

## Appendix A. Characterization of equilibrium

### Appendix A.1. Final good producer problem

From the final good producer problem (3) the first order conditions with respect to  $x(i)$  for any  $i \in [0, N]$ ,  $k$ ,  $l'$ , and  $v$  are respectively given by:

$$(1 - \psi) x(i)^{-\psi} [k^\alpha l'^{1-\alpha}]^\psi - p(i) = 0 \quad (\text{A.1})$$

$$\alpha \psi z^{1-\psi} [k^\alpha l'^{1-\alpha}]^{\psi-1} k^{\alpha-1} l'^{1-\alpha} - (r + \delta_k) = 0 \quad (\text{A.2})$$

$$-J(\omega, \Omega) + \beta E \{ \Lambda(\Omega, \Omega') V_l(l', \Omega') | l, \Omega \} = 0 \quad (\text{A.3})$$

$$J(\omega, \Omega) q - \kappa N'^{\frac{1}{1-\alpha}} = 0. \quad (\text{A.4})$$

The associated envelope condition reads:

$$V_l(l, \Omega) = (1 - \alpha) \psi z^{1-\psi} [k^\alpha l'^{1-\alpha}]^{\psi-1} k^\alpha l'^{-\alpha} - \omega + (1 - \delta_l) J(\omega, \Omega). \quad (\text{A.5})$$

Combining Equations (A.3) and (A.5) gives the following equation for the marginal value to the producer of having another worker (vacancy posting condition):

$$J(\omega, \Omega) = \beta E \left\{ \Lambda(\Omega, \Omega') \left[ (1 - \alpha) \psi z'^{1-\psi} \left( k'^\alpha l'^{1-\alpha} \right)^{\psi-1} k'^\alpha l'^{-\alpha} - w' + (1 - \delta_l) J(\omega', \Omega') \right] | l, \Omega \right\}. \quad (\text{A.6})$$

According to [Gertler and Trigari \(2009\)](#), the Lagrange multiplier  $J(\omega, \Omega)$  can be interpreted as aggregate producers' surplus. Equation (A.6) states that a firm optimally posts vacancies until the marginal cost  $J(\omega, \Omega) = N'^{\frac{1}{1-\alpha}} \kappa / q$  is equalized with the marginal product of labor net of the wage bill, plus the continuation value of a match. As  $\kappa$  tends to zero and  $\delta_l$  tends to unity, the vacancy posting condition approaches the standard RBC labor demand condition. Finally the free entry condition in the labor market implies that the vacancy posting cost,  $\kappa$ , proportional to productivity has to be equal in equilibrium to the expected firm value.

### Appendix A.2. Intermediate good producer problem

The problem of an intermediate good producer  $i \in [0, N]$  is:

$$W(i; \Omega) = \max_{p(i)} \{ p(i) x(i) - \mu x(i) + (1 - \delta_N) \beta E \{ \Lambda(\Omega, \Omega') W(i; \Omega') | \Omega \} \}, \quad (\text{A.7})$$

subject to:

$$x(i) = \left[ \frac{1 - \psi}{p(i)} \right]^{\frac{1}{\psi}} k^\alpha l'^{1-\alpha}, \quad (\text{A.8})$$

where  $x(i)$  is the quantity of the  $i$ th input demanded by the final goods firm and  $-1/\psi$  is the price elasticity of demand for each type of intermediate. The first order condition with respect to  $p(i)$  for any  $i \in [0, N]$  is given by:

$$p(i) = \mu (1 - \psi)^{-1} > \mu. \quad (\text{A.9})$$

Hence, the price  $p(i)$  is constant over time and across  $i$ . The intermediate good's price is equal to the marginal cost of production multiplied by the markup  $(1 - \psi)^{-1}$ . The price is identical for all intermediate goods because the cost of production is constant, and each good enters symmetrically into the production function in equation (1).

Substituting for  $p(i)$  from equation (A.9) into equation (A.8), we obtain the aggregate quantity produced of each intermediate good  $i \in [0, 1]$ :

$$x(i) = \left[ \frac{\mu}{(1 - \psi)^2} \right]^{-\frac{1}{\psi}} k^\alpha l^{1-\alpha}, \quad (\text{A.10})$$

which is the same for all intermediate goods  $i \in [0, 1]$ . Since the intermediate good's price exceeds the marginal cost of production, the quantity  $x(i)$  is smaller than it would be if intermediate goods were priced at marginal cost.

The aggregate quantity of intermediate goods, denoted by  $z$ , is given by:

$$z = N^{\frac{1}{1-\psi}} \left[ \frac{\mu}{(1 - \psi)^2} \right]^{-\frac{1}{\psi}} k^\alpha l^{1-\alpha}. \quad (\text{A.11})$$

Now substituting for  $z$  from Equation (A.11) into (1) yields the following derived production function for the final good:

$$y = A_{\mu,\psi} N k^\alpha l^{1-\alpha}, \quad (\text{A.12})$$

where  $A_{\mu,\psi} = [\mu/(1 - \psi)^2]^{-(1-\psi)/\psi} > 0$ . Notice that Equation (A.12) is fundamental for at least two reasons. First, it shows that even though the aggregate production function exhibits constant returns to scale from the viewpoint of final good producers, there are increasing returns to scale for the entire economy. Second, from Equation (A.12) it follows that an increase in the variety of intermediate goods,  $N$ , raises capital and labor productivity.

Each intermediate good firm  $i \in [0, N]$  produces a total amount of varieties given by (A.10). Since there are  $N$  producers, the total expenditure on intermediate goods is:

$$e = B_{\mu,\psi} N k^\alpha l^{1-\alpha}, \quad (\text{A.13})$$

where  $B_{\mu,\psi} = \mu [\mu/(1 - \psi)^2]^{-1/\psi} > 0$ . Per period value of owning a blueprint is the profit per period of the monopolist producing variety  $i$  of intermediate goods,  $\Pi(i, \Omega)$ . Hence Equations (A.9) and (A.10) implies that:

$$\Pi(i, \Omega) = C_{\mu,\psi} k^\alpha l^{1-\alpha}, \quad (\text{A.14})$$

where  $C_{\mu,\psi} = \mu\psi/(1 - \psi) [\mu/(1 - \psi)^2]^{-1/\psi} > 0$ . Equation (A.14) implies that profit  $\Pi(i, \Omega)$  is the same for all intermediate good producers. Profits that the intermediate good firms make from producing a variety  $i \in [0, N]$  arise from monopolistically competitive pricing. Finally substituting for  $\Pi(i, \Omega)$  from (A.14) into (A.7), the value of an R&D firm with a blueprint reads:

$$W(i, \Omega) = \Pi(i, \Omega) + (1 - \delta_N) \beta E \{ \Lambda(\Omega, \Omega') W(i; \Omega') | \Omega \}. \quad (\text{A.15})$$

Since  $\Pi(i, \Omega)$  is the same for all intermediate good producers, it follows that  $W(i, \Omega) = W(\Omega)/N$ . An entrepreneur will find R&D investment attractive if this present value is

at least as large as the R&D cost. There is free entry into research sector. The following no-arbitrage condition must be satisfied in equilibrium:

$$(1 - \delta_N) \beta E \{ \Lambda (\Omega, \Omega') W (\Omega') | \Omega \} \cdot [N' - (1 - \delta_N) N] = (1 + r') a' N'. \quad (\text{A.16})$$

The left-hand side of the free entry condition denotes the benefit from developing a new intermediate good, while the right-hand side is the R&D cost. Notice that Equation (A.16) shows how the model generates procyclical R&D. For instance, during a boom, the value of a new intermediate good,  $W$ , increases. Since the profit flow from intermediate goods rises, the benefit to creating new types of these products goes up. R&D spending will increase in response. Now combining (22) and (A.16), we obtain after some elementary manipulations:

$$g_N = (1 - \delta_N) + \frac{a'}{K'} \eta, \quad (\text{A.17})$$

where  $\eta$  is a random shock to the innovation process.

### Appendix A.3. Representative household problem

The household's first-order conditions with respect to  $c$  and  $s'$  imply the following equations must hold in equilibrium:

$$u_c(c) - \lambda^c(\Omega) = 0 \quad (\text{A.18})$$

$$-\lambda^c(\Omega) + \beta E \{ Q_{k'}(k', a', \Omega') | k, a, \Omega \} = 0 \quad (\text{A.19})$$

$$-\lambda^c(\Omega) + \beta E \{ Q_{a'}(k', a', \Omega') | k, a, \Omega \} = 0, \quad (\text{A.20})$$

where  $\lambda^c(\Omega)$  is the Lagrange multiplier on the budget constraint. The associated envelope conditions with respect to  $s$  and  $L$  respectively read:

$$Q_k(k, a, \Omega) = \lambda^c[R + (1 - \delta_k)] \quad (\text{A.21})$$

$$Q_a(k, a, \Omega) = \lambda^c(1 + r) \quad (\text{A.22})$$

$$Q_L(k, a, \Omega) = \lambda^c(\omega - b) + (1 - \delta_l - f) H^l(\omega, \Omega), \quad (\text{A.23})$$

where  $H^l(\omega, \Omega)$  is the Lagrange multiplier on the law of motion for labor. Combining Eq. (A.18), (A.19) and (A.20) with Eq. (A.21) and (A.22) gives the following Euler equations for consumption:

$$1 - \beta E \{ [R' + (1 - \delta_k)] \Lambda (\Omega, \Omega') | k, a, \Omega \} = 0 \quad (\text{A.24})$$

$$1 - \beta E \{ (1 + r') \Lambda (\Omega, \Omega') | k, a, \Omega \} = 0, \quad (\text{A.25})$$

where  $\Lambda(\Omega, \Omega') = u_c(c')/u_c(c)$ . Moreover, Eq. (A.24) and (A.25) imply the following no-arbitrage condition:

$$1 + r' = R' + (1 - \delta_k). \quad (\text{A.26})$$

Given Equations (A.23), and deriving the objective function (23) with respect to  $L'$ , we obtain:

$$H(\omega, \Omega) = \beta E \left\{ \Lambda(\Omega, \Omega') \left\{ (\omega' - b) + (1 - \delta_l - f') H'(\omega', \Omega') \right\} \mid \omega, \Omega \right\}. \quad (\text{A.27})$$

As in Gertler and Trigari (2009),  $H(\omega, \Omega) = H^l(\omega, \Omega)/\lambda^c(\Omega)$  denotes the marginal utility value of a job to household in terms of consumption good.

#### Appendix A.4. Wage determination

Workers and final good producers use the Nash (1950) bargaining approach to determine the contract wage,  $\omega$ . The bargaining problem is to choose the contract wage such that:

$$\begin{aligned} \omega &= \arg \max_{\omega} G \{ H(\omega, \Omega), J(\omega, \Omega); \xi \} \\ &= \arg \max_{\omega} \left\{ H(\omega, \Omega)^\xi J(\omega, \Omega)^{1-\xi} \right\}, \end{aligned} \quad (\text{A.28})$$

where  $\xi \in (0, 1)$  is the relative bargaining power. The first order condition is given by:

$$\xi J(\omega, \Omega) = (1 - \xi) H(\omega, \Omega). \quad (\text{A.29})$$

After manipulations, the surplus sharing rule can be rewritten as follows:

$$\begin{aligned} &\xi \beta E \left\{ \Lambda(\Omega, \Omega') \left\{ (1 - \alpha) \psi \left( \frac{y'}{l'} \right) - \omega' + (1 - \delta_l) \left( \frac{\kappa N''^{\frac{1}{1-\alpha}}}{q'} \right) \right\} \mid \omega, \Omega \right\} \\ &= (1 - \xi) \beta E \left\{ \Lambda(\Omega, \Omega') \left\{ (\omega' - b) + (1 - \delta_l - f') \left( \frac{\xi}{1 - \xi} \right) \left( \frac{\kappa N''^{\frac{1}{1-\alpha}}}{q'} \right) \right\} \mid \omega, \Omega \right\}, \end{aligned} \quad (\text{A.30})$$

where from (A.4) and (A.29):  $J(\omega', \Omega') = \left( \kappa N''^{1/1-\alpha} / q \right)$  and  $H'(\omega', \Omega') = (\xi / (1 - \xi)) (\kappa / q')$ . After rearrangements, we find:

$$\omega = (1 - \xi)b + \xi \left[ (1 - \alpha) \psi \left( \frac{y}{l} \right) + \kappa N'^{\frac{1}{1-\alpha}} \theta \right], \quad (\text{A.31})$$

where  $y/l$  denotes the labor productivity.

## Appendix B. Full set of equilibrium conditions

The entire set of equilibrium conditions of the model are given by:

$$y = A_{\mu, \psi} N k^\alpha l^{1-\alpha} \quad (\text{B.1})$$

$$e = B_{\mu, \psi} N k^\alpha l^{1-\alpha} \quad (\text{B.2})$$

$$W = C_{\mu,\psi} N k^\alpha l^{1-\alpha} + (1 - \delta_N) g_N^{-1} \beta E \{ \Lambda W' \} \quad (\text{B.3})$$

$$(1 - \delta_N) \beta E \{ \Lambda W' \} \cdot [N' - (1 - \delta_N) N] = (1 + r') a' N' \quad (\text{B.4})$$

$$\alpha \psi \frac{y}{k} - (r + \delta_k) = 0 \quad (\text{B.5})$$

$$u_c(c) - \beta E \{ [R' + (1 - \delta_k)] u_c(c') \} = 0 \quad (\text{B.6})$$

$$u_c(c) - \beta E \{ (1 + r') u_c(c') \} = 0 \quad (\text{B.7})$$

$$u = 1 - l \quad (\text{B.8})$$

$$\kappa N'^{\frac{1}{1-\alpha}} = qJ \quad (\text{B.9})$$

$$J = \beta E \Lambda \left[ \pi^{\#'} + (1 - \delta_l) J' \right] \quad (\text{B.10})$$

$$\pi^{\#} = (1 - \alpha) \psi \left( \frac{y}{l} \right) - \omega \quad (\text{B.11})$$

$$\omega = (1 - \xi) b + \xi \left[ (1 - \alpha) \psi \left( \frac{y}{l} \right) + \kappa N'^{\frac{1}{1-\alpha}} \theta \right] \quad (\text{B.12})$$

$$\theta = \frac{v}{u} \quad (\text{B.13})$$

$$m = m(v, u) \quad (\text{B.14})$$

$$f = m(\theta, 1) \quad (\text{B.15})$$

$$q = m \left( 1, \frac{1}{\theta} \right) \quad (\text{B.16})$$

$$bu + T = 0 \tag{B.17}$$

$$l' = (1 - \delta_l)l + qv \tag{B.18}$$

$$N' = (1 - \delta_N)N + \chi a' N \tag{B.19}$$

$$g_N = \frac{N'}{N} \tag{B.20}$$

$$y = c + k' - (1 - \delta_k)k + a' + e + \kappa v, \tag{B.21}$$

where  $y$  denote the gross output and  $e$  intermediate inputs. As a result, the net value added output is equal to  $y$  minus  $e$ .

### Appendix C. Full set of stationarized equilibrium conditions

Denoting by *tilde* the normalized variable, we can rewrite the model in terms of only stationary variables as follows:

$$\tilde{y} = \tilde{k}^\alpha l^{1-\alpha} \tag{C.1}$$

$$\tilde{e} = (1 - \psi)^2 \tilde{y} \tag{C.2}$$

$$\tilde{W} = \psi (1 - \psi) \tilde{y} + (1 - \delta_N) \beta g_N^{\frac{\alpha-\sigma}{1-\alpha}} E \left\{ \tilde{\Lambda} \tilde{W}' \right\} \tag{C.3}$$

$$(1 - \delta_N) \beta g_N^{\frac{\alpha-\sigma}{1-\alpha}} E \left\{ \tilde{\Lambda} \tilde{W}' \right\} \eta = (1 + r') \tilde{k} \tag{C.4}$$

$$\alpha \psi \frac{\tilde{y}}{\tilde{k}} - (r + \delta_k) = 0 \tag{C.5}$$

$$\tilde{c}^{-\sigma} - \beta g_N^{\frac{-\sigma}{1-\alpha}} E \left\{ [R' + (1 - \delta_k)] \tilde{c}'^{-\sigma} \right\} = 0 \tag{C.6}$$

$$\tilde{c}^{-\sigma} - \beta g_N^{\frac{-\sigma}{1-\alpha}} E \left\{ (1 + r') \tilde{c}'^{-\sigma} \right\} = 0 \tag{C.7}$$

$$u = 1 - l \tag{C.8}$$

$$\kappa = g_N^{\frac{-1}{1-\alpha}} \tilde{J} q \tag{C.9}$$

$$\tilde{J} = \beta g_N^{\frac{1-\sigma}{1-\alpha}} E \tilde{\Lambda} \left[ \tilde{\pi}^{\#'} + (1 - \delta_l) \tilde{J}' \right] \tag{C.10}$$

$$\tilde{\pi}^{\#} = (1 - \alpha) \psi \left( \frac{\tilde{y}}{l} \right) - \tilde{\omega} \tag{C.11}$$

$$\tilde{\omega} = (1 - \xi) \tilde{b} + \xi \left[ (1 - \alpha) \psi \left( \frac{\tilde{y}}{l} \right) + \kappa \tilde{g}_N^{\frac{1}{1-\alpha}} \theta \right] \tag{C.12}$$

$$\theta = \frac{v}{u} \tag{C.13}$$

$$m = \gamma_m v^{1-\gamma} u^\gamma \tag{C.14}$$

$$f = \frac{m}{u} \tag{C.15}$$

$$q = \frac{m}{v} \tag{C.16}$$

$$l' = (1 - \delta_l) l + m \tag{C.17}$$

$$\tilde{y} = \tilde{c} + g_N^{\frac{1}{1-\alpha}} \tilde{k}' - (1 - \delta_k) \tilde{k} + g_N^{\frac{1}{1-\alpha}} \tilde{a}' + \tilde{e} + \kappa g_N^{\frac{1}{1-\alpha}} v \tag{C.18}$$

$$\tilde{\Lambda} = \Lambda g_N^{\frac{\sigma}{1-\alpha}} = \frac{\tilde{c}'^{-\sigma}}{\tilde{c}^{-\sigma}} \tag{C.19}$$

$$g_N = 1 - \delta_N + \frac{\tilde{a}'}{\tilde{k}'} \eta. \tag{C.20}$$

## Appendix D. Steady state

In what follows, we make reference only to detrended variables. We neglect *tilde* for ease of notation. An *overline* denotes the steady state value of a variable, which is determined as in the conventional neoclassical growth model.

$$\bar{y} = \bar{k}^\alpha \bar{l}^{1-\alpha} \quad (\text{D.1})$$

$$\bar{e} = (1 - \psi)^2 \bar{y} \quad (\text{D.2})$$

$$\bar{\Lambda} = 1 \quad (\text{D.3})$$

$$\left[ 1 - \beta (1 - \delta_N) \bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}} \right] \bar{W} = \psi (1 - \psi) \bar{y} \quad (\text{D.4})$$

$$\beta (1 - \delta_N) \bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}} \bar{W} = (1 + \bar{r}) \bar{k} \quad (\text{D.5})$$

$$\alpha \psi \frac{\bar{y}}{\bar{k}} - (\bar{r} + \delta_k) = 0 \quad (\text{D.6})$$

$$\bar{g}_N^{\frac{\sigma}{1-\alpha}} - \beta [\bar{R} + (1 - \delta_k)] = 0 \quad (\text{D.7})$$

$$1 + \bar{r} = \bar{R} + (1 - \delta_k) \quad (\text{D.8})$$

$$\bar{u} = 1 - \bar{l} \quad (\text{D.9})$$

$$\kappa = \bar{g}_N^{\frac{-1}{1-\alpha}} \bar{J} \bar{q} \quad (\text{D.10})$$

$$\left[ 1 - \beta (1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \right] \bar{J} = \beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \bar{\pi}^\# \quad (\text{D.11})$$

$$\bar{\pi}^\# = (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{\omega} \quad (\text{D.12})$$

$$\bar{\omega} = (1 - \xi) \bar{b} + \xi \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{\theta} \right] \quad (\text{D.13})$$



$$\bar{\theta} = \frac{\bar{v}}{\bar{u}} \quad (\text{D.14})$$

$$\bar{m} = \gamma_m \bar{v}^{1-\gamma} \bar{u}^\gamma \quad (\text{D.15})$$

$$\bar{f} = \frac{\bar{m}}{\bar{u}} \quad (\text{D.16})$$

$$\bar{q} = \frac{\bar{m}}{\bar{v}} \quad (\text{D.17})$$

$$0 = -\delta_l \bar{l} + \bar{m} \quad (\text{D.18})$$

$$\bar{y} = \bar{c} - \left(1 - \delta_k - \bar{g}_N^{\frac{1}{1-\alpha}}\right) \bar{k} + \bar{g}_N^{\frac{1}{1-\alpha}} \bar{a} + \bar{e} + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{v} \quad (\text{D.19})$$

$$\bar{g}_N = 1 - \delta_N + \bar{g}_N^{\frac{1}{1-\alpha}} \frac{\bar{a}}{\bar{k}} \quad (\text{D.20})$$

$$\bar{\eta} = 1. \quad (\text{D.21})$$

In the steady state, the gross growth rate  $\bar{g}_N$  is the zeros of the following polynomial:

$$\mathbf{A} \bar{g}_N^{-\frac{\sigma}{1-\alpha}} + \mathbf{B} \bar{g}_N^{-\frac{\alpha-\sigma}{1-\alpha}} + \mathbf{C} = 0 \quad (\text{D.22})$$

where  $\mathbf{A} = \beta(1 - \delta_N)(1 - \psi)(1 - \delta_k)$ ,  $\mathbf{B} = \alpha(1/\beta)$ , and  $\mathbf{C} = -(1 - \delta_N)(1 - \psi)(1 - \delta_k)$ . To pin down numerical values for  $\bar{g}_N$  we use the U.S capital-output ratio. Hence, given  $\bar{r}$ , the gross growth rate of the innovation possibilities frontier reads:

$$\bar{g}_N = \left[ \alpha \left( \frac{1 + \bar{r}}{\bar{r} + \delta_k} \right) \right]^{\frac{1-\alpha}{\alpha-\sigma}} \left\{ \beta(1 - \delta_N) \left[ (1 - \psi) + \alpha \left( \frac{1 + \bar{r}}{\bar{r} + \delta_k} \right) \right] \right\}^{-\frac{1-\alpha}{\alpha-\sigma}}. \quad (\text{D.23})$$

(see Technical Appendix [Appendix F](#) for details.)

## Appendix E. Log-Linear Model

In order to examine the model's dynamic behavior, we derive the complete log-linear model. We implement the Taylor approximation around the steady state. Hereafter a *hat* the log-deviation from the steady state value.

Production technology:

$$0 \approx \hat{y} - \alpha \hat{k} - (1 - \alpha) \hat{l} \quad (\text{E.1})$$

Expenditure on intermediates:

$$0 \approx \widehat{e} - \widehat{y} \quad (\text{E.2})$$

Value of blueprints:

$$0 \approx \overline{W}\widehat{W} - \psi(1-\psi)\overline{y}\widehat{y} - (1-\delta_N)\beta\overline{W}\overline{g}_N^{\frac{\alpha-\sigma}{1-\alpha}}E\left\{\widehat{\Lambda} + \widehat{W}' + \frac{\alpha-\sigma}{1-\alpha}\widehat{g}_N\right\} \quad (\text{E.3})$$

Free entry condition in R&D:

$$0 \approx (1-\delta_N)\beta\overline{W}\overline{g}_N^{\frac{\alpha-\sigma}{1-\alpha}}E\left\{\widehat{\Lambda} + \widehat{W}' + \frac{\alpha-\sigma}{1-\alpha}\widehat{g}_N + \widehat{\eta}\right\} - \overline{k}\overline{r}\widehat{r}' - (1+\overline{r})\overline{k}\widehat{k} \quad (\text{E.4})$$

Capital renting:

$$0 \approx \alpha\psi\frac{\overline{y}}{\overline{k}}(\widehat{y} - \widehat{k}) - \overline{r}\widehat{r} \quad (\text{E.5})$$

Euler equations for consumption:

$$0 \approx E\left\{\beta\overline{g}_N^{\frac{-\sigma}{1-\alpha}}\overline{R}\widehat{R}' - \sigma(\widehat{c}' - \widehat{c}) - \frac{\sigma}{1-\alpha}\widehat{g}_N\right\} \quad (\text{E.6})$$

$$0 \approx E\left\{\beta\overline{g}_N^{\frac{-\sigma}{1-\alpha}}\overline{r}\widehat{r}' - \sigma(\widehat{c}' - \widehat{c}) - \frac{\sigma}{1-\alpha}\widehat{g}_N\right\} \quad (\text{E.7})$$

As a result, the log-linearized no-arbitrage condition reads:

$$\widehat{R}' = \frac{\overline{r}}{\overline{R}}\widehat{r}' \quad (\text{E.8})$$

Unemployment:

$$0 \approx \overline{u}\widehat{u} + \overline{l}\widehat{l} \quad (\text{E.9})$$

Free entry condition:

$$0 \approx \widehat{J} + \widehat{q} - \frac{1}{1-\alpha}\widehat{g}_N \quad (\text{E.10})$$

Vacancy posting condition:

$$0 \approx \overline{J}\widehat{J} - \beta\overline{\pi}^\# \overline{g}_N^{\frac{1-\sigma}{1-\alpha}}E\left\{\widehat{\Lambda} + \widehat{\pi}^\# + \frac{1-\sigma}{1-\alpha}\widehat{g}_N\right\} \\ - \beta(1-\delta_l)\overline{J}\overline{g}_N^{\frac{1-\sigma}{1-\alpha}}E\left\{\widehat{\Lambda} + \widehat{J}' + \frac{1-\sigma}{1-\alpha}\widehat{g}_N\right\} \quad (\text{E.11})$$

$$0 \approx \overline{\pi}^\# \widehat{\pi}^\# - (1-\alpha)\psi\left(\frac{\overline{y}}{\overline{l}}\right)(\widehat{y} - \widehat{l}) + \overline{\omega}\widehat{\omega} \quad (\text{E.12})$$

Aggregate wage:

$$0 \approx \bar{\omega} \hat{\omega} - (1 - \xi) \bar{b} \hat{b} - \xi \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) (\hat{y} - \hat{l}) + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{\theta} \left( \frac{1}{1-\alpha} \hat{g}_N + \hat{\theta} \right) \right] \quad (\text{E.13})$$

Labor market tightness:

$$0 \approx \hat{\theta} - \hat{v} + \hat{u} \quad (\text{E.14})$$

Matching:

$$0 \approx \hat{m} - (1 - \gamma) \hat{v} - \gamma \hat{u} \quad (\text{E.15})$$

Transition probabilities:

$$0 \approx \hat{f} - \hat{m} + \hat{u} \quad (\text{E.16})$$

$$0 \approx \hat{q} - \hat{m} + \hat{v} \quad (\text{E.17})$$

Employment dynamics:

$$0 \approx \bar{l} \hat{l}' - (1 - \delta_l) \bar{l} \hat{l} - \bar{m} \hat{m} \quad (\text{E.18})$$

Accounting identity:

$$0 \approx \bar{y} \hat{y} - \bar{c} \hat{c} - \bar{g}_N^{\frac{1}{1-\alpha}} \bar{k} \left( \frac{1}{1-\alpha} \hat{g}_N + \hat{k}' \right) + (1 - \delta_k) \bar{k} \hat{k} - \bar{a} \hat{a}' - \bar{e} \hat{e} - \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{v} \left( \frac{1}{1-\alpha} \hat{g}_N + \hat{v} \right) \quad (\text{E.19})$$

Lambda factor:

$$0 \approx \hat{\Lambda} + \hat{c}' - \hat{c} \quad (\text{E.20})$$

Innovation possibilities frontier:

$$0 \approx \bar{g}_N \hat{g}_N - \frac{\bar{a}}{\bar{k}} \left( \hat{a}' - \hat{k}' + \hat{\eta} \right). \quad (\text{E.21})$$

## Appendix F. Characterization of the gross growth rate at the steady state

From (D.4), we have:

$$\bar{W} = \frac{\psi(1-\psi)}{1-\beta(1-\delta_N)\bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}}\bar{y}}. \quad (\text{F.1})$$

Then combining (F.1) and (D.5) yields:

$$\left[ \frac{\beta(1-\delta_N)\psi(1-\psi)}{1-\beta(1-\delta_N)\bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}}} \right] [\bar{g}_N - (1-\delta_N)] \bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}} \bar{y} = (1+\bar{r})\bar{a}. \quad (\text{F.2})$$

Eq. (D.20) can be rewrite as follows:

$$\frac{\bar{a}}{k} = \bar{g}_N - (1 - \delta_N). \quad (\text{F.3})$$

Next substituting (F.3) into Eq. (F.2), we find:

$$\left[ \frac{\beta(1 - \delta_N)\psi(1 - \psi)}{1 - \beta(1 - \delta_N)\bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}}} \right] [\bar{g}_N - (1 - \delta_N)] \bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}} \frac{\bar{y}}{k} = (1 + \bar{r}) [\bar{g}_N - (1 - \delta_N)]. \quad (\text{F.4})$$

After some elementary manipulations, we obtain:

$$\left[ \frac{\beta(1 - \delta_N)\psi(1 - \psi)}{1 - \beta(1 - \delta_N)\bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}}} \right] \bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}} \frac{\bar{y}}{k} = (1 + \bar{r}). \quad (\text{F.5})$$

Then using (D.6), it follows:

$$\left[ \frac{\beta(1 - \delta_N)\psi(1 - \psi)}{1 - \beta(1 - \delta_N)\bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}}} \right] \bar{g}_N^{\frac{\alpha - \sigma}{1 - \alpha}} = \alpha\psi \left( \frac{1 + \bar{r}}{\bar{r} + \delta_k} \right). \quad (\text{F.6})$$

Usin Eq. (D.7) and after some elementary manipulations, we find that  $\bar{g}_N$  is the zeros of the following polynomial:

$$\mathbf{A}\bar{g}_N^{-\frac{\sigma}{1 - \alpha}} + \mathbf{B}\bar{g}_N^{-\frac{\alpha - \sigma}{1 - \alpha}} + \mathbf{C} = 0 \quad (\text{F.7})$$

where  $\mathbf{A} = \beta(1 - \delta_N)(1 - \psi)(1 - \delta_k)$ ,  $\mathbf{B} = \alpha(1/\beta)$ , and  $\mathbf{C} = -(1 - \delta_N)(1 - \psi)(1 - \delta_k)$ .

To pin down numerical values for  $\bar{g}_N$  we use the U.S capital-output ratio. Hence, given  $\bar{r}$ , the gross growth rate of the innovation possibilities frontier reads:

$$\bar{g}_N = \left[ \alpha \left( \frac{1 + \bar{r}}{\bar{r} + \delta_k} \right) \right]^{\frac{1 - \alpha}{\alpha - \sigma}} \left\{ \beta(1 - \delta_N) \left[ (1 - \psi) + \alpha \left( \frac{1 + \bar{r}}{\bar{r} + \delta_k} \right) \right] \right\}^{-\frac{1 - \alpha}{\alpha - \sigma}}. \quad (\text{F.8})$$

## Appendix G. Non-stochastic steady state

After some manipulation, each variable can be now expressed as a function of model parameters. steady state value of unemployment rate.<sup>11</sup>

$$\bar{u} = u_{Shimer\_data} \quad (\text{G.1})$$

$$\bar{l} = 1 - \bar{u} \quad (\text{G.2})$$

$$\bar{m} = \delta_l \bar{l} \quad (\text{G.3})$$

Eq. (G.3) allows to identify the deep parameter  $\delta_l$

$$\bar{f} = \frac{\bar{m}}{\bar{u}} \quad (\text{G.4})$$

<sup>11</sup>Shimer data <https://sites.google.com/site/robertshimer/data>. Period: 1953–2003.

$$\bar{v} = \left( \frac{\bar{m}}{\gamma_m \bar{u}^\gamma} \right)^{\frac{1}{1-\gamma}} \quad (\text{G.5})$$

where  $\gamma_m$  must be set to pin down steady state vacancies at  $\bar{m}/\bar{q}$ , that is Eq. (G.5) and Eq. (G.6) must be consistent

$$\bar{q} = \frac{\bar{m}}{\bar{v}} \quad (\text{G.6})$$

$$\bar{\theta} = \frac{\bar{v}}{\bar{u}} \quad (\text{G.7})$$

$$\bar{\Lambda} = 1 \quad (\text{G.8})$$

Given the steady state U.S capital-output ratio U.S ( $\bar{k}/\bar{y}$ ), it follows from Eq. (D.6):

$$\bar{r} = -\delta_k + \alpha \psi \left( \frac{\bar{y}}{\bar{k}} \right)_{US\_data} \quad (\text{G.9})$$

$$\bar{R} = \bar{r} + \delta_k \quad (\text{G.10})$$

$$\bar{g}_N = [\beta (\bar{R} + 1 - \delta_k)]^{\frac{1-\alpha}{\sigma}} \quad (\text{G.11})$$

Given  $\bar{l}$ , combining Eq. (D.1) with (D.6) yields:

$$\bar{k} = \left( \frac{\alpha \psi}{\bar{r} + \delta_k} \right)^{\frac{1}{1-\alpha}} \bar{l} \quad (\text{G.12})$$

$$\bar{a} = [\bar{g}_N - (1 - \delta_N)] \bar{k} \quad (\text{G.13})$$

$$\bar{y} = \bar{k}^\alpha \bar{l}^{1-\alpha} \quad (\text{G.14})$$

$$\bar{W} = \frac{\psi (1 - \psi)}{1 - \beta (1 - \delta_N) \bar{g}_N^{\frac{\alpha-\sigma}{1-\alpha}}} \bar{y} \quad (\text{G.15})$$

$$\bar{\omega} = (1 - \xi) \bar{b} + \xi \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{\theta} \right] \quad (\text{G.16})$$

$$\bar{\pi}^\# = (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{\omega} \quad (\text{G.17})$$

$$\bar{J} = \frac{\beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}}}{1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}}} \bar{\pi}^\# \quad (\text{G.18})$$

Given Eq. (G.18),  $\kappa$  must be set to pin down  $\bar{J} = \bar{g}_N^{\frac{1}{1-\alpha}} \kappa / \bar{q}$ . From Eq. (D.10), we have:

$$\bar{J} = \bar{g}_N^{\frac{1}{1-\alpha}} \frac{\kappa}{\bar{q}}. \quad (\text{G.19})$$

From Eq. (D.11):

$$\bar{J} = \beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \left[ 1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \right]^{-1} \bar{\pi}^\#. \quad (\text{G.20})$$

Given (D.12), Eq. (G.20) now reads:

$$\bar{J} = \beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \left[ 1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \right]^{-1} \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{\omega} \right]. \quad (\text{G.21})$$

Given (D.13), it follows:

$$\bar{J} = \beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \left\{ \frac{(1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - (1 - \xi) \bar{b} - \xi \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{\theta} \right]}{1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}}} \right\}. \quad (\text{G.22})$$

Now combining Eq. (G.22) with Eq. (G.21), we obtain:

$$\bar{g}_N^{\frac{1}{1-\alpha}} \frac{\kappa}{\bar{q}} = \beta \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \left\{ \frac{(1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - (1 - \xi) \bar{b} - \xi \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) + \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{\theta} \right]}{1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}}} \right\}. \quad (\text{G.23})$$

After some elementary manipulations, we finally find the desired *kappa* value:

$$\kappa = \beta \bar{q} (1 - \xi) \left[ (1 - \alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - b \right] \times \left\{ \bar{g}_N^{\frac{\sigma}{1-\alpha}} \left[ 1 - \beta(1 - \delta_l) \bar{g}_N^{\frac{1-\sigma}{1-\alpha}} \right] + \beta \bar{q} \xi \bar{\theta} \bar{g}_N^{\frac{1}{1-\alpha}} \right\}^{-1}. \quad (\text{G.24})$$

Moreover, we have:

$$\bar{e} = (1 - \psi) \bar{y} \quad (\text{G.25})$$

$$\bar{c} = (\bar{y} - \bar{e}) + \left( 1 - \delta_k - \bar{g}_N^{\frac{1}{1-\alpha}} \right) \bar{k} - \bar{a} - \kappa \bar{g}_N^{\frac{1}{1-\alpha}} \bar{v}. \quad (\text{G.26})$$

Appendix G.1. Endogenous productivity process and labor market dynamic

Following Shimer (2005), we combine the job creation condition (C.9 – C.10) and the wage equation (C.11 – C.12) to evaluate the sensitivity of the  $v$ - $u$  ratio to deviation of productivity from unemployment benefits in the steady state:

$$\bar{g}_N^{\frac{-1}{1-\alpha}} \frac{\bar{\pi}^\#}{\kappa} = + \left( \frac{1-\xi}{\kappa} \right) \bar{g}_N^{\frac{-1}{1-\alpha}} \left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right] - \xi \bar{\theta} \quad (\text{G.27})$$

$$\bar{g}_N^{\frac{-1}{1-\alpha}} \frac{\bar{\pi}^\#}{\kappa} = \left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{1}{q(\bar{\theta})}. \quad (\text{G.28})$$

Substituting (G.28) into (G.27) yields the following relation between  $\theta$  and the net labor productivity:

$$\left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{1}{q(\bar{\theta})} + \xi \bar{\theta} = \left( \frac{1-\xi}{\kappa} \right) \bar{g}_N^{\frac{-1}{1-\alpha}} \left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right]. \quad (\text{G.29})$$

Now totally differentiate Eq. (G.29) with respect to  $\bar{\theta}$  and  $(1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b}$  to get:

$$- \left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{q'(\bar{\theta})}{q(\bar{\theta})^2} d\bar{\theta} + \xi d\bar{\theta} = \left( \frac{1-\xi}{\kappa} \right) \bar{g}_N^{\frac{-1}{1-\alpha}} d \left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right] \quad (\text{G.30})$$

After some elementary manipulations, we find:

$$\varepsilon_{\theta,p} = \left\{ \frac{(1-\xi) \left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right]}{\left[ \xi - \left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{q'(\bar{\theta})}{q(\bar{\theta})^2} \right] \kappa \bar{\theta}} \right\} \bar{g}_N^{\frac{-1}{1-\alpha}}, \quad (\text{G.31})$$

where the elasticity  $\varepsilon_{\theta,p}$  is given by:

$$\varepsilon_{\theta,p} \equiv \frac{d\bar{\theta}}{d \left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right]} \frac{\left[ (1-\alpha) \psi \left( \frac{\bar{y}}{\bar{l}} \right) - \bar{b} \right]}{\bar{\theta}}.$$

Using Eq. (G.29) it follows that:

$$\begin{aligned} \varepsilon_{\theta,p} &= \left\{ \frac{\left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{1}{\bar{\theta} q(\bar{\theta})} + \xi}{\left[ \xi - \left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{q'(\bar{\theta})}{q(\bar{\theta})^2} \right]} \right\} \bar{g}_N^{\frac{-1}{1-\alpha}} \\ &= \left\{ \frac{\left( \frac{1-\beta}{\beta} + \delta_l \right) + \xi f(\bar{\theta})}{\left[ \xi - \left( \frac{1-\beta}{\beta} + \delta_l \right) \frac{q'(\bar{\theta})}{q(\bar{\theta})^2} \right] f(\bar{\theta})} \right\} \bar{g}_N^{\frac{-1}{1-\alpha}} \\ &= \left\{ \frac{\beta^{-1}(1-\beta) + \delta_l + \xi f(\bar{\theta})}{[\beta^{-1}(1-\beta) + \delta_l] (1 - \varepsilon_{f,\theta}) + \xi f(\bar{\theta})} \right\} \bar{g}_N^{\frac{-1}{1-\alpha}}, \end{aligned} \quad (\text{G.32})$$

where from (30) and (31) the finding rate can be expressed as follows  $f(\bar{\theta}) = \bar{\theta} q(\bar{\theta})$ . The elasticity  $\varepsilon_{f,\theta}$  is given by :

$$\begin{aligned} - \frac{q'(\bar{\theta})}{q(\bar{\theta})^2} f(\bar{\theta}) &= \frac{[f(\bar{\theta})]^2 - f'(\bar{\theta}) \bar{\theta} f(\bar{\theta})}{[\bar{\theta} q(\bar{\theta})]^2} \\ &= 1 - \frac{f'(\bar{\theta}) \bar{\theta}}{f(\bar{\theta})} \\ &= 1 - \varepsilon_{f,\theta}, \end{aligned} \quad (\text{G.33})$$

where  $q'(\bar{\theta}) = f'(\bar{\theta})/\bar{\theta} - f(\bar{\theta})/\bar{\theta}^2$ . Thus  $\varepsilon_{f,\theta}$  denotes the elasticity of  $f(\bar{\theta})$  with respect to  $v-u$  ratio. When  $\bar{g}_N = 1$  (no endogenous productivity process) and  $\alpha = 0$ , our model becomes identical to the Shimer setup.

## Appendix H. Data

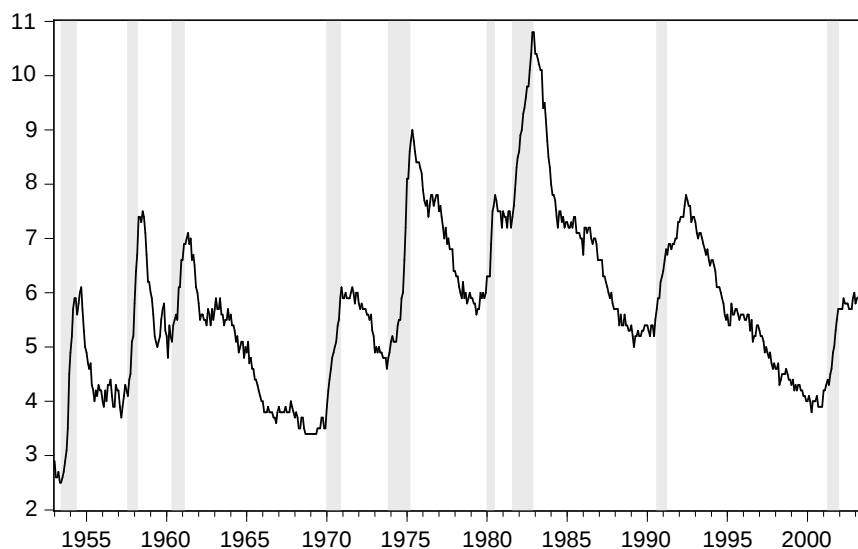
Table H.1 associates each variable and its source of data.

Table H.1: Key variables. Data 1953–2003

|                                 | Source                         |
|---------------------------------|--------------------------------|
| Unemployment                    | Shimer Database                |
| Vacancies                       | Shimer Database                |
| Per capita output               | U.S Bureau of Labor Statistics |
| Labor productivity              | U.S Bureau of Labor Statistics |
| Total Factor Productivity (TFP) | U.S Bureau of Labor Statistics |
| Research and Development (R&D)  | National Science Foundation    |

Figure H.1 shows the evolution of the U.S unemployment rate between 1953 and 2003

Figure H.1: U.S Unemployment rate. Data 1953–2003



Figures H.2 and H.3 respectively shows Beveridge curves in the business cycles frequencies and medium-term frequencies for the U.S labor market over the 1953–2003 period.



Figure H.2: Beveridge curve in the business cycles. Data 1953–2003

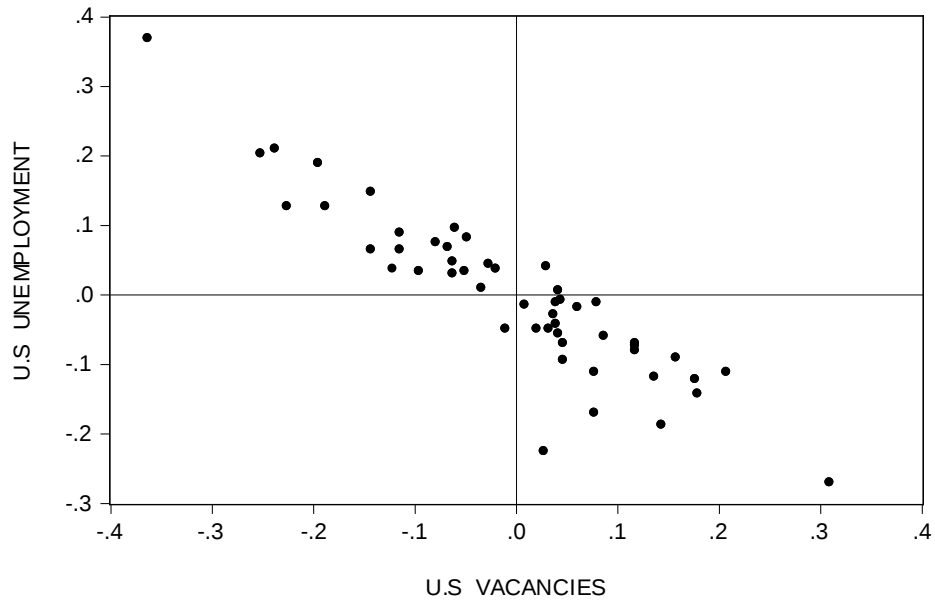
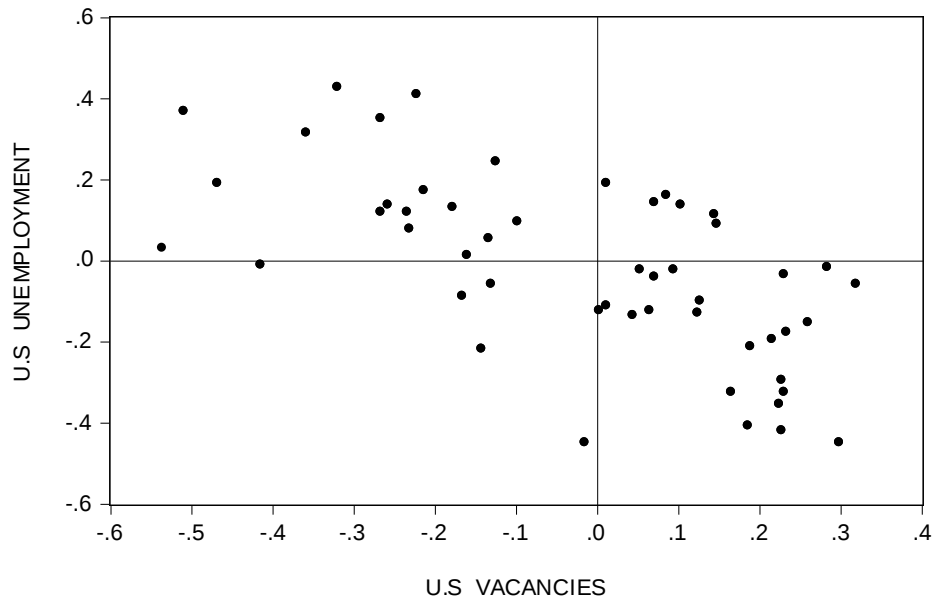


Figure H.3: Beveridge curve in the medium-term cycles. Data 1953–2003



## Appendix I. Empirical Cross Correlations

Table I.1: Correlation matrix (Medium-term Cycle). Data 1953–2003

| Correlation<br><i>t</i> -Statistic | GDP                    | TFP                    | R&D                    | $P_h$                  | $P_p$                  | $u$                    | $v$                    | $v/u$                  | $f$                    | $s$    |
|------------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|--------|
| GDP                                | 1                      |                        |                        |                        |                        |                        |                        |                        |                        |        |
|                                    | —                      |                        |                        |                        |                        |                        |                        |                        |                        |        |
| TFP                                | 0.788800<br>8.983327   | 1<br>—                 |                        |                        |                        |                        |                        |                        |                        |        |
| R&D                                | 0.106153<br>0.747291   | -0.020588<br>-0.144148 | 1<br>—                 |                        |                        |                        |                        |                        |                        |        |
| $P_h$                              | 0.671805<br>6.348665   | 0.893094<br>13.89656   | -0.150515<br>-1.065743 | 1<br>—                 |                        |                        |                        |                        |                        |        |
| $P_p$                              | 0.670956<br>6.334073   | 0.894862<br>14.03415   | 0.153974<br>1.090825   | 0.894688<br>14.02043   | 1<br>—                 |                        |                        |                        |                        |        |
| $u$                                | -0.701988<br>-6.899745 | -0.455180<br>-3.578465 | -0.491045<br>-3.945794 | -0.144967<br>-1.025604 | -0.381108<br>-2.885522 | 1<br>—                 |                        |                        |                        |        |
| $v$                                | 0.723104<br>7.327981   | 0.424150<br>3.278573   | 0.164386<br>1.166569   | 0.169521<br>1.204076   | 0.225942<br>1.623580   | -0.646150<br>-5.926341 | 1<br>—                 |                        |                        |        |
| $v/u$                              | 0.804713<br>9.488502   | 0.500338<br>4.045097   | 0.342442<br>2.551354   | 0.205835<br>1.472370   | 0.355136<br>2.659300   | -0.883971<br>-13.23474 | 0.925643<br>17.12347   | 1<br>—                 |                        |        |
| $f$                                | 0.826026<br>10.25879   | 0.456288<br>3.589462   | 0.399066<br>3.046560   | 0.205250<br>1.468003   | 0.355852<br>2.665438   | -0.921074<br>-16.55810 | 0.791331<br>9.060229   | 0.940724<br>19.41515   | 1<br>—                 |        |
| $s$                                | -0.343415<br>-2.559566 | -0.349373<br>-2.610085 | -0.452070<br>-3.547703 | -0.066284<br>-0.465013 | -0.415069<br>-3.193572 | 0.784591<br>8.858085   | -0.287280<br>-2.099455 | -0.555959<br>-4.681985 | -0.529942<br>-4.374341 | 1<br>— |

Table I.2: Correlation matrix (High-frequency component). Data 1953–2003

| Correlation<br><i>t</i> -Statistic | GDP                    | TFP                    | R&D                    | $P_h$                  | $P_p$                  | $u$                    | $v$                    | $v/u$                  | $f$                    | $s$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-----|
| GDP                                | 1                      |                        |                        |                        |                        |                        |                        |                        |                        |     |
|                                    | —                      |                        |                        |                        |                        |                        |                        |                        |                        |     |
| TFP                                | 0.811184<br>9.709947   | 1                      |                        |                        |                        |                        |                        |                        |                        |     |
| R&D                                | 0.347395<br>2.593276   | 0.021946<br>0.153656   | 1                      |                        |                        |                        |                        |                        |                        |     |
| $P_h$                              | 0.513298<br>4.186721   | 0.840719<br>10.86858   | -0.070639<br>-0.495709 | 1                      |                        |                        |                        |                        |                        |     |
| $P_p$                              | 0.707279<br>7.003409   | 0.939708<br>19.23495   | -0.030015<br>-0.210203 | 0.929154<br>17.59324   | 1                      |                        |                        |                        |                        |     |
| $u$                                | -0.848639<br>-11.23023 | -0.503608<br>-4.080473 | -0.454538<br>-3.572097 | -0.129558<br>-0.914611 | -0.325903<br>-2.413067 | 1                      |                        |                        |                        |     |
| $v$                                | 0.936676<br>18.72301   | 0.629112<br>5.665385   | 0.437981<br>3.410365   | 0.264624<br>1.920845   | 0.499077<br>4.031513   | -0.913880<br>-15.75713 | 1                      |                        |                        |     |
| $v/u$                              | 0.914659<br>15.83916   | 0.579705<br>4.980120   | 0.460149<br>3.627943   | 0.205243<br>1.467949   | 0.426540<br>3.301142   | -0.972341<br>-29.14142 | 0.983058<br>37.54239   | 1                      |                        |     |
| $f$                                | 0.868043<br>12.23853   | 0.512450<br>4.177338   | 0.492941<br>3.965901   | 0.133035<br>0.939600   | 0.344710<br>2.570521   | -0.960306<br>-24.09824 | 0.937048<br>18.78394   | 0.968854<br>27.38719   | 1                      |     |
| $s$                                | -0.718319<br>-7.227437 | -0.670717<br>-6.329957 | -0.089835<br>-0.631401 | -0.448174<br>-3.509399 | -0.595680<br>-5.191291 | 0.727100<br>7.413633   | -0.686517<br>-6.609170 | -0.715228<br>-7.163622 | -0.584662<br>-5.044684 | 1   |

Table I.3: Correlation matrix (Medium-frequency component). Data 1953–2003

| Correlation<br><i>t</i> -Statistic | GDP                    | TFP                    | R&D                    | $P_h$                  | $P_p$                  | $u$                    | $v$                    | $v/u$                  | $f$                    | $s$ |
|------------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-----|
| GDP                                | 1                      |                        |                        |                        |                        |                        |                        |                        |                        |     |
|                                    | —                      |                        |                        |                        |                        |                        |                        |                        |                        |     |
| TFP                                | 0.778995<br>8.696506   | 1                      |                        |                        |                        |                        |                        |                        |                        |     |
|                                    |                        | —                      |                        |                        |                        |                        |                        |                        |                        |     |
| R&D                                | 0.048168<br>0.337567   | 0.004191<br>0.029340   | 1                      |                        |                        |                        |                        |                        |                        |     |
|                                    |                        |                        | —                      |                        |                        |                        |                        |                        |                        |     |
| $P_h$                              | 0.712599<br>7.110029   | 0.900074<br>14.45946   | -0.137654<br>-0.972839 | 1                      |                        |                        |                        |                        |                        |     |
|                                    |                        |                        |                        | —                      |                        |                        |                        |                        |                        |     |
| $P_p$                              | 0.674096<br>6.388289   | 0.875918<br>12.70866   | 0.208899<br>1.495287   | 0.888503<br>13.55396   | 1                      |                        |                        |                        |                        |     |
|                                    |                        |                        |                        |                        | —                      |                        |                        |                        |                        |     |
| $u$                                | -0.658069<br>-6.117845 | -0.455402<br>-3.580667 | -0.506674<br>-4.113865 | -0.160479<br>-1.138102 | -0.407014<br>-3.119146 | 1                      |                        |                        |                        |     |
|                                    |                        |                        |                        |                        |                        | —                      |                        |                        |                        |     |
| $v$                                | 0.664972<br>6.232429   | 0.402716<br>3.079790   | 0.030608<br>0.214354   | 0.179614<br>1.278085   | 0.175689<br>1.249256   | -0.544170<br>-4.540295 | 1                      |                        |                        |     |
|                                    |                        |                        |                        |                        |                        |                        | —                      |                        |                        |     |
| $v/u$                              | 0.781379<br>8.764667   | 0.510167<br>4.152153   | 0.281257<br>2.051619   | 0.236211<br>1.701631   | 0.362670<br>2.724154   | -0.854648<br>-11.52243 | 0.897044<br>14.20850   | 1                      |                        |     |
|                                    |                        |                        |                        |                        |                        |                        |                        | —                      |                        |     |
| $f$                                | 0.823763<br>10.17110   | 0.467704<br>3.704021   | 0.356515<br>2.671125   | 0.247651<br>1.789292   | 0.384528<br>2.915890   | -0.908939<br>-15.26057 | 0.730664<br>7.491377   | 0.932666<br>18.09799   | 1                      |     |
|                                    |                        |                        |                        |                        |                        |                        |                        |                        | —                      |     |
| $s$                                | -0.254980<br>-1.845874 | -0.273299<br>-1.988811 | -0.572176<br>-4.883652 | 0.002664<br>0.018645   | -0.383431<br>-2.906135 | 0.816055<br>9.883468   | -0.180786<br>-1.286702 | -0.532520<br>-4.404025 | -0.540124<br>-4.492557 | 1   |
|                                    |                        |                        |                        |                        |                        |                        |                        |                        |                        | —   |