Competing for Entrepreneurial Ideas: Matching and Contracting in the Venture Capital Market

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Abstract

We propose an equilibrium model of the VC market with two-sided matching between heterogeneous VCs and entrepreneurs in a principal-agent framework. Each VC matches endogenously with an entrepreneur, offering an equity share and investment. There is positive assortative matching (PAM), which enhances investment and effort. Entry at the bottom of the ladder induces improvements across all ventures in the market, but exit at the bottom may trigger a wave of failures as it travels up the ladder. Entry ameliorates match qualities, but entry by an entrepreneur or entrepreneur-VC pair dampens investment and effort, while entry by a VC enhances them.

Keywords: Entrepreneurship, venture capital, matching, contracts, moral hazard, incentives

JEL classifications: C78, D86, G24, L26, M13

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1 Introduction

The venture capital market has long been recognized as a fundamental source for entrepreneurs to finance their ventures, and thus to bring their innovative ideas to market (Kaplan, Sensoy, and Strömberg (2009)). In addition to providing entrepreneurs with the required financial resources to start their own businesses, venture capitalists (VCs) bring in specific expertise, which is particularly invaluable for entrepreneurs with little or no business-related skills. Specifically, VCs monitor and are commonly represented on the boards of directors (Lerner (1995), and Gompers and Lerner (1999)); provide technical and commercial advice (Bygrave and Tymmons (1992)); provide access to valuable networks and meet with suppliers and customers (Gorman and Sahlman (1989), and Hochberg, Ljungqvist, and Lu (2007, 2010)); help in designing business strategy and human resource policies, and have contacts with potential managers (Hellman and Puri (2002)); and assist with the creation of strategic alliances (Lindsey (2008)). The expertise of VCs can therefore be crucial for the exploration of innovative ideas with market potential, and hence for the success of new ventures.[1] Given their specific expertise, the key challenge for VCs is not only to identify entrepreneurs with suitable ideas, but also to attract them by offering mutually beneficial contracts.

This paper emphasizes the importance of *heterogeneity* across entrepreneurs (in terms of the quality of their business ideas) and VCs (in terms of their expertise), the *matching* process between entrepreneurs and VCs, and the inter-relationship between the VC market and contractual arrangements. VCs are significantly heterogeneous in terms of their characteristics, skills, and reputation (see Kaplan and Strömberg (2003, 2004), Hsu (2004), Kaplan and Schoar (2005), and Kaplan, Sensoy, and Strömberg (2009)). Moreover, Sørensen (2007) provides empirical evidence highlighting the contributions of heterogeneity across VCs and matching between entrepreneurs and VCs in determining the eventual success or failure of VC-backed start-ups. He shows that start-ups backed by more experienced VCs are more likely to go public, for two reasons: first, experienced VCs provide more value added (which we label the "heterogeneity" effect); and second, experienced VCs select superior (or more promising) ventures to begin with (which we label the "matching" effect). Although both effects have a significant impact on

[1] Indeed, Kaplan and Schoar (2005) provide empirical evidence suggesting that the experience of an individual general partner (which can be considered as a proxy for the expertise of the financier) has a positive effect on the overall performance of equity partnerships.

the success rate of start-ups, Sørensen finds that the matching effect is almost twice as important as the heterogeneity effect in explaining observed differences in IPO rates across VC investors.[2]

Inspired by this evidence, we propose a two-sided matching model in a principal-agent framework with moral hazard consisting of (i) a collection of VCs that are heterogeneous in terms of their expertise; and (ii) a collection of entrepreneurs that are heterogeneous with respect to the quality (or market potential) of their business ideas. Each VC matches endogenously with an entrepreneur, whereby the expertise of the VC and the quality of the entrepreneur's idea jointly determine the *match quality* of the venture. The entrepreneur needs to implement (unobservable) effort, which determines the success or failure of the venture, while the specific match quality as well as the VC's investment determine the profitability of the venture. Once matched with an entrepreneur, each VC offers a contract which stipulates an allocation of equity and the VC's capital investment. Due to endogenous matching in our framework, each contract must not only satisfy the usual incentive compatibility constraint associated with the moral hazard problem, but also a condition that guarantees that an entrepreneur cannot be better-off by contracting with another VC. In this sense, VCs are competing for entrepreneurs with promising business ideas, which – as we will show – plays a key role in the design of optimal VC contracts, and defines the specific impact of entry and exit in this market. In particular, the matching condition implies that the reservation utility of an entrepreneur is endogenous, as it is determined by the payoff he would receive upon contracting with the next best VC.[3]

While the analysis of isolated entrepreneur-VC relationships in a contractual context undoubtedly provides important insights, a main implication of our study is that matching in the VC market significantly affects the structure and properties of VC contracts. We first derive conditions under which we obtain *positive assortative matching* (PAM), the situation in which VCs with high expertise are matched to entrepreneurs with high quality ideas. We find that PAM arises if there is (i) complementarity between the entrepreneur's idea quality and the VC's expertise; (ii) complementarity between the match quality and the investment by the VC; and (iii) diminishing returns to the VC's investment. We refer to

[2]Matching is clearly not only relevant in VC markets, but also matters in general principal-agent relationships (see Ackerberg and Botticini (2002), Besley and Ghatak (2005), and Serfes (2005, 2008)).

[3]Dam (2007) also studies matching between venture capitalists and firms. However, his setting and predictions are substantially different from ours. Specifically, he considers a market where venture capitalists are heterogeneous with respect to their monitoring ability, and firms are heterogeneous with respect to their levels of initial wealth. The matching equilibrium in his framework is negative assortative (NAM): venture capitalists with higher monitoring ability invest in firms with lower initial wealth.

the collection of entrepreneur-VC matches as a *ladder*, wherein higher rungs in the ladder correspond to higher quality matches. As discussed above, a higher match quality stems from a higher expertise of the VC as well as from a higher quality of the entrepreneur's business idea. We show that endogenous matching affects the characteristics of the contracts in each entrepreneur-VC pair, which is rooted in the competition that takes place among VCs for promising entrepreneurial ideas. Specifically, we find that each VC transfers more surplus – in the form of greater equity in conjunction with a higher investment – to its matched entrepreneur than would occur in the absence of matching considerations (as in an isolated principal-agent relationship). This contractual adjustment, which is rooted in endogenous matching, concurrently motivates the entrepreneur to implement higher effort, thereby enhancing the venture's likelihood of success. Nevertheless, even when taking matching considerations into account, in equilibrium the investment by a VC and the effort exerted by the corresponding entrepreneur are below their socially efficient levels. We therefore argue that any managerial or policy implications should not be based on an isolated analysis of VC contracts; rather, it is indispensable to consider the two-sided VC market as a whole in order to account for the effects of competition for entrepreneurial ideas.

The effects of entry and exit in the market for VC are particularly interesting. We begin by examining *endogenous entry and exit*, under which fixed costs of entry determine the equilibrium number of entrepreneurs and VCs that enter the market. We find that entry at the bottom of the ladder (as brought about by a decrease in entry costs, for example) induces an increase in investment, effort, and success probability across all ventures in the market. This follows from the fact that entry at the bottom enhances the outside options of all entrepreneurs, forcing all VCs to provide more capital and equity aimed at attracting the most suitable entrepreneurs. We refer to this mechanism brought about by endogenous matching as the *ripple effect* since it propagates up the ladder by inductive reasoning. Because an entrepreneur-VC pair that enters the market does not internalize this positive externality, the equilibrium number of ventures is inefficiently low. A clear policy implication is that entry into the VC market should be encouraged and possibly subsidized by lowering entry barriers.

The ripple effect in the context of entry can lead the VC market to unravel in the context of exit. Exit at the bottom of the ladder potentially triggers a wave of failures in the VC market as it travels up the ladder. The reason is as follows. Ventures at the bottom of the ladder have the lowest probability of success. Upon failing and subsequently exiting the market, by reversing the inductive logic behind the ripple effect, we can infer that all matched pairs of superior quality respond by diminishing their investments and levels of effort, which in turn raises their probability of failure. Thus, from a policy standpoint, low quality ventures may be "too small to fail".[4] Paradoxically, it may be better for the VC market to experience failures at the top of the ladder (i.e., failures of large firms) since such failures do not impose a negative externality on all entrepreneur-VC pairs below.

We then investigate the effects of *exogenous entry and exit* in the VC market. We find that entry improves match qualities in the VC market whenever the entrants are of relatively high quality. However, entry can also negatively affect the outside option of an entrepreneur as it depends on the maximum willingness to pay for that entrepreneur among all VCs which are matched with lower quality entrepreneurs. In this case, VCs transfer less surplus to entrepreneurs, which is reflected by lower equity shares and equilibrium investments, and hence lower effort levels. Overall, due to the ripple effect caused by endogenous matching, we find that entry (exit) by an entrepreneur-VC pair leads to lower (higher) investment, effort, and success probability for ventures of superior match quality to the entering (exiting, respectively) pair; entry (exit) by a VC leads to higher (lower, respectively) investment, effort, and success probability for all ventures in the market; and entry (exit) by an entrepreneur leads to lower (higher, respectively) investment, effort, and success probability for all ventures in the market. Thus, an important lesson to be learned from our study is that entry in general does not necessarily improve efficiency in the VC market, even though it tends to ameliorate the quality of matches. Indeed, surprisingly, enhanced competition in the form of entry by an entrepreneur-VC pair worsens the contractual arrangements of all ventures of superior quality, though entry at the bottom of the ladder is beneficial to the entire VC market. Nevertheless, because VCs do not internalize the impact of their entry and exit decisions on the VC market, policymakers should consider subsidizing VC entry and preventing VC exit.

Our paper primarily contributes to the theoretical literature on the contractual relationship between entrepreneurs and VCs.[5] A number of papers have examined isolated entrepreneur-VC principal-agent problems, each of which emphasizes different aspects of that relationship. Casamatta (2003) considers the managerial incentives of investors and entrepreneurs as a double-sided moral hazard problem.

[4]Note that a venture with a low match quality is characterized by a low equilibrium capital investment, and is therefore by nature "small".

[5]See Hart (2001) for a review of the literature on financial contracting up to that point.

Kirilenko (2001) studies the entrepreneur-VC relationship when the entrepreneur derives non-pecuniary benefits from having some control over the firm. Repullo and Suarez (2004) study the design of optimal securities in an environment with multiple investment stages and double-sided moral hazard. Schmidt (2003) shows the optimality of convertible securities in an environment in which investment is sequential and observable but not contractible. Ueda (2004) examines the source of finance when there are weak property rights. Hellmann (1998) focuses on the allocation of control rights between VCs and entrepreneurs. Our model does not account for different allocations of control rights, the source of finance, or dynamic interactions between entrepreneurs and VCs. Instead, we examine in an equilibrium model of the VC market the allocation of equity, the magnitude of the investment by the VC, and the roles of entry and exit when matching considerations are taken into account.

Our paper is closest in spirit to the equilibrium models of the VC market in Inderst and Müller (2004) and Sørensen (2007). Our paper differs in two important aspects from Inderst and Müller. First, Inderst and Müller consider a *search* equilibrium, whereas we study *matching*. The search equilibrium in their model is characterized by the ratio of the number of VCs to the number of entrepreneurs, commonly referred to as "market thickness" in the search theory literature. Using a matching framework, however, we show that it is not just the market thickness that matters; the qualities of entrepreneurial ideas and the expertise of VCs are also critical in the optimal design of VC contracts, especially since they determine the (endogenous) outside options of entrepreneurs. Second, Inderst and Müller consider homogenous VCs and entrepreneurs, whereas we explicitly account for different match qualities by allowing entrepreneurs to be heterogenous with respect to the market potential of their ideas, and VCs with respect to their expertise.

Sørensen (2007) devises a structural model based on a two-sided matching model of VCs and entrepreneurs. The matching model in Sørensen differs from ours in the following respects. First, in Sørensen, contracts between VCs and entrepreneurs are not modeled. By contrast, we derive the optimal contract featuring an equity stake and investment by the VC. Second, Sørensen does not model the link among all entrepreneur-VC pairs that occurs via their outside options. We show that this link is crucial towards understanding the inter-relationship between the VC market as a whole and the properties of individual VC contracts. Third, Sørensen does not consider the roles of entry and exit, whereas we examine exogenous entry and exit by entrepreneurs and VCs along multiple points in the ladder as well as endogenous entry and exit brought about by changes in the costs of entering the VC market. We find that such considerations reveal the ripple effect that permeates throughout the VC market as ventures enter and exit, which in turn generates insights into the functioning and possible failure of the VC market.

The remainder of the paper is structured as follows. Section 2 introduces the features of the model. Section 3 derives the optimal contract of an entrepreneur-VC pair in the absence of matching considerations. Section 4 incorporates two-sided matching in the VC market and characterizes the optimal contracts. Section 5 investigates the roles of entry and exit in the VC market, and how they affect the contractual relationships between entrepreneurs and VCs. Section 6 concludes.

2 Preliminaries

2.1 The Structure and Timing of the Model

Consider a market of $n_E \geq 2$ risk-neutral entrepreneurs and $n_{VC} \geq 2$ risk-neutral VCs. We assume that there are more entrepreneurs than venture capitalists, i.e., $n_E \geq n_{VC}$. All entrepreneurs are wealth constrained and their reservation utilities are normalized to zero.[6]

There are four dates:

1. Entrepreneurs are endowed with ideas for new ventures.
2. Each VC matches with an entrepreneur and offers him a contract that consists of a share of the venture and an investment in the venture.
3. Entrepreneurs exert unobservable effort.
4. Profits of ventures are realized and payments are made.

At date 1, each entrepreneur i, $i \in E = \{1, ..., n_E\}$, conceives an innovative business idea, denoted μ_i, where a higher value of μ_i reflects an idea of superior quality. The entrepreneurial ideas can be ranked according to their respective quality, i.e., $\mu_{n_E} \geq ... \geq \mu_2 \geq \mu_1$. To commercially exploit his idea, each

[6]While we initially assume a zero outside option for every entrepreneur, a strictly positive reservation utility for entrepreneurs with high-quality projects arises endogenously in our matching framework.

entrepreneur relies on a capital investment K_i as well as on managerial and marketing expertise, both provided by a VC. Let x_j denote the expertise of VC j, $j \in V = \{1, ..., n_{VC}\}$, where a superior expertise is reflected by a higher x_j, i.e., $x_{n_{VC}} \geq ... \geq x_2 \geq x_1$. Entrepreneurial ideas and VC expertise are common knowledge. At date 2, each VC matches with exactly one entrepreneur.[7] A VC j then offers its entrepreneur i a contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ that specifies a share λ_{ij} of the venture's total profit for the entrepreneur as well as the VC's capital investment K_{ij}.[8] The risk-adjusted cost of capital faced by each VC is denoted $r > 0$.

At date 3, after signing the contract Γ_{ij}, entrepreneur i needs to exert effort e_{ij} in order to turn his idea into a marketable product. Implementing effort e_{ij} imposes the cost $c(e_{ij}) = e_{ij}^2/2$. The entrepreneur's effort choice e_{ij} – which cannot be observed by the VC – eventually determines the likelihood of whether the venture succeeds ($Y_{ij} = 1$) or fails ($Y_{ij} = 0$). Specifically, we assume that $\text{Prob}[Y_{ij} = 1|e_{ij}] = e_{ij}$.[9] Finally, at date 4, the profit of each venture is realized, and payments according to the respective contracts are made.

A key property of our framework is the reliance of entrepreneurs on the expertise of VCs to turn promising ideas into marketable products. Intuitively, the value of this expertise is closely related to the quality of the entrepreneur's specific idea. To capture this notion, let $\Omega_{ij} \equiv \Omega(\mu_i, x_j) > 0$ denote the *match quality* between an entrepreneur with idea μ_i and a VC with expertise x_j. The match quality Ω_{ij} is strictly increasing in both the entrepreneur's idea quality μ_i and the VC's expertise x_j, where ideas and expertise are complements (i.e., $\partial^2\Omega(\mu_i, x_j)/\partial x_i \partial \mu_j > 0$). This in turn allows us to also rank ventures according to their respective match quality, where we henceforth refer to the resulting collection of ventures – consisting of entrepreneurs each matched with one VC – as a *ladder*.

Let $\Pi_{ij} \in \{0, \pi(K_{ij}, \Omega_{ij})\}$ denote the gross profit of the venture started by entrepreneur i and financed by VC j, where $\pi(K_{ij}, \Omega_{ij}) \geq 0$ denotes the gross profit of a *successful* venture (i.e., $Y_{ij} = 1$).[10]

[7] To keep our framework as simple as possible, we focus on one-to-one matching. However, our analysis can easily be extended to a many-to-one matching, where a venture capital firm can contract with multiple entrepreneurs. We discuss this in greater detail in the conclusion.

[8] Alternatively, we may interpret λ_{ij} as the fraction of shares for the entrepreneur, whereas the remaining shares, $1 - \lambda_{ij}$, are held by the venture capitalist in exchange for its investment K_{ij}.

[9] To ensure interior solutions, we will assume that the venture's potential profit, denoted by π, is always smaller than one, so that even the first-best effort level e_{ij}^{fb} guarantees that $\text{Prob}[Y_{ij} = 1|e_{ij}^{fb}] < 1$.

[10] The assumption of Π being non-negative implies that potential losses are only incurred by the venture capitalist up to its investment K_{ij}, but not by the entrepreneur, which is consistent with his limited wealth.

Given the entrepreneur's effort e_{ij}, the VC's capital investment K_{ij}, and the match quality Ω_{ij}, the expected profit of entrepreneur i's venture financed by VC j is

$$\mathrm{E}[\Pi_{ij}|e_{ij}, K_{ij}, \Omega_{ij}] = \pi(K_{ij}, \Omega_{ij})e_{ij}. \tag{1}$$

The realization of Π_{ij} is uncertain from an *ex ante* perspective, implying that the VC cannot perfectly infer from the realized profit Π_{ij} the entrepreneur's effort level e_{ij}, leading to a typical moral hazard problem.

We now elaborate on the underlying properties of the gross profit $\pi(K_{ij}, \Omega_{ij})$ of a *successful* venture. Intuitively, a venture's profit $\pi(K_{ij}, \Omega_{ij})$ can only be strictly positive if (i) the entrepreneur implements effort ($e_{ij} > 0$); (ii) the VC provides capital ($K_{ij} > 0$); (iii) the entrepreneur has a marketable idea ($\mu_i > 0$); and (iv) the VC is equipped with expertise ($x_i > 0$). Technically, we assume

$$\pi(0, \Omega(\mu_i, x_j)) = \pi(K_{ij}, \Omega(0, x_j)) = \pi(K_{ij}, \Omega(\mu_i, 0)) = 0.$$

Furthermore, we assume that $\pi(K_{ij}, \Omega_{ij})$ is concave increasing in the VC's investment K_{ij} and in the match quality Ω_{ij}, i.e., $\partial^2\pi(K_{ij}, \Omega_{ij})/\partial z^2 \leq 0$ with $z = K_{ij}, \Omega_{ij}$ and $\pi_{K_{ij}K_{ij}}\pi_{\Omega_{ij}\Omega_{ij}} \geq (\pi_{\Omega_{ij}K_{ij}})^2$, where subscripts denote partial derivatives. Finally, we make the following assumptions to ensure interior solutions:

$$\left.\frac{\partial\pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}}\right|_{K_{ij}=0} = \infty\,, \qquad \left.\frac{\partial\pi(K_{ij}, \Omega_{ij})}{\partial \Omega_{ij}}\right|_{K_{ij}=0} = \left.\frac{\partial\pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}}\right|_{\Omega_{ij}=0} = 0.$$

These assumptions imply that the first unit of capital K_{ij} is very productive, but only as long as the match quality in question is strictly positive. We further assume that a higher match quality Ω_{ij} makes every unit of capital K_{ij} more productive, $\partial^2\pi(K_{ij}, \Omega_{ij})/\partial K_{ij}\partial\Omega_{ij} \geq 0$.

2.2 The Efficient (First-Best) Contract

So as to provide policy implications with respect to VC markets, we derive the contract between entrepreneur i and VC j which maximizes the total surplus of the venture. To do so, suppose a social planner can dictate entrepreneur i's effort level e_{ij} as well as the investment K_{ij} by VC j. The social planner would choose the (first-best) effort level e_{ij}^{fb} and the (first-best) investment K_{ij}^{fb} that maximize each venture's expected total surplus S_{ij}. The problem of the social planner can thus be stated as follows for a venture with a match quality Ω_{ij}:

$$\max_{\{e_{ij},K_{ij}\}} S_{ij}(e_{ij},K_{ij}) = \pi(K_{ij},\Omega_{ij})e_{ij} - e_{ij}^2/2 - rK_{ij}. \tag{2}$$

Notice that $S_{ij}(e_{ij},K_{ij})$ is not affected by the entrepreneur's share λ_{ij} of the venture's profit $\pi(K_{ij},\Omega_{ij})$ (if successful) since λ_{ij} determines only the allocation of surplus. The following lemma characterizes the solution to this problem.[11]

Lemma 1 (First-Best Contract) *The socially efficient investment K_{ij}^{fb} by VC j is implicitly characterized by*

$$\frac{\partial \pi(K_{ij}^{fb},\Omega_{ij})}{\partial K_{ij}} = \frac{r}{\pi(K_{ij}^{fb},\Omega_{ij})}; \tag{3}$$

and the socially efficient effort level e_{ij}^{fb} of entrepreneur i is

$$e_{ij}^{fb} = \pi(K_{ij}^{fb},\Omega_{ij}). \tag{4}$$

To ensure that $e_{ij}^{fb} < 1$, and hence $\text{Prob}[Y_{ij} = 1|e_{ij}^{fb}] < 1$, we assume that $\pi(K_{ij}^{fb},\Omega_{ij}) < 1$ for the socially optimal capital investment K_{ij}^{fb} and any possible match quality Ω_{ij}.[12] We will draw on the socially efficient investment K_{ij}^{fb} and effort level e_{ij}^{fb} as emphasized by Lemma 1 in subsequent sections to characterize the effect of matching considerations on the efficiency of VC contracts in a second-best environment.

[11] Unless otherwise stated, all proofs are contained in the Appendix.

[12] Alternatively, one could incorporate a parameter $\gamma > 0$ in the entrepreneur's effort cost function so that $c(e_{ij}) = \gamma e_{ij}^2/2$. Assuming a sufficiently high cost parameter γ would then also ensure that $\text{Prob}[Y_{ij} = 1|e_{ij}^{fb}] < 1$.

3 The Venture Capital Market in the Absence of Matching

To establish a benchmark, we first derive the optimal VC contract in the absence of any matching considerations. To do so, we proceed in two steps. First, in Section 3.1, we characterize the entrepreneur's effort choice for a *given* VC contract. Second, we derive the optimal VC contracts in Section 3.2 by accounting for the entrepreneur's specific effort choice.

3.1 An Entrepreneur's Effort Choice

We begin our analysis by investigating entrepreneur i's effort choice for a given contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by VC j, with $i \in \{1, ..., n_{VC}\}$ and $j \in \{1, ..., n_E\}$. For a given match quality Ω_{ij}, capital investment K_{ij}, and share of the venture's profit λ_{ij}, the entrepreneur chooses his effort e_{ij} to maximize his expected utility

$$\max_{\{e_{ij}\}} U_i(e_{ij}, \lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij} - e_{ij}^2/2. \quad (5)$$

The entrepreneur's effort choice is thus

$$e_{ij}^* = \lambda_{ij}\pi(K_{ij}, \Omega_{ij}). \quad (6)$$

Clearly, the entrepreneur's effort e_{ij}^* is increasing in his profit share λ_{ij}, the investment K_{ij}, his idea quality μ_i, and finally, in the specific expertise of the VC x_i. This is very intuitive: a higher capital investment or a better match quality makes every unit of the entrepreneur's effort more productive. In response, it is optimal for the entrepreneur to put in more effort in order to maximize his expected utility.

Finally, notice that the entrepreneur's participation constraint is always satisfied because potential losses (up to K_{ij}) are only incurred by the VC firm – which is rooted in the entrepreneur's limited wealth. His limited liability further implies that the entrepreneur extracts an economic rent, which can be shown (using the Envelope Theorem) to be strictly increasing in his profit share λ_{ij}, the VC's capital investment K_{ij}, and in the specific match quality Ω_{ij} of the venture.

3.2 A Venture Capitalist's Contract Choice

In this section, we derive the optimal contract offered by a VC in the absence of any matching considerations. This contract is not only aimed at securing the VC an adequate return on his investment, but also at motivating the entrepreneur to implement sufficient effort in order to turn his idea into a profitable product. Clearly, to indirectly control the entrepreneur's effort choice, the VC can simply adjust his share λ_{ij} of the venture's profit. Less obvious, however, is the fact that the VC can also utilize his investment K_{ij} to influence the entrepreneur's effort choice. As discussed in Section 3.1, a higher capital investment makes every unit of the entrepreneur's effort more productive, which in turn induces him to implement a higher effort level.

Consider the contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by VC j to entrepreneur i. The VC adjusts this contract to maximize its expected profit, taking the entrepreneur's effort choice e_i^* into account. The VC's expected profit is thus given by

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, e_{ij}^*, \Omega_{ij}) = (1-\lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij}. \tag{7}$$

By accounting for e_{ij}^* as defined by (6), the expected profit of VC j becomes

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}(1-\lambda_{ij})\pi^2(K_{ij}, \Omega_{ij}) - rK_{ij}. \tag{8}$$

Before deriving the optimal contract Γ_{ij}^*, it is insightful to unravel the relationship between the allocation of shares and the VC's expected profit. We start by briefly discussing the extreme cases: all shares are either allocated to the VC ($\lambda_{ij} = 0$) or to the entrepreneur ($\lambda_{ij} = 1$). Due to the lack of any prospective compensation, it is obvious that the entrepreneur refuses to implement effort whenever the VC holds all shares (i.e., $\lambda_{ij} = 0$), implying a zero profit for the VC as $\text{Prob}[Y_{ij} = 1 | e_{ij} = 0] = 0$, i.e., $\Pi_{ij}^{VC}(\lambda_{ij} = 0, K_{ij}, \Omega_{ij}) = 0$.[13] On the other hand, if the entire profit of the venture accrues to the entrepreneur (i.e., $\lambda_{ij} = 1$), the VC clearly does not benefit from the new enterprise. We can thus conclude that financing the new venture can only be profitable for the VC as long as *both* parties hold some shares (i.e., $\lambda_{ij} \in (0,1)$).

[13] Note that in this case, however, it is optimal for the venture capitalist to refrain from investing in the venture (i.e., $K_{ij}^* = 0$)

The optimal VC contract Γ^*_{ij} – comprising the entrepreneur's share λ^*_{ij} and the VC's investment K^*_{ij} – is dictated by two objectives: (i) to provide the entrepreneur with sufficient effort incentive to turn his idea into a profitable product; and (ii) to equip the new venture with sufficient capital, not only aimed at ensuring its survival, but also at generating an adequate return on investment for the VC. The following proposition describes the VC contract that maximizes (8).

Proposition 1 (VC Contract without Matching) *Suppose there is no matching in the VC market. The optimal contract between entrepreneur i and VC j then comprises the profit share $\lambda^*_{ij} = 1/2$ for the entrepreneur, and the capital investment K^*_{ij} which is implicitly defined by*

$$\pi(K^*_{ij}, \Omega_{ij}) \frac{\partial \pi(K^*_{ij}, \Omega_{ij})}{\partial K_{ij}} = 2r. \tag{9}$$

In the absence of any competition between entrepreneurs as well as between VCs, it is optimal for the financiers to equally split the shares of new ventures.[14] We will demonstrate in the next section that the split of shares is *not* even when accounting for endogenous matching in the VC market. Finally, by comparing (9) with (3), it becomes clear that the optimal contract $\{\lambda^*_{ij}, K^*_{ij}\}$ entails an under-provision of capital from a social perspective, i.e., $K^*_{ij} < K^{fb}_{ij}$. A comparison of (6) with (4) further reveals that entrepreneur i's effort level under this contract is also inefficiently low, i.e., $e^*_{ij} < e^{fb}_{ij}$.

Using the optimal contract $\Gamma^*_{ij} = \lambda^*_{ij}, K^*_{ij}$ as emphasized by Proposition 1, one can derive the expected utility of entrepreneur i matched with VC j:

$$U^V_{ij}(K^*_{ij}, \Omega_{ij}) = \pi^2(K^*_{ij}, \Omega_{ij})/8. \tag{10}$$

The superscript V indicates that the entire bargaining power in this relationship rests with the VC.[15] Entrepreneur i's expected utility $U^V_{ij}(K^*_{ij}, \Omega_{ij})$ constitutes his lowest possible expected utility level in our matching framework, which will play a fundamental role in our subsequent analysis.

[14] Note that this result is rooted in our specific assumptions about the entrepreneur's effort cost function $c(e_{ij})$, and how his effort affects the performance of the venture. However, the specific value of λ^*_{ij} is of minor importance for our subsequent analysis as we are mainly interested in how matching in the market for venture capital alters the *relative* allocation of equity between entrepreneurs and their financiers.

[15] We derive in the Appendix the entrepreneur's expected utility in case he holds the entire bargaining power. Using the superscript V will therefore ease the distinction between both cases.

The next proposition elaborates on how the optimal contract $\Gamma_{ij}^* = \{\lambda_{ij}^*, K_{ij}^*\}$ responds to different match qualities, measured by Ω_{ij}.

Lemma 2 (Properties of the VC Contract without Matching) *Suppose there is no matching in the VC market. The entrepreneur's share λ_{ij}^* is independent of the specific match quality Ω_{ij}, while the capital investment K_{ij}^* is increasing in Ω_{ij}.*

A better match quality—stemming either from a more promising entrepreneurial idea (μ_i) or superior VC expertise (x_j)—enhances the payoff to the VC of supplying the entrepreneur with more capital K_{ij}^*. On the other hand, providing the entrepreneur with the share $\lambda_{ij}^* = 1/2$ already maximizes the VC's return on investment, and is thus independent of the specific match quality.

Applying the Envelope Theorem to (8) and (10) yields the following lemma, which emphasizes the effect of the match quality Ω_{ij} on the expected utility of entrepreneur i and the expected profit of VC j.

Lemma 3 (Impact of Match Quality without Matching) *Suppose there is no matching in the VC market. Entrepreneur i, $i = 1, ..., n$, and VC j, $j = 1, ..., n$, strictly benefit from a superior match quality Ω_{ij}, i.e., $dU_{ij}^V / d\Omega_{ij} > 0$ and $d\Pi_{ij}^{VC} / d\Omega_{ij} > 0$.*

Both parties strictly benefit from a superior match quality, rooted either in a more promising entrepreneurial idea or in greater VC expertise. This observation will have important implications for the properties of the matching process in the VC market.

4 The Matching of Entrepreneurs with Venture Capitalists

4.1 Properties of Matching in the Venture Capital Market

After having derived the optimal VC contract in the absence of any matching considerations, we are now well equipped to investigate the optimal contracts offered by VCs when forming partnerships with entrepreneurs in a two-sided heterogeneous market. To do so, we proceed in two steps. In this section, we first identify the general properties of the matching process in the VC market. Then, in the subsequent section, we elaborate on the optimal adjustment of VC contracts when taking matching considerations into account.

In a traditional principal-agent setting, an entrepreneur would be forced to accept the offer of a specific VC. However, if entrepreneurs are free to choose the VC with the most attractive offer, it is intuitively clear that optimal VC contracts must take into consideration the best alternative available to an entrepreneur. VC j thus designs the contract $\{\lambda_{ij}, K_{ij}\}$ to maximize its expected profit subject to the entrepreneur receiving at least his outside value u_{ij}, which we will characterize later. The constrained optimization problem of VC j can thus be expressed as follows:

$$\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij}) \equiv \max_{\{\lambda_{ij}, K_{ij}\}} (1 - \lambda_{ij}) \pi(K_{ij}, \Omega_{ij}) e^*_{ij} - rK_{ij}$$

s.t.

$$\lambda_{ij} \pi(K_{ij}, \Omega_{ij}) e^*_{ij} - (e^*_{ij})^2/2 \geq u_{ij}, \tag{11}$$

where $e^*_{ij} = \lambda_{ij} \pi(K_{ij}, \Omega_{ij})$. The maximized objective function $\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ defines the bargaining frontier between VC j and entrepreneur i. Whether the entrepreneur's individual rationality (IR) constraint in (11) is binding clearly hinges on his specific reservation utility u_{ij}. We can infer from our previous analysis that U^V_{ij}, as defined by (10), constitutes the lower bound of u_{ij}. This is the utility level entrepreneur i receives when contracting with VC j in the absence of matching, assuming that the entire bargaining power rests with the VC. The maximum value of u_{ij}, on the other hand, is denoted by U^E_{ij}, which constitutes entrepreneur i's expected utility in case the entire bargaining power rests with him, and not with VC j as considered above.[16] The next lemma emphasizes an important property of the bargaining frontier defined by $\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij})$.

Lemma 4 (Bargaining Frontier) *The bargaining frontier $\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ is decreasing in the entrepreneur's reservation utility $u_{ij} \in [U^V_{ij}, U^E_{ij}]$.*

[16] As will become clear, our next analysis does not rely on the specific functional form of U^E_{ij}. For completeness purposes, we formally derive U^E_{ij} in the Appendix.

Figure 1 visualizes the result exposed by lemma 4. The bold curve depicts the bargaining frontier $\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij})$, which consists of the Pareto-optimal utilities for the (i, j) pair.

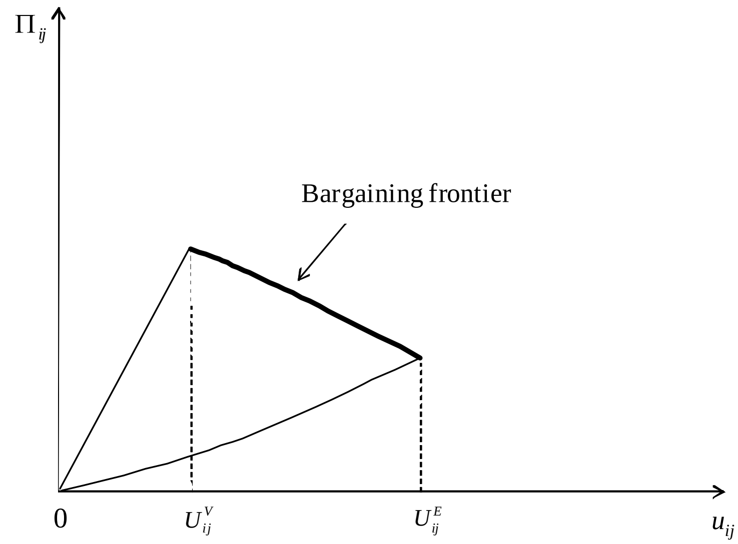

Figure 1: Bargaining Frontier

We can now define equilibrium in the VC market.

Definition 1 (VC Market Equilibrium) *An equilibrium in the VC market consists of a one-to-one matching function $m : E \to V$ and payoff allocations $\Pi^* : V \to \mathbb{R}$ and $u^* : E \to \mathbb{R}_+$ that satisfy the following two conditions:*

(i) Feasibility of (Π^*, u^*) with respect to m: For all $i \in E$, $\{\Pi^*(m(i)), u^*(i)\}$ is on the bargaining frontier $\overline{\Pi}(u_{i,m(i)}, \Omega_{i,m(i)})$.

(ii) Stability of m with respect to $\{\Pi^*, u^*\}$: There do not exist a pair $(i, j) \in E \times V$, where $m(i) \neq j$, and outside value $u > u^*(i)$ such that $\overline{\Pi}_{ij}(u, \Omega_{ij}) > \Pi^*(j)$.

The two conditions guarantee the existence of a stable matching equilibrium in the VC market. Specifically, the first condition requires that the payoffs for VCs and entrepreneurs must be attainable, which is guaranteed whenever the payoffs of any $(i, m(i))$ pair are on the bargaining frontier

$\overline{\Pi}_{i,m(i)}(u_{i,m(i)}, \Omega_{i,m(i)})$. Moreover, the stability condition ensures that all matched VCs and entrepreneurs cannot become strictly better off by breaking their current partnership (and matching with a new VC or entrepreneur).

We obtain *positive assortative matching* (PAM) whenever entrepreneurs with high quality ideas are matched with VCs enjoying high expertise. The opposite occurs with *negative assortative matching* (NAM). The next definition formalizes the characteristics of positive versus negative assortative matching.

Definition 2 (Assortative Matching in the VC Market) *Consider two entrepreneurs i and i' with idea qualities $\mu_i > \mu_{i'}$, and suppose that entrepreneur i is matched with VC $j = m(i)$ and entrepreneur i' is matched with VC $j' = m(i')$. Then, the matching equilibrium is positive assortative (PAM) if the VCs' expertise satisfy $x_j > x_{j'}$; and negative assortative (NAM) if $x_{j'} > x_j$.*

Applying the criteria identified by Legros and Newman (2007) to our framework, we can infer that the matching equilibrium is positive assortative if: (i) the cross-partial derivative of the bargaining frontier $\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ with respect to the entrepreneur's idea quality μ_i and the VC's expertise x_j is positive, i.e., $\partial^2 \overline{\Pi}_{ij}/\partial \mu_i \partial x_j \geq 0$; and (ii) it is relatively easier for a high (versus low) expertise VC to transfer surplus to an entrepreneur, i.e., $\partial^2 \overline{\Pi}_{ij}/\partial u_{ij} \partial x_j \geq 0$. The first condition is the standard complementarity condition that guarantees positive assortative matching in models with transferable utility (see Shapley and Shubik (1972) and Becker (1973)). However, as demonstrated by Legros and Newman, this is a sufficient condition to guarantee PAM whenever utility is non-transferable, as in our framework.[17]

The next proposition derives sufficient conditions for PAM to arise in the VC market, which are assumed henceforth.

Proposition 2 (PAM in the VC Market) *The matching between VCs and entrepreneurs is positive assortative (PAM) if the following two conditions are satisfied:*

(i) *The profit function $\pi(K_{ij}, \Omega_{ij})$ is increasing and concave in the capital investment K_{ij} and match quality Ω_{ij}, with complementarity between the two, i.e. $\partial^2 \pi(K_{ij}, \Omega_{ij})/\partial K_{ij} \partial \Omega_{ij} \geq 0$.*

[17] In our framework, utility can be transferred via the investment K and profit share λ. These two instruments, however, transfer surplus imperfectly as they also affect the size of the surplus. Due to entrepreneurs' liability limit, side payments from entrepreneurs to venture capitalists are not feasible, which is an important characteristic of venture capital markets; see Sørensen (2007) for a discussion.

(ii) *The match quality $\Omega_{ij} = \Omega(\mu_i, x_j)$ is increasing in the entrepreneur's idea quality μ_i and the VC's expertise x_j, with complementarity between the two, i.e. $\partial^2\Omega(\mu_i, x_j)/\partial\mu_i\partial x_j \geq 0$.*

Our assumption of the venture's gross profit $\pi(K_{ij}, \Omega_{ij})$ and the match quality function $\Omega(\mu_i, x_j)$ exhibiting diminishing marginal returns is not only very intuitive, but also implies (in addition to the two emphasized complementarity conditions) that matching in the VC market is positive assortative. Moreover, PAM is also efficient from a social perspective. Our conditions guarantee that a social planner would also prefer the matching to be positive assortative. Interestingly, the conditions that guarantee that PAM is socially efficient are weaker than the ones stated in Proposition 2 because it is ultimately irrelevant for the social planner how easily the matched parties can transfer surplus.

Given the PAM, the top n_{VC} entrepreneurs will match with the n_{VC} venture capitalists. The remaining $n_E - n_{VC}$ entrepreneurs will remain unmatched. Therefore, the market consists of n_{VC} matched VC-entrepreneur pairs. The highest quality entrepreneur is n_E and the lowest quality is 1. Hence, the last matched entrepreneur is $n_E - n_{VC} + 1$, see Figure 2.

4.2 Matching and the Design of Venture Capital Contracts

Before we can draw any inferences about matching and the optimal design of VC contracts, it is essential to first derive conditions for a given entrepreneur-VC match being affected by an improved alternative for the entrepreneur as captured by a higher reservation utility u_{ij}. This is important because some matched entrepreneurs might be "out of reach" of other (competing) VCs. For these partnerships, alternative matches are irrelevant, and the optimal VC contract is thus identical to the one derived in Section 3 in the absence of matching considerations.

Consider two adjacent entrepreneur-VC matches $(i, m(i))$ and $(i-1, m(i-1))$. Proposition 2 provides a useful characterization of the matching problem. Accordingly, entrepreneur i is the most desirable entrepreneur among all VCs that are ranked below VC $m(i)$. Nevertheless, it is VC $m(i-1)$ who has the highest willingness to pay for entrepreneur i.[18] Therefore, VC $m(i)$ directly competes for

[18] The positive cross-partial derivative of the bargaining frontier with respect to the entrepreneur's idea quality and VC expertise, and the fact that it is easier to transfer surplus to an entrepreneur as the quality of the specific venture increases, ensure that the highest bid for entrepreneur i, among all venture capitalists below $m(i)$, originates from venture capitalist $m(i-1)$.

entrepreneur i only with VC $m(i-1)$ (i.e., the one ranked directly below $m(i)$), which in turn allows us to explicitly define the outside option of entrepreneur i when he is matched with VC $m(i)$.

Figure 2 is a graphical depiction of the VC market equilibrium with PAM. Solid lines represent (stable) matches, while the dashed line represents the outside option of entrepreneur i, which is to contract with VC $m(i-1)$.

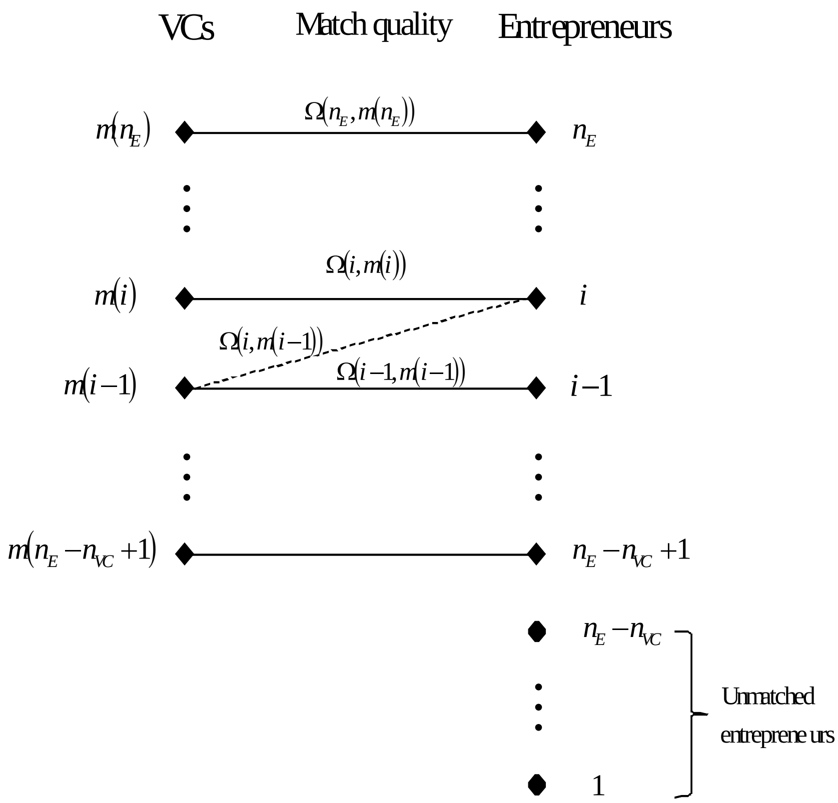

Figure 2: The VC Market Equilibrium with Positive Assortative Matching (PAM)

Let $u_{i,m(i-1)}$ denote the maximum utility that entrepreneur i can obtain from VC $m(i-1)$, provided that VC $m(i-1)$ does not become worse off relative to his current match with entrepreneur $i-1$. Formally, $u_{i,m(i-1)}$ is defined by

$$u_{i,m(i-1)} \equiv \max_{\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}} \lambda^2_{i,m(i-1)} \pi^2(K_{i,m(i-1)}, \Omega_{i,m(i-1)})/2 \tag{12}$$

s.t.

$$\lambda_{i,m(i-1)}(1-\lambda_{i,m(i-1)})\pi^2(K_{i,m(i-1)}, \Omega_{i,m(i-1)}) - rK_{i,m(i-1)} = \Pi^*(m(i-1)).$$

Consequently, the equilibrium utility of entrepreneur i is $u^*(i) = \max\left\{u_{i,m(i-1)}, U^V_{i,m(i)}\right\}$.[19] According to the properties of the bargaining frontier as stated in Lemma 4, $u_{i,m(i-1)}$ is increasing in $\Omega_{i,m(i-1)}$, and decreasing in $\Pi^*(m(i-1))$. Hence, $u_{i,m(i-1)}$ depends negatively on the equilibrium profit of the VC who is right below entrepreneur i in the ladder (i.e., $m\,(i-1)$), and positively on the match quality $\Omega_{i,m(i-1)}$ with VC $m\,(i-1)$.

Due to PAM in the VC market, it is clear that the higher expertise VC $m(i)$ can always outbid the lower expertise VC $m(i-1)$. Nevertheless, the improved alternative for the higher idea quality entrepreneur i might require the higher expertise VC $m(i)$ to adjust its contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in order to transfer more surplus to the entrepreneur, and thus to ensure entrepreneur i accepts the contract. Technically, the higher expertise VC $m(i)$ needs to adjust the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ whenever $U^V_{i,m(i)} < u_{i,m(i-1)}$.

When is the higher expertise VC $m(i)$ compelled to offer the higher idea quality entrepreneur i a contract that is less profitable to the VC? Put differently, when does the condition $U^V_{i,m(i)} < u_{i,m(i-1)}$ hold? To answer this question, we consider the following two cases.

Definition 3 (Case I: The VC Matters) ***Matching rent is zero.*** *The expertise of the VC enhances the match quality, while the quality of the entrepreneurial idea does not:*

$$\Omega_{i,m(i)} > \Omega_{i,m(i-1)} = \Omega_{i-1,m(i-1)}. \tag{13}$$

Under Case I, the lower expertise VC $m(i-1)$ does not gain by contracting with the higher idea quality entrepreneur i. We can thus infer that $U^V_{i,m(i)} = u^*(i)$, which in turn implies that matching considerations are irrelevant for designing the contract between VC $m(i)$ and entrepreneur i. The optimal VC contract is therefore identical to the one derived in Section 3 in the absence of matching considerations, namely $\{\lambda^*_{i,m(i)}, K^*_{i,m(i)}\}$.

[19]Note that $u_{i,m(i-1)}$ is the quasi-inverse of the bargaining frontier, $\overline{\Pi}(u_{i,m(i-1)}, \Omega_{i,m(i-1)})$, between entrepreneur i and venture capitalist $m(i-1)$. See Legros and Newman (2007) for the formal definition of the quasi-inverse of the bargaining frontier.

Definition 4 (Case II: The Entrepreneur Matters) ***Matching rent is positive.*** *The quality of the entrepreneurial idea enhances the match quality, while the expertise of the VC does not:*

$$\Omega_{i,m(i)} = \Omega_{i,m(i-1)} > \Omega_{i-1,m(i-1)}. \tag{14}$$

Under Case II, it follows that $U^V_{i,m(i)} < u_{i,m(i-1)}$, implying that VC $m(i)$ must offer the higher idea quality entrepreneur i a contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ that guarantees the entrepreneur an expected utility which exceeds $U^V_{i,m(i)}$. To see this, suppose for a moment that VC $m(i)$ offers the entrepreneur the contract $\{\lambda^*_{i,m(i)}, K^*_{i,m(i)}\}$ as derived in Section 3 in the absence of matching considerations. Since the lower expertise VC $m(i-1)$ is clearly better off being matched with the higher idea quality entrepreneur i, it would offer a contract $\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}$ aimed at guaranteeing entrepreneur i an expected utility level above $U^V_{i,m(i)}$. However, we can infer from Lemma 3 that the higher expertise VC $m(i)$ can profitably match $m(i-1)$'s contract offer $\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}$, while also making the higher idea quality entrepreneur $m(i)$ strictly better off. To summarize, while the high idea quality entrepreneur-high expertise VC match $(i, m(i))$ is mutually beneficial, and – according to Proposition 2 – constitutes the equilibrium outcome, competition for the higher idea quality entrepreneur nonetheless alters the equilibrium contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$.

Our previous discussion of both cases in conjunction with Lemma 3 lead to the following result:

Lemma 5 (The Relevance of Matching Considerations) *Consider the entrepreneur-VC matches* $(i, m(i))$ *and* $(i-1, m(i-1))$*, with* $i \geq 2$*. There exists a threshold match quality* $\overline{\Omega}_{i,m(i-1)} \in (\Omega_{i,m(i)}, \Omega_{i-1,m(i-1)})$ *such that for*

- (i) $\Omega_{i,m(i-1)} > \overline{\Omega}_{i,m(i-1)}$*, matching considerations alter the contract* $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ *between entrepreneur* i *and VC* $m(i)$*, i.e.,* $U^V_{i,m(i)} < u_{i,m(i-1)}$*;*
- (ii) $\Omega_{i,m(i-1)} \leq \overline{\Omega}_{i,m(i-1)}$*, matching considerations do not alter the contract* $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$*, i.e.,* $U^V_{i,m(i)} \geq u_{i,m(i-1)}$.

Lemma 5 provides an important implication about the role of matching in the VC market. If the predominant force behind a match quality improvement is the idea quality μ_i of entrepreneur i (i.e., Case

II arises), then matching considerations alter the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in his favor. Put differently, competition for entrepreneurs with high quality business ideas forces VCs to cede some of the surplus in order to attract those entrepreneurs. This in turn suggests that entrepreneurs generally benefit from the matching process – and thus the competition for "talent" – in the VC market at the expense of VCs. In other words, if Case II holds, each entrepreneur (except for the one with the lowest idea quality) earns a *matching rent*.

So far, we have examined whether VCs are forced to adjust their contracts due to matching considerations. We now investigate in detail how matching alters the profit share $\lambda_{i,m(i)}$ for the entrepreneur as well as the capital investment $K_{i,m(i)}$. According to Lemma 5, we need to distinguish between the two cases defined above. Under Case I, entrepreneur i's participation constraint (11) (see Section 4.1) does not bind for the contract $\{\lambda^*_{i,m(i)}, K^*_{i,m(i)}\}$ as derived in Section 3 in the absence of matching considerations. Thus, there is clearly no need for the corresponding VC $m(i)$ to adjust its contract as entrepreneur i is better-off accepting this contract rather than pursuing his best alternative (i.e., contracting with $m(i-1)$). As a result, under Case I, the matching process does not affect the optimal VC contracts as emphasized by Proposition 1.

Under Case II, however, competition for entrepreneur i results in an adjusted contract offer $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ from VC $m(i)$ which arises because, without such an adjustment, VC $m(i-1)$ would outbid VC $m(i)$ in order to attract entrepreneur i. Technically, entrepreneur i's participation constraint (11) is violated for the contract $\{\lambda^*_{i,m(i)}, K^*_{i,m(i)}\}$ (see Proposition 1). Matching in the VC market has therefore – under Case II – an impact on the contractual arrangements between VCs and entrepreneurs. The following proposition exposes the optimal VC contract under Case II.

Proposition 3 (VC Contracting with Matching) *Suppose Case II holds, i.e.,* $\Omega_{i,m(i-1)} > \overline{\Omega}_{i,m(i-1)}$*. Then, the optimal contract between entrepreneur* i *and VC* $m(i)$ *comprises the profit share*

$$\lambda^M_{i,m(i)} = \frac{\sqrt{2u^*(i)}}{\pi(K^M_{i,m(i)}, \Omega_{i,m(i)})} \qquad (15)$$

for the entrepreneur, and the capital investment $K^M_{i,m(i)}$ by the VC, which is implicitly defined by

$$\frac{\partial \pi(K^M_{i,m(i)}, \Omega_{i,m(i)})}{\partial K_{i,m(i)}} = \frac{r}{\sqrt{2u^*(i)}}. \tag{16}$$

The optimal VC contract in the presence of matching considerations, $\{\lambda^M_{i,m(i)}, K^M_{i,m(i)}\}$, as exposed by Proposition 3 will play an important role in the next section when we investigate how entry and exit in the VC market affect the efficiency and allocation of corresponding contracts. From a closer inspection of (16) it becomes clear that VC $m(i)$'s investment $K^M_{i,m(i)}$ is increasing in the value of entrepreneur i's outside option $u^*(i)$. Since a higher investment also motivates the entrepreneur to implement more effort (see (6)), it follows that his outside option $u^*(i)$ also has an indirect and positive effect on his equilibrium effort choice e^*_{ij}, and thus on the probability that his venture will be successful. To summarize, matching in the VC market improves the incentives of VCs to invest in new ventures, and at the same time, enhances entrepreneurs' incentives to exert effort in order to turn their innovative ideas into marketable products. However, does the competition for high quality entrepreneurial ideas eventually lead to the first-best investment K^{fb}_{ij} and effort level e^{fb}_{ij}? The next proposition proves that, despite the adjustments engendered by the matching process, the investment and effort levels remain sub-efficient from a social perspective.

Proposition 4 (VC Contracting with versus without Matching) *Suppose Case II holds, i.e., $\Omega_{i,m(i-1)} > \overline{\Omega}_{i,m(i-1)}$. Then, the optimal contract $\{\lambda^M_{i,m(i)}, K^M_{i,m(i)}\}$ between entrepreneur i and VC $m(i)$ with endogenous matching exhibits greater investment and effort levels compared to the optimal contract $\{\lambda^*_{i,m(i)}, K^*_{i,m(i)}\}$ in the absence of matching considerations, i.e., $K^M_{i,m(i)} > K^*_{i,m(i)}$ and $e^M_{i,m(i)} > e^*_{i,m(i)}$. However, the investment $K^M_{i,m(i)}$ and effort $e^M_{i,m(i)}$ with endogenous matching are still below their socially efficient levels, i.e., $K^M_{i,m(i)} < K^{fb}_{i,m(i)}$ and $e^M_{i,m(i)} < e^{fb}_{i,m(i)}$.*

Our previous analysis suggests that the consequences of matching on the optimal VC contracts are drastically different depending on whether the expertise of VCs is relatively more important for the specific match quality (Case I), or the quality of entrepreneurs' business ideas is relatively more important (Case II). Now, which case is more plausible for the VC market? Consider first Case I. Intuitively, it arises under two scenarios: (i) the match quality (and, by extension, the profitability of the venture) is significantly more sensitive to the expertise of a VC relative to the quality of an entrepreneur's business idea; or (ii) the distribution of expertise held by VCs is considerably more heterogenous than the distribution of entrepreneurial idea qualities. In contrast, Case II can arise under the following two scenarios: (i) the match quality is significantly more sensitive to the quality of an entrepreneurial idea relative to the expertise of a VC; or (ii) the distribution of the qualities of business ideas is considerably more heterogenous than the distribution of expertise held by VCs. We argue that the second scenario of Case II is the most plausible one, especially in light of empirical evidence suggesting that the matching process is of relatively high importance for the success of VC investments (Sørensen (2007)). We therefore focus exclusively on Case II in the subsequent section examining the roles of entry and exit, such that VCs need to take alternative matching outcomes into account when designing their contracts.

Finally, the following lemma shows that, as one would expect, VCs and entrepreneurs with superior ideas are better off:

Lemma 6 (Superior Entrepreneurial Ideas Yield Higher Payoffs) *Suppose Case II holds. The expected profit of VCs and the expected utility of entrepreneurs are increasing in the quality of entrepreneurial ideas, i.e.,* $\Pi^*(m(i)) \geq \Pi^*(m(i-1))$ *and* $u^*(i) \geq u^*(i-1)$ *for* $i \in E = \{n_E - n_{VC} + 1, ..., n_E\}$.

5 The effect of economic expansions and contractions on the VC market

We use our matching model to examine how economic expansions (booms) and contractions (busts) affect the equilibrium contracts in the VC market. Our approach takes the form of *comparative statics* with respect to the number of venture capitalists. More specifically, in an economic expansion, because there is more capital available for investments, the number of venture capitalists is higher than in an economic contraction.

5.1 Economic expansion

We postulate that due to an economic expansion more VCs enter the market at the bottom of the ladder. For simplicity, let's assume that one more VC enters and is matched with entrepreneur $n_E - n_{VC}$, see Figure 3. What is the effect on the entire VC market? The new entrant exerts a upward pressure on the outside option of entrepreneur $n_E - n_{VC} + 1$, who before entry had a zero outside option. This will force the VC that is matched with $n_E - n_{VC} + 1$, i.e., VC $m(n_E - n_{VC} + 1)$, to offer more capital. In turn this implies that this VC's profit will decrease, which from (12), follows that he is willing to bid more aggressively for entrepreneur $n_E - n_{VC} + 2$. The matched VC with entrepreneur $n_E - n_{VC} + 2$ will now be forced to provide more capital and so on. This effect travels up the ladder and all ventures receive a higher capital investment. The probability of success increases in all ventures.

Interestingly, our analysis reveals that entry of VCs, following an economic boom, benefits all entrepreneurs in the VC market and not only the ones that receive capital from the new entrants. The key mechanism is the interdependence of all ventures through the outside options of the entrepreneurs. Entry by VCs boosts the entrepreneurs' outside options and forces the VCs to increase their capital investments. Endogenous matching amplifies the effects of a boom.

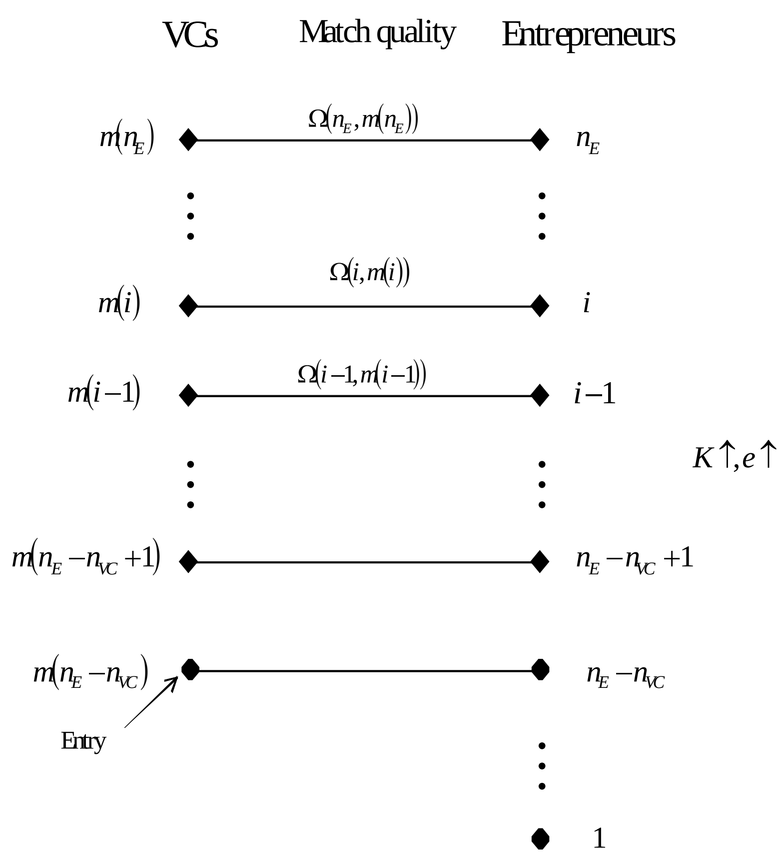

Figure 3: The VC Market in an Economic Expansion

5.2 Economic contraction

An economic contraction results in some VCs exiting the market. We reasonably assume that it is the lowest quality VCs that exit. Again, for simplicity we assume that only one VC exits, i.e., VC $m(n_F - n_E + 1)$, see Figure 4. Entrepreneur $n_E - n_{VC} + 2$'s outside option goes down to zero, allowing VC $m(n_E - n_{VC} + 2)$ to provide less capital which lowers effort and the probability of a successful venture. This effect influences negatively all investments and effort levels in the VC market, by simply reversing the logic we outlined in Section 5.1. Endogenous matching makes an economic downturn more severe.

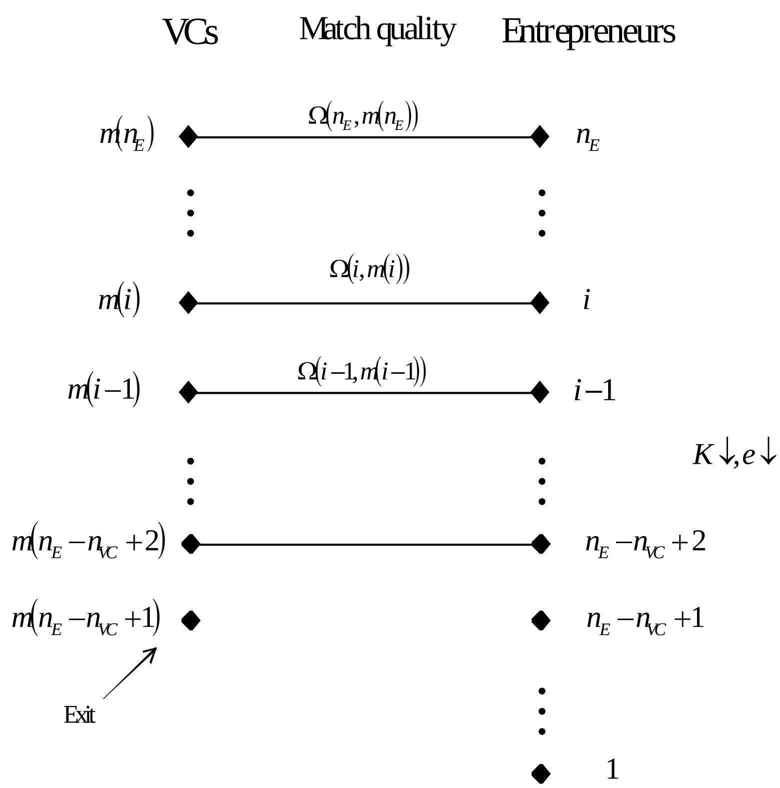

Figure 4: The VC Market in an Economic Contraction

6 Discussion and Conclusion

We consider a VC market that is comprised of VCs that are heterogeneous with respect to their expertise, and entrepreneurs that are heterogeneous with respect to the quality of their ideas. The success of a new venture depends crucially on the qualities that both sides bring to the contractual relationship. Within this market, we study how the two sides are matched endogenously. Our main interest is how endogenous matching affects the terms of the equilibrium contracts, in particular the investments made by the VCs and equity shares offered the entrepreneurs. We show that, under reasonable conditions, matching is positive assortative (PAM): VCs with high expertise match with entrepreneurs that have high quality business ideas. The key property of our matching framework is that each entrepreneur's outside option is endogenous as he could contract with an alternative VC. Because VCs compete for entrepreneurs with promising ideas, matching increases entrepreneurs' outside options, relative to what entrepreneurs would have received in the absence of matching (as occurs in a traditional stand-alone principal-agent model). Higher outside options imply that VCs must alter the terms of their contracts in order to transfer more surplus to entrepreneurs. This is achieved via greater investments, which also boost effort levels and success probabilities. While investments and effort levels are always sub-efficient, we find that matching improves the efficiency of the VC market from a social perspective. Finally, we show that entry into the VC market does not always improve efficiency. Interestingly, entry can impair efficiency even if it leads to higher match qualities in the market. Entry can deteriorate entrepreneurs' outside options, allowing VCs to attract and retain suitable entrepreneurs by transferring less of the generated surplus.

The model yields a series of empirically testable predictions, specifically with regards to the roles of entry and exit in the VC market. We make predictions about the impact of different types of entry on entrepreneur-VC pairs above and below the match quality of the entrants; these predictions pertain to the magnitude of the investment and the likelihood of success (or survival) of individual ventures. For such tests to be feasible, it is necessary to rank incumbent and entrant ventures according to their match quality. Sørensen (2007) resolved the econometric issues associated with two-sided matching in the VC market, while Hochberg, Ljungqvist, and Lu (2010) examined the properties of entry and exit in the VC market. Thus, the required ingredients are available to test our empirical predictions.

There are numerous potential theoretical extensions of the model. First, as in Sørensen (2007), suppose that each VC can finance multiple projects. One could reasonably assume that the expertise of a VC is the same for all projects it finances. Our analysis and main findings should follow through. The most significant change is that a VC will not compete with itself to attract entrepreneurs. For many entrepreneurs, their outside option would be the same VC and as a result their utility would not improve. More importantly, capital provision and effort would decrease. The implication is that, from a social perspective, it is more efficient to have numerous small VCs than fewer large ones. Second, suppose there are more entrepreneurs than VCs. Then the entrepreneurs with the lower quality ideas will not be matched, and our main results should still hold. Finally, our model may be extended by taking into account that VCs typically finance new ventures over the course of multiple stages (Kaplan and Strömberg (2003, 2004), and Kaplan, Sensoy, and Strömberg (2009)). However, such dynamic considerations would significantly complicate the matching process.

Appendix

Proof of Lemma 1

The socially efficient effort level e_i^{fb} as well as the efficient capital investment K_{ij}^{fb} are characterized by the following two first-order conditions:

$$\pi(K_{ij}, \Omega(\mu_i, x_j)) = e_i \tag{17}$$

$$\frac{\partial \pi(\cdot)}{\partial K_{ij}} e_i = r. \tag{18}$$

Substituting (17) into (18) yields the lemma. □

Proof of Proposition 1

By accounting for the effort policy given by (6), the optimal contract components λ_{ij}^* and K_{ij}^* are implicitly characterized by the following two first-order conditions:

$$2\lambda_{ij}^*(1-\lambda_{ij}^*)\pi(K_{ij}^*, \Omega_{ij})\frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = r; \tag{19}$$

$$(1-2\lambda_{ij}^*)\pi^2(K_{ij}^*, \Omega_{ij}) = 0. \tag{20}$$

Solving (20) for λ_{ij}^*, and substituting the resulting expression in (19), yields the proposition. □

Proof of Lemma 2

Recall from Proposition 1 that $K_{ij}^*(\Omega_{ij})$ is implicitly characterized by

$$\pi(K_{ij}, \Omega_{ij})\frac{\partial \pi(\cdot)}{\partial K_{ij}} = 2r. \tag{21}$$

Implicit differentiating yields

$$\frac{dK_{ij}^*(\Omega_{ij})}{d\Omega_{ij}} = -\frac{\frac{\partial \pi(\cdot)}{\partial \Omega_{ij}}\frac{\partial \pi(\cdot)}{\partial K_{ij}} + \pi(K_{ij}, \Omega_{ij})\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}\partial \Omega_{ij}}}{\frac{\partial}{\partial K_{ij}}\left[\pi(K_{ij}, \Omega_{ij})\frac{\partial \pi(\cdot)}{\partial K_{ij}} - 2r\right]} \tag{22}$$

Note first that the denominator is strictly negative due to the second-order condition. Moreover, recall that $\partial^2\pi(\cdot)/(\partial K_{ij}\partial\Omega_{ij}) \geq 0$. Thus, $dK^*_{ij}(\Omega_{ij})/d\Omega_{ij} > 0$. Finally, observe that $d\lambda^*_{ij}/d\Omega_{ij} = 0$. □

Proof of Lemma 3

Applying the Envelope Theorem to (8) yields the result for the VC. Next, the entrepreneur's expected utility under the optimal unconstrained contract $\Gamma^*_{ij}(\Omega_{ij}) = \{\lambda^*_{ij} = 1/2, K^*_{ij}(\Omega_{ij})\}$ is

$$U_i(e^*_{ij}, K^*_{ij}, \Omega_{ij}) = \frac{1}{2}\pi(K^*_{ij}, \Omega_{ij})e^*_{ij} - \frac{(e^*_{ij})^2}{2}. \tag{23}$$

Applying the Envelope Theorem yields

$$\frac{dU_i}{d\Omega_{ij}} = \frac{1}{2}\left[\frac{\partial\pi}{\partial K^*_{ij}}\frac{dK^*_{ij}}{d\Omega_{ij}} + \frac{\partial\pi}{\partial\Omega_{ij}}\right]. \tag{24}$$

Recall from Lemma 2 that $dK^*_{ij}/d\Omega_{ij} > 0$. Thus, $dU_i/d\Omega_{ij} > 0$. □

Proof of Lemma 4

The bargaining frontier is the solution to the following constrained maximization problem

$$\overline{\Pi}_{ij}(u_{ij}, \Omega_{ij}) = \max_{\{\lambda, K\}} (1-\lambda)\pi(K, \Omega_{ij})e^*_i - rK$$
$$\text{subject to: } \lambda\pi(K, \Omega_{ij})e^*_i - \frac{(e^*_i)^2}{2} \geq u_{ij}$$

where e^* is given by (6) and $u_{ij} \in [U^V_{ij}, U^E_{ij}]$.

At the bargaining frontier, the constraint must be binding. Using (6), the binding constraint becomes

$$\frac{\lambda^2(\pi(K, \Omega_{ij}))^2}{2} = u_{ij}.$$

Hence, λ must satisfy

$$\lambda^* = \frac{\sqrt{2u_{ij}}}{\pi(K, \Omega_{ij})}. \tag{25}$$

Using (25) and (6) the VC's problem becomes

$$\max_{\{K\}} \sqrt{2u_{ij}}\pi(K, \Omega_{ij}) - 2u_{ij} - rK.$$

The first order condition is

$$\sqrt{2u_{ij}}\pi_K(K, \Omega_{ij}) - r = 0. \tag{26}$$

The solution is denoted by $K^*(u_{ij}, \Omega_{ij})$. Thus, the frontier contract $\{\lambda^*, K^*\}$ satisfies

$$\pi_K(K^*, \Omega_{ij}) = \frac{r}{\sqrt{2u_{ij}}} \text{ and } \lambda^* = \frac{\sqrt{2u_{ij}}}{\pi(K^*, \Omega_{ij})}. \tag{27}$$

From (26) it follows that

$$\frac{dK^*}{du} = -\frac{\pi_K}{2u\pi_{KK}} > 0 \text{ and } \frac{dK^*}{d\Omega} = -\frac{\pi_{K\Omega}}{\pi_{KK}} > 0. \tag{28}$$

Higher outside option or higher match quality imply higher capital investment. Using (25) and (28) we have

$$\frac{d\lambda^*}{du} = \frac{\pi + (\pi_K)^2/\pi_{KK}}{\sqrt{2u}\pi^2}. \tag{29}$$

The maximized profit function $\overline{\Pi}(u_{ij}, \Omega_{ij}) = \sqrt{2u_{ij}}\pi(K^*(u_{ij}, \Omega_{ij}), \Omega_{ij}) - 2u_{ij} - rK^*(u_{ij}, \Omega_{ij})$ is the bargaining frontier. We differentiate the bargaining frontier with respect to u_{ij}, which yields

$$\frac{d\overline{\Pi}}{du_{ij}} = \frac{1}{\sqrt{2u_{ij}}}\pi + \sqrt{2u_{ij}}\pi_K\frac{dK^*}{du_{ij}} - 2 - r\frac{dK^*}{du_{ij}} \tag{30}$$

$$\text{using (26)} \quad = \frac{1}{\sqrt{2u_{ij}}}\pi - 2. \tag{31}$$

When the above derivative is evaluated at the lowest possible u_{ij} which is U_{ij}^V, see (10), it is zero. Hence, any higher u_{ij} implies that the slope of the frontier, as expected, is negative. □

Proof of Proposition 2

We check whether the sufficient conditions for PAM are satisfied. We differentiate the VC's constrained profit function $\overline{\Pi} \equiv \sqrt{2u}\pi(K^*, \Omega) - 2u - rK^*$ with respect to x

$$\Pi_x = \sqrt{2u_{ij}}\pi_K \frac{\partial K^*}{\partial \Omega}\frac{\partial \Omega}{\partial x} + \sqrt{2u_{ij}}\pi_\Omega \frac{d\Omega}{dx} - r\frac{dK^*}{\partial \Omega}\frac{\partial \Omega}{\partial x} = \tag{32}$$

$$\text{using (26)} = \sqrt{2u_{ij}}\pi_\Omega \frac{\partial \Omega}{\partial x}. \tag{33}$$

Now we differentiate Π_x with respect to μ

$$\begin{aligned}
\Pi_{x\mu} &= \sqrt{2u}\left(\pi_{\Omega\Omega}\frac{\partial \Omega}{\partial \mu}\frac{\partial \Omega}{\partial x} + \pi_{\Omega K}\frac{\partial K^*}{\partial \Omega}\frac{\partial \Omega}{\partial \mu}\frac{\partial \Omega}{\partial x} + \pi_\Omega \frac{\partial^2 \Omega}{\partial x \partial \mu}\right) \\
\text{using (28)} &= \sqrt{2u}\left(\pi_{\Omega\Omega}\frac{\partial \Omega}{\partial x}\frac{\partial \Omega}{\partial \mu} - \pi_{\Omega K}\frac{\pi_{K\Omega}}{\pi_{KK}}\frac{\partial \Omega}{\partial \mu}\frac{\partial \Omega}{\partial x} + \pi_\Omega \frac{\partial^2 \Omega}{\partial x \partial \mu}\right)
\end{aligned}$$

Given that $\partial^2\Omega / (\partial x \partial \mu) \geq 0$, the above is positive provided that

$$\sqrt{2u}\left(\pi_{\Omega\Omega} - \pi_{\Omega K}\frac{\pi_{K\Omega}}{\pi_{KK}}\right)\frac{\partial \Omega}{\partial \mu}\frac{\partial \Omega}{\partial x} \tag{34}$$

is positive. The positive sign follows from the assumption that $\pi(K, \Omega)$ is concave in both of its arguments and hence the Hessian determinant is positive, i.e., $\pi_{KK}\pi_{\Omega\Omega} - \pi_{\Omega K}\pi_{K\Omega} \geq 0$.

Next, we differentiate (33) with respect to u_{ij}

$$\Pi_{xu} = \frac{1}{\sqrt{2u_{ij}}}\pi_\Omega \frac{\partial \Omega}{\partial x} + \sqrt{2u_{ij}}\pi_{\Omega K}\frac{\partial K^*}{\partial u}\frac{\partial \Omega}{\partial x}. \tag{35}$$

Under our assumptions Π_{xv} is positive. □

Proof of Proposition 3

The result follows directly from the proof of Lemma 4. □

Proof of Lemma 6

By way of contradiction suppose that for some i, $\Pi^*(m(i)) < \Pi^*(m(i-1))$. Then, VC $m(i)$ can offer to entrepreneur $i-1$ exactly the same contract VC $m(i-1)$ offers to entrepreneur $i-1$. Because $x_{m(i)} > x_{m(i-1)}$ and $\Omega(i-1, m(i)) > \Omega(i-1, m(i-1))$, the utility of entrepreneur $i-1$ is greater than $u^*(i-1)$ and the profit of VC $m(i)$, is greater than $\Pi^*(m(i-1))$, which, given $\Pi^*(m(i-1)) > \Pi^*(m(i))$, implies that the matching m is not stable, contradiction. A similar argument can be used to show that $u^*(i)$ is increasing in i. □

Proof of Proposition 4

First, we can infer from (28) that $K_{ij}^M > K_{ij}^*$. Moreover, from (6), we have

$$\frac{de_i^*}{du_{ik}} = \frac{d\lambda_{ij}^*}{du_{ik}}\pi(\cdot) + \lambda_{ij}^* \frac{\partial \pi(\cdot)}{\partial K_{ij}} \frac{dK_{ij}^*}{du_{ik}}. \tag{36}$$

Using (25), (28), and (29), it follows that

$$\frac{de_i^*}{du_{ik}} = \frac{1}{\sqrt{2u_{ij}}} > 0. \tag{37}$$

Recall that if $U_i(\cdot) = u_{ik}$, K_{ij}^M, satisfies (27). This condition becomes equal to (3) only when $u_{ij} = \pi^2(K_{ij}, \Omega_{ij})/2$. However, on the bargaining frontier, we have

$$u_{ij} = \frac{1}{2}\lambda_{ij}^2 \pi^2(K_{ij}, \Omega_{ij}) < \frac{1}{2}\pi^2(K_{ij}, \Omega_{ij}), \tag{38}$$

because $\lambda_{ij} < 1$. This implies that $K_{ij}^M < K_{ij}^{fb}$, and hence, $e_i^M < e_i^{fb}$. From (28), higher outside option u_{ij}, due to the endogenous matching, implies higher capital investment. □

Proof of Proposition ??

A free entry equilibrium exists because, from Lemma 6, $\Pi^*(m(i))$ and $u^*(i)$ decrease as i decreases. Hence, if fixed costs of entry are high enough then there exists an $n^* < N$, meaning that only a subset of potential VCs and entrepreneurs enter the market. □

Proof of Proposition ??

Let entrepreneur i' and VC $m(i'-1)$ enter between $(i-1, m(i-1))$ and $(i, m(i))$. The matchings below and above are not affected. From Lemma 6, we have $\Pi^*(m(i')) > \Pi^*(m(i-1))$, suggesting that $u^*(i)$ decreases after entry for all $i > i'$. This follows from (12) and the fact that nothing else, except Π^*, changes for $i > i'$. This implies, following Proposition 3, less investment for all matched pairs above where entry occurred and less effort. There will be no change in the pairs below i'. □

Proof of Proposition ??

Assume that VC j enters between VC $m(i)$ and VC $m(i-1)$. For concreteness, and without any loss of generality, let's assume that j enters between VCs $m(3)$ and $m(4)$. VC 1 is unmatched in the new matching m'. In the new matching m', $m'(1) = 2$, $m'(2) = 3$ and $m'(3) = j$. Above the last pair there is no effect, i.e., $m'(4) = 4$, $m'(5) = 5$ and so on. In the new matching, m', the utility of the first entrepreneur $u^*(1)$ will increase, because his outside option is VC 1, as opposed to zero before entry. VC 2, with $m'(1) = 2$, provides more capital to entrepreneur 1 than the investment from $m(1)$. This is because $u^*(1)$ has increased and $\Omega(1, m'(1))$ is higher than $\Omega(1, m(1))$, although not much higher, given that we are in Case II. Entrepreneur 1 is now matched with a higher quality VC, which boosts the profit of the VC relative to m (first effect), but at the same time entrepreneur 1 has a better outside option under m' than under m which lowers the VC's profit (second effect). Moreover, the second effect is much stronger under Case II because the main driving force in a matched pair is the entrepreneur who has not changed. Hence, $\Pi^*(m'(1)) < \Pi^*(m(1))$, suggesting that VC 2 (who is now matched with entrepreneur 1) will bid more for entrepreneur 2 (than the bid of VC 1 under the old matching) and hence $u^*(2)$ increases. Following a similar logic, $u^*(3)$ increases as well. Hence, $\Pi^*(m'(3)) < \Pi^*(m(3))$, again given Case II. Thus, VC j is willing to bid more for entrepreneur 4 than VC 3 under the old matching, i.e., $u^*(4)$ increases. By following this logic, this change travels up the ladder and hence all outside options increase. This forces VCs to provide more capital, leading to higher effort and probability of a successful venture. □

Proof of Proposition ??

Assume that entrepreneur i' enters between entrepreneur i and $i-1$, with $i > 2$. For concreteness, and without any loss of generality, let's assume that i' is between entrepreneurs 3 and 4. Entrepreneur 1 is unmatched under the new matching m'. In the new matching, $m'(2) = 1$, $m'(3) = 2$ and $m'(i') = 3$. Above the last pair there is no effect, i.e., $m'(4) = 4$, $m'(5) = 5$ and so on. Let's first compare $\Pi^*(m(2))$ with $\Pi^*(m'(2))$. In both cases, the entrepreneur is the same. A negative effect on Π^*, under matching m', is that $\Omega(2, m(2)) > \Omega(2, m'(2))$, because $m'(2) = 1$, while $m(2) = 2$. A positive effect on Π^* under m' is that there is no VC below $m'(2)$, which implies a lower $u^*(2)$ under m', relative to $u^*(2)$ under m. Under Case II, we have that $\Omega(2, m(2)) \approx \Omega(2, m'(2))$ and thus $\Pi^*(m(2)) > \Pi^*(m(2))$. This also implies less investment in the pair $(2, m'(2))$ relative to $(2, m(2))$. Because $\Pi^*(m(2)) > \Pi^*(m(2))$, following from (12), $u^*(3)$ is lower under m'. Next, let's look at the pair $(3, m'(3))$. Because $u^*(3)$ has decreased, capital investment decreases (again, given that we are in Case II, match quality is not much different between $(3, m(3))$ and $(3, m'(3))$). For similar reasons as before, $\Pi^*(m(3)) > \Pi^*(m(3))$. The next entrepreneur higher up in the ladder is the new entrant i'. The question is what happens to $u^*(4)$, the outside option of the entrepreneur right above the new entrant. This depends on the comparison between $\Pi^*(m'(i'))$ and $\Pi^*(m(3))$. We know, from Lemma 6, that $\Pi^*(m'(i')) > \Pi^*(m'^*(3))$ and since $\Pi^*(m(3)) > \Pi^*(m(3))$ we have $\Pi^*(m'(i')) > \Pi^*(m(3))$. Hence, $u^*(4)$ decreases. The same is true for all matched pairs higher up in the ladder. Investment and effort decrease. $\square$

Entrepreneurs' Preferences

Suppose that the entire bargaining power rests with the entrepreneur. When completely ignoring the policy of the VC, it is intuitively clear that the entrepreneur would prefer (i) to be the residual claimant of the entire profit of the new venture (i.e., $\lambda_{ij} = 1$); and (ii) an unlimited investment of capital (i.e., $K_{ij} \to \infty$). However, without any adequate revenues for its capital investment, the VC would never invest in the new venture. Technically, the participation constraint of the VC would be violated. This in turn implies that we need to account for the policies of the VC when considering the contractual preferences of the entrepreneur.

To better understand the competition of ideas in the market for VC and its effect on the corresponding contracts, consider the following scenario: The entrepreneur chooses his preferred share λ_{ij}^E on the venture's profit, while taking the corresponding investment $K_{ij}^*(\lambda_{ij}^E)$ by the VC into account. Let U_{ij}^E denote entrepreneur i's expected utility, which constitutes his highest expected utility level when forming a partnership with VC j. Then, λ_{ij}^E is defined by

$$\lambda_{ij}^E \in \text{argmax}_{\widetilde{\lambda}_{ij}} \, U_i^E(e_i^*, \widetilde{\lambda}_{ij}, K_{ij}^*(\widetilde{\lambda}_{ij}), \Omega_{ij}) = \widetilde{\lambda}_{ij}\pi(K_{ij}^*(\widetilde{\lambda}_{ij}), \Omega_{ij})e_i^* - \frac{(e_i^*)^2}{2}.$$

where $K_{ij}^*(\lambda_{ij})$ is implicitly characterized by (19) (ignoring condition (20)). Obviously, it is in the entrepreneur's best interest to leave some shares for the VC (i.e., $\lambda_{ij}^E < 1$). Otherwise, without any share on the generated revenue of the new venture (i.e., $\lambda_{ij} = 1$), the VC would not invest at all (i.e., $K_{ij}^*(1) = 0$), thus rendering the entrepreneur's pursuit of his business idea impossible.[20]

Drawing on the entrepreneur's contractual preference—as characterized above—eventually allows us to identify whether the entrepreneur favors a higher share on the venture's profit—compared to the unconstrained contract $\Gamma^*(\Omega_{ij})$—at the expense of a higher capital investment, or vice versa. These insights are in turn critical for our subsequent analysis concerned with how contracts between VCs and entrepreneurs respond to matching considerations in the market for VC. To unravel potential conflicts of interest between both involved parties with respect to the allocation of shares, the next proposition elaborates on the relationship between the entrepreneur's preferences and those of the VC, and how this relationship is affected by the specific match quality $\Omega(\mu_i, x_j)$.

Proposition 5 *The entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ on the venture's profit always lies above the share $\lambda_{ij}^* = 1/2$ preferred by the venture capitalist, even though this would concurrently imply a lower capital investment $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij})$ (i.e., $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$ and $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$). Moreover, the entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ is strictly increasing in the match quality $\Omega(\mu_i, x_j)$ (i.e., $d\lambda_{ij}^E(\Omega_{ij})/d\Omega_{ij} > 0$).*

[20] In fact, one can show that there exists an inverse U-shaped relationship between the entrepreneur's expected utility $U_i^E(\cdot)$ and his share λ_{ij} on the venture's profit, similar to the relationship between the venture capital firm's expected profit $\Pi_{ij}(\cdot)$ and λ_{ij}.

Proof: By accounting for the entrepreneur's effort policy e_i^*, we get his expected utility as a function of λ_{ij}:

$$U_i(\lambda_{ij}, K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) = \frac{1}{2}\lambda_{ij}^2 \pi^2(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}).$$

From the first-order condition, the share $\lambda_{ij}^E(\Omega_{ij})$ preferred by the entrepreneur is implicitly characterized by

$$\pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij}\frac{\partial \pi(\cdot)}{\partial K_{ij}^*}\frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = 0. \tag{39}$$

The first-order condition implies that the optimal $\lambda_{ij}^E(\Omega_{ij})$ is where $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$. Implicit differentiating $K_{ij}^*(\cdot)$ as defined by (19) yields

$$\frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = -\frac{2(1-2\lambda_{ij})\pi(\cdot)\frac{\partial \pi(\cdot)}{\partial K_{ij}}}{\frac{\partial}{\partial K_{ij}}\left[2\lambda_{ij}(1-\lambda_{ij})\pi(K_{ij}, \Omega_{ij})\frac{\partial \pi(\cdot)}{\partial K_{ij}} - r\right]} \tag{40}$$

Note first that the denominator is strictly negative due to the second-order condition. Clearly, $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ requires that $\lambda_{ij} > 1/2$. Thus, $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$. Since $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ for $\lambda_{ij} > 1/2$, it follows that $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$.

Next, define

$$F \equiv \pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij}\frac{\partial \pi(\cdot)}{\partial K_{ij}^*}\frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}}$$

$$G \equiv 2\lambda_{ij}(1-\lambda_{ij})\pi(K_{ij}, \Omega_{ij})\frac{\partial \pi(\cdot)}{\partial K_{ij}} - r$$

Applying Cramer's rule gives

$$\frac{\partial \lambda_{ij}^E(\cdot)}{\partial \Omega_{ij}} = \frac{\det(A)}{\det(B)} \tag{41}$$

where

$$A = \begin{pmatrix} -\frac{\partial F}{\partial \Omega_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ -\frac{\partial G}{\partial \Omega_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix} \qquad B = \begin{pmatrix} \frac{\partial F}{\partial \lambda_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ \frac{\partial G}{\partial \lambda_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix}$$

Note that

$$\det(A) = -\frac{\partial F}{\partial \Omega_{ij}} \frac{\partial G}{\partial K_{ij}} + \frac{\partial G}{\partial \Omega_{ij}} \frac{\partial F}{\partial K_{ij}}. \tag{42}$$

Clearly, $\partial G/\partial K_{ij} < 0$ due to the second-order condition. Moreover,

$$\begin{aligned}
\frac{\partial F}{\partial \Omega_{ij}} &= \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} + \lambda_{ij} \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \frac{dK_{ij}^*}{d\lambda_{ij}} > 0 \\
\frac{\partial G}{\partial \Omega_{ij}} &= 2\lambda_{ij}(1-\lambda_{ij}) \left[\frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \pi(\cdot) \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \right] > 0 \\
\frac{\partial F}{\partial K_{ij}} &= \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \lambda_{ij} \underbrace{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}}_{\leq 0} \underbrace{\frac{dK_{ij}^*}{d\lambda_{ij}}}_{<0} > 0.
\end{aligned}$$

Thus, $\det(A) > 0$. Next, we have

$$\det(B) = \frac{\partial F}{\partial \lambda_{ij}} \frac{\partial G}{\partial K_{ij}} - \frac{\partial G}{\partial \lambda_{ij}} \frac{\partial F}{\partial K_{ij}}. \tag{43}$$

Notice that $\partial F/\partial \lambda_{ij}, \partial G/\partial K_{ij} < 0$ due to the respective second-order conditions. Moreover, recall that $\partial F/\partial K_{ij} > 0$. Furthermore,

$$\frac{\partial G}{\partial \lambda_{ij}} = 2(1-2\lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} < 0. \tag{44}$$

Therefore, $\det(B) > 0$. Combining previous observations, we have $\partial \lambda_{ij}^E(\cdot)/\partial \Omega_{ij} > 0$.

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