient city sizes and compositions – for different values of the utility level U. The contract curve connects the type optimal cities for the two types. Recall that our assumptions about the technology ensure that $n^{*1}_1 > n^{*1}_2$ and $n^{*2}_2 > n^{*2}_1$ – type-i workers are in the majority in a type-i optimal city. Under these conditions, the contract curve slopes down, as shown in Figure 2.

While all points on the contract curve are Pareto efficient, not all points are consistent with the given aggregate numbers of type-1 and type-2 workers in the economy. Restricting our attention to symmetric allocations and assuming that the economy is large enough that integer problems can be ignored, allocations that satisfy the constraints $N_i = \sum_j n_{ij}$ must lie on the "ratio line"

$$n_2 = (N_2/N_1)n_1. \quad \text{(III.13)}$$

The intersection of the ratio line in (III.13) and the contract curve characterized by (III.12) defines a symmetric constrained Pareto efficient city size and composition (n^{CPE}_1, n^{CPE}_2) for this economy. Such an allocation will exist provided $n^{*2}_2/n^{*2}_1 > N_2/N_1 > n^{*1}_2/n^{*1}_1$ – that is, provided the ratio line intersects the contract curve between the type optimal allocations. The constrained Pareto efficient allocation is labeled CPE in Figure 2.

D. Migration and equilibrium coagglomeration

Following Henderson (1974) and others, we assume that city sizes and compositions are determined by individual migration. Under individual migration, the conditions that must be satisfied in order for an allocation (n_1,n_2) to constitute a symmetric Nash equilibrium are:

i. Utility maximization. There must be nowhere a worker could move – either to another city or to an unsettled location – that offers a higher utility. In particular, it must be the case that that both types are at least as well off as they would be under autarky: $V_i(n_1,n_2) \geq V_i(0,0)$, i = 1,2.

ii. Adding up. An equilibrium must satisfy the aggregate population constraints and therefore lie on the ratio line.

iii. Stability. Neither type of worker has an incentive to move. This requires $\partial V_i(n_1,n_2)/\partial n_i \leq 0$ for all i. Thus, an equilibrium must be on the downward sloping portion of $V_i(n_1,n_2)$ for each type. Recalling (III.5), and invoking symmetry, stability requires $n_1 > n^{*1}{}_1(n_2)$ and $n_2 > n^{*2}{}_2(n_1)$.

These conditions are standard. The utility maximization condition requires that no agent can migrate to another city that would give higher utility. This condition does not require that there be no conceivable city that would give higher utility. The second condition is obvious. The third condition requires that utility not be increasing for any agent in in the number of own-type agents. In a model with one type of agent, this condition would require that the equilibrium population be on the non-increasing part of the utility frontier.

The first point to note is that the constrained Pareto efficient allocation in Figure 2 is not an equilibrium because it is not stable. At $(n^{CPE}{}_1,n^{CPE}{}_2)$, both types have an incentive to move (upward, toward point 2, for the type 2's and to the right, toward point 1, for the type 1's), since an increase in n_i will result in higher utility for a type-i worker. As noted above, stability requires $n_1 > n^{*1}{}_1(n_2)$ and $n_2 > n^{*2}{}_2(n_1)$. To see this, note that all points along $n^{*2}{}_2(n_1)$ satisfy $\partial V_2/\partial n_2 = \partial w_2/\partial n_2 - c'(n_1 + n_2) = 0$. Then, holding n_1 fixed, an increase (decrease) in n_2 must decrease (increase) $\partial V_2/\partial n_2$, since $\partial^2 V_2/\partial n_2{}^2 = \partial w_2/\partial n_2 - c'(n_1 + n_2) < 0$. Thus, points above $n^{*2}{}_2(n_1)$ satisfy $\partial V_2/\partial n_2 < 0$ and are stable from the perspective of a type-2 worker, while points below $n^{*2}{}_2(n_1)$ satisfy $\partial V_2/\partial n_2 > 0$ and are unstable. Similarly, points to the right of $n^{*1}{}_1(n_2)$ satisfy $\partial V_1/\partial n_1 < 0$, while points to the left of $n^{*1}{}_1(n_2)$ satisfy $\partial V_1/\partial n_1 > 0$. Thus, when a constrained Pareto efficient allocation exists, that is, when the ratio line intersects the contract curve between the type optimal cities, the efficient allocation must lie in the unstable region where both types have an incentive to move, and therefore cannot be an equilibrium. Candidates for a symmetric equilibrium that satisfy the aggregate population constraints consist of the portion of the ratio line above the intersection of $n^{*1}{}_1(n_2)$ and $n^{*2}{}_2(n_1)$, as shown in Figure 3.

It is striking that the constrained Pareto efficient allocation is not an equilibrium in this setting. This result is in marked contrast with the traditional systems of cities models that we have adapted here to deal with coagglomeration. The intuition for the difference is that there are externalities present in the coagglomeration case that are not present in models of a single industry's agglomeration. A worker of type-1, for instance, would enjoy a higher level of utility than at the CPE by joining a city already at CPE population levels. This would, however, reduce the utility enjoyed by a type-2 worker. This is because the type-2 worker incurs the same additional congestion as a type-1 worker, but receives a smaller benefit of agglomeration.

The second point to note is that, since utility for both types declines as one moves up the ratio line for (n_1,n_2) such that $n_1 > n^{*1}{}_1(n_2)$ and $n_2 > n^{*2}{}_2(n_1)$, the constraints $V_i(n_1,n_2) \geq V_i(0,0)$, i = 1,2, will eventually bind. This will limit the range of equilibrium allocations. Let $n^{iMax}{}_2$ be the smallest value of n_2 such that $V_i((N_1/N_2)n_2,n_2) = V_i(0,0)$, and let $n^{Max}{}_2 = \mathrm{Min}\{n^{1Max}{}_2,n^{2Max}{}_2\}$. Then, the set of symmetric Nash equilibria consists of the portion of the ratio line between the intersection of $n^{*1}{}_1(n_2)$ and $n^{*2}{}_2(n_1)$ and $(n^{Max}{}_1,n^{Max}{}_2)$, where $n^{Max}{}_1 = (N_1/N_2)n^{Max}{}_2$. Figure 3 shows the case where the autarky constraint binds for the type 2's first.

Third, since utilities for both types decline as one moves up the ratio line for $n_1 > n^{*1}{}_1(n_2)$ and $n_2 > n^{*2}{}_2(n_1)$, the intersection of the ratio line and $n^{*2}{}_2(n_1)$ is Pareto dominant in the sense that it offers higher utility than any other equilibrium allocation. For this reason, this intersection is labeled $(n^{PDSNE}{}_1,n^{PDSNE}{}_2)$ in Figure 2.

This analysis closely parallels conventional treatments of the role of individual migration in an open system of single industry cities. Moving up the ratio line is analogous to increasing the population of a single industry city beyond the size that the agglomeration economies versus congestion costs tradeoff would dictate. The key result here, again closely following the single industry case, is that individual migration leads to excessive city sizes. Stated somewhat more carefully, the key result is that, with free individual migration, equilibrium coagglomeration is not efficient. In this open system setting, where we have required that equilibria satisfy the aggregate population constraints, the mix of types in a city is fixed, and hence the same in the efficient and equilibrium allocations. However, individual migration leads to excessive numbers of both types of workers.

E. Asymmetric equilibrium

In addition to the symmetric interior allocations considered above, there can be asymmetric corner equilibria as well. Suppose that the system is comprised of two types of specialized cities, one with only type-1 workers and the other with only type-2 workers. Let all the type-1 cities have population n_1^+, while the type-2 cities all have population n_2^+. Under what circumstances are such specialized cities be consistent with the equilibrium conditions characterized above? First, the stability condition precludes $n_1^+ < n^{*1}{}_1(0)$ and $n_2^+ < n^{*2}{}_2(0)$. Second, the cities must provide at least as great utility as under autarky, $V_1(n_1^+,0) \geq V_1(0,0)$ and $V_2(0,n_2^+) \geq V_2(0,0)$. This defines a range of values of n_1 and n_2 consistent with equilibrium.

The welfare properties of these specialized-city equilibria are straightforward. The equilibria are consistent with the Henderson (1974) analysis in the sense that there exist multiple equilibrium city sizes offering a range of utility levels. At worst, the cities provide the autarky level. At best, they provide the highest level of utility that can be attained in a specialized city, $V_1(n^{*1}{}_1(0),0)$ or $V_2(0, n^{*2}{}_2(0))$ respectively. Unlike the analysis of Henderson, these latter two utility levels do not correspond to an optimal city, since optimality requires both types of agent in a city by assumption here (i.e., interior type optimal cities). The global utility comparison between a specialized equilibrium and an interior equilibrium is ambiguous. The utility in a specialized equilibrium may be greater or smaller than in the interior equilibrium.[4]

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[4] We are continuing to ignore integer problems. If we were not to do so, then a specialized equilibrium would have additional requirements. Suppose that the number of type-1 cities is M_1, while the number of type-2 cities is M_2. We then require that in equilibrium $M_1n_1^+ = N_1$ and $M_2n_2^+ = N_2$, ensuring that the aggregate population of type-1 and type-2 workers be located in some city.

In fact, one can construct hybrid equilibria that include both specialized and mixed cities. If there are M_1 cities containing type-1 workers and M_2 cities containing type-2 workers, then the constraint on mixed cities is that they be located on a residual ratio line, giving possible mixtures having removed the workers located in the specialized cities. Suppose that the cities containing type-1 workers have populations n_1^+ and those containing type-2 workers have populations n_2^+. Then the mixed cities must have populations that keep the ratio of type-1 to type-2 workers at $N_1 - M_1 n_1^+$ to $N_2 – M_2 n_2^+$. Utility for type-1 workers must be equal in any city containing type-1 workers. Similarly, utility for type-2 workers must be equal in any city containing type-2 workers. Clearly, there are many potential equilibria of this sort. The most favorable of these for both types of worker would give the level of utility at the intersection of the $n_1(n_2)$ and $n_2(n_1)$ loci.

In this section's model, coagglomeration is valuable in the sense that both types prefer locating in cities containing a mix of agent types. The key conclusion of this entire section is this: under the standard assumptions in models of systems of cities, any equilibrium where cities contain both types of agent is inefficient. It must contain more of both types than the constrained Pareto efficient allocation. And the equilibrium may be inferior to an allocation where cities specialize, which is already inefficient by assumption. Coagglomeration, thus, suffers from a more serious version of the migration pathology characteristic of system of cities models. All of the analysis has been couched in a model with two worker types. In the next section, we will work with a model of many worker types and industries. The key result will be that the equilibrium mix of types may be inefficient as well.

IV. Matching

A. General case

Suppose that there are I types of workers with aggregate populations N_i. This allows very rich possible patterns of coagglomeration. Industries could potentially locate in specialized cities, in completely diverse cities, and in various sorts of intermediate cases. Despite the range of possible outcomes, characterizing the equilibrium is a straightforward extension of the previous I = 2 analysis.

Let cities be indexed by j = {0,1,2,...J}. The occupied cities are those for which $j \in \{1,2,...,J\}$. City 0 will by convention be unoccupied. It will be relevant for the equilibrium conditions because in addition to equal utility among occupied cities, equilibrium must also involve the condition that no worker would deviate to an unoccupied city. Let the population of city j be given by N^j. Let J^i denote the set of cities where $n_{ij} > 0$. Let $J\backslash J^i$ denote the complement of this (e.g., the set of cities where $n_{ij} = 0$).

In this situation, equilibrium requires that there exist utility levels U_i^* such that

$$w_i(n_{1j},n_{2j},...n_{Ij}) - c(N^j) = U_i^* \;\forall\, j \in J^i \text{ and for } i \in \{1,2,...,I\} \qquad \text{(IV.1)}$$

$$w_i(n_{1j},n_{2j},...n_{Ij}) - c(N^j) \leq U_i^* \;\forall\, j \in J\backslash J^i \text{ and for } i \in \{0,2,...,I\} \qquad \text{(IV.2)}$$

$$\Sigma_j n_{ij} = N_i \text{ for } i \in \{1,2,...,I\} \qquad \text{(IV.3)}$$

$$\partial/\partial n_i \,[w_i(n_{1j},n_{2j},...,n_{Ij}) - c(N^j)\,] \leq 0 \;\forall\, j \in J^i \text{ and for } i \in \{1,2,...,I\} \qquad \text{(IV.4)}$$

(IV.1) requires that all cities that contain a particular type of worker must offer that worker the same level of utility. (IV.2) requires that the cities that do not contain a particular type of worker must offer weakly lower utility. This includes the empty city 0, so this condition also requires that no worker be able to obtain a higher level of utility under autarky. (IV.3) is the aggregate population constraint. (IV.4) is the stability condition. As in the previous analysis of the I = 2 case, it requires that workers be on the downward sloping part of their utility possibilities frontiers.

Although (IV.1) through (IV.4) characterize an equilibrium precisely and directly, there is not much more that can be learned in the general case. Our objective here is to explore the efficiency of various equilibrium coagglomeration patterns. In order to achieve that objective, we will work with a series of stylized examples that respect the structure of the general problem but are much more transparent.

B. Complete mixing and complete separation

We begin with the case of complete mixing. Suppose that there are four worker types (I = 4) and two occupied cities (J = 2). With aggregate populations N_i, the equal utility condition (IV.1) and the adding up condition (IV.3) require

$$w_i(N_1/2,N_2/2,N_3/2,N_4/2) - c(N/2) = U_i^* \text{ for } i = \{1,2,3,4\}. \quad \text{(IV.5)}$$

Both of the cities are occupied by all four types, so the no-incentive-to-move condition (IV.2) reduces to

$$w_i(0,0,0,0) - c(0) \leq U_i^* \text{ for } i = \{1,2,3,4\}. \quad \text{(IV.6)}$$

The stability conditions require that

$$\partial/\partial n_i[w_i(N_1/2,N_2/2,N_3/2,N_4/2) - c(N/2)] \leq 0 \text{ for } i = \{1,2,3,4\}. \quad \text{(IV.7)}$$

This equilibrium involves completely diverse coagglomeration, which is parallel to the diverse cities case in Abdel-Rahman and Fujita (1993).

The contrasting completely specialized equilibrium has four occupied cities (J = 4). The utility conditions are trivial:

$$w_1(N_1,0,0,0) - c(N_1) = U_1^* \quad \text{(IV.8a)}$$
$$w_2(0,N_2,0,0) - c(N_2) = U_2^* \quad \text{(IV.8b)}$$
$$w_3(0,0,N_3,0) - c(N_3) = U_3^* \quad \text{(IV.8c)}$$
$$w_4(0,0,0,N_4) - c(N_4) = U_4^* \quad \text{(IV.8d)}$$

The no-incentive-to-move condition requires that no individual prefers autarky, see above, and that no individual of any type prefers to coagglomerate by moving to a specialized city inhabited by another type of worker. For worker type-1, this requires that

$$w_1(0,N_2,0,0) - c(N_2) \leq U_1^* \quad \text{(IV.9a)}$$

$$w_1(0,0,N_3,0) - c(N_3) \leq U_1^* \quad \text{(IV.9b)}$$

$$w_1(0,0,0,N_4) - c(N_4) \leq U_1^* \quad \text{(IV.9c)}$$

The conditions for the other types of worker are parallel. The stability condition for worker 1 is identical to the above, except for the population level at which the derivative is evaluated:

$$\partial/\partial n_1[w_i(N_1,0,\ 0,0) - c(N_1)] \leq 0 \text{ for } = i\ \{1,2,3,4\}. \quad \text{(IV.10)}$$

The stability conditions for the other workers are also parallel. There are other forms that the equilibrium might take. They will be discussed in detail below.

The crucial point to take from this characterization of the two sorts of equilibria is that individual migration does not impose very strong discipline on the efficiency of multi-type coagglomeration. Complete diversity can be an equilibrium as long as the equilibrium allocations offer workers higher utility than under autarky, a very weak condition. Similarly, complete separation can emerge as an equilibrium as long as it is preferred to autarky and to any possible first step towards pairwise coagglomeration. Complete diversity may be an equilibrium even if it is Pareto inferior to separation. And vice-versa. Equilibrium requires only that no individual be able to obtain higher utility by moving.

In the mixed cities case, there is a further possible inefficiency that is exactly parallel to the migration pathology discussed in Section III. Suppose now that there are an arbitrary number, J, of identically mixed cities in equilibrium. In this case, an interior unconstrained Pareto-optimum must satisfy the higher order equivalent of the equal marginal rate of technical substitution condition given in (III.11).[5] These conditions define a contract surface, combinations of city sizes and compositions that maximize the

[5] For the record, the conditions for J = 4 require that $(dn_i/dn_j)|_{Vk = \text{constant}}$ be equal for all $k \in \{1,2,3,4\}$ and for all i and $j \in \{1,2,3,4\}$, $i \neq j$. So, for example, for i = 1 and j = 2, the conditions are $(dn_1/dn_2)|_{V1 = \text{constant}} = (dn_1/dn_2)|_{V2 = \text{constant}} = (dn_1/dn_2)|_{V3 = \text{constant}} = (dn_1/dn_2)|_{V4 = \text{constant}}$. Eliminating reciprocal cases, there are six sets of such equalities for J = 4.

utility of one type subject to fixed utility levels for the others. Following the analysis for J = 2, we can define the type-optimal points n^{*i}_k for all $k \neq i$. If the contract surface has downward sloping level sets for all pairs and the ratio hyperplane intersects the contract surface, then there exists a unique constrained Pareto efficient point where the contract surface intersects the ratio hyperplane, and is therefore consistent with the population adding up constraint.

In this more general case, the argument from Section III that the constrained Pareto optimum is not consistent with equilibrium continues to hold. There exists a subset of the ratio hyperplane in which allocations are stable ($\partial w_i/\partial n_i - c'(\cdot) \leq 0$ for all i) and utility is greater than under autarky ($V_i^* > V_i(0)$). The traditional result in a one-worker-type model is that cities cannot have populations lower than the level that maximizes worker utility. Larger populations can be sustained as equilibria as long as they offer utility greater than under autarky (in a newly formed city). The result here is the stronger one that completely diverse cities must be larger than the constrained Pareto optimal level (where smaller means along the ratio hyperplane).

C. Matching

The above situation has considered all or nothing coagglomeration. As noted in the Introduction, there are a lot of intermediate possibilities between complete diversity and complete specialization, or, put another way, between urbanization and localization. To explore these possibilities, continue to suppose that there are four worker types (I = 4) and two occupied cities (J = 2). Suppose that the type 1 and 2 workers coagglomerate in city 1, while the type 3 and 4 workers coagglomerate in city 2.

In this case, the equal utility and adding up conditions become:

$$w_1(N_1,N_2,0,0) - c(N_1+N_2) = U_1^* \qquad \text{(IV.11a)}$$

$$w_2(N_1,N_2,0,0) - c(N_1+N_2) = U_2^* \qquad \text{(IV.11b)}$$

$$w_3(0,0,N_3,N_4) - c(N_3+N_4) = U_3^* \qquad \text{(IV.11c)}$$

$$w_4(0,0,N_3,N_4) - c(N_3+N_4) = U_4^* \qquad \text{(IV.11d)}$$

The no-incentive to move conditions are:

$$w_1(0,0,N_3,N_4) - c(N_3+N_4) \leq U_1^* \tag{IV.12a}$$

$$w_2(0,0,N_3,N_4) - c(N_3+N_4) \leq U_2^* \tag{IV.12b}$$

$$w_3(N_1,N_2,0,0) - c(N_1+N_2) \leq U_3^* \tag{IV.12c}$$

$$w_4(N_1,N_2,0,0) - c(N_1+N_2) \leq U_4^* \tag{IV.12d}$$

The stability conditions, and the requirements that utility be greater than that available under autarky, are exactly parallel to those given above.

We have already observed that migration is a weak instrument of competitive discipline. In this case, the coagglomerated equilibrium need not be Pareto superior to separation or to complete mixing. In fact, the coagglomerated equilibrium described above need not be Pareto superior to other coagglomerated equilibria. Consider the alternate allocation of worker types to cities with worker types 1 and 3 in one city and worker types 2 and 4 in the other. The conditions for this situation to be an equilibrium are exactly parallel to those laid out immediately above. Now suppose that

$$w_1(N_1,N_2,0,0) - c(N_1+N_2) \geq w_1(N_1,0,N_3,0) - c(N_1+ N_3) \tag{IV.13a}$$

$$w_2(N_1,N_2,0,0) - c(N_1+N_2) \geq w_2(0,N_2,0,N_4) - c(N_2+N_4) \tag{IV.13b}$$

$$w_3(0,0,N_3,N_4) - c(N_3+N_4) \geq w_3(N_1,0,N_3,0) - c(N_1+ N_3) \tag{IV.13c}$$

$$w_4(0,0,N_3,N_4) - c(N_3+N_4) \geq w_4(0,N_2,0,N_4) - c(N_2+N_4) \tag{IV.13d}$$

In this situation, coagglomeration between types 1 and 2 and between types 3 and 4 is preferable by all to coagglomeration between types 1 and 3 and between types 2 and 4. But the inefficient coagglomeration is consistent with equilibrium.

This inefficient matching is parallel to Bewley's (1981) analysis of sorting in a system of local governments. He presents a number of situations where efficiency fails for local public goods, despite the presence of mobility that might be thought to allow the sort of shopping that takes place in Tiebout (1956). Bewley's general result is that efficiency requires more than simply allowing households to "vote with their feet." It requires also, among other things, entrepreneurial incentives on the part of public goods providers.[6]

D. Discussion

This section has extended the standard system of cities model in a natural way to look at coagglomeration among multiple types. Parallel to the last section's results on two-type coagglomeration, inefficient coagglomeration is consistent with equilibrium. This section will argue that the inefficiency is not an artifact of an unrealistic assumption in the model but a direct consequence of the well-known weakness of migration as a competitive mechanism.

The first point to make is that the various demonstrations of the possibility of inefficiency do not depend on the number of industries. The choice of J = 4 for the examples was purely for illustrative purposes. It is easy to see how excess coagglomeration could persist in more general models. It is equally easy to see how unrealized coagglomeration economies could exist in more general models as well.

The second issue to consider is the dynamics of city formation. In the analysis here, we have characterized equilibrium allocations that cannot be upset by individual migration decisions. It is natural to consider how such an equilibrium might arise. Articulating the dynamics more fully might suggest that some of the many equilibria are more plausible than others. The standard dynamic story that is told in the traditional single industry systems of cities literature (i.e., Henderson, 1974) involves one-by-one location decisions. In this heuristic dynamic, workers arrive one at a time and migrate to the city that offers the highest utility. The relationship of utility to population is assumed to have an inverted U-shape. Workers then fill up the first city to the point that the city offers maximum utility. They continue to migrate to the first city beyond this point, since it offers greater utility than under autarky (i.e., in a city of size zero). This continues until the city offers no greater utility than under autarky. The process continues. This

[6] For extensions of Bewley's normative analysis, see Ellickson et al (1999) and the many papers surveyed by Scotchmer (2002). As noted by Scotchmer, the literature contains both welfare theorems, showing that a price system can lead to efficiency, as well as parallel results showing that constraints on pricing lead to failures of efficiency. Our analysis of co-agglomeration obviously falls in the latter category.

dynamic story supports the most inefficient of the many possible inefficient equilibria in a one-type system-of-cities model.

The parallel here would be to assume that workers arrive one-at-a-time. If their arrival occurred in proportion to population shares, then the outcome would be very similar to the one-type outcome. Workers would fill the first city to the point where the utility in the mixed city equals the level under autarky. This would give a mixed city even if such a city were Pareto inferior to some other pattern of coagglomeration or to specialization.

This is not the only possible dynamic. One could instead consider a regime where workers arrive according to a multinomial random process, with probabilities equal to population shares. As long as the migration continued to be one-by-one, the potential for inefficiency would remain. Another possibility is to assume that the workers arrive instead in lumps, where the lumps correspond to firms. In this case, if the firms were large or in the multinomial model if there were a run of firms of a particular type, it could break the pattern of mixed cities that has arisen thus far.

Although this one-by-one dynamic of the urbanization of a national economy has a lot of appeal, it is not the only dynamic process at work. Another involves changes in the nature of agglomeration economies. These changes certainly have occurred in the past, and they seem to be occurring with great frequency now. For instance, it was once the case that a firm's back office activities (e.g., accounting and billing) needed to be located near the firm's front office. Improvements in communications technology have changed this relationship, and one now observes back offices located in different cities than their affiliated front offices. One way to interpret the demonstrations of sustainable inefficiencies is as capturing circumstances that could have arisen as part of the process of technological change and that would not have upset the existing pattern of coagglomeration. So the inefficiency may not be arising as a result of active migration but instead as a result of a failure of adaptation. While we are not able to provide direct evidence of this failure, it does seem to be quite plausible.

As noted previously, the failure to realize efficient coagglomeration in this analysis depends entirely on the assumption that individual migration is the only force that might correct an inefficient pattern of coagglomeration. An alternative approach

(Henderson, (1974)) is to suppose that cities are formed by "developers." In some conceptions, developers are private agents seeking to maximize land value. In others, they are city or regional governments attempting to maximize local welfare. These agents must have the power to exclude workers from the cities that they manage and the ability to offer incentive compatible city contracts (see Helsley and Strange, 1990, for formal details), and this can have profound impacts on the realization of efficient allocations of population. In Section III's two-type model, the ability to restrict migration can allow realization of the constrained Pareto efficient allocation. In this section's analysis, developers could engineer the group deviations and exclusions that would realize efficiency.

So the question becomes: are there real world institutions that correspond to such powerful city developers? Our answer is: no. Ascribing entrepreneurial incentives to city governments seems to be at odds with actual city government behavior. Even if local governments were fully entrepreneurial, the fragmentation of local government means that exclusion of the sort required for efficiency is not possible. Private developers are rarely large enough to be reasonably seen as controlling the development of entire cities. See Helsley and Strange (1997) for a formal model of limited developers and institutional details, including the few situations where developers have come closest to total control. Furthermore, as shown in Helsley and Strange (1994), if developers were powerful enough to compete as firms, the nature of urban development means that the competition would be imperfect, leading to an equilibrium that falls short of first best.

V. Empirical implications

One of the most important questions in the field of urban economics is: what are the sources of agglomeration economies? There are many ways that this question has been answered. One approach has been to examine the forces that lead to the agglomeration of particular industries or groups of industries. Another is to look in isolation at individual agglomerative forces. These approaches have demonstrated persuasively the existence of a number of individual agglomeration economies in a range of industries. A third approach is to look at many agglomeration economies and many industries together, and to look for evidence of the relative importance of various microfoundations. This approach is taken by Audretsch and Feldman (1996) and Rosenthal and Strange (2001), who regress measures of the spatial concentration of industries on proxies for various Marshallian localization economies. These approaches are based the idea that observed patterns of locations reflect the benefits that can be realized from agglomeration.

A particularly creative way to look for the relative importance of various microfoundations is carried out in Ellison et al (2010). Their approach is to infer the importance from patterns of coagglomeration. Suppose that industries that tend to coagglomerate also tend to draw from the same patents. This suggests that knowledge spillovers are important. Suppose instead that industries that tend to coagglomerate tend to employ the same sorts of manufactured inputs. This suggests that input sharing is important. And so on. Ellison et al (2010) implement this idea by regressing coagglomeration between industry pairs on indices capturing the industries' tendencies to draw from related resources. This approach is based on the idea that observed location patterns reflect the benefits that can be realized from coagglomeration.

The efficiency results above have implications for both approaches. The first key efficiency result in Section IV shows that it is possible that there be inefficient diversity. A worker from a particular industry chooses a big and diverse city not for its diversity *per se* but instead because big city has a lot of activity in the same industry. The agglomeration benefit is by virtue of size and not by virtue of composition. In this unfortunate situation, one could see workers choosing to coagglomerate even when they do not share the same orientations with respect to ideas, people, or goods.

The second important efficiency result in Section IV is that there may be unrealized coagglomeration benefits. In this case, a worker from a particular industry is not attracted to another industry *per se* but instead is attracted to workers from the own industry. In this case, one may fail to observe in equilibrium the most efficient possible coagglomeration.

It is important to recognize, however, that in the complete absence of agglomeration economies of any kind, there would be neither agglomeration nor coagglomeration in equilibrium. Furthermore, there is a range of equilibria with differing degrees of inefficiency. While highly inefficient equilibria are possible, it is not at all clear that they are probable. The question posed by this section's demonstration of non-uniqueness for equilibrium patterns of coagglomeration is how to select among the many candidate equilibria.

VI. Conclusion

Traditional models of systems of cities focus almost entirely on the polar cases completely mixed and fully specialized cities. There has been little consideration of coagglomeration. In the absence of formal analysis, the folk wisdom about coagglomeration is that it is efficient. If there are localization economies only, then cities will be specialized. Only if there are sufficiently strong urbanization economies will diverse cities emerge.

This paper challenges the idea that coagglomeration can be expected to be efficient. In some situations, industries will coagglomerate inefficiently. In others, there will be unrealized coagglomeration that could have enhanced efficiency. These results are important as extensions to the theory of systems of cities and as bases for urban policy analysis. They are also important for shedding light on what can be learned about the microfoundations of agglomeration economies from observed patterns of coagglomeration.

There is one additional point about coagglomeration that this paper has not considered, and it bears making here. In much of the analysis, we have employed a reduced form relationship between agglomeration and productivity. The relationship is consistent with various microfoundations of agglomeration economies. The inefficiency results arise from the weakness of migration as a disciplining mechanism regardless of the agglomeration forces at work.

In fact, the likelihood of inefficiency is even greater. While some of the benefits of agglomeration arise as a result of the essentially accidental contact of nearby agents, most agglomeration benefits require some sort of action in order for benefits to accrue. In the case of knowledge spillovers, there can be no exchange unless, at a minimum, agents have sufficient levels of human capital to have knowledge to transfer and to learn from other agents. In addition, agents must, in many situations, be willing participants in the exchange (Helsley and Strange, 2004). In the case of input sharing, firms must decide to outsource in order for there to be an increasing return (Helsley and Strange, 2007). For labor market pooling, both employers and workers make decisions that impact the efficiency of agglomeration (Combes-Duranton, 2006). That the benefits of agglomeration depend on what are effectively public goods provision decisions further increases the likelihood of inefficient coagglomeration. If there are no local inputs available because firms have chosen to in-source, then this will further decrease the likelihood of that efficient coagglomeration motivated by input sharing.

Clearly, if there are systematic inefficiencies in coagglomeration, then interventions may be beneficial. But who should intervene? Governments have police powers that may allow them obtain improved matches. However, it is questionable that they have the information needed to recognize the matches and the incentives to correct them if they are recognized. Land developers are likely to have the right incentives, but they are not large enough to internalize all the relevant externalities. The same is true for firms. Given these information issues, this paper's analysis should not be seen as calling for planning solutions to inefficiencies in the system of cities.

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Figures

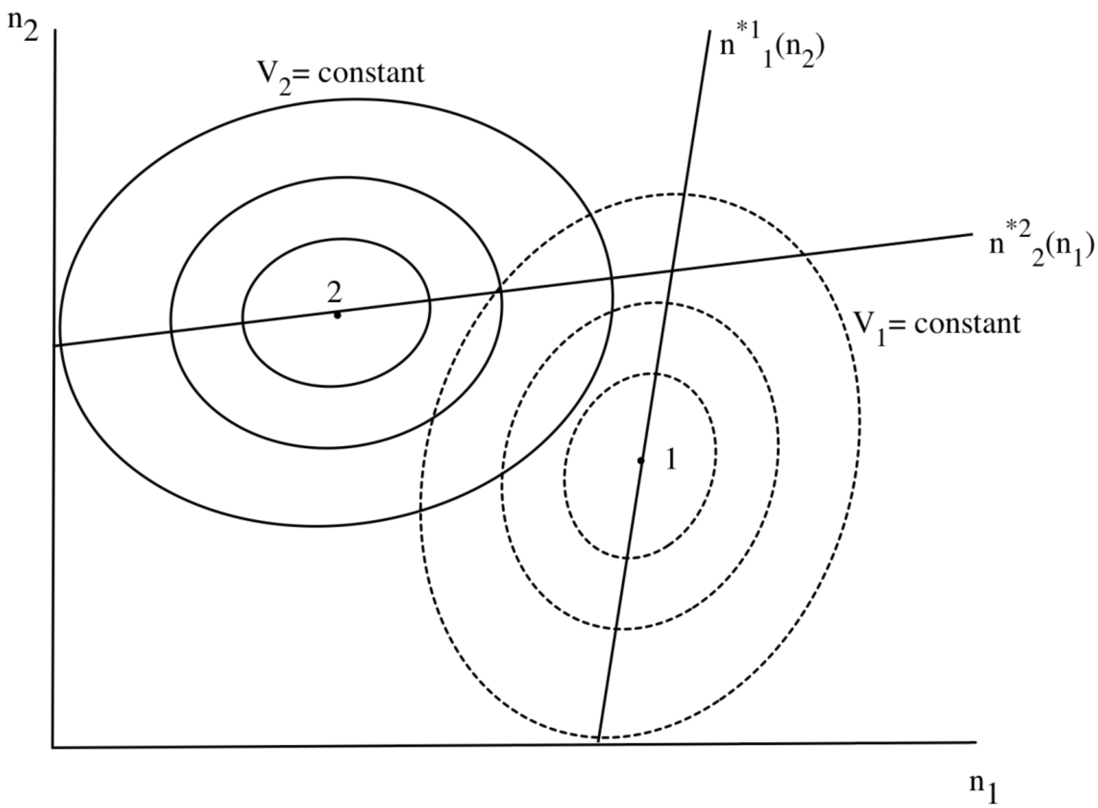

Figure 1: Type Optimal Cities and their Level Sets

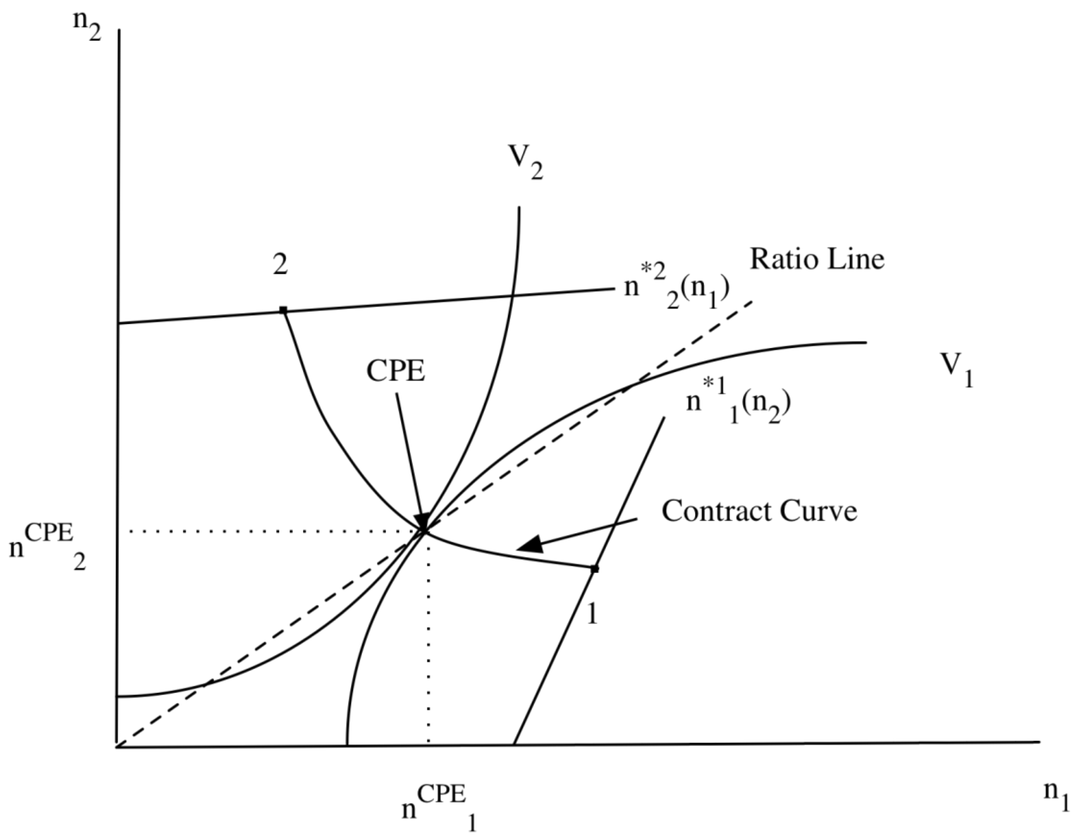

Figure 2: Constrained Pareto Efficient Coagglomeration

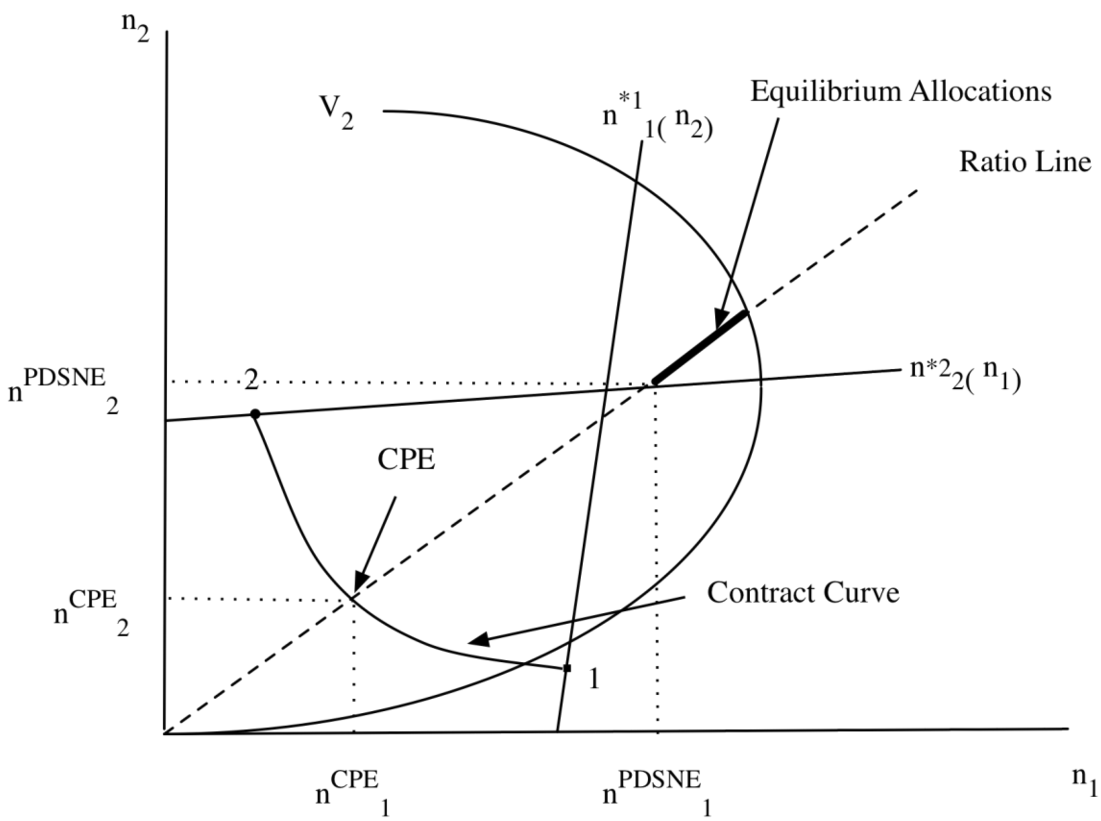

Figure 3: Equilibrium Coagglomeration