Financial Development and Capital flows: The Schumpeterian growth on the "Allocation puzzle"

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Abstract

We explore the role of financial development on the negative correlation between capital inflows and productivity catch-up. As observed in the data, countries with higher productivity growth rate export capital while countries with lower productivity growth rate receive positive capital inflows; this is contradictory with the predictions of the standard neoclassical growth model. Gourinchas and Jeanne (2013) called this paradox the "Allocation puzzle". We propose an extension of the puzzle to a larger sample, both for the period and for the countries. We then introduce credit constraint in a calibrated schumpeterian growth model to address this paradox; our main result indicates that, when the level of financial development prevent countries to catch-up the world technological frontier, countries import capital to compensate their domestic saving as they fall behind the frontier.

KEYWORDS: Capital flows, Financial Development, Productivity, Schumpeterian growth

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1 Introduction

According to the neoclassical growth theory, capital must flow into countries where its marginal product is higher, in contrast with what we observe with data.[1] Starting with Lucas (1990) who showed that little capital flows from rich to poor countries, some articles are emphasized on the role of financial frictions to explain the discrepancy between theoretical predictions and observed data. By using a calibrated neoclassical growth model, Gourinchas and Jeanne (2013) showed that the "Allocation puzzle" may be explained by a wedge affecting saving decision; the "Allocation puzzle" is related to the "upstream" capital flows from high growth to low growth countries while the "Lucas puzzle" is related to the upstream capital flows for high income to low income countries.

In this paper, we introduce an endogenous imperfect creditor protection *à la* Aghion, Howitt and Mayer-Foulkes (2005) in a calibrated schumpeterian growth model to address the "Allocation puzzle". Our model shows that countries above a certain threshold of financial development will catch-up the world technological frontier while the others fall behind. Then, following Gourinchas and Jeanne (2013), we decompose the theoretical net capital inflows and focus on the contributions of investment and saving in change of external debt. We find an interesting prediction for countries which fall behind the world technological frontier because of their level of financial development: predicted net capital inflows going toward domestic saving is strongly and negatively correlated with the productivity catch-up. Our model is able to replicate partially the direction of capital inflows as observed in the data, for group of countries with low level of financial development. Our choice of domestic credit constraint rather than a friction in the international credit market is mainly motivated by two reasons: first, as mentioned by Gourinchas and Jeannes (2013), international financial frictions can just mute capital flows by increasing the cost of external finance relative to domestic finance, but these frictions cannot reverse the direction of the capital flows. Therefore, most of the articles using financial constraint to deal with the allocation puzzle are more focused on domestic distortions. According to Gertler and Rogoff (1990), domestic financial frictions, determined endogenously and depending on the country wealth can mute and possibly reverse capital flows direction from rich to poor countries. A country level of financial development gives the capacity of borrowing of private agents who would like to invest in a risky project. Second, as we show in this paper[2], financial development measured by the credit to private sector by domestic banks is positively correlated with productivity catch-up[3] (as suggested by Gourinchas and Jeanne (2013)) but also negatively correlated with capital inflows. The more a country is financially developed, the more likely it catches up the technological frontier and the least it imports capital.

We show in this paper that the "Allocation puzzle" can be generalized actually to developed economies, and also that the schumpeterian growth model predicts a positive relationship between capital inflows and productivity catch-up when entrepreneurs have unlimited access to

[1] Prasad, Rajan and Subramanian (2007) showed that capital tends to flow from high growth to low growth non-industrialized countries

[2] See figures 5, 6, 7 and 8

[3] See Aghion *et al* (2005) for theoretical support and empirical evidence

credit. With an extended sample including OECD and non-OECD countries, we show in this article that observed net capital inflows are negatively correlated with productivity growth as most of the works done in the literature with samples of developing countries (Prasad *et al*. (2007), Aguiar and Amador (2011)). As we do not perform decomposition into public and private debts of observed net capital inflows as done in most of empirical works, we are able to consider any country over the world in the composition of our sample[4]. We are only interested in the general pattern of total net capital inflows, in other words, the negative correlation of observed capital flows and productivity growth, which we use to assess the prediction of our model. Alfaro *et al*. (2014) argued that the sign of the correlation between net capital flows and productivity changes depending on the selected sample; with a sample dominated by Asian and African countries, they find a robust negative correlation while this correlation is weakly positive with a larger sample. Gourinchas and Jeanne (2013) used a sample of developing countries and found that total net capital inflows is negatively correlated with productivity growth; we generalize their results to the whole world. Our result is robust to several changes as a limitation of the sample to developing countries and limitation of the period to 1980-2000 instead 1980-2010. We also use 2 different measures for the total net capital inflows but the sign of the correlation between capital flows and productivity growth remains the same.

Intuitively, we can interpret the predictions of our model as followed. When agents have unlimited access to credit, the country will catch-up the world technological frontier; agent will anticipate higher future income, and then they increase their consumption. Given that their current income is unchanged and that they have access to international financial market, they will borrow abroad. When agents have limited access to credit and the level of financial development is sufficient, we observe the same pattern but the predicted capital inflows going toward saving is higher; this is attributable to the credit constraint which reduces wealth. In contrast, a financially underdeveloped country will likely fall behind the world technological frontier. Agents will borrow on the international financial market to finance the increase of their consumption and their debt will also increase due to the reduction of their wealth created by the expenditure on new projects.

This article is related to several strands in the literature. First, this paper is linked to the literature on financial development and economics growth. Since Goldsmith (1969), the literature in development economics has established evidences on the strong positive relationship between financial development and economic growth. We show in this paper that, depending on their level of financial development, productivity of countries grows at a higher or lower rate than the technological frontier productivity growth rate. Our findings corroborate the conclusions of Aghion *et al*. (2005) who show that countries converge to the technological frontier growth rate if and only if their level of financial development is above a critical level.

Second, this paper is related to the literature on the determinants of capital flows. Many commentators have focused on determinants which affect relationship saving-investment-growth to explain the lack of the standard neoclassical growth model to predict the negative correlation

[4]The World Bank data does not report details on foreign debt for classified developed countries. Therefore, empirical articles which perform decomposition of net debt can only focus on developing countries.

between productivity growth and net capital inflows. It is well known in the literature that saving is strongly positively correlated with growth across countries (Modigliani (1970), Carroll and Weil (1994), Carroll and Weil (2000)) and also with investment (Feldstein and Horioka (1980), Attanasio *et al*. (2000)). Among determinants affecting these relationships, friction on the domestic financial market seems to be a potential candidate. A financial friction can reduce the capacity of agents to borrow against future income (Caballero, Farhi and Gourincha (2008)), and with a lack of social insurance could also increase their precautionary savings (Mendoza *et al*. (2009), Carroll and Jeanne (2009)). Aghion *et al*. (2009) showed that distortions on domestic financial market prevent domestic savings to substitute foreign savings perfectly. Therefore, they found that agents in poor countries have to increase their saving to be able to invest in a new project and that saving matters for growth in these countries. Some articles in the literature are also focused on distortions affecting physical capital to explain the puzzle (Buera (2016), Caselli and Feyrer 2007)). We propose a model with an endogenous credit constraint which affects the consumption-saving behaviour of agents. We show that credit constraint measured by the level of financial development tends to reduce the total wealth of agents; its effects are more severe in countries which likely fall behind the technological frontier and in particular, increases external debt.

Third, this article is linked to the wide literature giving evidences on the negative correlation between capital flows and growth. Our analysis is close to Gourinchas and Jeanne (2013). We show that the schumpeterian growth model under perfect financial market (Howitt (2000), Aghion and Howitt (1998a, b)) also yields to the same predictions as the standard neoclassical growth model. In addition to Gourinchas and Jeanne (2013), our model endogenizes the productivity catch-up; we also show that this negative correlation holds for developing countries and is also strong when extended to the whole world.

The rest of the article is organized as follows. In Section 2 we present the schumpeterian growth model under perfect financial market and propose the theoretical ratio of cumulated net capital inflows to initial output. In section 3, we introduce the imperfect creditor protection in our model and make predictions. In section 4, we discuss the data and calibration, and compare the predictions of the model to the observed data. Section 5 concludes the article.

2 The Schumpeterian growth framework

We use the Schumpeterian growth paradigm including physical capital accumulation developed by Howitt (2000) or Aghion and Howitt (1998a, b). Time is discrete and there is a continuum of individuals in each country. There are J small open countries, indexed by $j = 1, ..., J$, which exchange goods and factors, and are technologically interdependent in the sense that they use technological ideas developed elsewhere in the world. Each country can borrow and lend at an exogenously given world real interest rate $r^{\star}$.

2.1 Household

The economy consists of a set of identical households (which number is normalized to 1), but where the number of infinite lifetime individuals in each household grows at the exogenous rate n, so that $L_j(t) = (1+n_j)^t L_j(0)$. Each individual supplies one unit of labor inelastically. The representative household maximizes the following constant relative risk aversion (CRRA) utility function:

$$\max_{\{c_j(t)\}_{t=0,1,\ldots}} U_j(0) = L_j(0) \sum_{t=0}^{\infty} \beta^t \frac{c_j(t)^{1-\gamma} - 1}{1-\gamma} \tag{2.1}$$

where $c_j(t) = C_j(t)/L_j(t)$ per worker consumption of country j at date t, $\beta \equiv \dfrac{1+n_j}{1+\rho}$ is the effective discount rate and ρ is the subjective discount rate, with $\rho > n_j$, and $\gamma > 0$ is the inverse of elasticity of intertemporal substitution. Denoting $\mathcal{A}_j(t)$ the asset holding of the representative household at time t. Therefore, the law of motion of total assets is given by:

$$\mathcal{A}_j(t+1) = w_j(t)L_j(t) + (1+r^\star)\mathcal{A}_j(t) - C_j(t) \tag{2.2}$$

and where we assume that the following no-Ponzi condition holds:

$$\lim_{t\to\infty} \left(\frac{1}{1+r^\star}\right)^t \mathcal{A}_j(t+1) \geq 0 \tag{2.3}$$

The Euler condition for this small open economy is given by:

$$c_j(t)^{-\gamma} = \beta(1+r^\star)c_j(t+1)^{-\gamma} \tag{2.4}$$

so that, we follow Gourinchas and Jeanne (2013) assuming that the world interest rate is:

$$1 + r^\star = \frac{(1+g)^\gamma}{\beta} \tag{2.5}$$

where we implicitly assume that the rest of the world is composed of advanced countries at their steady state level, and sharing the same preference parameters of the J small countries under consideration here.

2.2 Production

Production of final good. Let us assume each country produces a single good which can be used for consumption, as capital good or invested as input for R&D. It is produced under perfect competition by labor and a continuum of intermediate products, according to the production function:

$$Y_j(t) = \left(\frac{L_j(t)}{Q_j(t)}\right)^{1-\alpha} \int_0^{Q_j(t)} A_j(\nu,t)^{1-\alpha} x_j(\nu,t)^\alpha d\nu \tag{2.6}$$

where $Y_j(t)$ is the country's j gross output at date t, $L_j(t)$ is the flow of raw labor used in production, $Q_j(t)$ measures the number of different intermediate products produced and used in the country j at date t, $x_j(\nu,t)$ is the flow output of intermediate product $\nu \in [0, Q_j(t)]$ used at date t and $A_j(\nu,t)$ is a productivity parameter attached to the latest version of intermediate product ν.

We assume that labor supply and population size are identical. They both grow exogenously at the fixed proportional rate n_j. The form of the production function, that is the presence of the term $Q_j(t)$ dividing the labor, ensures that growth in product variety does not affect aggregate productivity. Therefore, we suppose as Aghion and Howitt (1998) and Howitt (2000) that the number of products grows as result of serendipitous imitation, not deliberate innovation. Imitation is limited to domestic intermediate products; thus each new product will have the same productivity parameter as a randomly chosen existing product within the country. Each agent has the same propensity to imitate $\xi > 0$, which we assume identical for each country j. Moreover, we assume that the exogenous fraction ψ of existing intermediate products disappears each period. Thus the aggregate flow of new products is: $Q_j(t+1) - Q_j(t) = \xi L_j(t) - \psi Q_j(t)$, so that the number of workers per product $l_j(t) \equiv L_j(t)/Q_j(t)$ converges monotonically to the constant:

$$l = \psi/\xi \tag{2.7}$$

Assume that this convergence has already occurred, so that: $L_j(t) = lQ_j(t)$ for all t. The form of the production function (2.6) ensures that growth in product variety does not affect aggregate productivity. This and the fact that population growth induces product proliferation guarantee that the model does not exhibit the sort of scale effect that Jones (1995) argues is contradicted by postwar trends in R&D spending and productivity. Without loss of generality, we set l=1 in the rest of the paper.

The final good is used for consumption, as an input into entrepreneurial innovation or invested to create new units of physical capital. Producers of the final good act as perfect competitors in all markets, so that the inverse demands for intermediate goods and labor are given by:

$$\text{(FOC)} \quad \begin{cases} p_j(\nu,t) = \alpha \left(\dfrac{A_j(\nu,t)L_j(t)}{Q_j(t)} \right)^{1-\alpha} x_j(\nu,t)^{\alpha-1} & \text{for all sectors } \nu \in [0, Q_j(t)] \\ w_j(t) = (1-\alpha)\dfrac{Y_j(t)}{L_j(t)} \end{cases} \tag{2.8}$$

Production of intermediate goods. Each intermediate good is produced with physical capital using a one-to-one technology as:

$$x_j(\nu,t) = K_j(\nu,t)$$

where $K_j(\nu,t)$ is the physical capital used in sector ν at date t in country j to produce $x_j(\nu,t)$ units of intermediate goods. For each intermediate good ν, there is an innovator who enjoys a monopoly power in the production of this intermediate good and he maximizes its profits according to:

$$\max_{\{x_j(\nu,t)\}} \pi_j(\nu,t) = p_j(\nu,t)x_j(\nu,t) - (r^\star+\delta)x_j(\nu,t) = \alpha\left(\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}\right)^{1-\alpha} x_j(\nu,t)^\alpha - (r^\star+\delta)x_j(\nu,t)$$

where δ is the depreciation rate of physical capital. The equilibrium quantity of intermediate good ν is given by:

$$x_j(\nu,t) = \alpha^{\frac{2}{1-\alpha}}(r^\star+\delta)^{-\frac{1}{1-\alpha}}\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}$$

Replacing in the inverse demand, we obtain the equilibrium price as: $p_j(\nu,t) = \alpha^{-1}(r^\star+\delta)$.

Aggregate stock of capital. The aggregate stock of physical capital demanded is:

$$K_j(t) = \int_0^{Q_j(t)} K_j(\nu,t)d\nu = \int_0^{Q_j(t)} x_j(\nu,t)d\nu = \alpha^{\frac{2}{1-\alpha}}(r^\star+\delta)^{-\frac{1}{1-\alpha}}A_j(t)L_j(t)$$

where $A_j(t) \equiv \dfrac{1}{Q_j(t)}\displaystyle\int_0^{Q_j(t)} A_j(\nu,t)d\nu$ is the productivity average in country j, so that the capital stock per efficient unit of labor, denoted by $\widehat{k}$, is constant and given by:

$$\widehat{k} = \left(\frac{\alpha^2}{r^\star+\delta}\right)^{\frac{1}{1-\alpha}}$$

Aggregate profits. Substituting the equilibrium quantity of intermediate good and the equilibrium price leads to the equilibrium profit of the monopoly in the sector ν of country j at date t, as:

$$\pi_j(\nu,t) = \frac{1-\alpha}{\alpha}(r^\star+\delta)x_j(\nu,t) = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(r^\star+\delta)^{-\frac{\alpha}{1-\alpha}}\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}$$

which we can rewrite in function of the per efficient unit of labor physical capital, $\widehat{k}$ as: $\pi(\hat{k})\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}$ where $\pi(\widehat{k}) \equiv \alpha(1-\alpha)\widehat{k}^\alpha$. The aggregate profits are therefore given by:

$$\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu = \pi(\widehat{k})A_j(t)L_j(t)$$

Equilibrium wage and output. Introducing equilibrium quantity of intermediate product in each sector ν in the production function of final good sector leads to the equilibrium quantity of per worker final good: $y_j(t) = A_j(t)\widehat{k}^\alpha$, where $y_j(t) \equiv Y_j(t)/L_j(t)$ is the per worker GDP. The equilibrium wages are $w_j(t) = (1-\alpha)A_j(t)\widehat{k}^\alpha = \omega(\widehat{k})A_j(t)$ where $\omega(\widehat{k}) \equiv (1-\alpha)\widehat{k}^\alpha$.

Per worker GDP is given by the sum of incomes (wages, profits of monopolists and rent of capital) in the economy as $y_j(t) = \pi(\widehat{k})A_j(t) + \omega(\widehat{k})A_j(t) + (r^\star+\delta)\widehat{k}A_j(t) = A_j(t)\widehat{k}^\alpha$ so that the growth rate of the economy is therefore given by the growth rate of the average productivity.[5]

2.3 Innovation and dynamics of aggregate productivity

Assume that each period and in each sector ν, there exists a large number of innovators which invest $Z_j(\nu, t)$ units of final goods in R&D. When successful with a probability $\mu_j(\nu, t)$, an innovator replaces the incumbent monopolist next period and reach the worldwide technological frontier denoted by $\overline{A}(t)$, and when there is no innovation in sector ν, the level of productivity remains at its previous level of $A_j(\nu, t)$. Therefore, the law of motion of productivity in each sector ν is given by:

$$A_j(\nu, t+1) = \begin{cases} \overline{A}(t+1) & \text{with probability} \quad \mu_j(\nu, t) \\ A_j(\nu, t) & \text{with probability} \quad (1 - \mu_j(\nu, t)) \end{cases} \tag{2.9}$$

The probability of innovation is linear and given by:

$$\mu_j(\nu, t) = \lambda \frac{Z_j(\nu, t)}{\overline{A}(t)}$$

where $\lambda > 0$ is the productivity of R&D, and where we deflate R&D expenditures in each sector by $\overline{A}(t)$ in order to recognize the force of increasing complexity; as technology advances, the resource cost of further advances increases proportionally. The leading-edge technological is the worldwide technology frontier denoted as $\overline{A}(t)$ and its growth rate is g, so that $\overline{A}(t) = (1+g)^t \overline{A}(0)$.

Moreover, incumbent monopolist which innovated at date t and still producing at date $t+1$, with a probability of $(1 - \mu_j(\nu, t))$, has the following firm value written in recursive form:

$$V_j(\nu, t) = \frac{1}{1 + r^\star} \left(\pi_j(\nu, t) + (1 - \mu_j(\nu, t)) V_j(\nu, t+1) \right) \tag{2.10}$$

so that the problem of innovator is given by:

$$\max_{\{Z_j(\nu, t)\}} \mu_j(\nu, t) V_j(\nu, t+1) - Z_j(\nu, t) \tag{2.11}$$

Therefore, the innovator invests $Z_j(\nu, t)$ units of final good in R&D and obtains the value $V_j(\nu, t+1)$ with probability $\mu_j(\nu, t)$. The FOC of the innovator's problem gives the schumpeterian non-arbitrage condition which is: $\lambda v_j(\nu, t+1) = \frac{1}{1+g}$, where $v_j(\nu, t+1) \equiv \dfrac{V_j(\nu, t+1)}{\overline{A}(t+1)}$ is the value of a firm in sector ν in efficient units, so that the equilibrium probability to innovate is given by:

$$\mu_j^\star = \lambda \pi(\widehat{k}) - \frac{r^\star - g}{1+g} \tag{2.12}$$

[5] $y_j(t)$ is indeed the per worker GDP of the country j, since the sum of value added in all sectors is given by: $\left(y_j(t) - \int_0^{Q_j(t)} p_j(\nu, t) x_j(\nu, t) \right) + \left(\int_0^{Q_j(t)} p_j(\nu, t) x_j(\nu, t) - 0 \right) = y_j(t)$.

Finally, the total R&D expenditures is given by:

$$Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu = \int_0^{Q_j(t)} \left[\frac{\overline{A}(t)}{\lambda}\mu_j^\star\right] d\nu = \frac{\overline{A}(t)}{\lambda}\mu_j^\star Q_j(t) = z(\widehat{k})\overline{A}(t)L_j(t)$$

where $z(\widehat{k}) = \left(\pi(\widehat{k}) - \frac{r^\star - g}{\lambda(1+g)}\right)$.

Denoting the proximity to the technological frontier by $a_j(t) = A_j(t)/\overline{A}(t)$, using equation (2.9), it evolves according to:

$$a_j(t+1) = \mu_j^\star + \frac{1-\mu_j^\star}{1+g} a_j(t) \equiv F_1(a_j(t)) \tag{2.13}$$

and the steady state equilibrium of the proximity to the frontier is given by:

$$a_j^\star = \frac{1+g}{g+\mu_j^\star}\mu_j^\star$$

where $\mu_j^\star$ is given by equation (2.12).

2.4 The net capital inflows

Using our Schumpeterian growth model, the net capital inflows can be decomposed, as in Gourinchas and Jeanne (2013), in terms of convergence, trend, investment and saving. We therefore write the volume of capital inflows in terms of the exogenous parameters of the model to be able to compare the prediction of the model to the data observed.

Market clearing implies that the assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is the country's j external debt, and $V_j(t) = \int_0^{Q_j(t)} V_j(\nu,t)d\nu$ is the total value of corporate assets. Therefore, the resources constraint can be rewritten as:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t))$$

Given the recursive from of the value of firms and the free entry condition in R&D sector $\mu_j(\nu,t)V_j(\nu,t+1) = Z_j(\nu,t)$, we have:

$$\begin{aligned} \int_0^{Q_j(t)} \left(r^\star V_j(\nu,t) - \Delta V_j(\nu,t)\right) d\nu &= \int_0^{Q_j(t)} \left(\pi_j(\nu,t) - Z_j(\nu,t)\right) d\nu \\ &= \Pi_j(t) - Z_j(t) \end{aligned}$$

where $\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu$ is the aggregate profits and $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu$ is the aggregate R&D expenditures. Therefore, the budget constraint of the representative household of country j at date t is written as:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + \Pi_j(t) - Z_j(t)$$

We can now rewrite the budget constraint with per effective worker variables as:

$$\widehat{c}_j(t)+(1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1)-\widehat{d}_j(t+1)\right)=(1+r^\star)\left(\widehat{k}_j(t)-\widehat{d}_j(t)\right)+\omega(\widehat{k})+\pi(\widehat{k})-\frac{z(\widehat{k})}{a_j(t)}$$

where, $\widehat{x}=x/A$ denotes the per worker variables in efficiency units and $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1)\equiv\dfrac{A_j(t+1)-A_j(t)}{A_j(t)}$.

As Gourinchas and Jeanne (2013), we assume that the economy reaches its steady state at a finite date $T<\infty$. The economy steady growth path is $g_j(t+1)=g$, $a_j(T)=a_j^\star$, $\widehat{k}_j(t+1)=\widehat{k}_j(t)=\widehat{k}$ and $\widehat{d}_j(t+1)=\widehat{d}_j(t)=\widehat{d}_j(T)$, so that the steady state debt value is given by:

$$\widehat{d}_j(T)=\widehat{k}+\frac{\omega(\widehat{k})+\pi(\widehat{k})-\frac{z(\widehat{k})}{a_j(T)}-\widehat{c}_j(T)}{r^\star-G_j} \tag{2.14}$$

where $1+G_j\equiv(1+g)(1+n_j)$. The steady state consumption in terms of the proximity to the technological frontier is given as:

$$\widehat{c}_j(T)=\frac{\widehat{c}_j(0)}{a_j(T)/a_j(0)} \tag{2.15}$$

and the initial consumption per efficient worker is given by:

$$\widehat{c}_j(0)=\frac{r^\star-G_j}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)-\frac{z(\widehat{k})}{a_j(0)}+(r^\star-G_j)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right) \tag{2.16}$$

Finally, we follow Gourinchas and Jeanne (2013) and use the change in external debt between dates 0 and T normalized by initial GDP as the measure of capital inflows:

$$\frac{\Delta D_j}{Y_j(0)}=\frac{D_j(T)-D_j(0)}{Y_j(0)} \tag{2.17}$$

Given equations (2.14), (2.15), (2.16) and (2.17), we obtain the volume of capital inflows in terms of the exogenous parameters of the model as follow:[6]

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)}=&\overbrace{\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0}+\overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)}^{\Delta D^t/Y_0}+\overbrace{\left(\frac{a_j(T)}{a_j(0)}-1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}\\&\underbrace{+\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(1-f(t))\right]}_{\Delta D^s/Y_0}\end{aligned} \tag{2.18}$$

where $f(t)\leq 1$ and $f(t)=1$ for $t\geq T$. It is worthy to note that the Schumpeterian framework

[6]See Appendix A.

developed in our paper allows to endogenize the productivity catch-up parameter which depends on the proximity to the technological frontier, unlike the one used by Gourinchas and Jeanne (2013). Because the evolution of the proximity to the technological frontier $a_j(T)$ is endogenous according to equation (2.13), we obtain an endogenous productivity catch-up and also a function $f(t)$ which is given by: $f(t) = 1 - \left(\frac{1-\mu^\star}{1+g}\right)^t$ and therefore depends explicitly on probability to innovate given by equation (2.12).

The decomposition of the capital inflows in Equation (2.18) leads to the following terms similar to Gourinchas and Jeanne (2013): the convergence term ($\Delta D^c/Y_0$) which represents the initial level of capital scarcity, the trend term ($\Delta D^t/Y_0$) which is the impact of initial debt on the capital inflows, the investment term ($\Delta D^i/Y_0$) and the saving term ($\Delta D^s/Y_0$) which both represent the effect of the productivity catch up. The investment term reflects the amount of external debt dedicated to domestic investment while the saving term is the impact of domestic saving on external debt.

3 The model with imperfect credit market

We now introduce a asymmetric information as in Aghion et *al*. (2005) in contrast to the previous section where we implicitly assumed perfect credit markets, innovators have therefore unlimited access to credit and all countries will converge to the technological frontier growth rate. The constraint will affect the total amount potential innovators could invest in R&D, the equilibrium probability to innovate and the evolution of the proximity to the technological frontier. As we will show, countries fall in three different groups depending on their own level of financial development: non-credit constrained group, credit constrained with convergence group and credit-constrained divergence group. Each group leads to a different predicted net capital inflows.

3.1 Innovation under credit constraints

Each period, a potential innovator with current total wealth $\mathcal{A}_j(t)$ decides to invest $Z_j(\nu,t)$ units of final goods in R&D in each sector ν. Assuming she invests a constant fraction $\mathcal{A}_j(\nu,t)$ of her total wealth in each sector, she needs to borrow from a financial institution the amount $Z_j(\nu,t) - \mathcal{A}_j(\nu,t)$ for each project. We also assume as in Aghion et *al* (2005), Bernanke and Gertler (1989) that she can pay a cost $H_j Z_j(\nu,t)$ to defraud her creditor. As shown in appendix B, the innovator could only invest up to a finite multiple of her total wealth in equilibrium:

$$Z_j(\nu,t) \leq \phi_j \mathcal{A}_j(\nu,t) \tag{3.1}$$

where $\phi_j = \frac{1+r^\star}{1+r^\star - H_j}$, $\phi_j \in [1,\infty)$ is the credit multiplier and H_j is the hiding cost. The innovator now chooses $Z_j(\nu,t)$ to maximize her expected net profit of being the incumbent in date t+1 with probability $\mu_j(\nu,t) = \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}$, namely:

$$\begin{aligned} \max_{\{Z_j(\nu,t)\}} \quad & \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)} V_j(\nu,t+1) - Z_j(\nu,t) \\ \text{subject to} \quad & Z_j(\nu,t) \leq \phi_j \mathcal{A}_j(\nu,t) \end{aligned} \tag{3.2}$$

The first order conditions of (3.2) give the Schumpeterian non-arbitrage condition and the expenditure in R&D for each sector ν:

$$\lambda V_j(\nu,t+1) = (1+\Gamma_j(\nu,t))\overline{A}(\nu,t) \quad \text{and} \quad Z_j(\nu,t) = \phi_j \mathcal{A}_j(\nu,t)$$

where $\Gamma_j(\nu,t) > 0$ is the Lagrange multiplier[7]. Since the probability of innovation is the same in all sectors at the equilibrium, the potential innovator will self-finance the same amount $\mathcal{A}_j(\nu,t)$ in each sector and the total expenditure in R&D in the economy is given therefore by $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu = \phi_j q_j(t)\mathcal{A}_j(t)$, , where $q_j(t) < 1$ represents the fraction of the total wealth of households devoted to the R&D. The entrepreneur will innovate now with $\mu(a_j(t)) < \mu^\star$ given by:

$$\mu(a_j(t)) = \lambda\phi\hat{\mathcal{F}}_j(t)a_j(t) \tag{3.3}$$

where $\hat{\mathcal{F}}_j(t) \equiv q(t)\hat{\mathcal{A}}_j(t)$ is the self-financing per efficient worker. The evolution of the proximity to the technological frontier is now given by:

$$a_j(t+1) = \mu(a_j(t)) + \frac{1-\mu(a_j(t))}{1+g}a_j(t) \equiv F_2(a_j(t)) \tag{3.4}$$

which is an increasing concave function and $F_2(0) = 0$.

3.2 Household

From the evolution of the assets of the representative household and the evolution of the value of the firms in equilibrium given by $r^\star V_j(t) - \Delta V_j(t) = \Pi_j(t) - (1+\Gamma_j(t))\phi_j\mathcal{F}_j(t)$, we can write the new budget constraint as:

$$C_j(t)+K_j(t+1)-D_j(t+1) = w_j(t)L_j(t)+(1+r^\star)(1-\tau_j(t))(K_j(t)-D_j(t))+\Pi_j(t)-(1+\Gamma_j(t))q_j(t)\phi_jV_j(t) \tag{3.5}$$

where $\tau_j(t) = \frac{(1+\Gamma_j(t))q_j(t)}{1+r^\star-H_j}$ is the saving wedge. This wedge will act as a "tax" on household saving and will be spent in R&D. The saving is increasing with the level of financial development H_j.

The representative household will now maximize her utility function given by equation (2.1), subject to his budget constraint (3.5). The Euler condition for the small open economy is now

[7] $\Gamma_j(\nu,t) = 0$ corresponds to the case where the entrepreneur is not financially constrained. We do not consider the case where $\Gamma_j(\nu,t) = 0$ and $Z_j(\nu,t) = \phi\mathcal{A}_j(\nu,t)$).

given by:

$$c_j(t)^{-\gamma} = \beta(1+r^\star)(1-\tau_j(t))c_j(t+1)^{-\gamma} \tag{3.6}$$

and finally

$$c_j(t) = c_j(0)(1+g)^t \Phi_j(t)^{min(t,T)} \tag{3.7}$$

The saving wedge will affect the consumption growth. Indeed, consumption will now grow by factor $(1+g)\Phi_j(t)$ in every period $t < T$ and by factor $(1+g)$ afterwards, with $\Phi_j(t) = (1-\tau_j(t))^{1/\gamma}$.

3.3 The net capital inflows under credit constraint

Once again, we need to write the volume of capital inflows in terms of the exogenous parameters. From the country's aggregate resource constraint, we write the steady state debt in terms of efficiency units as:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j \hat{\mathcal{F}}_j(T) - \widehat{c}_j(T)}{r^\star - G_j} \tag{3.8}$$

Because of the Euler equation given by (3.6), the steady state consumption in terms of the proximity to the technological frontier becomes:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)\Phi_j^T}{a_j(T)/a_j(0)} \tag{3.9}$$

and the initial consumption per efficiency worker will be:

$$\begin{aligned} \widehat{c}_j(0) = \quad & (r^\star - g)\Theta\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t \left(\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right) a_j(t)\right) \\ & + (r^\star - g)\Theta\left(\widehat{k}_j(0) - \widehat{d}_j(0)\right) \end{aligned} \tag{3.10}$$

where $\Theta_j = \dfrac{(1+r^\star) - (1+G_j)\Phi_j}{r^\star - G_j + \left(\dfrac{1+G_j}{1+r^\star}\right)^T \Phi_j^T (1+G_j)(1-\Phi_j)}$

Finally, using equations (3.8), (3.9), (3.10) and the definition of the change in external debt given by (2.17), we can write the volume of capital inflows with credit constraint as:

$$\begin{aligned} \frac{\Delta D_j}{Y_j(0)} = & \overbrace{\left(\frac{\widehat{k} - \Theta_j \Phi_j^T \widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0} \\ & \underbrace{+\left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\Theta_j\Phi_j^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{a_j(T)\Phi_j^{(t-T)} - a_j(t)}{a_j(T) - a_j(0)}\right)\right]}_{\Delta D^s/Y_0} \end{aligned} \tag{3.11}$$

Our model with credit constraint leads to equation (3.11), which gives a general form of the volume and direction of the capital inflows. We can thus identify again the four terms as above: the convergence term $\Delta D^c/Y_0$ representing the part of the international borrowing going toward investment to reach the steady state, the trend term $\Delta D^t/Y_0$ representing the part of the initial debt on capital inflows, the investment term $\Delta D^i/Y_0$ and the saving term $\Delta D^s/Y_0$ representing the effect of productivity growth on capital inflows. The imperfection on the domestic financial market will affect all the terms except the investment term; the capital scarcity is higher in the case of credit constraint, $\left(\widehat{k} - \Theta_j \Phi_j^T \widehat{k}_j(0)\right) > \left(\widehat{k} - \widehat{k}_j(0)\right) \forall\ \Theta_j \Phi_j^T > 1$ because a fraction of the initial capital is used in R&D; the gap of capital needed to be financed by external debt is therefore higher. The impact of initial debt on external debt is reduced because a fraction of the initial debt goes toward to R&D. The trend term will be therefore lower under imperfect financial market. The investment term does not change as shown in equation (3.11) because the domestic friction affects only the saving. The country could always borrow on the international financial market to finance domestic investment as its productivity grows.

In the next section, we discuss on the particular forms of equation (3.11) depending on the level of financial development.

3.4 Theoretical predictions

From the above, we can separate economies into 3 different groups, according to the level of financial development, and therefore predict the direction and the volume of capital inflows with respect to the productivity evolution between 0 and T.

3.4.1 Capital inflows under perfect credit market

Looking at the condition $\phi_j \in [1, \infty)$ where $\phi_j = \infty$ correspond to the case without financial friction so entrepreneurs have a unlimited access to credit, it follows that countries with a high financial development level, $H_j \geq (1 + r^\star)$ are assumed to be financially unconstrained. Therefore, one shows that $\Phi_j = \Theta_j = 1$, $q_j = 0$ and thus equation (3.11) becomes similar to equation (2.18). The economy proximity to the technological frontier will evolve according $F_1(a_j(t))$ and will converge to:

$$a_j^\star = \frac{1+g}{g+\mu_j^\star}\mu^\star$$

The growth rate of the productivity will be the same as the technological frontier productivity growth rate $g = \left(\frac{1+r^\star}{\beta}\right)^{1/\gamma} - 1$ for $t \geq T$. For this group of countries, the model predictions are similar to Gourinchas and Jeanne (2013) as follows:

1. Without capital scarcity and initial debt, a country will have a positive net capital inflows if and only if it converges.

$$\Delta D_j/Y_j(0) > 0 \quad \text{if and only if} \quad a_j(T) > a_j(0)$$

2. Consider two identical countries, except for their productivity catch-up, country j will receive more capital inflows than i if and only if j catches up the technological frontier faster than i.

$$\Delta D_j/Y_j(0) > \Delta D_i/Y_i(0) \quad \text{if and only if} \quad \left(\frac{a_j(T)}{a_j(0)} - 1\right) > \left(\frac{a_i(T)}{a_i(0)} - 1\right)$$

Interpretation: The positive slope of $\Delta D_j/Y_j(0)$ indicate that countries which catch up the technological frontier will import capital. As the country catches up the frontier, the productivity grows at a higher rate and households, by anticipating higher future incomes, will thus increase their consumption. The saving will decrease and the external debt has to increase since current income $(\omega(\widehat{k}) + \pi(\widehat{k}))$ does not change. Also, external debt has to increase with the productivity catch up to finance domestic investment. Both effects of the productivity catch up on the saving and the investment lead to the positive relation between the capital inflows and the productivity catch up.

3.4.2 Capital inflows under imperfect credit market and convergence

In countries where the level of financial development is low ($H_j < (1+r^\star)$), entrepreneurs have a limited access to credit and thus cannot invest more than a certain multiple of their total wealth in R&D. They will innovate with probability $\mu(a_j(t)) < \mu^\star$ and the economy proximity to the technological frontier will evolve according to $F_2(a_j(t))$ which is an increasing concave function. With a sufficient level of financial development so that $\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) > 1$, the economy will converge to a limit $\widehat{a}_j < a^\star$ given by:[8]

$$\widehat{a}_j = (1+g) - \frac{g}{\lambda\phi_j\hat{\mathcal{F}}_j}$$

The volume and direction of capital inflows between 0 and T (equation (3.11)), depending on the exogenous parameters of the model is therefore given by:

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\left(\frac{\widehat{k} - \Theta_j\Phi_j^T\widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$

$$\underbrace{+\left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T\Theta_j\Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{\widehat{a}_j\Phi_j^{(t-T)} - a_j(0)\left(\frac{1+g_j}{1+g}\right)^t}{\widehat{a}_j - a_j(0)}\right)\right]}_{\Delta D^s/Y_0}$$

where $g_j = (1+g)\lambda\phi_j\hat{\mathcal{F}}_j$ and $g_j > g$ is the average growth rate of productivity, specific to the economy j for $t < T$, $g_j = g$ for $t \geq T$ and $a_j(0)$ is its initial proximity to the technological

[8] Indeed, we have $F_2(0) = 0$ and $F_2'(0) = \lambda\phi_j\hat{\mathcal{F}}_j + \frac{1}{1+g}$ so that countries with the condition $\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$ converges to a positive proximity to the technological frontier whereas the others diverge and belong to the third group presented below.

frontier. As the economy converges to $\widehat{a}_j > a_j(t) > a_j(0)$ and $\Phi_j < 1$, we can easily show the positive relationship between $\Delta D_j/Y_j(0)$ and $(a_j(T)/a_j(0) - 1)$.

For this group, the model also predicts a similar net capital inflows as the previous one: without capital scarcity and initial debt, one observes a positive capital inflows if and only if the country converges. Considering two identical countries except for their productivity catch-up, a country will receive more capital inflows than the second one if and only if it catches up the technological frontier faster.

Interpretation. The behaviour of the investment term is the same as the group of countries with perfect financial market: external debt has to increase to finance domestic investment as the country has a positive productivity growth. On the other hand, we observe two effects on the direction and the volume of the saving component when the economy experiences an imperfection on the domestic financial market, but has a sufficient level of financial development to catch up the world technological frontier.

The first effect is driven by the expenditure in R&D. Indeed, a positive fraction of wealth is used to self-finance a part of a new project because of the credit constraint and it decreases the households current income. The second effect is due to the permanent income hypothesis. Indeed, as the country invests in R&D and expects a higher growth rate of its productivity, households will anticipate higher future income; thus they will increase their consumption and saving will decrease. The country will borrow on the international financial market to finance this increase of the consumption. Because of the decrease of the current income induced by the first effect, the volume of external debt going toward domestic saving is amplified.

Theoretically, both of these effects added to the effect of the investment component imply a positive relationship between capital inflows and productivity catch-up and also a higher volume of capital inflows compared to the group of countries with perfect domestic financial market. To sum up briefly, countries in this group will catch up the technological frontier because of their level of financial development, and also have a positive net capital inflows. We observe a positive correlation between net capital inflows predicted by the saving component and productivity catch up; a sufficient level of financial development amplifies the volume of the saving component, compared to an economy with perfect credit market.

3.4.3 Capital inflows under imperfect credit market

This group is also characterized by countries with limited access to credit. Then, the proximity to the technological frontier will evolve according to $F_2(a_j(t))$. In opposite to the previous group, countries in this group have an insufficient level of financial development so that $\phi_j < g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) < 1$. The economy proximity to the technological frontier will therefore converge to 0 at a lower growth rate, $g_j \in (0, g)$. By the l'Hôpital's rule, one shows that the productivity growth rate $g_j(t)$ will approach:

$$\lim_{t\to\infty} g_j(t) = (1+g)\lim_{t\to\infty}\left(\frac{a_j(t+1)}{a_j(t)}\right) - 1 = (1+g)\lim_{a\to 0} F_2'(a) = (1+g)\lambda\phi_j l\hat{\mathcal{F}}_j$$

Equation (3.11) representing the volume and the direction of capital inflows between 0 and T, with respect to the exogenous parameters of the model can be written as:

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\left(\frac{\widehat{k} - \Theta_j \Phi_j^T \widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T \Theta_j \Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$\underbrace{+ \left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j \hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T \Theta_j \Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t \left(\frac{\left(\frac{1+g_j}{1+g}\right)^T \Phi_j^{(t-T)} - \left(\frac{1+g_j}{1+g}\right)^t}{\left(\frac{1+g_j}{1+g}\right)^T - 1}\right)\right]}_{\Delta D^s/Y_0}$$

Given that the economy converges to $0 < a_j(t) < a_j(0)$ at a growth rate lower than the productivity growth rate of the technological frontier and $\Phi_j < 1$, we find a negative relation between the capital inflows and the productivity catch-up[9]. The prediction is therefore different for this group of countries. Without capital scarcity and initial debt, we can observe the following directions and volume of capital inflows with respect to the productivity catch-up:

1. Capital will flow into a country if and only if the country diverges.

$$\Delta D_j/Y_j(0) > 0 \quad \text{if and only if} \quad a_j(T) < a_j(0)$$

2. Consider two identical countries belong to this group, except for their productivity catch-up, country i will receive more capital inflows than j if and only if i falls behind the technological frontier faster than j.

$$\Delta D_j/Y_j(0) < \Delta D_i/Y_i(0) \quad \text{if and only if} \quad \left(\frac{a_j(T)}{a_j(0)} - 1\right) > \left(\frac{a_i(T)}{a_i(0)} - 1\right)$$

Interpretation. As the credit constraint does not affect the fraction of international borrowing going toward domestic investment, we can observe the same pattern of the investment term as for the two groups above. As the previous credit-constrained group, countries with an insufficient level of financial development experience same effects of productivity growth and R&D expenditures on the direction and volume of the saving component of net capital inflows.

Total effect will be a positive net capital inflows, denoting that the volume of net capital inflows will also be exaggerated by the extra external debt due to the decrease of the current income. In contrast, countries in this group fail to innovate and will fall behind the technological frontier. Also, the more a country is financially underdeveloped, the more its wealth decreases because to the wedge, and it likely falls behind the technological frontier. We therefore observe a negative correlation between net capital inflows predicted by the saving component and productivity catch-up; as above, the credit constraint imply a higher volume of capital inflows

[9] For $g_j < g$ and $\Phi_j < 1$, we can show that $\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t \left(\frac{\left(\frac{1+g_j}{1+g}\right)^T \Phi_j^{(t-T)} - \left(\frac{1+g_j}{1+g}\right)^t}{\left(\frac{1+g_j}{1+g}\right)^T - 1}\right) < 0$

predicted by the saving component than an economy with perfect credit market.

We have shown that the schumpeterian growth model allows to have several predictions on the relation between productivity growth and net capital inflows, depending on the level of financial development. The next section confronts our model predictions and the data.

4 Empirical assessment of the model

4.1 Data

We follow the standard calibration of the development accounting literature proposed by Caselli (2005) and set the depreciation rate of physical capital to δ=0.06 and the capital share to α=0.3. We set the discount factor to β=0.96 and assume log preference ($\gamma = 1$) as in Gourinchas and Jeanne (2013). The growth rate of the world technological frontier productivity is set to (1+g) = 1.017 which corresponds to the observed average growth rate of U.S.A total factor productivity between 1980 and 2010. Therefore, the world interest rate is calculated using (2.5) and given by $1 + r^{\star} = 1.04$.

Regarding the measure of the productivity, we use data from Version 9.0 of the Penn World Tables[10] (Feenstra *et al* (2015)) and from the World Bank's World Development Indicators for national accounts[11], population, GDP, price levels, income classification and investment; productivity is obtained by using $A_j(t) = (y_j(t)/k_j(t)^{\alpha})^{1/1-\alpha}$ and level of capital stock per efficient unit of labour $\widehat{k}_j(t)$ as $k_j(t)/A_j(t)$, where $y_j(t)$ and $k_j(t)$ are respectively the per capita output and capital stock. Using the trend component of the productivity $A_j(t)^{hp}$ obtained with the Hodrick-Prescott filter, we then construct the proximity of each country to the technological frontier, which is the USA, as $a_j(t) = \left(A_j(t)^{hp}/\overline{A}(t)^{hp}\right)$.

We use the External Wealth of Nations Mark II database (EWN) of Lane and Milesi-Ferretti (2007) to calculate the net capital inflows as the opposite of the ratio of the change in net foreign assets to initial GDP between 1980 and 2010. We normalize the series by GDP to control for the relative size of countries. We also use data on current account from the IMF's International Financial Statistics as an alternative measure of the net capital inflows. Since EWN and IFS data are in current US dollars, we use the PPP-adjustment method in Hsieh and Klenow (2007) to convert NFA from current US dollars to constant international dollars[12]. Since PWT 9.0 reports data on price of investment $P_j(t)$, we use this last as price index. The PPP-adjustor is therefore computed as $\tilde{P}_j(t) = P_j(t)\left(\dfrac{CGDPO_j(t)}{RGDPO_j(t)}\right)$ where $CGDPO(RGDPO)$ is the real GDP at current (constant) US dollars (international dollars). As in Gourinchas and Jeanne (2013), the volume of capital inflows normalized by initial GDP between 1980 and 2010 for each

[10] http://www.aeaweb.org/articles?id=10.1257/aer.20130954

[11] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD

[12] According to Gourinchas and Jeanne (2013), this adjustment method does not affect the results and allow to compare the capital flows measure to the capital accumulation or the output measure used in the development accounting literature.

country is constructed using:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(2010) - D_j(1980)}{Y_j(1980)} \tag{4.1}$$

As we do not have a direct measure of the level of financial development, we follow what is standard in the literature by using private credit as a proxy of the parameter H. Data on private credit comes from Beck *et al* (2000) and represents the ratio of credits by financial intermediaries to the private sector, to GDP.[13]

The parameters regarding R&D for the countries with perfect credit market are calibrated to match observed data in the United States; the probability of innovation $\mu^\star$ is set to 3.6%, which corresponds to the steady state rate of creative destruction of firms in the U.S.A economy[14] and the productivity of R&D λ to 0.41 by using equation (2.12). Table 1 summarizes the main parameters for the 3 groups of countries. ϕ_{conv} and ϕ_{div} are the average credit multipliers for each group. The average productivity growth rate is computed using $g_{conv} = (1+g)\lambda\phi_{conv}\hat{\mathcal{F}}_j$ and $g_{div} = (1+g)\lambda\phi_{div}\hat{\mathcal{F}}_j$. We use the World Bank data on R&D expenditure and data on country's total wealth to set q = 0.025 identical to each country with credit constraint. Therefore, we assume that 2.5% of total wealth in the economy is dedicated to R&D self-financing.

Table 1 around here

After excluding outliers countries, the final sample consist of 109 countries over the world, 89 non-OECD and 19 OECD countries. Initial period is 1980 except for few countries (Angola, Burundi and Brunei (1985), Belize and Laos (1984), China (1982)) and final period is 2010.

4.2 The "Allocation puzzle": Evidence and Predictions

Figure 1 gives a quick illustration of the negative relationship between the average growth rate of the total factor productivity (TFP) and the average ratio of net capital inflows measured by the negative of the ratio of the average current account to GDP for the whole sample over the period 1980-2010. In our sample, we include developed countries but the pattern of the capital inflows remains similar to the one illustrated by Gourinchas and Jeanne (2013); several countries with a negative average growth rate of TFP (total factor productivity) receive a positive capital inflows while others with a positive average growth rate of TFP export capital. Also, one can notice that capital inflows decreases with productivity growth as shown by the regression line; the slope is negative (-0.91) and significant at 1% with a standard error of (0.20). In figure 1, we can also observe that the volume of capital flows varies between -5% of the GDP for countries which export capital up to 15% of the GDP for countries which import capital. This reveals the high volume of the capital movement across countries. The standard neoclassical growth

[13] We use private credit by deposit money banks and other financial institutions to GDP alternatively private credit by deposit money banks to GDP and results still qualitatively unchanged.

[14] Caballero and Jaffe (1993).

model predicts that a lower capital should flow from rich to poor countries, but also a positive correlation between productivity growth and capital inflows. Figure 1 highlights that what we observe in data is the opposite of the predictions of the neoclassical growth model; therefore, we find the "Allocation puzzle" (Gourinchas and Jeanne (2013)) and the "Lucas puzzle" (Lucas (1990)) in the same figure.

Figure 1 around here

To further explore the relationship between net capital inflows and productivity catch-up, we group countries following the level of financial development in table 2. In group 1 we find countries with a high level of financial development, so they are not financially constrained ($H_j \geq (1 + r^\star)$). In group 2 we have countries with medium level of financial development ($H_j < (1 + r^\star)$ and $\phi_j > g/((1 + g)\lambda\hat{\mathcal{F}}_j)$). Finally, countries with a low level of financial development are in group 3 ($H_j < (1 + r^\star)$ and $\phi_j \leq g/((1 + g)\lambda\hat{\mathcal{F}}_j)$). In average, countries in our sample fell behind the technological frontier ($(a_j(T)/a_j(0) - 1) = -0.19$) and receive a positive capital inflows (63.80%) measured by the ratio of change in external debt to initial output over the period 1980-2010. We observe the same pattern on average for non-OECD countries (productivity catch-up = -0.25 and net capital inflows = 70.64% of initial output); our finding for developing countries is similar to the one Gourinchas and Jeanne (2013) who found an average productivity catch up of -0.10 and an average net capital inflows of 31.49% with a sample of 68 developing countries. Compared to Non-OECD countries, OECD countries likely catch up on average the technological frontier (0.10) and also receive a positive but lower volume of capital inflows (31.41%). Among the three groups mentioned above, we observe a negative relationship between productivity catch up and capital inflows only with the third group[15]. With a negative productivity catch-up (-0.27) on average, countries in this group borrow about 77.53% of their initial output. The pattern is different for the two other groups; with a positive productivity catch-up, countries with high (0.12) and sufficient (0.005) level of financial development borrow respectively on average 42.02% and 18.08% of their initial output abroad. Our results are, without surprise, in line with the conclusion of Gourinchas and Jeanne (2013); in opposite with the prediction of the neoclassical growth model, net capital inflows is negatively correlated with productivity growth in observed data.

Table 2 around here

In figure 2 we confront the predictions of the schumpeterian growth model with perfect financial market against the observed capital inflows in data. Assuming that there is no capital

[15]See table (3): By grouping countries with a low level of financial development following their income, one can observe that external debt decreases with productivity catch-up

scarcity, initial debt and common population growth[16], we present only the predicted investment (D^i/Y_0) and saving terms (D^s/Y_0). On the one hand, the negative relationship with the observed data shows up sharply in the graph; the slope of the regression line has a negative and significant (-1.19 significant at 5%).[17] One can observe that most of countries which fell behind the technological frontier (countries on the left panel) receive a positive net capital inflows; according to our threshold of financial development, it is countries which likely diverge because of their low level of financial development. In this panel, we find African and Latin-American countries which are characterized by low long run productivity growth rates and indeed low level of financial development. Asian and some European countries characterized by higher productivity growth rates and higher level of financial development are in the right panel.

Figure 2 around here

On the other hand, we represent net capital inflows predicted by the schumpeterian model under perfect financial market. Under the assumption of absence of capital scarcity and initial debt, total capital inflows is the sum of the investment term and the saving term. We observe in the graph that both these terms have a positive slope for the reasons we invoked above. The volume of capital inflows predicted by the saving term is much greater than the one of investment term. Indeed, the slope of the saving term is (36.42) while the one of the investment term is (2.49).[18] This seems to be normal because of the assumption of infinite life of consumers in the model; according to Gourinchas and Jeanne (2013), households can perfectly smooth their consumption, so the saving term is more sensitive to the productivity growth. Gourinchas and Jeanne (2013) have predicted approximately the same magnitude for the investment term but a lower one for the saving term. Considering the positive correlation predicted by our model under perfect financial market and the negative correlation observed in the data, we conclude that the schumpeterian growth model also faces the “Allocation puzzle” as the neoclassical growth model. We now compare in the next paragraph the prediction of a schumpeterian growth model with an imperfection in the domestic financial market with the observed data.

Figure 3 around here

Figures 3 and 4 give capital flows predicted by the saving and investment components against the observed data for the group of countries which are credit constrained. In Figure 3, we group countries with a sufficient level of financial development. As we discussed in the previous section,

[16]Gourinchas and Jeanne (2013) show with multiple regressions and robustness checks that initial capital scarcity and population growth do not enter significantly in observed capital inflows; they also found that initial debt has a positive and significant coefficient as predicted by their model.

[17]The slope is also negative (-1.12) and significant (5%) when we keep only non-OECD countries

[18]Gourinchas and Jeanne (2013) predicts a slope of the investment term=2.14 and a slope for the saving term=5.25. The slope of saving is higher in our model because we normalize the saving and investment terms by the initial income (as indicated in equation (2.18)) instead capital as they did.

countries belong to this group catch up the technological frontier in the long run. As with the model without credit constraint, one can observe that net capital inflows predicted by saving and investment terms have positive slopes; because of the credit constraint, the saving component predicted by the model will be greater as we also explained in the previous section but the predicted investment component remains unchanged. The saving term has indeed a greater positive slope (164.73%). Regarding the observed data, productivity catch-up seems to have no significant effect on the capital inflows or at the best a positive effect.[19] As we do not have a wider sample of convergent countries with credit constraint, the correlation between capital flows and productivity catch-up observed with the data is ambiguous. Thenceforth, we conclude that the "Allocation puzzle" for this group of countries is related to the positive correlation predicted by the model and the positive (at most null) correlation observed with the data.

Figure 4 around here

Our main finding is presented in figure 4; countries in this graph are those which likely diverge because of a low level of financial development. When we look at the observed net external debt and productivity catch up, most of the countries are indeed located in the left panel. Few of them catch up the frontier despite their low level of financial development. The whole pattern is a decrease of the net capital inflows with the productivity catch up. We also draw the saving and investment terms predicted by the model with imperfection in domestic financial market. The predicted investment terms increases with the productivity catch up as we argued above. An increase of one percent point of the productivity catch-up implies an increase of capital inflows predicted by investment term by 2.08% of initial output. This is identical to the model without credit constraint and also the model with sufficient level of financial development because the imperfection we introduce in the model does not affect the investment but only the saving component of the net capital inflows. Our model prediction about the investment component of capital flows are similar to the conclusion of Gourinchas and Jeanne (2013). Concerning the saving component, the low level of financial development in addition preventing the countries to catch-up the technological frontier, also increases in general external borrowing going toward saving. The more a country is financially constrained, the more it fell behind the technological frontier and the more its net capital inflows predicted by the saving component higher. This leads to the negative correlation between productivity catch-up and net external debt predicted by the saving term.

Introducing a friction on the domestic financial market allows the schumpeterian growth model to replicate the direction the saving part of net capital inflows as observed with the data for countries which fail to catch-up the technological frontier because of their level of financial development. However, our model fails to replicate the volume of net capital inflows; even if predicted investment component succeeds to replicate in absolute value the magnitude of net

[19]Given that, there is only few countries belonging to this group, we find that the regression of the net capital inflows on productivity catch-up gives is non significant.

capital inflows as with the observed net external debt[20], our predicted saving component decreases by 202.94 % of initial output for an increase of one percentage point in the productivity catch-up for countries with insufficient level of financial development; predicted saving component increases by 164.73 % of initial output for an increase of one percentage point in the productivity catch-up for countries with sufficient level of financial development. To be able to replicate perfectly the volume of observed net capital inflows, Gourinchas and Jeanne (2013) estimates trivially an average saving wedge about 1% of aggregate saving, which is relatively small. In Gourinchas and Jeanne (2013), the saving wedge τ_s for each country is computed such the predicted net capital inflows perfectly matches the observed net external debt. We propose in our model a endogenous wedge of about 5% of the aggregate saving, which depend on the level of financial development. We assume that there is no initial debt or capital scarcity and use this wedge to predict separately the saving and the investment components. This is not the case in Gourinchas and Jeanne (2013) who also use an additional wedge in the physical capital; the saving wedge is estimated in their model using the whole model (with initial debt and capital scarcity).

Schumpeterian growth model gives the same prediction as the neoclassical growth model about capital inflows when there is not friction on the domestic financial market. We show that the observed net capital inflows is negatively correlated with the average growth rate of the productivity whereas the theoretical model predicts a positive correlation; under perfect financial market, our model also faces the allocation puzzle and fail to explain the negative correlation between productivity growth and net capital inflows. However, by assuming a friction on the domestic financial market which reduces the capacity of entrepreneurs to borrow and invest in a new project, the model is able to replicate the negative relationship as seen in the data for countries which likely diverge.

5 Concluding remarks

We addressed in this paper the "Allocation puzzle" by looking at the movement of capital across countries according their level of financial development. We introduced a credit constraint in a schumpeterian growth model and showed that this constraint reduces countries total wealth. When the level of financial development allows a country to catch-up the world technological frontier, we find that net capital inflows increases with the productivity catch-up. In contrast, we find that the saving component of net capital inflows decreases with productivity catch-up when countries grow at a lower rate than the technological frontier; from the fact that the "Allocation puzzle" is more related to domestic saving (Gourinchas and Jeanne (2013) and Alfaro *et al* (2014)), our model contributes to explain the negative correlation between productivity catch-up and capital inflows. We also showed that the "Allocation puzzle" holds in a schumpeterian growth framework and can be generalized with recent data to a larger sample including developed

[20]Net capital inflows predicted by the investment component increases by 2.08% and 1.10% of initial output for an increase of one percentage point in the productivity catch-up respectively for countries with insufficient and countries with sufficient level of financial development. According to the data, net capital inflows decreases by 1.19% of initial output for an increase of one per cent of productivity catch-up

countries.

Despite these interesting findings, our model is unable to replicate the volume of the external debt; little capital flows from rich to poor countries (Lucas (1990)) according to the data in contrast with the predicted capital flows. This lack of our model to replicate the volume of capital flows can be due to the fact that the saving wedge in our model is high.[21] In addition, our model does not take into account human capital which is an important determinant of capital flows (Lucas (1990)) and saving (Aghion *et al* (2009)) in developing countries. We believe that these features can help to improve the predictions of our model. We will propose an approach in this direction in a future research.

References

[1] Aghion P. and Howitt P. 1998a. "Endogenous Economic Growth Theory." Cambridge MA: MIT Press.

[2] Aghion P. and Howitt P. 1998b. "Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth." *Journal of Economic Growth*: 3, 111-130.

[3] Aghion, P., Comin, D. and Howitt, P. 2009. "When does Domestic Saving Matter for Economic Growth?" (Working Paper 12275, NBER).

[4] Aghion P., Howitt P. and Mayer-Foulkes D. 2005. "The Effect of Financial Development on Convergence: Theory and Evidence." *Quarterly Journal of Economics*: 120(1), 323-351.

[5] Aguiar, M and Amador, M. 2011. "Growth in the Shadow of Expropriation." *Quarterly Journal of Economics*: 126, 651-697.

[6] Alfaro, L., Kalemli-Ozcan, S. and Volosovych, V. 2014. "Sovereigns, upstream capital flows, and global imbalances." *Journal of the European Economic Association*: 82(1), 182-211.

[7] Attanasio, O., Picci, L. and Scorcu, A. 2000. "Saving, Growth, and Investment: A Macroeconomic Analysis Using a Panel of Countries." *Review of Economics and Statistics*: 12(5), 1240-1284.

[8] Beck T., Levine R. and Loayza N. 2000."Finance and the Sources of Growth." *Journal of Financial Economics*: 58, 261-300.

[9] Bernanke B., and Gertler M. 1989. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*: 79, 14-31.

[10] Buera F. and Shin Y. 2017. "Productivity Growth and Capital Flows: The Dynamics of Reforms." *American Economic Journal: Macroeconomics*: 9(3), 147-185.

[11] Caballero, J.R., Farhi, E. and Gourinchas, P. 2008. "An Equilibrium Model of Global Imbalances and Low Interest Rates." *American Economic Review*: 98, 358-393.

[21] Gourinchas and Jeanne (2013) estimated a average wedge of 1% of aggregate saving to replicate perfectly the volume of observed net capital inflows in opposite of a average wedge of 5% in our model.

[12] Caballero, J.R. and Jaffe, A.B. 1993. "How High Are the Giants' Shoulders: An Empirical Assessment of Knowledge and Creative Destruction in a Model of Economic Growth," in Blanchard, O. and Fisher, S. (eds) *National Bureau of Economic Research Macroeconomics Annual 8* MIT Press.

[13] Carroll, C., Overland, J. and Weil, D. 2000. "Saving and Growth with Habit Formation." *American Economic Review*: 90, 341-355.

[14] Carroll, C. and Jeanne, O. 2009. "A Tractable Model of Precautionary Reserves or Net Foreign Assets." (Manuscript, Johns Hopkins University).

[15] Carroll, C. and Weil, D. 1994. "Saving and Growth: A Reinterpretation." *Carnegie-Rochester Series on Public Policy*: 133-192.

[16] Caselli, F. 2005. "Accounting for Cross-Country Income Differences", in Aghion, P. and Durlauf, S. N. (eds) *Handbook of Economic Growth*: Vol. 1, Part 1 (Elsevier) 679-741.

[17] Caselli, F. and Feyrer, J. 2005. "The Marginal Product of Capital." *Quarterly Journal of Economics*: 122, 535-568.

[18] Feenstra, R., Inklaar, R. and Timmer, P. 2015. "The Next Generation of the Penn World Table." *American Economic Review*: 105(10), 3150-3182.

[19] Feldstein, M. and Horioka, C. 1980. "Domestic Saving and International Capital Flows." *The Economic Journal*: 358, 314-329.

[20] Gertler, M.and Rogoff, K. 1990. "North-south Lending and Endogenous Domestic Capital Market Inefficiencies." *Journal of Monetary Economics*: 26, 245-266.

[21] Goldsmith R. 1969. *Financial Structure and Development.* New Haven, CT: Yale University Press.

[22] Gourinchas, P-O.and Jeannes, K. 2013. "Capital Flows to Developing Countries: The Allocation Puzzle." *Review of Economic Studies*: 80, 1484-1515.

[23] Howitt, P. 2000. "Endogenous growth and cross-country income differences." *American Economic Review*: 90(4), 829-846.

[24] Hsieh, C.-T. and Klenow, P. 2007. "Relative Prices and Relative Prosperity." *American Economic Review*: 97, 562-585.

[25] Jones, C.I. 1995. "R&D-Based Models of Economic Growth." *Journal of Political Economy*: 103, 759-784.

[26] Lane, P. and Milesi-Ferretti, G. 2007. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities,1970-2004." *Journal of International Economics*: 73, 223-250.

[27] Lucas, R. 1990. "Why doesnt Capital Flow from Rich to Poor Countries?" *American Economic Review*: 80, 92-96.

[28] Mendoza, E. Quadrini, V. and Rìos-Rull,J. 2009. "Financial Integration, Financial Development, and Global Imbalances." *Journal of Political Economy*: 117, 371-416.

[29] Modigliani, F. 1970. "The Life Cycle Hypothesis of Savings and Intercountry Differences in the Savings Ratio," in Scott, J. V. M. FG. and Eltis, W. (eds) Induction, Growth and Trade. Essays in Honor of Sir Roy Harrod (Oxford: Clarendon Press): 197-225.

[30] Prasad, E., Rajan, R. and Subramanian,A. 2007. "Foreign Capital and Economic Growth." Brookings Papers on Economic Activity, 1, 153-230.

A Appendix

A.1 *Ratio of cumulated net capital inflows to initial output under perfect financial market*

Ratio of the debt to initial GDP. We first write the ratio of the debt to initial GDP in terms of per efficient worker variables.

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(T) - D_j(0)}{Y_j(0)} = \frac{\widehat{d}_j(T)\dfrac{A_j(T)L_j(T)}{A_j(0)L_j(0)} - \widehat{d}_j(0)}{\widehat{y}_j(0)}$$

where $\widehat{d}_j(t) \equiv \dfrac{D_j(t)}{A_j(t)L_j(t)}$ is the per efficient worker debt and $\widehat{y}_j(t) \equiv \dfrac{Y_j(t)}{A_j(t)L_j(t)}$ is the per efficient worker GDP, for all $t \geq 0$. Using $a_j(t) \equiv \dfrac{A_j(t)}{\overline{A}(t)}$ the proximity to the frontier, $L_j(T) = L_j(0)(1+n_j)^T$ and $\overline{A}(T) = \overline{A}(0)(1+g)^T$, we can write the debt ratio as:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{\dfrac{a_j(T)}{a_j(0)}\widehat{d}_j(T)(1+G_j)^T - \widehat{d}_j(0)}{\widehat{y}_j(0)} \tag{A.1}$$

where $1 + G_j \equiv (1+g)(1+n_j)$ without loss of generality.

Steady state debt per efficient worker. The law of motion of total assets is given by:

$$\mathcal{A}_j(t+1) = w_j(t)L_j(t) + (1+r^\star)\mathcal{A}_j(t) - C_j(t)$$

where market clearing implies that the assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is the country's j external debt, and $V_j(t) = \displaystyle\int_0^{Q_j(t)} V_j(\nu, t)d\nu$ is the total value of corporate assets. We have:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t))$$

Given the recursive from of the firm value $V_j(\nu,t) = \frac{1}{1+r^\star}\left(\pi_j(\nu,t) + (1-\mu_j(\nu,t))V_j(\nu,t+1)\right)$ and the free entry condition in R&D sector $\mu_j(\nu,t)V_j(\nu,t+1) = Z_j(\nu,t)$, we have:

$$\begin{aligned}\int_0^{Q_j(t)} \left(r^\star V_j(\nu,t) - \Delta V_j(\nu,t)\right) d\nu &= \int_0^{Q_j(t)} \left(\pi_j(\nu,t) - Z_j(\nu,t)\right) d\nu \\ &= \Pi_j(t) - Z_j(t)\end{aligned}$$

so that:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + \Pi_j(t) - Z_j(t)$$

where $\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu$ is the aggregate profits and $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu$ is the aggregate R&D expenditures.

We can now write the budget constraint with per effective worker variables as:

$$\widehat{c}_j(t)+(1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1) - \widehat{d}_j(t+1)\right) = (1+r^\star)\left(\widehat{k}_j(t) - \widehat{d}_j(t)\right) + \omega(\widehat{k}) + \pi(\widehat{k}) - \frac{z(\hat{k})}{a_j(t)}$$

where $z(\hat{k}) = \left(\pi(\hat{k}) - \frac{\xi}{\psi\lambda}\frac{r^\star - g}{1+g}\right)$ and $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1) \equiv \dfrac{A_j(t+1) - A_j(t)}{A_j(t)}$.

After time T, the economy steady growth path is $g_j(t+1) = g$, $\widehat{k}_j(t+1) = \widehat{k}_j(t) = \widehat{k}$ and $\widehat{d}_j(t+1) = \widehat{d}_j(t) = \widehat{d}_j(T)$, so that the steady state debt value is given by:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - \frac{z(\widehat{k})}{a_j(T)} - \widehat{c}_j(T)}{r^\star - G_j} \tag{A.2}$$

Steady state consumption per effective worker. We now compute the steady state consumption in terms of the proximity to the technological frontier. The Steady state consumption per effective worker is defined by:

$$\widehat{c}_j(T) = \frac{c_j(T)}{A_j(T)}$$

We can therefore define the average productivity $A_j(t) = a_j(t)\overline{A}(t) = \overline{A}(0)a_j(t)(1+g)^t$.

Using $c_j(T) = c_j(0)(1+g)^T$ and the average productivity definition, we can write:

$$\widehat{c}_j(T) = \frac{c_j(0)(1+g)^T}{a_j(T)\overline{A}_j(0)(1+g)^T}$$

And finally:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)}{a_j(T)/a_j(0)} \tag{A.3}$$

Initial consumption per effective worker. The per worker intertemporal budget constraint is:

$$\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t c_j(t) = (1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t w_j(t)+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t (\pi_j(t)-z_j(t))$$

Using $c_j(t) = \widehat{c}_j(0)A_j(0)(1+g)^t$, $w_j(t) = \omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$, $\pi_j(t) = \pi(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$ and $z_j(t) = z(\widehat{k})\overline{A}_j(0)(1+g)^t$ we can write:

$$\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t \widehat{c}_j(0)A_j(0)(1+g)^t = (1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t \omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$$
$$+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t \left(\pi(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t - z(\widehat{k})\overline{A}_j(0)(1+g)^t\right)$$

It follows that:

$$\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t \widehat{c}_j(0)A_j(0) = (1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t \omega(\widehat{k})a_j(t)\overline{A}_j(0)$$
$$+\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t \left(\pi(\widehat{k})a_j(t)\overline{A}_j(0) - z(\widehat{k})\overline{A}_j(0)\right)$$

Let us denote again $(1+g)(1+n_j) = (1+G_j)$ without loss of generality and divide both sides by $A_j(0)$:

$$\begin{aligned}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t \widehat{c}_j(0) &= (1+r^\star)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)+\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t \frac{a_j(t)}{a_j(0)}\omega(\widehat{k})\\ &+\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t \left(\frac{a_j(t)}{a_j(0)}\pi(\widehat{k}) - \frac{z(\widehat{k})}{a_j(0)}\right)\end{aligned}$$

Using :

$$\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t = \frac{1+r^\star}{r^* - G_j}$$

We have:

$$\widehat{c}_j(0) = \frac{r^\star - G_j}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t \left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t) - \frac{z(\widehat{k})}{a_j(0)} + (r^\star - G_j)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right) \tag{A.4}$$

Ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$. Given equations (A.1),(A.2),(A.3) and (A.4), we can finally compute the volume of capital inflows in terms of the exogenous parameters of the model as follow: Using the initial consumption per effective worker equation, we can write the steady state consumption per effective worker as:

$$\widehat{c}_j(T)=\frac{r^\star-G_j}{a_j(T)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)-\frac{z(\widehat{k})}{a_j(T)}+\frac{a_j(0)}{a_j(T)}(r^\star-G_j)\left(\widehat{k}_j(0)-\widehat{d}_j(0))\right)$$

We then introduce this last into the steady state debt per worker equation. It follows that:

$$\widehat{d}_j(T)=\widehat{k}+\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{r^\star-G_j}\right)-\frac{a_j(0)}{a_j(T)}\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)-\frac{1}{a_j(T)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)$$

Multiplying both side by $\frac{a_j(T)}{a_j(0)}(1+G_j)^T$, we have:

$$\begin{aligned}\frac{a_j(T)}{a_j(0)}(1+G_j)^T\widehat{d}_j(T)&=\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{r^\star-G_j}\right)-(1+G_j)^T\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\\&+\frac{a_j(T)}{a_j(0)}(1+G_j)^T\widehat{k}-\frac{(1+G_j)^T}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)\end{aligned}$$

Introducing this last into equation (A.1), we finally obtain:

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)}&=\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right)-(1+G_j)^T\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)}-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right)-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\\&+\frac{a_j(T)}{a_j(0)}(1+G_j)^T\frac{\widehat{k}}{\widehat{y}_j(0)}-\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)\end{aligned}$$

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)}&=\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)+\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T+\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right)\\&+\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T-\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)\end{aligned}$$

Let us denote:

$$\begin{aligned}A&=\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)\right]\\&=\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T)+a_j(T))\right]\\&=\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T))\right]\\&+\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(T)\right]\end{aligned}$$

Using:

$$\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t = \frac{1+r^\star}{r^*-G_j}$$

We can write:

$$\begin{aligned}
A &= \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right) \\
&= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right) \\
&= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))+\sum_{t=T}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right)
\end{aligned}$$

Using $a_t(j)=a_j(T)\ \forall t\geq T$; then we can write A as:

$$\begin{aligned}
A &= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right) \\
&= \left(1-\frac{a_j(T)}{a_j(0)}\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right)
\end{aligned}$$

We then reintroduce this expression into the capital inflows equation. That gives:

$$\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} &= \frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T+\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)+\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T \\
&\quad +\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right]
\end{aligned}$$

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)}^{\Delta D^t/Y_0} + \overbrace{\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$+\underbrace{\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right]}_{\Delta D^s/Y_0}$$

In addition, we can use the evolving of the distance to the technological frontier to write:

$$\begin{aligned}
a_j(t+1) &= \mu_j(t)+\frac{1-\mu_j(t)}{1+g}a_j(t)\\
a_j(t+1)-a_j(T) &= \mu_j(t)-a_j(T)+\frac{1-\mu_j(t)}{1+g}a_j(t)\\
a_j(t+1)-a_j(T) &= \mu_j(t)-a_j(T)+\frac{1-\mu_j(t)}{1+g}(a_j(t)-a_j(T))+\frac{1-\mu_j(t)}{1+g}a_j(T)\\
a_j(t+1)-a_j(T) &= \mu_j(t)-a_j(T)+\frac{1-\mu_j(t)}{1+g}a_j(T)+\frac{1-\mu_j(t)}{1+g}(a_j(t)-a_j(T))\\
a_j(t+1)-a_j(T) &= \mu_j(t)-\frac{g+\mu_j(t)}{1+g}a_j(T)+\frac{1-\mu_j(t)}{1+g}(a_j(t)-a_j(T))\\
a_j(t+1)-a_j(T) &= \mu_j(t)-\frac{g+\mu_j(t)}{1+g}\frac{1+g}{g+\mu_j(t)}\mu_j(t)+\frac{1-\mu_j(t)}{1+g}(a_j(t)-a_j(T))\\
a_j(t+1)-a_j(T) &= \frac{1-\mu_j(t)}{1+g}(a_j(t)-a_j(T))
\end{aligned}$$

By induction, it follows that:

$$\begin{aligned}
a_j(t)-a_j(T) &= \left(\frac{1-\mu_j(t)}{1+g}\right)^t(a_j(0)-a_j(T))\\
\left(\frac{a_j(t)-a_j(T)}{a_j(0)-a_j(T)}\right) &= \left(\frac{a_j(T)-a_j(t)}{a_j(T)-a_j(0)}\right)=\left(\frac{1-\mu_j(t)}{1+g}\right)^t\\
1-\left(\frac{a_j(T)-a_j(t)}{a_j(T)-a_j(0)}\right) &= \left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)=1-\left(\frac{1-\mu_j(t)}{1+g}\right)^t\equiv f(t)
\end{aligned}$$

where $f(t)\leq 1$ and $f(t)=1$ for $t\geq T$.

Thus, the ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$ becomes:

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G)^T-1)}^{\Delta D^t/Y_0} + \overbrace{\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$+\underbrace{\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(1-f(t))\right]}_{\Delta D^s/Y_0} \quad \text{(A.5)}$$

A.2 *Ratio of cumulated net capital inflows to initial output under imperfect financial market*

Ratio of the debt to initial GDP. We first write the ratio of the debt to initial GDP in terms of per effective worker variables.

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(T) - D_j(0)}{Y_j(0)} = \frac{\widehat{d}_j(T)\frac{A_j(T)L_j(T)}{A_j(0)L_j(0)} - \widehat{d}_j(0)}{\widehat{y}_j(0)}$$

where $\widehat{d}_j(t) \equiv \frac{D_j(t)}{A_j(t)L_j(t)}$ is the per efficient worker debt and $\widehat{y}_j(t) \equiv \frac{Y_j(t)}{A_j(t)L_j(t)}$ is the per effective worker GDP, for all $t \geq 0$. Using $a_j(t) \equiv \frac{A_j(t)}{\overline{A}(t)}$ the proximity to the frontier, $L_j(T) = L_j(0)(1+n)^T$ and $\overline{A}(T) = \overline{A}(0)(1+g)^T$, we can rewrite the debt ratio as:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{\frac{a_j(T)}{a_j(0)}\widehat{d}_j(T)(1+G)^T - \widehat{d}_j(0)}{\widehat{y}_j(0)} \tag{A.6}$$

where $1 + G \equiv (1+g)(1+n)$ without loss of generality.

Steady state debt per effective worker. The law of motion of total assets is given by:

$$\mathcal{A}_j(t+1) = w_j(t)L_j(t) + (1+r^\star)\mathcal{A}_j(t) - C_j(t)$$

where market clearing implies that the assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is the country's j external debt, and $V_j(t) = \int_0^{Q_j(t)} V_j(\nu, t)d\nu$ is the total value of corporate assets. We have:

$$C_j(t) + K_j(t+1) + V_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) + V_j(t) - D_j(t))$$

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t))$$

The evolution of the value of the firms in equilibrium is given by $r^\star V_j(t) - \Delta V_j(t) = \Pi_j(t) - (1+\Gamma_j(t))\phi_j\mathcal{F}_j(t)$. We can now write the budget constraint per efficient worker variables as:

$$\widehat{c}_j(t) + (1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1) - \widehat{d}_j(t+1)\right) = (1+r^\star)\left(\widehat{k}_j(t) - \widehat{d}_j(t)\right) + \omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j(T)$$

where $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1) \equiv \frac{A_j(t+1) - A_j(t)}{A_j(t)}$.

After time T, the economy steady growth path is $g_j(t+1) = g$, $\widehat{k}_j(t+1) = \widehat{k}_j(t) = \widehat{k}$, $\widehat{v}_j(t+1) = \widehat{v}_j(t) = \widehat{v}_j(T)$ and $\widehat{d}_j(t+1) = \widehat{d}_j(t) = \widehat{d}_j(T)$, so that the steady state debt value is given by:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j \hat{\mathcal{F}}_j(T) - \widehat{c}_j(T)}{r^\star - G_j} \tag{A.7}$$

Steady state consumption per effective worker. We now compute the steady state consumption in terms of the proximity to the technological frontier.

The Steady state consumption per effective worker is defined by:

$$\widehat{c}_j(T) = \frac{c_j(T)}{A_j(T)}$$

We can therefore define the average productivity $A_j(t) = a_j(t)\overline{A}(t) = \overline{A}(0)a_j(t)(1+g)^t$.

It follows from the Euler equation:

$$\left(\frac{c_j(t+1)}{c_j(t)}\right)^\gamma = \beta(1+r^\star)(1-\tau_j(t))$$

where $\tau_j(t) = \frac{(1+\Gamma_j(t))q_j(t)}{1+r^\star - H_j}$. Then:

$$c_j(t) = c_j(0)(1+g)^t \Phi_j(t)^{min(t,T)}$$

where $\Phi_j(t) = (1-\tau_j(t))^{1/\gamma}$. Using $c_j(T) = c_j(0)(1+g)^T\Phi^T$ and the average productivity definition, we can write:

$$\widehat{c}_j(T) = \frac{c_j(0)\Phi^T(1+g)^T}{a_j(T)\overline{A}_j(0)(1+g)^T}$$

And finally:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)\Phi^T}{a_j(T)/a_j(0)} \tag{A.8}$$

Initial consumption per effective worker. The per worker intertemporal budget constraint is:

$$\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t c_j(t) = (1+r^\star)(k_j(0)-d_j(0)) + \sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t (w_j(t)+\pi_j(t)-(1+\Gamma_j)\phi_j\mathcal{F}_j(t))$$

Using $c_j(t) = \widehat{c}_j(0)A_j(0)(1+g)^t\Phi^{min(t,T)}$, we show that the left hand side is given by:

$$\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t c_j(t) = \frac{A_j(0)\widehat{c}_j(0)}{\left(1-\frac{1+G}{1+r^\star}\right)\Theta}$$

where $\Theta = \dfrac{(1+r^\star)-(1+G)\Phi}{r^\star - G + \left(\dfrac{1+G}{1+r^\star}\right)^T \Phi^T(1+G)(1-\Phi)}$ and using $w_j(t) = \omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$, it follows that:

$$\widehat{c}_j(0) = (r^\star - g)\Theta\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)+\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\right) \tag{A.9}$$

Ratio of cumulated net capital inflows to initial output between $t = 0$ and $t = T$. Given equation (A.6),(A.7),(A.8) and (A.9), we can finally compute the volume of capital inflows in terms of the exogenous parameters of the model as follow. First, using the initial consumption per effective worker equation, we can write the steady state consumption per effective worker as:

$$\widehat{c}_j(T) = \frac{a_j(0)}{a_j(T)}(r^\star - g)\Theta\Phi^T\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)+\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\right)$$

We then introduce this last into the steady state debt per worker equation. It follows that:

$$\begin{aligned}\widehat{d}_j(T) \quad &= \widehat{k} + \frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G} - \frac{a_j(0)}{a_j(T)}\Theta\Phi^T\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\\ &-\frac{a_j(0)}{a_j(T)}\Theta\Phi^T\left[\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)\right]\end{aligned}$$

Multiplying both sides by $\dfrac{a_j(T)}{a_j(0)}(1+G)^T$, we have:

$$\begin{aligned}\frac{a_j(T)}{a_j(0)}(1+G)^T\widehat{d}_j(T) &= \frac{a_j(T)}{a_j(0)}(1+G)^T\widehat{k} + \frac{a_j(T)}{a_j(0)}(1+G)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G}\right)\\ -(1+G)^T\Theta\Phi^T\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right) &- \frac{(1+G)^T\Theta\Phi^T}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)\end{aligned}$$

Introducing in equation (A.6), we finally obtain:

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)} &= \frac{a_j(T)}{a_j(0)}(1+G)^T\frac{\widehat{k}}{\widehat{y}_j(0)} + \frac{a_j(T)}{a_j(0)}(1+G)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G}\right) - \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\\ &-(1+G)^T\Theta\Phi^T\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)} - \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right) - \frac{(1+G)^T\Theta\Phi^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\omega(\widehat{k})a_j(t)\end{aligned}$$

Let us write:

$$
\begin{aligned}
B &= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t} a_j(t) \\
&= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)+(a_j(T)-a_j(T))\,\Phi^{min(0,t-T)}\right) \\
&= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{min(0,t-T)}\right)-\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t} a_j(T)\Phi^{min(0,t-T)} \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+\sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(T-T)}\right) \\
&\quad +\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t} a_j(T)\Phi^{min(0,t-T)}
\end{aligned}
$$

Using $a_j(t)=a_j(T)\ \forall t\geq T$; then we have:

$$
\begin{aligned}
B &= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+a_j(T)\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{min(0,t-T)} \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+a_j(T)\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{t-T}+\sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{T-T}\right) \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+a_j(T)\Phi^{-T}\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{t}+\sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{T}\right)
\end{aligned}
$$

Finally, using:

$$
\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{t}+\sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\Phi^{T}\right)=\frac{1+r^{\star}}{r^{\star}-G}\Theta^{-1}
$$

we obtain:

$$
B=\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+a_j(T)\Phi^{-T}\Theta^{-1}\frac{1+r^{\star}}{r^{\star}-G}
$$

We then reintroduce this expression into the capital inflows equation. That gives:

$$
\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} &= \frac{a_j(T)}{a_j(0)}(1+G)^{T}\frac{\widehat{k}}{\widehat{y}_j(0)}-(1+G)^{T}\Theta\Phi^{T}\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)}-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right)-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)} \\
&-\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)}{a_j(0)\widehat{y}_j(0)(1+r^{\star})}\right)(1+G)^{T}\Theta\Phi^{T}\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^{\star}}\right)^{t}\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)
\end{aligned}
$$

Thus, the ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$ becomes:

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\left(\frac{\widehat{k} - \Theta_j \Phi_j^T \widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T \Theta_j \Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$\underbrace{+\left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j \hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T \Theta_j \Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t \left(\frac{a_j(T)\Phi_j^{(t-T)} - a_j(t)}{a_j(T) - a_j(0)}\right)\right]}_{\Delta D^s/Y_0}$$

A.3 *Problem of the entrepreneur with credit constraint*

Assume an entrepreneur in country j wants to undertake a new project and have a limited access to credit. She can only borrow the amount $B_j(\nu,t)$ from a financial institution and self-finance a fraction $\mathcal{A}_j(\nu,t)$ of her total wealth with respectively returns $\eta(t)$ and $\upsilon(t)$ to invest in R&D. She pays back the loan and recovers her self-financed amount if and only if she succeeds with probability $\mu(\nu,t)$. The entrepreneur cannot invest more than her self-finance plus the amount borrowed from the financial institution. Therefore, we can write:

$$B_j(\nu,t) + \mathcal{A}_j(\nu,t) \geq Z(\nu,t) \tag{A.10}$$

We also assume that the expected return of the loan and the expected return of the self-financed are not greater than a risk-free return to ensure a non-arbitrage between the two returns. This is represented by these equations:

$$\lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t) \geq (1+r^\star)B_j(\nu,t) \tag{A.11}$$

and

$$\lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\upsilon(t))\mathcal{A}_j(\nu,t) \geq (1+r^\star)\mathcal{A}_j(\nu,t) \tag{A.12}$$

Finally, we assume that the entrepreneur can defraud the financial institution; she can pay a cost $H_j Z_j(\nu,t)$ to hide her successful result to the financial institution and if she does, she will not pay back his loan $\lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t)$. To ensure she will pay the loan, we assume that the cost of defraud is greater than the repayment of the loan:

$$H_j(t) Z_j(\nu,t) \geq \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t) \tag{A.13}$$

The entrepreneur problem is to maximize the expected net profit of becoming the incumbent in the next period. It is given by the expected value of being the incumbent minus the discounted refund of the total investment in R&D, namely:

$$\max_{\{Z_j(\nu,t),B_j(\nu,t),\mathcal{A}_j(\nu,t),\rho(t),\upsilon(t)\}} \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}\left(V_j(\nu,t+1) - \frac{1}{1+r^\star}((1+\eta(t))B_j(\nu,t) + (1+\upsilon(t))\mathcal{A}_j(\nu,t))\right)$$

$$(A.14)$$

subject to A.10, A.11, A.12 and A.13.

By combining constraints A.10, A.11, A.12 and A.13, the innovator problem can be rewrite as:

$$\begin{aligned} \max_{\{Z_j(\nu,t)\}} \quad & \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)} V_j(\nu,t+1) - Z_j(\nu,t) \\ \text{subject to} \quad & Z_j(\nu,t) \leq \phi \mathcal{A}_j(\nu,t) \end{aligned} \tag{A.15}$$

where $\phi_j = \frac{1+r^\star}{1+r^\star - H_j}$ and $\phi_j \in [1,\infty)$.

	Perfect credit market	Imperfect credit market	
		Convergence	Divergence
Probability of innovation	$\mu^{\star}$=3.6%	$\mu(\widehat{a})$= 1.3%	$\mu(\widehat{a})$= 0%
Lagrange multiplier	$\Gamma = 0$	$\Gamma_{conv} = 0.13$	$\Gamma_{div} = 0.86$
Credit multiplier	$\phi^{\star} = \infty$	$\phi_{conv} = 3.99$	$\phi_{div} = 1.28$
Productivity growth rate	g=0.017	$g_{conv} = 0.028$	$g_{div} = 0.0091$

Table 1: Parameter Calibration

	Productivity $\left(\frac{a_j(T)}{a_j(0)} - 1\right)$	Financial development H_j	Capital flows $\Delta D_j/Y_j(0)$	Obs
Total sample	-0.19	41.56	63.80	109
OECD countries	0.10	94.06	31.41	19
Non-OECD countries (Developing countries)	-0.25	30.48	70.64	90
By financial development level:				
High	0.12	135.39	42.02	7
Medium	0.005	79.44	18.08	21
Low	-0.27	23.63	77.53	81

Table 2: Productivity catch-up and Capital Inflows (1980-2010) by level of financial development.

	Productivity $\left(\frac{a_j(T)}{a_j(0)} - 1\right)$	Financial development H_j	Capital flows $\Delta D_j/Y_j(0)$	Obs
Group of countries with level of financial development	-0.27	23.63	77.53	81
Low income	-0.40	14.40	130.43	39
Lower middle income	-0.19	28.47	103.51	23
Upper middle income	-0.09	30.98	-7.98	14
High income	-0.12	52.83	-215.10	5

Table 3: Productivity catch-up and Capital Inflows (1980-2010), Low level of Financial Development.

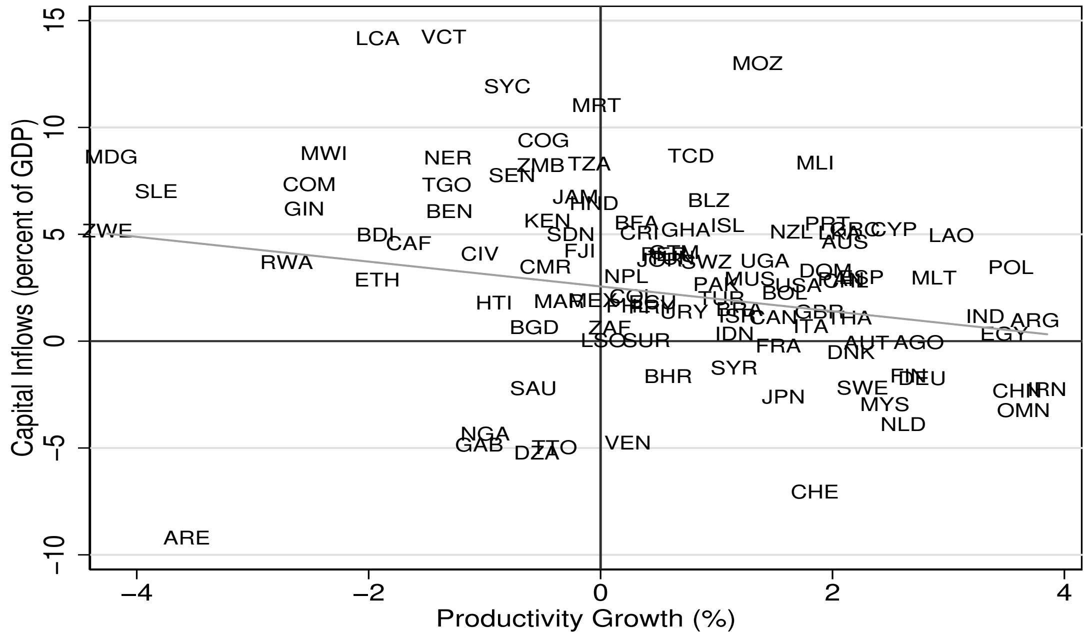

Figure 1: Productivity growth and average Capital Inflows (Negative of Current Account) between 1980 and 2010

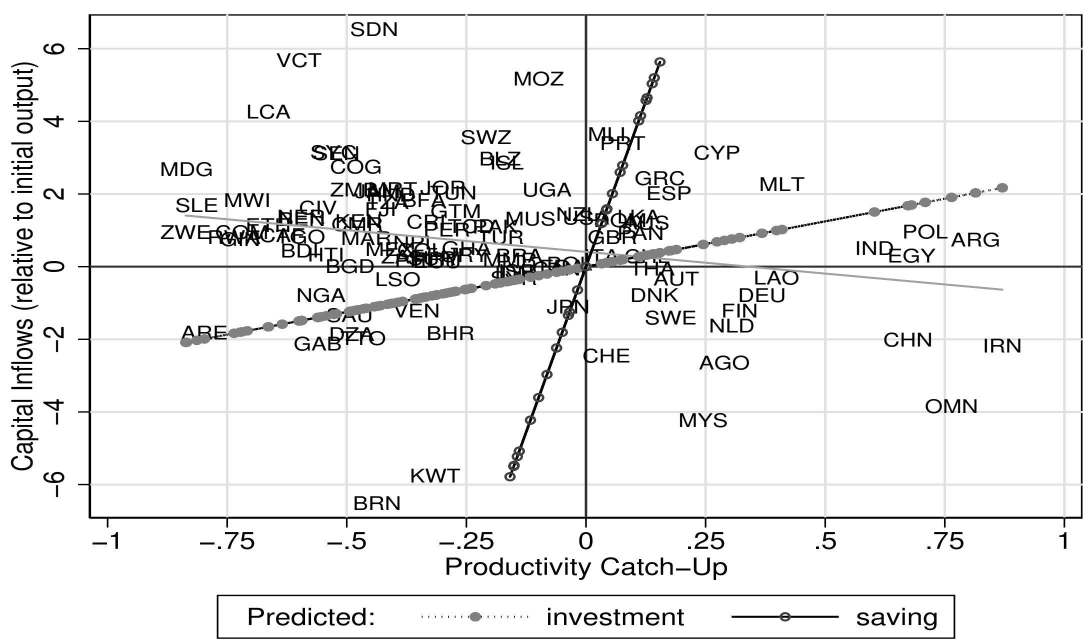

Figure 2: Productivity catch-up and average Capital Inflows (NFA) between 1980 and 2010

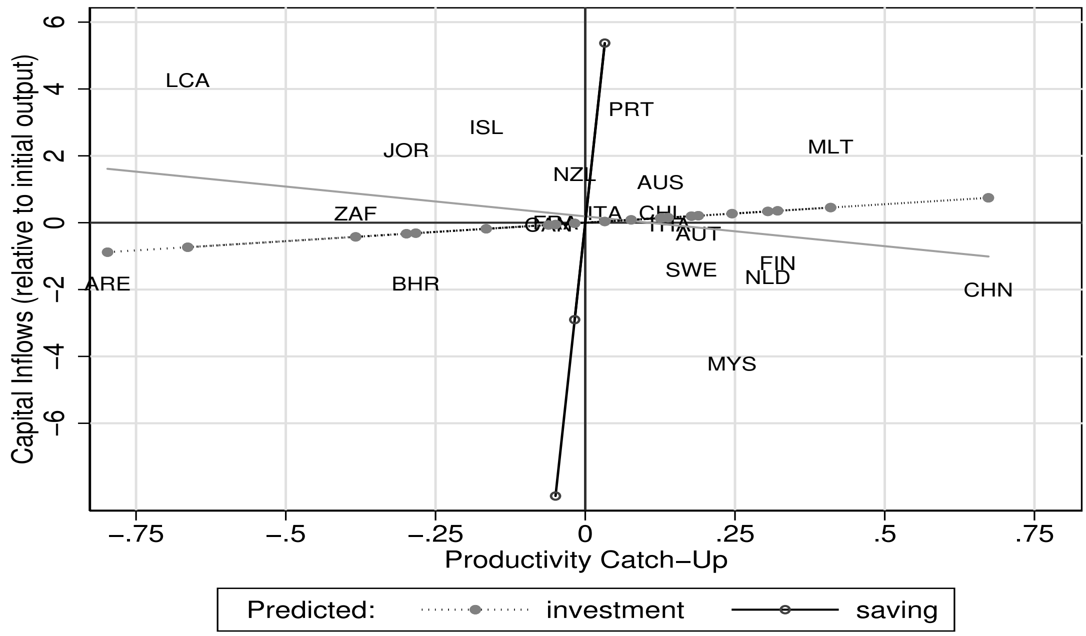

Figure 3: Productivity catch-up and average Capital Inflows between 1980 and 2010, Medium level of Financial Development

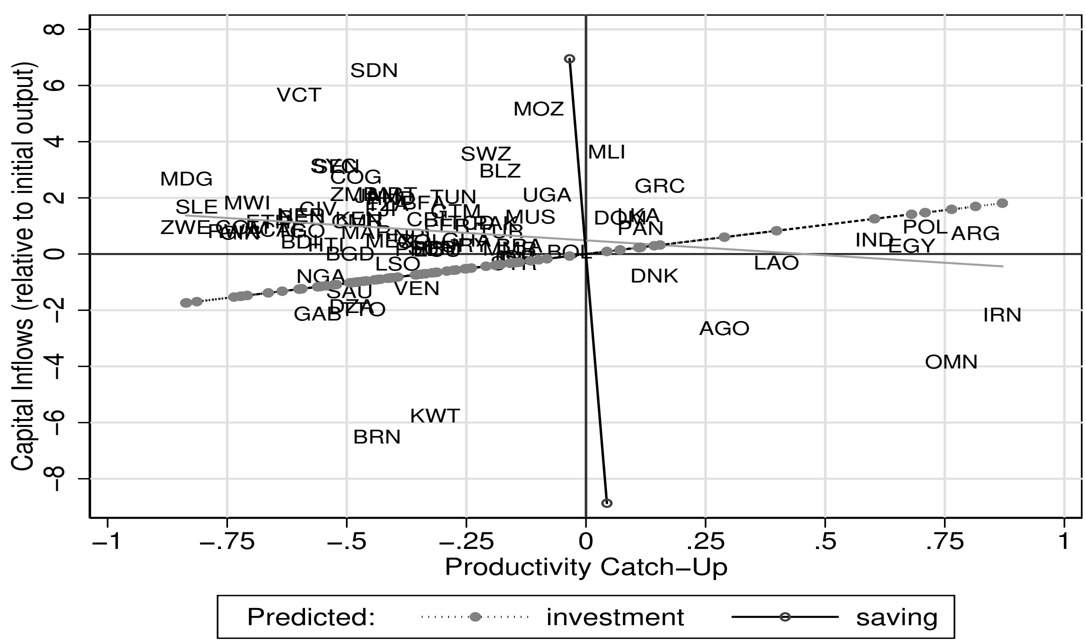

Figure 4: Productivity catch-up and average Capital Inflows between 1980 and 2010, Low level of Financial Development

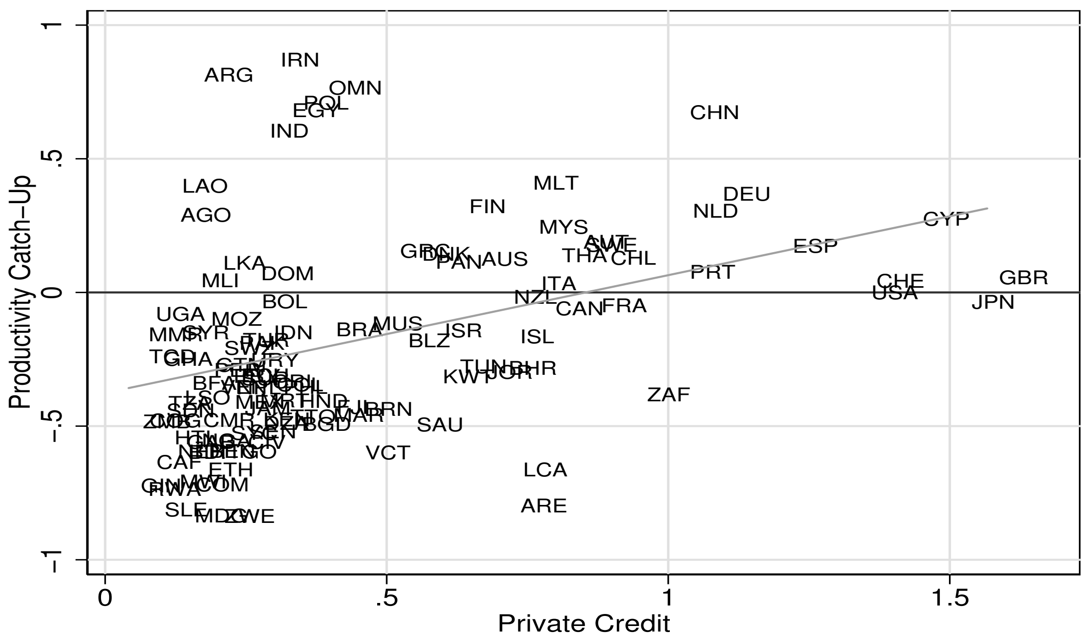

Figure 5: Average Private Credit and Productivity catch-up between 1980 and 2010

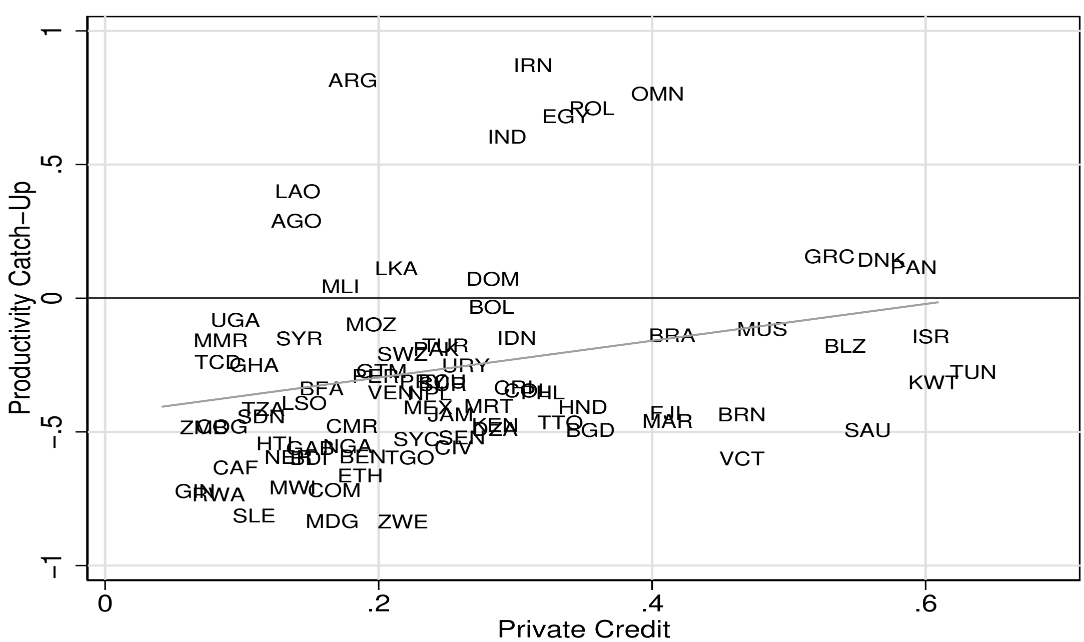

Figure 6: Average Private Credit and Productivity catch-up between 1980 and 2010, Low level of Financial Development

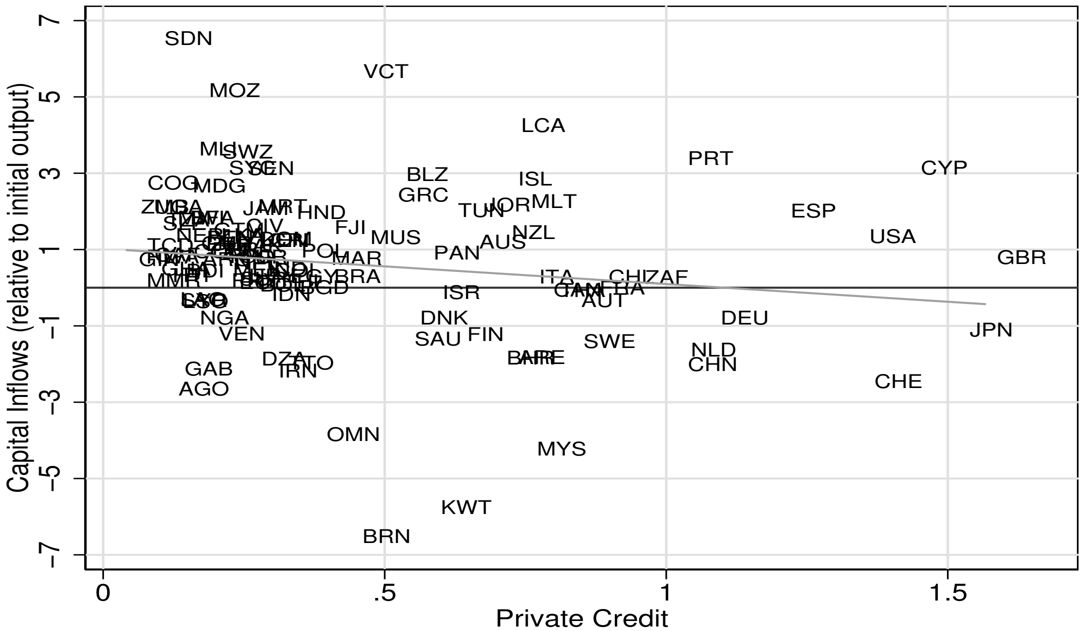

Figure 7: Average Private Credit and average Capital Inflows between 1980 and 2010

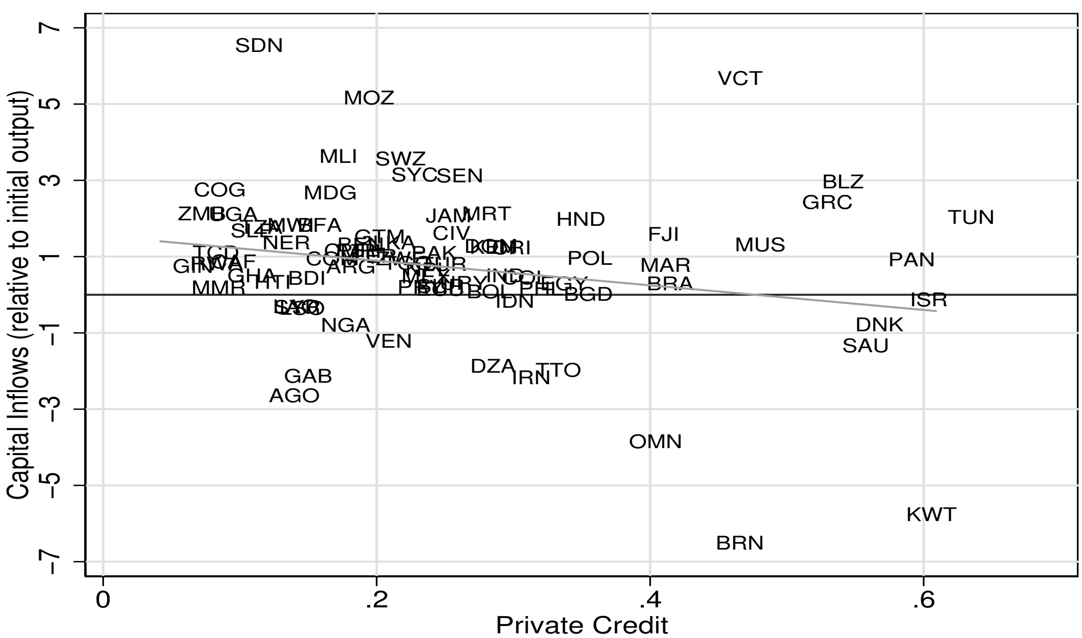

Figure 8: Average Private Credit and average Capital Inflows between 1980 and 2010, Low level of Financial Development