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Accountability and Political Competition*

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Abstract

Is increasing political competition good for voters? We address this question in the political career concerns framework. We model political competition as the cost of challenging an incumbent politician. Our results show that there is no clear relationship between political competition and a politician's incentive to behave in the voters' interest. Analogous results hold for the effect of an increase in political competition on voter welfare, where selection of politicians into office is also taken into account. So, unlike in economic markets, competition in political markets may not always work to produce the most favorable outcome for voters. We tie our results to a contractual incompleteness that is typical of political markets.

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1 Introduction

"An ideal political democracy is defined as: an institutional arrangement for arriving at political decisions in which individuals endeavor to acquire political office through perfectly free competition for the votes of a broadly based electorate." (Becker 1958, p.106).

Democracy, or what Schumpeter calls the democratic method, is procedural in the sense that it is about the method of selecting politicians for office (Schumpeter 1942). Some authors have argued that analogously to economic markets, where competition among firms for consumers produces efficient outcomes, competition among politicians for votes disciplines their behavior, so that an invisible hand may also be at work in political markets. For instance, according to Becker (1958), in an ideal democracy free competition for votes ensures that politicians in power act in the best interest of the electorate and only the best politicians survive.[1] However, in the same way that imperfections in economic markets lead to market failures, Becker asserts that there are two major imperfections that undermine efficiency in political markets: the high entry cost due to the large scale of operation in the political sector and informational problems. Nonetheless, Wittman (1989, 1995) argues that political institutions may respond to the presence of principal-agent and informational issues and mitigate them, with competition for political office and the implied risk of political takeover playing an important role in this respect.

In this paper, we investigate whether competition for political office and the implied risk of political takeover can indeed motivate politicians to act in a way that is beneficial to voters. We do so in the political career concerns framework, which captures in a parsimonious way the presence of principal-agent and informational problems in political markets. In this framework, an office-motivated incumbent politician privately chooses how much effort to exert. The incumbent's performance depends positively on his effort and unknown ability. A voter observes the incumbent's performance, which provides information about the incumbent's ability, and then decides whether to retain or replace the incumbent. So, what motivates the politician to exert effort is his desire to influence the voter's assessment of his ability, thus increasing the likelihood that he is retained.

[1]For comparisons of competition in the economic and political markets, see also Stigler (1972) and Mulligan and Tsui (2006). McGuire and Olson (1996) argue that an invisible hand is at work whenever those detaining power have a strong interest "in the domain over which the power is exercised."

We introduce political competition in the career concerns framework in the form of a citizen who decides whether or not to run for office against the incumbent. The citizen's ability is also unknown and the voter observes a signal about this ability before making his retention decision. Running for office is costly, though. This cost affects the amount of political competition faced by the incumbent. A low cost of entry implies that mounting a challenge to the incumbent's position is easy. On the other hand, a high cost of entry implies that even highly qualified citizens, i.e., citizens with a strong signal about their ability, may be discouraged from running for office.

Our results show that there is no clear relationship between political competition and effective accountability, i.e., the incumbent's incentive to exert effort in office. In order to understand this result, notice that the incumbent's incentive to exert effort is tied to his likelihood of retention given his performance in office. By making it easier for the citizen to run for office against the incumbent, an increase in political competition has an ambiguous impact on effective accountability. On the one hand, it reduces the likelihood that an incumbent with a good performance is retained, which is bad for incentives. On the other hand, it reduces the likelihood that an incumbent with a poor performance is retained, which is good for incentives. We show that while there are cases in which an increase in political competition always increases effective accountability, there are other cases in which an increase in political competition always decreases effective accountability. Moreover, we show that typically, an increase in political competition can be associated with both an increase and a decrease in effective accountability, so that a priori there is no unequivocal relationship between effective accountability and political competition. Furthermore, we show that effective accountability need not be maximized when the cost of entry is zero.

Ultimately, the question of interest is whether more political competition improves voter welfare. An increase in political competition affects voter welfare not only by changing effective accountability but also by affecting electoral selection. Indeed, a change in political competition affects the citizen's decision to run for office, so changing the set of politicians available for the voter to choose. Moreover, as emphasized by Ashworth et al. (2017), changes in effective accountability have an impact on the informational content of the incumbent's performance in office, further affecting electoral selection.

We show that our results about the impact of political competition on effective accountability extend to voter welfare. In particular, voter welfare need not be maximized when the cost of entry is zero. So, unlike in economic markets, competition in political markets may not always work to produce the most favorable outcome for voters.

Our negative results about the relationship between political competition and voter welfare beg the question of what is the feature of political markets that is responsible for these results. A key feature of political markets in the political career concerns framework is that retention is the only incentive device that the voter has to motivate a politician to exert effort in office. We show in the paper that this contractual incompleteness is at the heart of our results: if the voter has a sufficiently rich set of instruments allowing her to tie a politician's payoff to his performance in office, then an increase in political competition is always beneficial to the voter. This result sheds light on another imperfection of political markets besides the imperfections typically emphasized in the literature, namely, that compensation in these markets is usually not directly tied to performance. To the extent that political markets cannot accomodate institutions allowing for such a link, our results show that an increase in political competition need not be associated with higher voter welfare.

The rest of the paper is organized as follows. We discuss the related literature in the remainder of this section. In Section 2, we introduce our model. In Section 3, we define equilibria and characterize equilibrium behavior. In Section 4, we study the impact of changes in political competition on effective accountability and voter welfare. We also show that our negative results about the impact of political competition on voter welfare are tied to the contractual incompleteness of political markets discussed in the previous paragraph. In Section 5, we discuss the robustness of our results to some of our modelling assumptions. We conclude in Section 6. The Appendix contains omitted proofs and details.

Related Literature There is a large literature that studies the consequences of increasing political competition on a number of outcomes of interest. A standard measure of political competition is the cost of entry, which reflects the ex-ante electoral advantage of an incumbent.[2] A related measure of political competition is the number of candidates or political parties, which depends on the

[2] Iaryczower and Mattozzi (2012) show that campaign limits are another variable that affects entry and, as such, should be taken into account together with the cost of entry to determine the overall competitiveness of elections. See also Avis et al. (2017).

cost entry. For majoritarian elections, a different measure of political competition is the difference in vote shares between an elected politician and his best voted competitor (electoral advantage).[3]

A number of papers study theoretically and empirically the relationship between political competition and politician quality and behavior in office. Iaryczower and Mattozzi (2008) study spatial models of political competition with proportional representation in which candidates can invest in their quality and find that increasing political competition leads to higher investment in candidate quality. Galasso and Nannicini (2011) introduce a model where ideological parties select and allocate high-valence (experts) and low-valence (party loyalists) candidates into electoral districts and voters care both about ideology and valence. They show that parties compete by selecting high valence candidates to the most competitive electoral districts and provide supportive evidence for their theory using data from Italian parliamentary elections. De Paola and Scoppa (2011), also using Italian data, study how political competition affects the quality of politicians, as measured by ex-ante characteristics such as educational level and previous occupation.

Polo (1998) studies the relationship between electoral competition and dissipation of political rents and shows that without commitment to electoral platforms, the incentive to grab rents gets stronger as the number of candidates increases. In the context of redistributive politics, Myerson (1993) shows that a higher number of candidates leads to more unequal redistribution, while Lizzeri and Persico (2005) show that increasing the number of candidates worsens the distortion in the provision of public goods. Ashworth et al. (2006), using data from Flemish municipalities, document that political competition has a beneficial effect on the efficiency of municipal administration. Besley and Preston (2007), using data on English local governments, show that electoral bias in favor of one party leads mayors from that party to offer policies that suit the party's core constituency rather than swing voters.

Theoretical and empirical studies of the relation between political competition and economic growth reach mixed conclusions.[4] For exemple, Acemoglu and Robinson (2006) propose a model where innovation by the incumbent politician may change the nature of political competition in subsequent elections and find a U-shaped relationship between pre-existing political competition

[3]See Bardhan and Tsung-Tao (2004) for a discussion of the notions of political competition used in the literature.
[4]See Alfano and Baraldi (2016) for a review of the literature.

and incentives to promote innovation. Padovano and Ricciuti (2009) examine the effects of political competition, as measured by the electoral margin between the two largest political parties, on the economic performance of Italian regions, and find that growth is higher in more competitive regions. Besley et al. (2010), using ex-ante electoral advantage as a measure of political competition, provide theoretical and empirical support for a positive effect of political competition on growth. Man (2014) empirically studies the relationship between political competition and growth and finds a non-linear relationship between the two. Alfano and Baraldi (2015, 2016) empirically study the same relationship and find an inverted U-shaped relationship.[5]

The political career concerns model is widely used to study how a politician's concern for his future career affects his behavior. Standard political career concerns models adapt the career concerns framework of Holmström (1999) to a political economy setting. These models typically assume that there is a single principal (a representative voter) and several agents of unknown ability who compete for office. The payoff to an agent from holding office is fixed and the payoff to the principal depends on the incumbent's output, which is a stochastic function his ability and effort. Incentives are provided by the retention decision. The incumbent has an incentive to exert effort in order to manipulate the principal's assessment of his ability, as this affects the likelihood that he is reappointed to office. Elections thus exert the double role of selecting politicians and of providing incentives for effort.[6] Our paper is the first, to our knowledge, that directly analyses the effect of political competition on political accountability in a political career concerns setting.[7]

Within the political career concerns literature, the paper most closely related to ours is Ashworth et al. (2017), who analyze the trade-off between effective accountability and electoral selection. They depart from the assumption in Holmström (1999) that output is additive in effort and ability and consider general production functions. They show that depending on the way effort

[5]These papers use a Herfindahl index based on vote shares as a measure of political competition.

[6]Models of political career concerns have also been used to study other aspects of the dynamic incentives faced by elected politicians; see, e.g., Besley and Case (1995), Ashworth (2005), Gowrisankaran et al. (2008), Martinez (2009), Alt et al. (2011), and Ferraz and Finan (2011).

[7]Several papers study political accountability outside of the career concerns framework; see, e.g., Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989), Banks and Sundaram (1998), Duggan (2000), Canes-Wrone et al. (2007), Smart and Sturm (2013), and Aghion and Jackson (2016). None of these papers consider political competition represented by barriers to entry, though.

and ability interact in the production function, increasing effective accountability can be associated with either more or less information about an incumbent' quality, affecting the selection of candidates for office. The tension between accountability and selection can be so serious that voter welfare may actually decrease with effective accountability. Our analysis differs from the analysis in Ashworth et al. (2017) in a number of ways. The most important is that our main objective to is analyze how increasing competition affects effective accountability and voter welfare.

2 Model

In this section we first present our model and then make a number of remarks about it.

Agents There is an incumbent, a citizen who can become a candidate and run for office against the incumbent, and a voter. The incumbent and the citizen, i.e., the politicians, are risk neutral and can be of two types: a low-ability type and a high-ability type. We denote a politician's type by τ, where $\tau = L$ if the politician is of low ability and $\tau = H$ otherwise. A politician's type is unknown to both him and the other agents and is independent of the other politician's type. The (ex-ante) probability that a politician is of high ability is $\pi_0 \in (0, 1)$. We refer to the probability that the other agents assign to a politician being of high ability as the politician's reputation.

Output A politician's output in office depends on his private choice of effort $a \in A = [0, \overline{a}]$ and type τ, and is either $y = h$, a 'success', or $y = \ell$, a 'failure'.[8] The probability that a politician of type τ succeeds when he exerts effort a is $f(a, \tau) \in (0, 1]$, where f is a twice continuously differentiable, strictly increasing, and strictly concave function of a. Moreover, $f(a, H) > f(a, L)$ for all $a \in A$, so that a high-ability incumbent is more likely to succeed than a low-ability incumbent regardless of effort. In what follows, we let $f(a, \pi) = \pi f(a, H) + (1 - \pi)f(a, L)$ be the expected probability that a politician of reputation π succeeds when he exerts effort a.

Learning The incumbent's performance in office is observable, and so can be used by all agents to update their beliefs about the incumbent's type.[9] Before deciding whether to become a candi-

[8]We relax the binary-output assumption in Section 5.

[9]Since politicians' types are independent, observing the incumbent's performance does not provide any information about the citizen.

date, the citizen observes a signal about his ability, which the voter also observes.[10] For the citizen, observing the signal about his ability and observing his reputation are equivalent. So, in what follows we treat the citizen's signal and reputation as being the same. Let Ω be the cumulative distribution function in the interval $[0, 1]$ describing the distribution of the citizen's reputation. The c.d.f. Ω has full support and a continuous density ω such that $\omega(0) = \omega(1) = 0$.[11]

Preferences Politicians only care about holding office and effort is costly for them. The benefit of holding office is $B > 0$ and the cost of effort a is $c(a)$. The function c is twice continuously differentiable, increasing and convex. It also satisfies the 'Inada' conditions $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. The voter's payoff from having a politician in office depends on the politician's output and on an additive politician-specific shock, ξ. Let $F = 0$ and $S > 0$ be the voter's payoff from a failure and a success, respectively. So, the voter's payoff from having a politician with shock ξ in office is $S + \xi$ if the politician succeeds and ξ otherwise. The shocks ξ represent a dimension of horizontal differentiation among politicians that co-exists with the dimension of vertical differentiation that is at the center of our analysis. They are independent across politicians and distributed according to a cumulative distribution function Λ in $\mathbb{R}$ with support $[-\eta, \eta]$, where $\eta > 0$. The c.d.f. Λ has a density λ that is strictly positive in $[-\eta, \eta]$ and continuous in $(-\eta, \eta)$.

Entry The citizen pays a cost $\kappa \in [0, \overline{\kappa}]$, with $\overline{\kappa} < B$, if he runs for office. This cost measures the extent of political competition faced by the incumbent; a higher κ decreases political competition by making a challenge to the incumbent more difficult. In what follows we use a change in the entry cost and a change in political competition interchangeably when referring to change in κ.

Timing Action takes place in two periods, one and two. In the first period, the incumbent privately chooses his effort, output is realized, and all agents update their beliefs about the incumbent's type. In the second period, the citizen observes his reputation and decides whether he runs for office. Following that, the voter observes the realization of the preference shocks for the incumbent and the citizen (in case the citizen runs for office) and chooses which politician to put in

[10] Our results remain the same if the voter only observes the citizen's signal in case the citizen runs for office.

[11] Our results extend to the case in which the support of Ω is $[\underline{\pi}_C, \overline{\pi}_C]$, with $0 < \underline{\pi}_C < \pi_0 < \overline{\pi}_C < 1$. We can also relax the assumption that $\omega(0) = \omega(1) = 0$ at some cost. We gain no insights by considering these extensions.

office. Finally, the politician in office in the second period chooses his effort.

Remarks The assumption that the voter observes the signal about the citizen's ability implies that there is no asymmetry of information between the citizen and the voter at the election stage. In particular, the citizen's decision to run for office against the incumbent has no signalling content.[12] This corresponds to the idealized situation in which electoral campaigns eliminate any asymmetry of information between candidates and voters regarding the candidates' abilities. The assumption that politicians are risk-neutral is done for simplicity. Our analysis easily extends to the case in which politicians are risk averse as long as their payoffs are separable in costs and rewards.

As in the canonical political career concerns model, the incumbent has an incentive to exert effort so as to manipulate the voter's assessment of his ability and increase the likelihood that he is retained in office. We depart from the canonical career concerns model by endogenizing the entry of challengers to the incumbent. Two features of our model are important in this regard, namely, the presence of horizontal differentiation and the assumption that the citizen's reputation is random. We discuss these features in the next paragraph.

Horizontal differentiation introduces uncertainty in the voter's retention decision in period two. Without this uncertainty, the citizen's entry decision does not respond to changes in the entry cost: either the citizen always enters or the citizen never enters. On the other hand, in the presence of horizontal differentiation, small changes in the entry cost do not change the incumbent's probability of retention conditional on his output if the citizen's reputation is not random. Indeed, with horizontal differentiation, the citizen who is indifferent between entering and not entering is not marginal with respect to the voter's retention decision in period two except in knife-edge cases.

3 Equilibria

We begin our analysis by defining and characterizing equilibria and showing that they exist. The main results in this section are the following. First, we establish a characterization of the incumbent's equilibrium choice of effort in the first period as a function of the model's primitives. Second, we show that any interior choice of effort can be sustained as an equilibrium choice of

[12] Signalling would introduce a further source of inefficiency in the political market.

effort for the incumbent in the first period for a suitably chosen cost function. These two results play a key role when the analyze the impact of changes in political competition on effective accountability and voter welfare.

3.1 Strategies and Equilibria

A strategy profile specifies the incumbent's effort choice in the first period, the citizen's 'entry' decision in the second period, i.e., the citizen's decision of whether to run for office or not, the voter's appointment decision in the second period, and the effort choice of the politician in office in the second period. Clearly, politicians have no incentive to exert effort in office in period two. So, in what follows we take the politicians' choice of effort in period two as given and omit it from our description of equilibria. Moreover, we refer to the incumbent's choice of effort in period one simply as the incumbent's effort.

We consider pure-strategy perfect bayesian equilibria. Notice that both the citizen's entry decision in period two and the voter's appointment decision in the same period depend on their conjectures about the incumbent's effort, as this choice affects how the citizen and the voter update their beliefs about the incumbent's type; we discuss this in detail below. In equilibrium, the citizen and the voter are correct about the incumbent's effort. We proceed by backward induction.

3.2 Appointment Decision

The voter's appointment decision is trivial when the citizen does not run for office: reappoint the incumbent. Suppose now that the citizen runs for office and let π_I and π_C be, respectively, the incumbent's and the citizen's period-two reputation. Moreover, let ξ_I and ξ_C be, respectively, the realization of the voter's preference shock for the incumbent and the citizen. Since politicians have no incentive to exert effort in office in period two, the voter replaces the incumbent if, and only if

$$Sf(0, \pi_I) + \xi_I \leq Sf(0, \pi_C) + \xi_C;$$

we assume, without loss, that the voter replaces the incumbent when indifferent between the incumbent and the citizen. Let $\theta = 2\eta/S[f(0, H) - f(0, L)]$. The next result is immediate.

Lemma 1. *Suppose the citizen runs for office. The voter replaces the incumbent if, and only if,* $\xi_I \leq \xi_C + (\pi_C - \pi_I)2\eta/\theta$.

We now use Lemma 1 to determine the probability that the incumbent is replaced as a function of his and the citizen's reputations. Let $G : \mathbb{R} \to [0, 1]$ be such that

$$G(x) = \int_{-\infty}^{+\infty} \Lambda\left(\xi + 2x\eta/\theta\right)\lambda(\xi)d\xi.$$

By Lemma 1, if the incumbent's and the citizen's period-two reputation are π_I and π_C, respectively, then the voter replaces the incumbent with probability $G(\pi_C - \pi_I)$. The function G is nondecreasing, with $G(-\theta) = 0$ and $G(\theta) = 1$, and is strictly increasing and continuously differentiable in $(-\theta, \theta)$.[13] So, the probability that the incumbent is retained is one if the reputation difference $\pi_C - \pi_I$ is sufficiently negative, decreases as this difference increases, and becomes zero when this difference becomes sufficiently positive.

The ratio θ measures the importance preference shocks relative to the importance of vertical differentiation among politicians. When θ is small, and vertical differentiation is important, the probability $G(\pi_C - \pi_I)$ is responsive even to small changes in the reputation difference $\pi_C - \pi_I$. On the other hand, when θ is large, it follows that $G(-1) > 0$ and $G(1) < 1$. In this second case, the voter can find it optimal to replace a high-ability incumbent even if the citizen is of low ability and keep a low-ability incumbent even if the citizen is of high ability. We focus our analysis in the case in which θ is small and discuss the large-θ case in Section 5.

Assumption 1. $\theta < \min\{\pi_0, 1 - \pi_0\}$.

3.3 Entry Decision

The citizen's expected payoff from becoming a candidate when his reputation is π_C and the incumbent's reputation is π_I is

$$BG(\pi_C - \pi_I) - \kappa.$$

We assume that the citizen does not become a candidate when indifferent between entering and not entering. This assumption is without loss when the entry cost is positive, as the ex-ante probability

[13] We include a proof of these facts in the Appendix for completeness.

that the citizen is indifferent between entering and not entering is zero when $\kappa > 0$. This assumption constrains the incumbent's behavior when the entry cost is zero, though. Indeed, when $\kappa = 0$, it is (weakly) optimal for the citizen to enter regardless of his reputation. In particular, it is optimal for the citizen to enter even if his reputation is low enough that his probability of being selected for office is zero. The assumption under consideration rules this out by imposing that the citizen enters only if his probability of winning is positive.

We view the assumption that the citizen enters only if it is strictly optimal for him to do so as a reasonable equilibrium refinement. It rules out situations in which the citizen's equilibrium behavior when $\kappa = 0$ cannot be approximated by his equilibrium behavior when $\kappa > 0$ regardless of how small the entry cost is. This assumption also implies that the citizen's equilibrium behavior is robust to a small uncertainty in the cost of entry: as long as the citizen assigns positive probability to the event that $\kappa > 0$, he enters only if the probability of being selected for office is positive.

Let $H : [0, 1] \to [-\theta, \theta]$ be the inverse of the restriction of G to $[-\theta, \theta]$. The function H is continuous, strictly increasing, and continuously differentiable in $(0, 1)$, with $H(0) = -\theta$ and $H(1) = \theta$. Since $\kappa/B \in [0, 1]$ for all $\kappa \in [0, \overline{\kappa}]$, the indifference condition $G(\pi_C - \pi_I) = \kappa/B$ is equivalent to $\pi_C = \pi_I + H(\kappa/B)$. The next result is now immediate.

Lemma 2. *The citizen enters if, and only if,*

$$\pi_C > \pi_C(\pi_I, \kappa) := \pi_I + H\left(\frac{\kappa}{B}\right). \tag{1}$$

By Lemma 2, $\pi_C(\pi_I, \kappa)$ is the cutoff reputation of entry for the citizen when the incumbent's reputation is π_I and the cost of entry is κ. Notice that $\pi_C(\pi_I, \kappa)$ need not be in the interval $[0, 1]$; when $\pi_C(\pi_I, \kappa) < 0$, the citizen always enters, and when $\pi_C(\pi_I, \kappa) \geq 1$, the citizen never enters.[14] It follows from (1) and the properties of H that $\pi_C(\pi_I, \kappa)$ is strictly increasing in π_I and κ. The citizen becomes a candidate with an interior probability if, and only if, $\pi_C(\pi_I, \kappa) \in (0, 1)$. In this case, Lemma 2 implies that the citizen's probability of entry is responsive to changes in political competition.[15] Notice that $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$ by Assumption 1.

[14] So, technically, $\pi_C(\pi_I, \kappa)$ is not a belief—the actual cutoff reputation is $\max\{0, \min\{\pi_C(\pi_I, \kappa), 1\}\}$. We still refer to $\pi_C(\pi_I, \kappa)$ as the citizen's cutoff reputation of entry as it is clear how he behaves when $\pi_C(\pi_I, \kappa) \notin [0, 1]$.

[15] When $\eta = 0$, and there is no horizontal differentiation among politicians, the citizen's entry decision does not respond to changes in κ. Indeed, if $\eta = 0$, then the citizen enters if, and only if, $\pi_C \geq \pi_I$.

3.4 Incumbent's Effort

To conclude our equilibrium characterization, we consider the incumbent's choice of effort in period one. We begin by determining the probability that the incumbent is retained as a function of his period-two reputation. We then discuss how the incumbent's period-two reputation responds to his performance in office in the first period. Together, these two pieces of information allow us to the determine the incumbent's payoff given his effort. This, in turn, allows us to characterize the incumbent's equilibrium effort.

Retention The incumbent is retained in two cases. Either $\pi_C \leq \pi_C(\pi_I, \kappa)$, and the citizen does not become a challenger, or $\pi_C > \pi_C(\pi_I, \kappa)$ but $\xi_I > \xi_C + (\pi_C - \pi_I)2\eta/\theta$, and the citizen becomes a challenger but is not chosen by the voter. Let $Q(\pi_I, \kappa)$ be the probability that the incumbent is retained as a function of his period-two reputation and the entry cost. Then

$$Q(\pi_I, \kappa) = \Omega(\pi_C(\pi_I, \kappa)) + \int_{\max\{0, \pi_C(\pi_I, \kappa)\}}^{1} [1 - G(\pi - \pi_I)]\omega(\pi)d\pi,$$

where we adopt the convention that the above integral is zero if $\pi_C(\pi_I, \kappa) > 1$. The next result establishes some properties of $Q(\pi_I, \kappa)$; see the Appendix for a proof.

Lemma 3. *The retention probability $Q(\pi_I, \kappa)$ is continuous in π_I for all $\kappa \in [0, \overline{\kappa}]$, nondecreasing in π_I and κ, and strictly increasing in π_I and κ if $\pi_C(\pi_I, \kappa) \in (0, 1)$. Moreover, $Q(\pi_I, \kappa)$ is continuously differentiable in $(0, 1) \times (0, \overline{\kappa})$, with*

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B^2}H'\left(\frac{\kappa}{B}\right) \tag{2}$$

for all $(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa})$ such that $\pi_C(\pi_I, \kappa) \in (0, 1)$.

The intuition behind (2) is simple. The marginal increase in the probability that the incumbent is retained following a marginal increase in the cost of entry is proportional to: (*i*) the probability that the citizen is on the margin between entering or not entering; and (*ii*) the marginal increase in the cutoff reputation of entry. It follows from (2) that the incumbent's probability of retention is responsive to changes in political competition if the citizen's probability of entry is interior.[16]

[16]When the citizen's reputation π_C is deterministic, it follows that $Q(\pi_I, \kappa) = 1$ if $\pi_C(\pi_I, \kappa) \geq \pi_C$, and $Q(\pi_I) = 1 - G(\pi_C - \pi_I)$ otherwise. Hence, unless κ is such that $\pi_C(\pi_I, \kappa) = \pi_C$, the probability $Q(\pi_I, \kappa)$ does not respond to small changes in the cost of entry.

Belief Updating We now determine how the incumbent's responds to his performance in the first period. Let $\pi^+(y|a)$ be the incumbent's period-two reputation conditional on producing output y when the other agents believe that his effort is a. Then

$$\pi^+(h|a) = \frac{f(a,H)\pi_0}{f(a,H)\pi_0 + f(a,L)(1-\pi_0)}$$

and

$$\pi^+(\ell|a) = \frac{(1-f(a,H))\pi_0}{(1-f(a,H))\pi_0 + (1-f(a,L))(1-\pi_0)}.$$

Since $f(a,H) > f(a,L)$ for all $a \in A$, it follows that $\pi^+(\ell|a) < \pi_0 < \pi^+(h|a)$ for all $a \in A$. So, the incumbent's performance in office is informative of his ability regardless of his effort. On the other hand, since $f(a,L) > 0$ for all $a \in A$ and $f(a,H) < 1$ for all $a \in [0,\overline{a})$, it follows that $\pi^+(\ell|a) > 0$ for all $a \in A$ and $\pi^+(h|a) < 1$ for all $a \in [0,\overline{a})$. So, except possibly when $a = \overline{a}$, the incumbent's perfomance in office is not completely informative of his ability.[17]

Payoffs and Effort We can now compute the incumbent's payoff as a function of his effort a, and from this determine his equilibrium effort. Notice that the incumbent's payoff also depends on the other agents' conjecture about his effort, a^e, as this conjecture determines how the incumbent's reputation responds to his performance in office. The incumbent's payoff given a and a^e is

$$U(a,a^e) = B\Big[f(a,\pi_0)Q(\pi^+(h|a^e),\kappa) + (1-f(a,\pi_0))Q(\pi^+(\ell|a^e),\kappa)\Big] - c(a),$$

where we omit the dependence of $U(a,a^e)$ on κ for ease of notation. Since $Q(\pi^+(h|a^e),\kappa) \geq Q(\pi^+(\ell|a^e),\kappa)$ regardless of a^e and κ by Lemma 3, $U(a,a^e)$ is concave in a for all $a^e \in A$.

The effort a^* is an equilibrium choice of effort for the incumbent if, and only if, a^* maximizes $U(a,a^e)$ when $a^e = a^*$. The next result provides a necessary and sufficient condition for $a^* \in A$ to be an equilibrium choice of effort for the incumbent. We use this condition when we analyze the impact of changes in political competition on effective accountability and voter welfare.

Lemma 4. *The effort a^* is an equilibrium choice of effort for the incumbent if, and only if,*

$$B\frac{\partial f}{\partial a}(a^*,\pi_0)\left[Q(\pi^+(h|a^*),\kappa) - Q(\pi^+(\ell|a^*),\kappa)\right] = c'(a^*). \tag{3}$$

[17] We show below that $\overline{a}$ is never an equilibrium choice of effort for the incumbent.

The interpretation of condition (3) is straightforward. The left-hand side of (3) is the marginal benefit of effort to the incumbent when he exerts effort a^* and the citizen and the voter correctly anticipate his behavior. The right-hand side of (3) is the marginal cost of effort to the incumbent when he exerts effort a^*. The concavity of $U(a, a^e)$ in a for all $a^e \in A$ implies that (3) is a sufficient condition for a^* to maximize $U(a, a^*)$. In the Appendix, we show that the Inada conditions on the cost function c imply that (3) is also necessary for a^* to maximize $U(a, a^*)$.

It follows from (3) and the Inada condition $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ that $a^* \in A$ can be an equilibrium choice of effort for the incumbent only if $a^* < \overline{a}$. It also follows from (3) and the Inada condition $c'(0) = 0$ that $a^* = 0$ is an equilibrium choice of effort for the incumbent only if the incumbent's probability of retention is the same regardless of his output when $a^e = 0$. In the Appendix we show that is not possible since $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$ by Assumption 1, and so the incumbent's probability of retention responds to his performance in office regardless of the conjecture about his behavior. We have thus established the following corollary to Lemma 4.

Corollary 1. *The incumbent's equilibrium choice of effort is always interior.*

3.5 Existence and Rationalization

We conclude this section by showing that an equilibrium always exists and that any interior choice of effort for the incumbent is consistent with equilibrium behavior. Let

$$\Delta(a, \kappa) = B\frac{\partial f}{\partial a}(a, \pi_0)\left[Q(\pi^+(h|a), \kappa) - Q(\pi^+(\ell|a), \kappa)\right] - c'(a).$$

It follows from Lemma 4 that an equilibrium exists when the entry cost is κ if $\Delta(a^*, \kappa) = 0$ for some $a^* \in A$. A straightforward argument shows that the equation $\Delta(a, \kappa) = 0$ has a solution a^* in A for all $\kappa \in [0, \overline{\kappa}]$; see the Appendix for a proof.

Proposition 1. *An equilibrium always exists.*

Before we state the last result in this section, we introduce some terminology. If follows from the proof of Lemma 3 is that $\Delta(a, \kappa)$ is differentiable in a for all $a \in (0, \overline{a})$ and $\kappa \in [0, \overline{\kappa}]$. Suppose the entry cost is κ and the incumbent's equilibrium choice of effort is a^*, which we know is interior. We say that the equilibrium is *regular* if $\partial\Delta(a^*, \kappa)/\partial a \neq 0$ and that the equilibrium is *stable* if $\partial\Delta(a^*,\kappa)/\partial a < 0$; these concepts are important when we discuss our approach to comparative statics in the next section. Moreover, we say that a cost function c is *admissible* if it is twice continuously differentiable, increasing and convex, and satisfies the Inada conditions $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0,\pi_0)/\partial a$.

We now show that every $a \in (0,\overline{a})$ can be an equilibrium choice of effort for the incumbent for a suitably chosen cost function. Moreover, we can choose this cost function in such a way that a is the incumbent's unique equilibrium choice of effort and the equilibrium is stable. Recall that besides the cost function, the primitives of the model are the probability π_0 that politicians are of high type, the production function f, the distribution of the citizen's reputation Ω, the benefit of holding office B, the cost of entry κ, the voter's payoff from a success S and the distribution of the politician-specific preference shocks Λ.

Lemma 5. *Fix all the model's primitives but the cost function. For each $a^* \in (0,\overline{a})$, there exists an admissible cost function c such that a^* is the unique equilibrium choice of effort for the incumbent and the equilibrium is stable when the cost function is c.*

Lemma 5 is not surprising. Indeed, fix all the model's primitives but the cost function c and let

$$MB(a,\kappa) = B\frac{\partial f}{\partial a}(a,\pi_0)\left[Q(\pi^+(h|a),\kappa) - Q(\pi^+(\ell|a),\kappa)\right].$$

Now let $a^* \in (0,\overline{a})$. We know from the proof of Corollary 1 that $MB(a^*,\kappa)$ is positive. Since $MB(a^*,\kappa)$ is bounded above by $B\partial f(0,\pi_0)/\partial a$, it is easy to see that there exists an admissible cost function c with $c'(a^*) = MB(a^*,\kappa)$ and $c'(\overline{a}) > B\partial f(0,\pi_0)/\partial a$.[18] In the Appendix, we show that we can take c to be such that $c'(a) \neq MB(a,\kappa)$ for all $a \neq a^*$ and $\partial MB(a^*,\kappa)/\partial a < c''(a^*)$, so that a^* is the unique equilibrium choice of effort for the incumbent and the equilibrium is stable.

4 Competition, Accountability, and Voter Welfare

In this section we study how effective accountability and voter welfare respond to changes in political competition. We begin by discussing our approach to comparative statics. We then present our

[18] Let $c_1(a) = (a^2/\overline{a})B\partial f(0,\pi_0)/\partial a$, $\alpha = MB(a^*,\kappa)/c_1'(a^*)$, and set $c(a) = \alpha c_1(a)$.

results about the impact of changes in political competition on effective accountability, followed by our results about the impact of changes in political competition on voter welfare. We conclude by tying our negative results about the impact of changes in political competition on voter welfare to the fact that retention is the only instrument the voter has to provide the incumbent with an incentive to exert effort.

4.1 Comparative Statics

It follows from the analysis of the previous section that the incumbent's incentive to exert effort when the other agents conjecture that his choice of effort is a is proportional to the increase

$$\delta(a,\kappa) = Q(\pi^+(h|a),\kappa) - Q(\pi^+(\ell|a),\kappa).$$

in the incumbent's probability of retention following a success. By affecting the citizen's entry decision, a change in political competition changes the incumbent's probability of retention conditional on his period-two reputation, $Q(\pi_I,\kappa)$, and thus changes $\delta(a,\kappa)$ for all $a \in A$. This, in turn, changes the incumbent's incentive to exert effort, and so has an impact on the incumbent's equilibrium choice of effort.

When the incumbent's equilibrium choice of effort is unique for every entry cost, his response to a change in political competition is unambiguous: if the incumbent's equilibrium choice of effort is a^*, then a change in κ that increases $\delta^* = \delta(a^*,\kappa)$ leads to higher effort and a change in κ that decreases $\delta^* = \delta(a^*,\kappa)$ leads to lower effort.[19] However, the incumbent's equilibrium choice of effort is globally unique only in some special cases.[20] Indeed, we also show in the Appendix that if there exist $0 < a_1 < a_2 < \overline{a}$ and $\kappa \in [0,\overline{\kappa}]$ such that $MB(a_1,\kappa)/a_1 < MB(a_2,\kappa)/a_2$, then there exists an admissible cost function c such that equilibria with different effort choices for the incumbent exist when the cost function is c and the entry cost is κ. Moreover, we can take c to be such that two of these equilibria are stable.

In the absence of global uniqueness of the incumbent's equilibrium choice of effort, we rely on

[19] These facts are straightforward to establish. We include a proof of them in the Appendix for completeness.

[20] One such case is when $\pi^+(h|a)$ is strictly decreasing in a and $\pi^+(\ell|a)$ is strictly increasing in a, so that an increase in the incumbent's effort decreases the dispersion in the distribution of his period-two reputation. In this case, $\delta(a,\kappa)$ is nondecreasing in a for all $\kappa \in [0,\overline{\kappa}]$, and so $\Delta(a,\kappa)$ is strictly decreasing in a regardless of κ.

local comparative statics analysis using the implicit function theorem. The equation

$$\Delta(a,\kappa) = B\frac{\partial f}{\partial a}(a,\pi_0)\delta(a,\kappa) - c'(a) = 0$$

defines the incumbent's equilibrium choice of effort implicitly as a function of the entry cost. Since the function Δ is continuously differentiable in $(0,\overline{a}) \times (0,\overline{\kappa})$ by Lemma 3, we can apply the implicit function theorem to study how the incumbent's equilibrium choice of effort in a given equilibrium responds to small, i.e., local, changes in the entry cost.[21] This requires us to restrict attention to regular equilibria, otherwise one cannot ensure local uniqueness of the incumbent's equilibrium choice of effort, a necessary condition for a meaningful comparative statics exercise.

On the other hand, as is well known, with multiple equilibria local comparative statics analysis using the implicit function theorem can lead to ambiguous results in the sense that a local change in the incumbent's incentive to exert effort can lead to opposite changes in the incumbent's effort depending on the equilibrium under play. Hence, as is standard when using the implicit function theorem for comparative statics analysis, we further restrict attention to stable equilibria. This ensures that a local change in political competition leads to an unambiguous and economically meaningful response in the incumbent's effort: if the incumbent's equilibrium choice of effort is a^*, then a local change in the cost of entry that increases $\delta(a^*,\kappa)$ leads to higher effort, while a local change in the cost of entry that decreases $\delta(a^*,\kappa)$ leads to lower effort.[22]

4.2 Political Competition and Effective Accountability

We now use our equilibrium characterization to study how changes in political competition affect effective accountability in stable equilibria. Given that we rely on local comparative statics analysis using the implicit function theorem, from now on we understand changes in political competition as local changes. Moreover, given our focus on stable equilibria, in the remainder of this section an equilibrium always means a stable equilibrium.

[21] Additional care is needed when $\kappa = 0$, as the implicit function theorem is not immediately applicable in this case.

[22] Indeed, if $\kappa_0 \in (0,\overline{\kappa})$ is the initial entry cost and $a^*(\kappa)$ is the incumbent's equilibrium choice of effort as a function of κ in a neighborhood of κ_0, then totally differentiating $\Delta(a^*(\kappa),\kappa)$ in a neighborhood of $(a^*(\kappa_0),\kappa_0)$ yields $(\partial\Delta(a^*(\kappa_0),\kappa_0)/\partial a)(da^*(\kappa_0)/d\kappa) + \partial\Delta(a^*(\kappa_0),\kappa_0)/\partial\kappa = 0$. Since $\partial\Delta(a^*(\kappa_0),\kappa_0)/\partial\kappa \propto \delta^* = \delta(a^*(\kappa_0),\kappa_0)$, the sign of $da^*(\kappa_0)/d\kappa$ is the same as the sign of δ^* in a stable equilibrium.

We know from Lemma 3 that regardless of his effort and output, the incumbent's probability of retention is nondecreasing in the entry cost and is strictly increasing in the entry cost as long as the citizen's probability of entry is interior. So, for any $a \in (0, \overline{a})$ and $\kappa \in (0, \overline{\kappa})$, the impact of a reduction in κ on $\delta(a, \kappa)$ is ambiguous: it depends on whether the reduction in the citizen's probability of retention after a success is greater or smaller than the reduction in the citizen's probability of retention after a failure. Thus, a priori, the impact of an increase in political competition on effective accountability can be either positive or negative.

As it turns out, there are conditions under which $\delta(a, \kappa)$ responds unambiguously to a reduction in κ regardless of a. For instance, suppose there exists $\kappa \in (0, \overline{\kappa}]$ such that

$$0 < \pi_C(\pi^+(\ell|a), \kappa) < 1 < \pi_C(\pi^+(h|a), \kappa) \text{ for all } a \in (0, \overline{a}). \tag{4}$$

Then, when the entry cost is κ, the citizen does not enter if the incumbent succeeds and enters with interior probability if the incumbent fails no matter the incumbent's effort. In this case,

$$\delta(a, \kappa) = 1 - Q(\pi^+(\ell|a), \kappa)$$

and an increase in political competition increases $\delta(a, \kappa)$ for all $a \in (0, \overline{a})$.

Now suppose that there exists $\kappa \in (0, \overline{\kappa}]$ such that

$$\pi_C(\pi^+(\ell|a), \kappa) < 0 < \pi_C(\pi^+(h|a), \kappa) < 1 \text{ for all } a \in (0, \overline{a}). \tag{5}$$

Then, when the entry cost is κ, the citizen enters with interior probability if the incumbent succeeds and enters for sure if the incumbent fails no matter the incumbent's effort. In this second case,

$$\delta(a, \kappa) = Q(\pi^+(h|a), \kappa) - \int_0^1 [1 - G(\pi - \pi^+(\ell|a))]\omega(\pi) d\pi,$$

and an increase in political competition decreases $\delta(a, \kappa)$ for all $a \in (0, \overline{a})$. The next result now follows immediately.

Proposition 2. *Suppose the entry cost is $\kappa \in (0, \overline{\kappa})$. If condition* (4) *holds, then an increase in political competition increases effective accountability. If, instead, condition* (5) *holds, then an increase in political competition decreases effective accountability.*

Let $\overline{\pi}^+(\ell) = \sup_{a\in(0,\overline{a})} \pi^+(\ell|a)$ be the highest reputation possible for the incumbent after a failure and $\underline{\pi}^+(h) = \inf_{a\in(0,\overline{a})} \pi^+(h|a)$ be the lowest reputation possible for the incumbent after a success; notice that $0 < \overline{\pi}^+(\ell) < \pi_0 < \underline{\pi}^+(h) < 1$. Suppose the entry cost is $\kappa \in (0, \overline{\kappa}]$. A necessary condition for (4) is that $\underline{\pi}^+(h) + H(\kappa/B) \geq 1$, otherwise there exists $a \in (0, \overline{a})$ such that $\pi^+(h|a) + H(\kappa/B) < 1$. Given that $H(\kappa/B)$ is bounded above by θ, we then have that a necessary condition for (4) is that $\theta \geq 1 - \underline{\pi}^+(h)$. Likewise, a necessary condition for (5) is that $\overline{\pi}^+(\ell) + H(\kappa/B) \leq 0$. Since $H(\kappa/B)$ is bounded below by $-\theta$, we then have that a necessary condition for (5) is that $\theta \geq \overline{\pi}^+(\ell)$. Thus, (4) and (5) are violated for all $\kappa \in (0, \overline{\kappa}]$ and $a \in (0, \overline{a})$ when $\theta < \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$.[23] In other words, the conditions of Proposition 2 are never satisfied if horizontal differentiation is small enough.

The next result we establish is that conditions (4) and (5) are essentiality necessary for an unambiguous response of effective accountability to an increase in political competition: if there exist $\kappa \in (0, \overline{\kappa}]$ and $a \in (0, \overline{a})$ such that the probability of entry for the citizen is interior regardless of the incumbent's performance when the incumbent's effort is a and the entry cost is κ, then the response of effective accountability to an increase in political competition becomes ambiguous. In what follows, we say that a cumulative distribution function Ω in $[0, 1]$ is an *admissible* distribution of the citizen's reputation if it has a continuous density ω such that $\omega(0) = \omega(1) = 0$.

Proposition 3. *Suppose the entry cost is $\kappa \in (0, \overline{\kappa})$ and fix all other primitives of the model but the cost function c and the distribution of the citizen's reputation Ω. If there exists $a^* \in (0, \overline{a})$ with*

$$0 < \pi_C(\pi^+(\ell|a^*), \kappa) < \pi_C(\pi^+(h|a^*), \kappa) < 1, \tag{6}$$

then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c, \Omega) = (c_1, \Omega_1)$ and when $(c, \Omega) = (c_2, \Omega_2)$; and (ii) an increase in political competition increases effective accountability in one case and decreases it in the other.

Proof. Suppose $a^* \in (0, \overline{a})$ is such that (6) holds. Let Ω_1 and Ω_2 be admissible distributions of the

[23] Notice that $\theta > \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$ is sufficient for either (4) or (5) to hold for some $\kappa \in (0, B]$. Indeed, since $\min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\} < \min\{\pi_0, 1 - \pi_0\}$, then either $\theta \in (\overline{\pi}^+(\ell), \pi_0)$ or $\theta \in (1 - \underline{\pi}^+(h), 1 - \pi_0)$ when $\theta > \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$. In the first case, (5) holds for κ small. In the second case, (4) holds for κ close to B.

citizen's reputation with densities ω_1 and ω_2 such that $\omega_1(\pi_C(\pi^+(h|a^*), \kappa)) < \omega_1(\pi_C(\pi^+(\ell|a^*), \kappa))$ and $\omega_2(\pi_C(\pi^+(h|a^*), \kappa)) > \omega_2(\pi_C(\pi^+(\ell|a^*), \kappa))$. By Lemma 5, for each Ω_i there exists an admissible cost function c_i such that a^* is the unique equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c_i and the distribution of the citizen's reputation is Ω_i. Now let $Q_i(\pi_I, \kappa)$ be the incumbent's probability of retention as a function of his period-two reputation and the entry cost when the distribution of the citizen's reputation is Ω_i and let $\delta_i(a, \kappa)$ be such that $\delta_i(a, \kappa) = Q_i(\pi^+(h|a), \kappa) - Q_i(\pi^+(\ell|a), \kappa)$. The desired result follows from the fact that Lemma 3 implies that

$$\frac{\partial \delta_i}{\partial \kappa}(a^*, \kappa) \propto \omega_i(\pi_C(\pi^+(h|a^*), \kappa)) - \omega_i(\pi_C(\pi^+(\ell|a^*), \kappa)) \qquad \square$$

A corollary of Proposition 3 is that without additional assumptions on the distribution of the citizen's reputation, one cannot predict how effective accountability responds to a change in political competition when the cost of entry is such that (6) holds for some effort level a. In particular, since $\theta < \overline{\pi}^+(\ell)$ implies that there exist $a \in (0, \overline{a})$ and $\kappa' \in (0, \overline{\kappa}]$ such that (6) holds for the effort level a when the entry cost is $\kappa \in (0, \kappa')$, one cannot predict how effective accountability responds to a change in political competition when horizontal differentiation is sufficiently small if the entry cost is also sufficiently small.

Proposition 3 does not concern how the set of possible equilibrium effort choices for the incumbent depends on the cost of entry holding everything else constant. The last two results in this section show that: (*i*) for any positive cost of entry there are cases in which multiple equilibria exist and effective accountability responds differently to an increase in political competition in these equilibria; and (*ii*) effective accountability need *not* be maximized when the cost of entry is zero if horizontal differentiation is sufficiently small.

Proposition 4. *For any entry cost $\kappa \in (0, \overline{\kappa}]$ there exist values of the other primitives of the model for which there are multiple equilibria and an increase in political competition increases effective accountability in one of these equilibria but decreases effective accountability in another.*

Proposition 4 relies on the result that if there exist $0 < a_1 < a_2 < \overline{a}$ and $\kappa \in [0, \overline{\kappa}]$ such that $MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$, then there are two stable equilibria for an appropriately chosen cost function. We show that for any entry cost $\kappa \in (0, \overline{\kappa}]$, there are values of the model's primitives for which the condition under consideration holds for some $0 < a_1 < a_2 < \overline{a}$. Moreover, similarly to the proof of Proposition 3, we can choose the distribution of the citizen's reputation in such a way that the incumbent's equilibrium choice of effort responds in opposite directions to an increase in political competition in these two equilibria. The proof of Proposition 4 is in the Appendix.

Proposition 5. *Fix all the model's primitives but the cost function, the distribution of the citizen's reputation, and the entry cost. If $\theta < \overline{\pi}^+(\ell)$, then there exist a cost function and a distribution of the citizen's reputation for which effective accountability is not maximized when entry is costless.*

Proof. Suppose $\theta < \overline{\pi}^+(\ell)$. Then $\theta < \pi^+(\ell|a^*)$ for some $a^* \in (0, \overline{a})$. Let Ω be an admissible distribution of the citizen's reputation with density ω satisfying $\omega(\pi^+(\ell|a^*) - \theta) < \omega(\pi^+(h|a^*) - \theta)$. Now let c be an admissible cost function such that a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable when the entry cost is zero, the cost function is c, and the distribution of the citizen's reputation is Ω. Take the cost function and the distribution of the citizen's reputation to be c and Ω, respectively.

We show in the Appendix that the uniqueness of the incumbent's equilibrium choice of effort and the stability of the equilibrium when $\kappa = 0$ together imply that there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique and differentiable in κ for all $\kappa \in (0, \underline{\kappa})$. For each $\kappa \in (0, \underline{\kappa})$, let $a^*(\kappa)$ be the incumbent's unique equilibrium choice of effort when the entry cost is κ. Since $\Delta(a^*(\kappa), \kappa) = 0$ for all $\kappa \in (0, \underline{\kappa})$, it then follows that

$$\frac{\partial \Delta}{\partial a}(a^*(\kappa), \kappa) \frac{da^*}{d\kappa}(\kappa) + \frac{\partial \Delta}{\partial \kappa}(a^*(\kappa), \kappa) = 0$$

for all $\kappa \in (0, \underline{\kappa})$. Now observe from Lemma 3 that

$$\frac{\partial \Delta}{\partial \kappa}(a^*(\kappa), \kappa) \propto \omega\Big(\pi^+(h|a^*(\kappa)) + H\left(\frac{\kappa}{B}\right)\Big) - \omega\Big(\pi^+(\ell|a^*(\kappa)) + H\left(\frac{\kappa}{B}\right)\Big)$$

for all $\kappa \in (0, \underline{\kappa})$. Since $\lim_{\kappa \to 0} \pi^+(y|a^*(\kappa)) + H(\kappa/B) = \pi^+(y|a^*) - \theta$ for each $y \in \{\ell, h\}$ and the density ω is continuous, it follows that $\partial\Delta(a^*(\kappa), \kappa)/\partial a > 0$ if κ is sufficiently close to zero. Reducing $\underline{\kappa}$ even further, we can then conclude that $da^*(\kappa)/d\kappa > 0$ for all $\kappa \in (0, \underline{\kappa})$. Thus, effective accountability is not maximized when $\kappa = 0$. $\square$

4.3 Political Competition and Voter Welfare

We conclude this section by studying the impact of changes in political competition on voter welfare. A change in political competition directly affects the voter's welfare by changing the citizen's cutoff reputation of entry, thus affecting the pool of politicians available for the voter to choose in the second period. A change in political competition also indirectly affects the voter's welfare by changing the incumbent's effort. This change in the incumbent's effort not only affects the voter's payoff in the first period, but also affects the voter's payoff in the second period by changing the informational content of the incumbent's performance, thus affecting the selection of politicians for office in this period. Moreover, this change in the incumbent's effort further affects the citizen's entry decision by changing the incumbent's reputation after each possible output.

Below we compute the voter's welfare and discuss how it responds to a change in political competition. As with effective accountability, we show that if horizontal differentiation is small enough, then one cannot predict how voter welfare responds to changes in political competition when the entry cost is also small enough. We also show that voter welfare need not be maximized when the entry cost is zero.

Voter Welfare Voter welfare is the sum of two terms, the first-period welfare and the second-period welfare. The first-period welfare is $W_1(a) = Sf(a, \pi_0)$, the voter's expected payoff when the incumbent's effort is a. In order to determine the second-period welfare, let $\mu(y|a)$ be the probability that the incumbent's output is y when his effort is a and z be the random variable such that $z = \xi_I - \xi_C$. Two events are possible if the incumbent's effort is a and his output is y. Either the citizen enters and the voter's expected payoff is

$$\mathbb{E}_z\big[\max\{Sf(0, \pi^+(y|a), \kappa) + z, Sf(0, \pi_C)\}\big]$$

or the citizen does not enter and the voter's expected payoff is $Sf(0, \pi^+(y|a))$; recall that ξ_C has zero mean. Since when the incumbent's effort is a and his output is y the citizen enters if, and only if, his reputation is greater than the cutoff reputation of entry $\pi_C(\pi^+(y|a), \kappa)$, it follows that the second-period welfare is

$$W_2(a,\kappa) = \sum_{y\in\{\ell,h\}} \mu(y|a)\Big[\int_{\pi_C(\pi^+(y|a),\kappa)}^1 \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\big]\omega(\pi)d\pi$$

$$+\Omega(\pi_C(\pi^+(y|a),\kappa))Sf(0,\pi^+(y|a))\Big],$$

where we adopt the convention that the above integral is zero if $\pi_C(\pi^+(y|a),\kappa) \geq 1$ and the citizen does not enter regardless of his reputation.

Let Γ denote the cumulative distribution function of the random variable z. We show in the Appendix that $W_2(a,\kappa) = W_2^+(a) - W_2^-(a,\kappa)$, where

$$W_2^+(a) = \sum_{y\in\{\ell,h\}} \mu(y|a)\int_0^1 \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\big]\omega(\pi)d\pi$$

and

$$W_2^-(a,\kappa) = \sum_{y\in\{\ell,h\}} \mu(y|a)\int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \Gamma(z)dz\right)\omega(\pi)d\pi.$$

The term $W_2^+(a)$ is the voter's period-two welfare when the citizen always enters, while the term $W_2^-(a,\kappa)$ is the voter's welfare loss in the second period due to the cost of entry—when the cost of entry is positive, the citizen may choose not to enter even if there is a positive probability that he is better for the voter than the incumbent, which lowers the voter's payoff in ex-ante terms.

Welfare Changes We now discuss how a change in political competition affects voter welfare. Suppose the entry cost is $\kappa_0 \in (0,\overline{\kappa})$ and let $a^*(\kappa)$ be the incumbent's equilibrium choice of effort as a function of the entry cost when this cost is in a neighborhood of κ_0; our restriction to stable, and thus regular, equilibria ensures that $a^*(\kappa)$ is well-defined and continuously differentiable in κ. Now let $W^*(\kappa) = W(a^*(\kappa),\kappa)$ be the voter's welfare when κ is in a neighborhood of κ_0. Then the (first-order) effect of a change in the entry cost on voter welfare is[24]

$$\frac{dW^*}{d\kappa}(\kappa_0) = \left[\frac{dW_1}{da}(a^*(\kappa_0)) + \frac{dW_2^+}{da}(a^*(\kappa_0)) - \frac{\partial W_2^-}{\partial a}(a^*(\kappa_0),\kappa_0)\right]\frac{da^*}{d\kappa}(\kappa_0) - \frac{\partial W_2^-}{\partial \kappa}(a^*(\kappa_0),\kappa_0)$$

The direct effect of a change in the entry cost on voter welfare is

$$\Delta W^*_{\text{direct}} = -\frac{\partial W_2^-}{\partial \kappa}(a^*(\kappa_0),\kappa_0)$$

[24]The differentiability of $W_1(a)$ and $W_2^-(a,\kappa)$ is straightforward to establish. The differentiability of $W_2^+(a)$ follows from standard results in measure theory. We include a proof of this latter fact in the Appendix for completeness.

The term $\partial W_2^-(a,\kappa)/\partial\kappa$ describes the effect on the voter's second-period welfare of the resulting change in the citizen's cutoff reputation of entry. In the Appendix, we compute $\partial W_2^-(a,\kappa)/\partial\kappa$ and show that this term is positive. Intuitively, a higher cost of entry increases the cutoff reputation of entry, which decreases voter welfare by reducing the pool of politicians available for the voter to choose in the second period. Thus, $\Delta W^*_{\text{direct}}$ is negative.

The indirect effect of a change in the entry cost on voter welfare due to the resulting change in the incumbent's effort is

$$\Delta W^*_{\text{indirect}} = \left[\frac{dW_1}{da}(a^*(\kappa_0)) + \frac{dW_2^+}{da}(a^*(\kappa_0)) - \frac{\partial W_2^-}{\partial a}(a^*(\kappa_0),\kappa_0)\right]\frac{da^*}{d\kappa}(\kappa_0)$$

The term $dW_1(a)/da = S\partial f(0,\pi_0)/\partial a$ describes the effect of a change in the incumbent's effort on the voter's first-period welfare. The terms $dW_2^+(a)/da$ and $\partial W_2^-(a,\kappa)/\partial a$ describe the effect of a change in the incumbent's effort on the voter's second-period welfare due to the change in the informational content of the incumbent's performance and the change in the citizen's entry decision conditional on the incumbent's performance, respectively. While the sign of $dW_1(a)/da$ is positive, the sign of the terms $dW_2^+(a)/da$ and $\partial W_2^-(a,\kappa)/\partial a$ can be either positive or negative. Thus, the sign of $\Delta W^*_{\text{indirect}}$ need not be the same as the sign of $da^*(\kappa_0)/da$. We now discuss conditions under which $\Delta W^*_{\text{indirect}}$ and $da^*(\kappa_0)/da$ have the same sign.

We say that effort increases the informativeness of output if an increase in effort decreases the distribution of the incumbent's period-two reputation strictly in the second-order stochastic sense. Since $\mathbb{E}_z[\max\{Sf(0,\pi^+)+z, Sf(0,\pi)\}]$ is convex in π^+ regardless of π, it follows that $dW_2^+(a)/da$ is nonnegative when effort increases the informativeness of output. Intuitively, if effort increases the dispersion in the incumbent's period-two reputation, then more effort improves the selection of politicians for office, which increases voter welfare. In the Appendix we compute $\partial W_2^-(a,\kappa)/\partial a$ and show that if $\partial f(0,L)/\partial a$ and $\partial f(0,H)/\partial a$ are both finite, then $\partial W_2^-(a,\kappa)/\partial a$ converges to zero in the limit as κ decreases to zero uniformly in a. So, when effort increases the informativeness of output and $\partial f(a,\tau)/\partial a$ is finite for $\tau = L, H$, the indirect effect of a change in political competition on voter welfare due to the resulting change in the incumbent's effort has the same sign at the change in the incumbent's effort if the entry cost is sufficiently small.

In general, a comparison of the direct and indirect effects on voter welfare of a change in political competition is not possible. However, the next result shows that when horizontal differentiation is sufficiently small there are values of the model's primitives for which a stable equilibrium exists in which the sign of $dW^*(\kappa_0)/d\kappa$ is the same as the sign of $\Delta W^*_{\text{indirect}}$ as long as the entry cost is also small enough. The proof of Lemma 6 is in the Appendix; recall from the proof of Proposition 5 that if $\theta < \overline{\pi}^+(\ell)$, then there exists $a^* \in (0, \overline{a})$ with $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$.[25]

Lemma 6. *Fix all the primitives of the model but the cost function, the distribution of the citizen's reputation, and the entry cost and suppose that $\theta < \overline{\pi}^+(\ell)$. For all $a^* \in (0, \overline{a})$ with the property that $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$, there exist $\kappa' \in (0, \overline{\kappa})$ such that if the entry cost is $\kappa_0 \in (0, \kappa')$, then: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium for a suitably chosen cost function and a suitably chosen distribution of the citizen's reputation; and (ii) $dW^*(\kappa_0)/d\kappa$ has the same sign as $\Delta W^*_{\text{indirect}}$ in this equilibrium.*

An immediate corollary of Lemma 6 and the discussion preceding it is that if effort increases the informativeness of output, then when horizontal differentiation is small enough there are choices of the model's primitives for which a stable equilibrium exists in which $dW^*(\kappa_0)/d\kappa$ and $da^*(\kappa_0)/da$ have the same sign if the entry cost is also small enough. The next result follows immediately from the proof of Lemma 6 and the discussion preceding it together with the proofs of Propositions 3 and 5. It shows that our main conclusions about the impact of changes in political competition on effective accountability extend to voter welfare.

Proposition 6. *Fix all the primitives of the model but the cost function c, the distribution of the citizen's reputation Ω, and the entry cost κ. Suppose that $\theta < \overline{\pi}^+(\ell)$, effort increases the informativeness of output, and $\partial f(0, \tau)/\partial a$ is finite for $\tau = L, H$. The following facts hold.*

1. Let $a^* \in (0, \overline{a})$ satisfy $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$. There exists $\kappa' \in (0, \overline{\kappa})$ such that if $\kappa \in (0, \kappa')$, then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c, \Omega) = (c_1, \Omega_1)$ and when $(c, \Omega) = (c_2, \Omega_2)$; and (ii) an increase in political competition increases voter welfare in one case and decreases it in the other.

[25]An implicit assumption in Lemma 6 is that $\Delta W^*_{\text{indirect}}$ is different from zero when the entry cost is small enough. As the discussion above shows, it is sufficient to assume that $\partial f(0, \tau)/\partial a$ is finite for $\tau = L, H$ for this to hold.

2. Voter welfare is not maximized when the entry cost is zero for a suitably chosen cost function and a suitably chosen distribution of the citizen's reputation.

We can dispense with the assumption that effort increases the informativeness of output in Proposition 6. The key observation is that we can take the production function to be such that while the incumbent's performance is sensitive to his effort, the informational content of the incumbent's performance is small. In this case, the sign of $dW_1(a)/da + dW_2^+(a)/da$ is positive even if effort does not increase the informativeness of output.[26]

To conclude our analysis of voter welfare, notice that while Proposition 6 shows that voter welfare need not be maximized when entry is costless, clearly it is not optimal for the voter to have the entry cost so high that the citizen never enters. However, Assumption 1 and the restriction that $\kappa \leq \overline{\kappa} < B$ together imply that in any equilibrium the probability that the citizen enters is positive at least after one output realization for the incumbent. It is possible to show that there are choices of the model's primitives for which welfare is not minimized when $\kappa = \overline{\kappa}$.

4.4 Contractual Incompleteness

An important feature of our environment is that the only mechanism the voter has to incentivize a politician is retention. To conclude this section, we discuss what happens when the voter has a richer set of instruments that she can use to provide incentives for a politician and show that the contractual incompleteness in our environment is at the heart of our results about the relationship between voter welfare and political competition.

Suppose the voter can provide the incumbent with an additional payoff $v(y)$ in case his ouput is y.[27] The incumbent's payoff as a function of his effort, a, and the other agents' conjecture about his effort, a^e, is now

$$U(a, a^e) = f(a, \pi_0)\big[BQ(\pi^+(h|a), \kappa) + v(h)\big] + (1 - f(a, \pi_0))\big[BQ(\pi^+(\ell|a), \kappa) + v(\ell)\big] - c(a).$$

A straightforward modification of the argument leading to Lemma 4 in Section 3 shows that a^* is

[26] See Appendix C for a discussion of this.

[27] We assume that the voter cannot provide the politician in office in period two with any incentive to exert effort. When output-contingent contracts are possible in period two, the voter can ensure that the politician in office in period two chooses the static first-best effort. It is straightforward to extend the analysis to this case.

an equilibrium choice of effort for the incumbent if, and only if,

$$\frac{\partial f}{\partial a}(a^*, \pi_0)\Big\{B\big[Q(\pi^+(h|a^*), \kappa) - Q(\pi^+(\ell|a^*), \kappa)\big] + v(h) - v(\ell)\Big\} = c'(a^*). \tag{7}$$

So, by adjusting the difference $v(h) - v(\ell)$, the voter can affect the incumbent's effort. How much the voter can do so depends on the values that $v(h)$ and $v(\ell)$ can assume.

Consider the case in which $v(h)$ and $v(\ell)$ can assume any positive value—so, there is limited liability in the sense that the voter cannot impose unbounded punishments on the incumbent. Let $W(a, \kappa)$ be the voter welfare when the incumbent's effort is a and the entry cost is κ and $a_{\max}(\kappa)$ be the choice of effort that maximizes $W(a, \kappa)$. Clearly, for any value of the entry cost κ, there exist choices of $v(h)$ and $v(\ell)$ for which $a_{\max}(\kappa)$ is a solution to (7), and so an equilibrium choice of effort the incumbent. Now let $W^{**}(\kappa) = W(a_{\max}(\kappa), \kappa)$. By the envelope theorem,

$$\frac{dW^{**}}{d\kappa}(\kappa) = -\frac{\partial W_2^-}{\partial \kappa}(a_{\max}(\kappa), \kappa) < 0.$$

Therefore, when the set of contracts that the voter can offer to incumbent is sufficiently rich, an increase in political competition is always beneficial to the voter.

An implicit assumption in the argument of the previous paragraph is that if (7) has more than one solution, then the incumbent's equilibrium choice of effort is the one most desired by the voter. It is important to emphasize that this assumption is consistent with the core of the preceding analysis in this section. Propositions 3 and 5 about the ambiguous effect of political competition on effective accountability and Proposition 6 about the ambiguous effect of political competition on voter welfare are derived under conditions that ensure equilibrium uniqueness.[28] Moreover, as we show in the next section in our analysis of the finite-output extension of our model, with output-contingent rewards the voter can uniquely pin down the incumbent's behavior when there are more than two output levels by suitably designing these rewards (under technical assumptions).

5 Robustness

We make two important assumptions in our analysis, namely, that output is binary and horizontal differentiation is small. Here, we show that our results extend to the case in which a politician's

[28] We also conjecture that there are primitives of the model for which the equilibrium multiplicity result of Proposition 4 holds and voter welfare is the same across equilibria. This would be knife-edge case, though.

output can assume any finite number of values and discuss what happens in the binary output case when horizontal differentiation is large.

Finite-Output Case We first consider what happens when we relax our binary-output assumption. Suppose a politician's output is now $y \in Y = \{y_0, \ldots, y_N\}$, with $y_0 < y_1 < \ldots < y_N$ and $N \geq 1$, and keep all else in the model the same. For each $y \in Y$, let $f(y|a,\tau) \in (0,1]$ be the probability that a politician of type $\tau \in \{L, H\}$ produces output y when he exerts effort a. For each $y \in Y$ and $\tau \in \{L, H\}$, the function $f(y|a,\tau)$ is twice continuously differentiable in a. Moreover, we make the following two assumptions:

1. The ratio $\dfrac{f(y|a,H)}{f(y|a,L)}$ is strictly increasing in y for all $a \in A$.

2. For all $a' > a$ and $\tau \in \{L, H\}$, the ratio $\dfrac{f(y|a',\tau)}{f(y|a,\tau)}$ is strictly increasing in y.

Assumptions 1 and 2 are standard monotone likelihood-ratio assumptions.[29] Assumption 1 implies that $\pi^+(y|a)$ is strictly increasing in y for all $a \in A$. Let $F(y|a,\tau) = \sum_{y' \leq y} f(y|a,\tau)$ be the probability that a politician of type τ produces output y or less when he exerts effort a. Assumption 2 implies that for each $\tau \in \{L, H\}$, the probability $F(y|a,\tau)$ is strictly decreasing in a for all $y < y_N$, i.e., an increase in effort increases the distribution of the politician's output strictly in the first-order stochastic sense for both types of politician.

Let $f(y|a,\pi_0) = \pi_0 f(y|a,H) + (1-\pi_0) f(y|a,L)$ be the ex-ante probability that the incumbent produces output y when exerts effort a. Then

$$U(a, a^e) = B \sum_{i=0}^{N} f(y_i|a,\pi_0) Q(\pi^+(y|a^e), \kappa) - c(a)$$

is the incumbent's payoff when he exerts effort a and the other agents believe that his effort is a^e. The function Q is the same as before: $Q(\pi_I, \kappa)$ is the probability that the incumbent is retained if his period-two reputation is π_I when the entry cost is κ. Now let $F(y|a,\pi_0) = \pi_0 F(y|a,H) + (1-\pi_0) F(y|a,L)$ be the ex-ante probability that the incumbent produces output y or less when he

[29] Assumptions 1 and 2 reduce to the assumptions made about the production function in the binary-output case.

exerts effort a. Straightforward algebra shows that

$$U(a,a^e) = B\left\{Q(\pi^+(y_N|a^e),\kappa) + \sum_{i=0}^{N-1} F(y_i|a,\pi_0)\big[Q(\pi^+(y_i|a^e),\kappa) - Q(\pi^+(y_{i+1}|a^e),\kappa)\big]\right\} - c(a)$$

In light of the above expression for $U(a,a^e)$, we make the following additional assumption:

3. $F(y|a,\tau)$ is convex in a for each $y \in Y$ and $\tau \in \{L,H\}$.

Assumption 3 implies that $U(a,a^e)$ is strictly concave in a for all $a^e \in A$, in which case the Inada conditions on the cost function c imply that the first-order condition $\partial U(a,a^e)/\partial a = 0$ is necessary and sufficient for a to maximize the incumbent's payoff given a^e.[30] From this it follows that

$$B\left\{\sum_{i=1}^{N-1} \frac{\partial F}{\partial a}(y_i|a^*,\pi_0)\left[Q(\pi^+(y_i|a^*),\kappa) - Q(\pi^+(y_{i+1}|a^*),\kappa)\right]\right\} = c'(a^*) \tag{8}$$

is necessary and sufficient for a^* to be an equilibrium choice of effort for the incumbent. The above equation extends condition (3) in the binary-output case to the finite-output case.

Since $\pi^+(y|a)$ is strictly increasing in y for all $a \in A$, the martingale property of beliefs implies that for each $a \in A$, there exists $j \in \{1,\ldots,N-1\}$ such that $\pi^+(y_j|a) < \pi_0 < \pi^+(y_{j+1}|a)$. So, given that $\pi_C(\pi_0,\kappa)$ is interior for all $\kappa \in [0,\overline{\kappa}]$ by Assumption 1, the left-hand side of (8) is positive for all $a^* \in A$; as before, $\pi_C(\pi_I,\kappa) = \pi_I + H(\kappa/B)$ is the cutoff reputation of entry for a citizen with reputation π_I when the entry cost is κ. Therefore, the incumbent's effort is positive in any equilibrium. The analysis now proceeds almost exactly as in Sections 3 and 4.

First, notice that we can immediately extend Lemma 5 to the finite-output case. We can then use this result to establish that if there exist $j \in \{0,\ldots,N-1\}$, $a^* \in (0,\overline{a})$, and $\kappa \in (0,\overline{\kappa})$ with

$$0 < \pi_C(\pi^+(y_j|a^*),\kappa) < \pi_C(\pi^+(y_{j+1}|a^*),\kappa) < 1,$$

then the following holds when the entry cost is κ: (*i*) there exist choices of the cost function and distribution of the citizen's reputation for which a^* is the incumbent's equilibrium choice of effort and a^* increases with an increase in political competition; and (*ii*) there exist choices of the cost function and distribution of the citizen's reputation for which a^* is the incumbent's equilibrium

[30] Assumption 3 is the counterpart in our environment of the assumption that the distribution of an agent's output is convex in effort used in standard moral problems to ensure the validity of the first-order approach.

choice of effort and a^* decreases with an increase in political competition. Moreover, if we let $\overline{\pi}^+(y) = \sup_{a\in A} \pi^+(y|a)$ be the highest reputation possible for the incumbent when his output is $y \in Y$, then we can also use the extension of Lemma 5 to show that if $\theta < \overline{\pi}^+(y_{N-1})$, then effective accountability need not be maximized when entry is costless.[31]

Welfare analysis in the finite-output case is taken almost verbatim from the previous section. The only difference with respect to the binary-output case is that now summations in y are over the set Y and $\mu(y|a) = f(y|a, \pi_0)$.

To conclude the analysis of the finite-output case, we discuss what happens when the voter can provide the incumbent with an additional non-negative payoff $v(y)$ in case the incumbent's output is y. It is straightforward to show that given a vector $v = (v(y_0), \ldots, v(y_N))$ of output-contingent rewards, a^* is an equilibrium choice of effort for the incumbent if, and only if,

$$\sum_{i=0}^{N-1} \frac{\partial F}{\partial a}(y_i|a^*, \pi_0)\Big\{B\big[Q(\pi^+(y_i|a^*), \kappa) - Q(\pi^+(y_{i+1}|a^*), \kappa)\big] + v(y_i) - v(y_{i+1})\Big\} = c'(a^*). \quad (9)$$

As in the binary-output case, by adjusting the vector of rewards, v, the voter can ensure that the welfare maximizing choice of effort, $a_{\max}(\kappa)$, is a solution to (9). From this, an envelope-theorem argument shows that welfare is strictly increasing in political competition.

We now show that under a technical assumption, the voter can choose v so that $a_{\max}(\kappa)$ is the unique solution to (9). For each $a \in A$, let $w(a) = (\partial F(y_0|a, \pi_0)/\partial a, \ldots, \partial F(y_{N-1}|a, \pi_0)/\partial a)$. Suppose the following holds:

4. For all $a, a' \in A$ such that $a \neq a'$, the vectors $w(a)$ and $w(a')$ are not colinear.

Notice that Assumption 4 can only be satisfied if $N \geq 2$. Now let v^* be a vector of rewards for which there exists $a' \neq a_{\max}(\kappa)$ such that $a_{\max}(\kappa)$ and a' are solutions to (9). For each vector of rewards v, define Δv to be such that $\Delta v = (v(y_0) - v(y_1), \ldots, v(y_{N-1} - v(y_N))$ and consider an alternative vector of rewards $\hat{v}$ such that $\Delta\hat{v} = \Delta v^* + \varepsilon$, where ε is orthogonal to $w(a^*)$. By construction, $a_{\max}(\kappa)$ is still a solution to (9) when the vetor rewards is $\hat{v}$. On the other hand, by Assumption 4, a' is not. Thus, by suitably adjusting rewards, the voter can ensure that $a_{\max}(\kappa)$ is the only solution to (9).

[31] For completeness, we include formal statements and proofs of the results in this paragraph in Appendix C.

The economic intuition behind the above result is straightforward. The voter can ensure that a given effort level a is the unique equilibrium choice of effort for the incumbent if he can adjust rewards in such a way that any effort $a' \neq a$ is not a best response to the conjecture $a^e = a'$. This, however, is only possible if there are at least three output levels. Indeed, with two output levels, (explicit) incentives are completely determined by the reward difference $v(y_1) - v(y_0)$, and any change in rewards that makes $a' \neq a$ suboptimal given the conjecture $a^e = a'$ also makes a suboptimal given the conjecture $a^e = a$. On the other hand, with three or more output levels, incentives are determined by the interaction between the payoff gains $v(y_{i+1}) - v(y_i)$, with $i = 0, \ldots, N-1$, and the marginal increases in output $\partial F(y_i|a, \pi_0)/\partial a$, with $i = 0, \ldots, N-1$. Assumption 4 ensures that these extra degrees of freedom for the voter can be used to pin down uniquely the incumbent's equilibrium behavior.

Large Horizontal Differentiation We now discuss the case in which horizontal differentiation is large. Suppose that $\theta > 1$, and so $G(-1) > 0$ and $G(1) < 1$, i.e., the voter can find it optimal to replace a high-ability incumbent even if the citizen is of low ability and keep a low-ability incumbent even if the citizen is of high ability.

First notice that since $H(\kappa/B) < -1$ if κ is sufficiently close to zero and $H(\kappa/B) > 1$ if κ is sufficiently close to B, then there exist $0 < \kappa' < \kappa'' < B$ such that $\pi_C(\pi_I, \kappa) \leq 0$ regardless of π_I if $\kappa \leq \kappa'$ and $\pi_C(\pi_I, \kappa) \geq 1$ regardless of π_I if $\kappa \geq \kappa''$. So, when the entry cost is sufficiently small, the citizen always enters and a local change in the entry cost does not affect effective accountability. Likewise, when the entry cost is sufficiently close to B, the citizen never enters and a local change in the entry cost also does not affect effective accountability. Therefore, unlike in the case of small horizontal differentiation, the impact of a change in political competition on effective accountability can be ambiguous only for intermediate values of the entry cost.

Now, for each $\pi_I \in [0, 1]$, let $\kappa_1 = \kappa_1(\pi_I)$ be such that $\pi_C(\pi_I, \kappa_1) = 0$ and $\kappa_2 = \kappa_2(\pi_I) > \kappa_1$ be such that $\pi_C(\pi_I, \kappa_2) = 1$. Both κ_1 and κ_2 are well-defined and strictly decreasing in π_I since H is strictly increasing with $H(0) = -\theta < -1$ and $H(1) = \theta > 1$. By construction, $\pi_C(\pi_0, \kappa) \in (0, 1)$ if $\kappa \in (\kappa_1(\pi_0), \kappa_2(\pi_0))$, in which case the incumbent's probability of retention is sensitive to his performance in office. Clearly, Proposition 3 (and Proposition 4 as well) holds when the entry cost is in this intermediate range.[32] We now show that both effective accountability and voter welfare need not be maximized when entry is costless.

Fix all the model's primitives but the entry cost κ and let $a_0 \in (0, \overline{a})$ the incumbent's optimal choice of effort when the citizen enters with probability one regardless the incumbent's performance. Then a_0 is the incumbent's (unique) equilibrium choice of effort when $\kappa \leq \kappa_1(\pi^+(h|a_0))$, and we can ensure that the equilibrium is stable by suitably choosing the cost function. Now consider the impact on the incumbent's behavior of a local increase in the entry cost when $\kappa = \kappa_1(\pi^+(h|a_0))$. Since the probability that the citizen enters if the incumbent succeeds drops below one after this change in the entry cost but the probability that the citizen enters if the incumbent fails remains equal to one, the marginal benefit of effort to the incumbent when he exerts effort a_0 and the other agents correctly anticipate his behavior increases. This, in turn, leads to a higher equilibrium effort for the incumbent. Moreover, given that there is full entry up to the point where the entry cost becomes equal to $\kappa_1(\pi^+(h|a_0))$, a local increase in the entry cost when κ assumes this value has a second-order effect on voter welfare. Thus, voter welfare is also not maximized when entry is costless if effort increases the informativeness of output.

6 Concluding Remarks

Political markets are markets in which there is a wedge between the private and social returns of effort. This creates the need of institutional arrangements to provide politicians with an incentive to behave in the voters' interest. In democracies, these incentives are provided by the possibility of reelection or, more generally, the prospect of a continuing political career. For this to work, however, voters need to credibly commit to reward good performance and punish bad performance. Political competition helps voters punish bad performance by providing them with viable alternatives to an incumbent. Political competition makes the promise of rewarding good performance through retention less credible, though. We show that this tension between the good and the bad aspects of political competition implies that there is no clear relationship between political com-

[32] The condition $0 < \pi_C(\pi^+(\ell|a), \kappa) < \pi_C(\pi^+(h|a), \kappa) < 1$ is equivalent to $\kappa_1(\pi^+(\ell|a)) < \kappa < \kappa_2(\pi^+(h|a))$. Notice that the interval $(\kappa_1(\pi^+(\ell|a)), \kappa_2(\pi^+(h|a)))$ might be empty if $\pi^+(h|a)$ and $\pi^+(\ell|a)$ are too far apart.

petition and voter welfare. In particular, we show that voter welfare need not be maximized when entry of politicians is free. We also show that if voters have additional instruments to reward politicians for their performance in office, then an increase in political competition is always beneficial to voters as long as these instruments are sufficiently rich.

Our analysis highlights an imperfection of political markets that is typically not emphasized in the literature, namely, that compensation in these markets is usually not directly tied to performance. To the extent that political markets cannot accomodate institutions allowing for a link between rewards and performance, our results show that increases in political competition need not be associated with higher voter welfare.

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Appendix A: Omitted Details

In this Appendix we provide omitted details from Section 4.

Equilibrium Multiplicity

Here we establish conditions under which multiple stable equilibria arise. The key result is Corollary 2, which we use to establish Proposition 4.

Lemma 7. *Fix all the model's primitives but the cost function and let κ be the entry cost. Suppose there exist $0 < a_1 < a_2 < \overline{a}$ with*

$$\frac{MB(a_1,\kappa)}{a_1} < \frac{MB(a_2,\kappa)}{a_2}.$$

There exist an admissible cost function c and $a_0 \in (0, a_1]$ such that if the cost function is c, then a_0 and a_2 are equilibrium effort choices for the incumbent in stable equilibria.

Proof. Let $\hat{\lambda}_2 : A \to \mathbb{R}$ be such that $\hat{\lambda}_2(a) = \max\{0, MB(a_2,\kappa) + \xi_2(a - a_2)\}$, where

$$\xi = \frac{1}{\overline{a} - a_2}\left[B\frac{\partial f}{\partial a}(0,\pi_0) - MB(a_2,\kappa)\right] + \max\left\{\frac{MB(a_2,\kappa) - MB(a_1,\kappa)}{a_2 - a_1}, \max_{s\in A}\frac{\partial MB}{\partial a}(s,\kappa)\right\}.$$

Moreover, let $\hat{\lambda}_1 : A \to \mathbb{R}$ be such that $\hat{\lambda}_1(a) = \max\{0, MB(a_1,\kappa) + \xi_1(a - a_1)\}$, where

$$\xi_1 = \frac{\hat{\lambda}_2(a_2 - \eta) - MB(a_1,\kappa)}{a_2 - a_1 - \eta}$$

and $0 < \eta < a_2 - a_1$. Since $\lim_{\eta\to 0}\hat{\lambda}_2(a_2 - \eta) = MB(a_2,\kappa)$ and

$$MB(a_1,\kappa) - \left(\frac{MB(a_2,\kappa) - MB(a_1,\kappa)}{a_2 - a_1}\right)a_1 \propto MB(a_1,\kappa)a_2 - MB(a_2,\kappa)a_1 < 0,$$

there exists $\overline{\eta} > 0$ such that if $\eta \in (0,\overline{\eta})$, then $\hat{\lambda}_1(a) = 0$ if $a \le \hat{a}$ for some $\hat{a} \in (0, a_1)$.

Let $\eta < \overline{\eta}$ and define $\hat{\lambda} : A \to \mathbb{R}$ to be such that $\hat{\lambda}(a) = \max\{\hat{\lambda}_1(a), \hat{\lambda}_2(a)\}$. By construction, $\hat{\lambda}_1(a_2 - \eta) = \hat{\lambda}_2(a_2 - \eta)$. Moreover,

$$\xi_2 > \frac{MB(a_2,\kappa) - MB(a_1,\kappa)}{a_2 - a_1} > \frac{\hat{\lambda}_2(a_2 - \eta) - MB(a_1,\kappa)}{a_2 - a_1 - \eta} = \xi_1,$$

where the second inequality follows from the fact that the second ratio is strictly decreasing in η. Thus, $\hat{\lambda}(a) = \hat{\lambda}_1(a)$ if, and only if, $a \le a_2 - \eta$. A straightforward modification of the argument in the proof of Lemma 5 now shows that there exists an admissible cost function c with $c'(a_1) = MB(a_1,\kappa)$, $c'(a_2) = MB(a_2,\kappa)$, and $c''(a_2) = \xi_2 > \partial MB(a_2,\kappa)/\partial a$. From Lemma 4, there exists a stable equilibrium in which a_2 is the incumbent's equilibrium choice of effort. Moreover, if $c''(a_1) = \xi_1 > \partial MB(a_1,\kappa)/\partial a$, then it also follows that there exists a stable equilibrium in which a_1 is the incumbent's equilibrium choice of effort.

Consider then the case in which $\xi_1 \leq \partial MB(a_1,\kappa)/\partial a$. We can assume without loss that $\xi_1 < \partial MB(a_1,\kappa)/\partial a$, otherwise we can increase η in the definition of $\hat{\lambda}_1$ slightly, reducing ξ_1. Given that $MB(a,\kappa) > 0$ for all $a \in A$ and $\hat{\lambda}_1(a) = 0$ if a is sufficiently close to zero, there exists $a_0 \in (0, a_1)$ such that $MB(a_0,\kappa) = \hat{\lambda}_1(a_0)$ and $MB(a,\kappa) > \hat{\lambda}_1(a)$ for all $a \in [0, a_0)$. Hence, $\partial MB(a_0,\kappa)/\partial a \leq \xi_1$, otherwise there exists $\tilde{a} \in (0, a_0)$ for which $MB(\tilde{a},\kappa) = \hat{\lambda}_1(\tilde{a})$, a contradiction. If $\partial MB(a_0,\kappa)/\partial a < \xi_1$, then we can take the cost function c in the first paragraph of the proof to be such that $c'(a_0) = \hat{\lambda}_1(a_0) = MB(a_0,\kappa)$. In this case, a_0 is an equilibrium choice of effort for the incumbent in a stable equilibrium.

Suppose then that $\partial MB(a_0,\kappa)/\partial a = \xi_1$ and define $\hat{\lambda}_0 : A \to \mathbb{R}$ to be such that $\hat{\lambda}_0(a) = \max\{0, MB(a_0,\kappa) + \xi_0(a - a_0)\}$, where

$$\xi_0 = \frac{\hat{\lambda}_2(a_2 - \eta) - MB(a_a,\kappa)}{a_2 - a_0 - \eta}$$

and η is the same as above. By construction, $\xi_1 = \xi_0$. Reducing η, and thus increasing ξ_0, we have that $\hat{\lambda}_0(a) = 0$ if a is sufficiently close to zero and $\partial MB(a_0,\kappa)/\partial a < \xi_0$. Let then $\lambda : A \to \mathbb{R}$ be such that $\lambda(a) = \max\{\hat{\lambda}_0(a), \hat{\lambda}_2(a)\}$. If the reduction in η is small enough, then it follows that $\xi_0 < \xi_2$, and so $\lambda(a) = \hat{\lambda}_0(a)$ if, and only if, $a \leq a_2 - \eta$. The same argument as in the first part of the proof now shows that there exists an admissible cost function with the desired properties. □

The next result follows immediately from the proof of Lemma 7

Corollary 2. *Fix all the model's primitives but the cost function and let κ be the entry cost. Suppose that there exist $0 < a_1 < a_2 < \overline{a}$ with*

$$\max\left\{\frac{\partial MB}{\partial a}(a_1,\kappa), \frac{MB(a_1,\kappa)}{a_1}\right\} < \frac{MB(a_2,\kappa)}{a_2}.$$

There exists an admissible cost function c such that if the cost function is c, then stable equilibria in which a_1 and a_2 are equilibrium effort choices for the incumbent exist.

Political Competition and Voter Welfare

Here we provide the omitted details from Section 4.3. We begin with the decomposition of $W_2(a,\kappa)$ in terms of $W_2^+(a)$ and $W_2^-(a,\kappa)$.

Recall from the main text that

$$W_2(a,\kappa) = \sum_{y\in\{\ell,h\}} \mu(y|a)\Bigg\{ \int_{\pi_C(\pi^+(y|a),\kappa)}^{1} \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\big]\omega(\pi)d\pi + \Omega(\pi_C(\pi^+(y|a),\kappa))\mathbb{E}_z\big[Sf(0,\pi^+(y|a))+z\big]\Bigg\},$$

where we used the fact that $\mathbb{E}_z[z]=0$. Given that $Sf(0,\pi^+(y|a))+z \geq Sf(0,\pi)$ if, and only if, $z \geq (\pi-\pi^+(y|a))2\eta/\theta$, it follows that $\Omega(\pi_C(\pi^+(y|a),\kappa))\mathbb{E}_z\big[Sf(0,\pi^+(y|a))+z\big]$ is equal to

$$\int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \big[Sf(0,\pi^+(y|a))+z\big]\gamma(z)dz \right)\omega(\pi)d\pi + \int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{(\pi-\pi^+(y|a))2\eta/\theta}^{2\eta} \max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\gamma(z)dz \right)\omega(\pi)d\pi,$$

where γ is the density of Γ and we used the fact that z has support in $[-2\eta,2\eta]$. Since

$$\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\} = Sf(0,\pi^+(y|a))+z+\max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta - z\big\},$$

we then have that $\Omega(\pi_C(\pi^+(y|a),\kappa))\mathbb{E}_z\big[Sf(0,\pi^+(y|a))+z\big]$ is equal to

$$\int_0^{\pi_C(\pi^+(y|a),\kappa)} \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\big]\omega(\pi)d\pi - \int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta - z\big\}\gamma(z)dz \right)\omega(\pi)d\pi.$$

Therefore,

$$W_2(a,\kappa) = \sum_{y\in\{\ell,h\}} \mu(y|a)\Bigg\{ \int_0^1 \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z, Sf(0,\pi)\}\big]\omega(\pi)d\pi - \int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta - z\big\}\gamma(z)dz \right)\omega(\pi)d\pi\Bigg\}.$$

The desired result now follows from the fact that

$$\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta - z\big\}\gamma(z)dz = \int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \Gamma(z)dz$$

using integration by parts.

We now compute $\partial W_2^-(a,\kappa)/\partial\kappa$ and $\partial W_2^-(a,\kappa)/\partial a$ and show that $\partial W_2^-(a,\kappa)/\partial\kappa$ is positive and $\lim_{\kappa\to 0}\partial W_2^-(a,\kappa)/\partial a = 0$ uniformly in a if $\partial f(0,\tau)/\partial a$ is finite for all $\tau\in\{L,H\}$.

It follows from the fundamental theorem of calculus that

$$\frac{\partial W_2^-}{\partial\kappa}(a,\kappa) = \frac{1}{B}H'\left(\frac{\kappa}{B}\right)\sum_{y\in\{\ell,h\}}\mu(y|a)\omega(\pi_C(\pi^+(y|a),\kappa))\int_{-2\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz,$$

and that

$$\begin{aligned}\frac{\partial W_2^-}{\partial a}(a,\kappa) &= \sum_{y\in\{\ell,h\}}\frac{\partial\mu}{\partial a}(y|a)\int_0^{\pi_C(\pi^+(y|a),\kappa)}\left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\Gamma(z)dz\right)\omega(\pi)d\pi \\ &\quad+\sum_{y\in\{\ell,h\}}\mu(y|a)\frac{\partial\pi^+}{\partial a}(y|a)\omega(\pi_C(\pi^+(y|a),\kappa))\int_{-2\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz \\ &\quad-\sum_{y\in\{\ell,h\}}\mu(y|a)\frac{2\eta}{\theta}\frac{\partial\pi^+}{\partial a}(y|a)\int_0^{\pi_C(\pi^+(y|a),\kappa))}\Gamma\left(\left(\pi-\pi^+(y|a)\right)\frac{2\eta}{\theta}\right)\omega(\pi)d\pi.\end{aligned}$$

Clearly, $\partial W_2^-(a,\kappa)/\partial\kappa$ is positive. We now show that $\lim_{\kappa\to 0}\partial W_2^-(a,\kappa)/\partial a = 0$ uniformly in a if $\partial f(0,L)/\partial a$ and $\partial f(0,H)/\partial a$ are finite. Notice that

$$\begin{aligned}&\int_0^{\pi_C(\pi^+(y|a),\kappa)}\left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\Gamma(z)dz\right)\omega(\pi)d\pi \\ &\qquad\le\int_0^{\pi_C(\pi^+(y|a),\kappa))}\Gamma\left(\left(\pi-\pi^+(y|a)\right)\frac{2\eta}{\theta}\right)2\eta\left[1+(\pi-\pi^+(y|a))\frac{1}{\theta}\right]\omega(\pi)d\pi \\ &\qquad\le 2\eta\left[1+H\left(\frac{\kappa}{B}\right)\frac{1}{\theta}\right]\int_0^{\pi_C(\pi^+(y|a),\kappa))}\Gamma\left(\left(\pi-\pi^+(y|a)\right)\frac{2\eta}{\theta}\right)\omega(\pi)d\pi\end{aligned}$$

and that

$$\omega(\pi_C(\pi^+(y|a),\kappa))\int_{-2\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz\le\overline{\omega}2\eta\left[1+H\left(\frac{\kappa}{B}\right)\frac{1}{\theta}\right]\Gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right),$$

where $\overline{\omega}=\sup_{\pi\in[0,1]}\omega(\pi)$ is finite since ω is continuous. Given that

$$\int_0^{\pi_C(\pi^+(y|a),\kappa))}\Gamma\left(\left(\pi-\pi^+(y|a)\right)\frac{2\eta}{\theta}\right)\omega(\pi)d\pi\le\Gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)$$

and $\lim_{\kappa\to 0}\Gamma\left(H(\kappa/B)2\eta/\theta\right)=0$, the desired result holds if $\partial\mu(y|a)/\partial a$ and $\mu(y|a)\partial\pi^+(y|a)/\partial a$ are uniformly bounded in a for every output y. Straightforward algebra shows that this is indeed the case when $\partial f(0,\tau)/\partial a$ is bounded for each $\tau\in\{L,H\}$; notice that the concavity in a of $f(a,\tau)$ implies that $\partial f(a,\tau)/\partial a\le\partial f(0,\tau)/\partial a$ for all $a\in(0,\overline{a}]$.

Appendix B: Omitted Proofs

In this Appendix we provide all proofs that were omitted from the main text.

Proof of Lemma 3

First notice that $Q(\pi_I, \kappa) = 1$ if $\pi_C(\pi_I, \kappa) \geq \overline{\pi}_C$ and that

$$Q(\pi_I, \kappa) = 1 - \int_{\max\{0, \pi_C(\pi_I, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi \tag{10}$$

otherwise. Given that $G(\pi - \pi_I)$ is nonincreasing in π_I for all $\pi \in [0, 1]$ and $\pi_C(\pi_I, \kappa)$ is strictly increasing in π_I and κ, it follows that $Q(\pi_I, \kappa)$ is nondecreasing in π_I and κ and strictly increasing in π_I and κ if $\pi_C(\pi_I, \kappa) \in (0, 1)$. The continuity of $Q(\pi_I, \kappa)$ in π_I for all $\kappa \in [0, \overline{\kappa}]$ follows from the dominated convergence theorem.

We now compute the partial derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$. Since $\pi_C(\pi_I, \kappa)$ is differentiable in π_I and κ for all $(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa})$, it follows from the fundamental theorem of calculus and the fact that $G(\pi_C(\pi_I, \kappa) - \pi_I) = \kappa/B$ that if $\pi_C(\pi_I, \kappa) \in (0, 1)$, then

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B} + \int_{\pi_C(\pi_I, \kappa)}^{1} G'(\pi - \pi_I)\omega(\pi)d\pi$$

and

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B^2}H'\left(\frac{\kappa}{B}\right).$$

On the other hand, it is immediate to see that both $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ are equal to zero if $\pi_C(\pi_I, \kappa) > 1$ and that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \int_0^1 G'(\pi - \pi_I)\omega(\pi)d\pi$$

and $\partial Q(\pi_I, \kappa)/\partial \kappa = 0$ if $\pi_C(\pi_I, \kappa) < 0$.

Now suppose that $\pi_C(\pi_I, \kappa) = 1$ and let $\{h_n\}$ be such that $h_n \to 0$. Then

$$\frac{Q(\pi_I + h_n, \kappa) - Q(\pi_I, \kappa)}{h_n} = -\frac{1}{h_n}\int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi$$

and

$$\frac{Q(\pi_I, \kappa + h_n) - Q(\pi_I, \kappa)}{h_n} = -\frac{1}{h_n}\int_{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi$$

Since $\pi_C(\pi_I + h_n, \kappa) = h_n + \pi_C(\pi_I, \kappa) = 1 + h_n$, it follows that

$$\left| \frac{1}{h_n} \int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi) d\pi \right| \leq A \frac{|\Omega(1) - \Omega(1 + h_n)|}{|h_n|},$$

where $A = \sup_{\pi \in [0,1]} |G(\pi - \pi_I)|$. Given that Ω is differentiable and $\omega(1) = \Omega'(1) = 0$, it then follows that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \lim_{n \to \infty} \frac{Q(\pi_I + h_n, \kappa) - Q(\pi_I, \kappa)}{h_n} = 0.$$

On the other hand, $\pi_C(\pi_I, \kappa + h_n) = \pi_C(\pi_I, \kappa) + H'(\kappa/B)(h_n/B) + o(h_n/B)$, and so

$$\left| \frac{1}{h_n} \int_{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}}^{1} G(\pi - \pi_I)\omega(\pi) d\pi \right| \leq A \frac{|\Omega(1) - \Omega(1 + \alpha(h_n))|}{|h_n|},$$

where the constant A is the same as above and $\alpha(h) = H'(\kappa/B)(h/B) + o(h/B)$. Given that $\Omega(1 + \alpha(h)) = \Omega(1) + o(\alpha(h))$ and $o(\alpha(h_n))/h_n \to 0$, since $\lim_{h \to 0} \alpha(h) = 0$ and $\alpha(h)/h$ is bounded when h is small, it then follows that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \lim_{n \to \infty} \frac{Q(\pi_I, \kappa + h_n) - Q(\pi_I, \kappa)}{h_n} = 0.$$

To finish the derivation of the partial derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$, suppose that $\pi_C(\pi_I, \kappa) = 0$. Once again let $\{h_n\}$ be such that $h_n \to 0$, and note that

$$\begin{aligned} \frac{Q(\pi_I + h_n, \kappa) - Q(\pi_I, \kappa)}{h_n} &= -\int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} \left[\frac{G(\pi - \pi_I - h_n) - G(\pi - \pi_I)}{h_n} \right] \omega(\pi) d\pi \\ &\quad + \frac{1}{h_n} \int_{0}^{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}} G(\pi - \pi_I)\omega(\pi) d\pi \end{aligned}$$

and

$$\frac{Q(\pi_I, \kappa + h_n) - Q(\pi_I, \kappa)}{h_n} = \frac{1}{h_n} \int_{0}^{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}} G(\pi - \pi_I)\omega(\pi) d\pi$$

The dominated convergence theorem implies that

$$\lim_{n \to \infty} \int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} \left[\frac{G(\pi - \pi_I - h_n) - G(\pi - \pi_I)}{h_n} \right] \omega(\pi) d\pi = -\int_{0}^{1} G'(\pi - \pi_I)\omega(\pi) d\pi.$$

On the other hand, since $\pi_C(\pi_I + h_n, \kappa) = h_n$, it follows that

$$\left| \frac{1}{h_n} \int_{0}^{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}} G(\pi - \pi_I)\omega(\pi) d\pi \right| \leq A \frac{|\Omega(h_n) - \Omega(0)|}{|h_n|},$$

where we again set $A = \sup_{\pi \in [0,1]} |G(\pi - \pi_I)|$. Given that $\Omega'(0) = \omega(0) = 0$, it then follows that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = -\int_0^1 G'(\pi - \pi_I)\omega(\pi)d\pi.$$

Now observe that

$$\left| \frac{1}{h_n} \int_0^{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}} G(\pi - \pi_I)\omega(\pi)d\pi \right| \leq A \frac{|\Omega(\alpha(h_n)) - \Omega(0)|}{|h_n|},$$

where A is the same constant as above and $\alpha(h)$ is the same as in the previous paragraph. From this inequality and the argument in the previous paragraph, it then follows that $\partial Q(\pi_I, \kappa)/\partial \kappa = 0$.

To conclude the proof, first notice that $\omega(\pi_C(\pi_I, \kappa))$ is jointly continuous in π_I and κ, with $\lim_{\pi_C(\pi_I, \kappa) \downarrow 0} \omega(\pi_C(\pi_I, \kappa)) = \lim_{\pi_C(\pi_I, \kappa) \uparrow 1} \omega(\pi_C(\pi_I, \kappa)) = 0$. Since

$$\int_{\pi_C(\pi_I, \kappa)}^1 G'(\pi - \pi_I)\omega(\pi)d\pi$$

is jointly continuous in π_I and κ by the dominated convergence theorem, it then follows that the partial derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ computed above are continuous.

Proof of Lemma 4

The sufficiency of (3) was established in the main text. Suppose now that a^* maximizes $U(a, a^*)$. Then $a^* < \overline{a}$, as the Inada condition $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ implies that $\partial U(\overline{a}, a^e)/\partial a < 0$ for all $a^e \in A$. A necessary condition for $a^* < \overline{a}$ to maximize $U(a, a^*)$ is that $\partial U(a^*, a^*)/\partial a \leq 0$ with $\partial U(a^*, a^*)/\partial a = 0$ if $a^* > 0$. Since $c'(0) = 0$ implies that $\partial U(0, a^e)/\partial a \geq 0$ for all $a^e \in A$, it also follows that $\partial U(a^*, a^*)/\partial a = 0$ when $a^* = 0$. Thus, condition (3) is necessary as well.

Proof of Corollary 1

Since $\pi^+(\ell|a^e) < \pi_0 < \pi^+(h|a^e)$ for all $a^e \in A$ and $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$, it follows that $\pi_C(\pi^+(\ell|a^e), \kappa) < 1$ and $\pi_C(\pi^+(h|a^e), \kappa) > 0$ regardless of a^e and κ. So, $Q(\pi^+(h|a^e), \kappa) > Q(\pi^+(\ell|a^e), \kappa)$ for all $a^e \in A$ and $\kappa \in [0, \overline{\kappa}]$ by Lemma 3. This, in turn, implies that the left-hand side of (3) is positive for all $a^* \in [0, \overline{a})$, in which case $a^* = 0$ cannot be a solution to (3); notice that $f(a, \tau)$ strictly increasing and strictly concave in a implies that $\partial f(a, \tau)/\partial a > 0$ for all $a \in [0, \overline{a})$.

Proof of Proposition 1

We know from the argument leading to Corollary 1 that $\Delta(0,\kappa) > 0 > \Delta(\overline{a},\kappa)$ regardless of the entry cost. Since $\pi^+(h|a)$ and $\pi^+(\ell|a)$ are continuous in a and $Q(\pi_I,\kappa)$ is continuous in π_I for all $\kappa \in [0,\overline{\kappa}]$, the difference $\Delta(a,\kappa)$ is continuous in a regardless of κ. Hence, for all $\kappa \in [0,\overline{\kappa}]$ there exists $a^* \in (0,\overline{a})$ such that $\Delta(a^*,\kappa) = 0$.

Proof of Lemma 5

Fix all the model's primitives but the cost function and define $\hat{\lambda} : A \to \mathbb{R}$ to be such that $\hat{\lambda}(a) = \max\{0, MB(a^*,\kappa) + \xi(a - a^*)\}$, where

$$\xi = \frac{1}{\overline{a} - a^*}\left[B\frac{\partial f}{\partial a}(0,\pi_0) - MB(a^*,\kappa)\right] + \max\left\{\frac{MB(a^*,\kappa)}{a^*}, \max_{s\in A}\frac{\partial MB}{\partial a}(s,\kappa)\right\}.$$

Notice that $\hat{\lambda}(a)$ is increasing in a, with $\hat{\lambda}(\overline{a}) > B\partial f(0,\pi_0)/\partial a$. Given that $\xi > MB(a^*,\kappa)/a^*$, there exists $a_0 \in (0,a^*)$ such that $\hat{\lambda}(a) = 0$ if $a \le a_0$ and $\hat{\lambda}(a) > 0$ otherwise. Moreover, since $\xi > \partial MB(s,\kappa)/\partial a$ for all $s \in A$, it also follows that $\hat{\lambda}(a) < MB(a,\kappa)$ for all $a < a^*$ and $\hat{\lambda}(a) > MB(a,\kappa)$ for all $a > a^*$.

Now fix $0 < \varepsilon < \min\{a_0, a^* - a_0\}$ and let $\lambda : A \to \mathbb{R}$ be such that $\lambda(a) = \hat{\lambda}(a)$ if $|a - a_0| \ge \varepsilon$ and $\lambda(a) = \alpha(a - a_0 + \varepsilon)^n$ if $|a - a_0| < \varepsilon$, where $\alpha(2\varepsilon)^n = MB(a^*,\kappa) + \xi(a_0 + \varepsilon - a^*)$ and $\alpha n(2\varepsilon)^{n-1} = \xi$.[33] By construction, $\lambda(a_0 - \varepsilon) = \hat{\lambda}(a_0 - \varepsilon)$, $\lambda(a_0 + \varepsilon) = \hat{\lambda}(a_0 + \varepsilon)$, $\lambda'(a_0 - \varepsilon) = 0$, and $\lambda'(a_0 + \varepsilon) = \xi$. So, $\lambda(a)$ is increasing and continuously differentiable in a. Moreover, by letting n be sufficiently large and ε be sufficiently small, we can ensure that $\lambda(a) < MB(a,\kappa)$ for all $a \in A$ such that $|a - a_0| < \varepsilon$.

To conclude, let $c(a) = \int_0^a \lambda(s)ds$. By construction, c is twice continuously differentiable, increasing, convex, and such that $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0,\pi_0)/\partial a$. So, c is an admissible cost function. Moreover, $c'(a^*) = MB(a^*,\kappa)$, $c'(a) \ne MB(a,\kappa)$ for all $a \ne a^*$, and $c''(a^*) = \xi > \partial MB(a^*,\kappa)/\partial a$. Thus, a^* is the unique equilibrium choice of effort for the incumbent when the cost function is c and the equilibrium is stable.

[33] We solve for the system of equations defining α and n as follows. First, substitute ξ by $\alpha n(2\varepsilon)^{n-1}$ in the first equation to obtain α; note that $\alpha > 0$ since $a^* > a_0 + \varepsilon$. Then obtain n residually by the second equation.

Proof of Proposition 4

Suppose the entry cost is $\kappa \in (0, \overline{\kappa})$ such that $H(\kappa/B) = 0$. Let Ω be an admissible distribution of the citizen's reputation with the property that its density ω is strictly decreasing in a neighborhood $\mathcal{N}$ of π_0 and satisfies $\omega\big(\pi_0/[\pi_0 + 2(1-\pi_0)]\big) < \omega(\pi)$ for all $\pi \in \mathcal{N}$. Take the distribution of the citizen's reputation to be Ω.

Now let $g : A \to (0, 1]$ be twice continuously differentiable and such that $g(\overline{a}) = 1$, $g'(a) > 0$ and $g''(a) < 0$ for all $a \in (0, \overline{a})$, and $\lim_{a\to\overline{a}} g'(a) > 0$. Suppose the production function f is such that $f(a, H) = g(a)$ and $f(a, L) = \mu g(a)$, where $\mu \in (0, 1)$. Notice that

$$\pi^+(h|a) \equiv \pi^+(h) = \frac{\pi_0}{\pi_0 + \mu(1-\pi_0)}.$$

Let then $a_1 = \overline{a}/2$ and define $a_2 \in A$ implicitly as a function of μ by $g(a_2) = \mu$. By construction, $a_2 \in (0, \overline{a})$ for all $\mu \in (0, 1)$ and there exists $\mu_1 \in (0, 1)$ such that $a_2 > a_1$ if $\mu > \mu_1$. Moreover,

$$\pi^+(\ell|a_1) = \frac{\pi_0}{\pi_0 + \dfrac{1-g(\overline{a}/2)}{1-\mu g(\overline{a}/2)}(1-\pi_0)} \quad \text{and} \quad \pi^+(\ell|a_2) = \frac{\pi_0}{\pi_0 + (1+\mu)(1-\pi_0)}.$$

Since $\lim_{\mu\to1} \pi^+(h|a) \equiv \pi_0 = \lim_{\mu\to1} \pi^+(\ell|a_1)$ and $\lim_{\mu\to1} \pi^+(\ell|a_2) = \pi_0/[\pi_0 + 2(1-\pi_0)]$, there exists $\mu_2 \in (0, 1)$ such that $\omega(\pi_C(\pi^+(\ell|a_1), \kappa)) > \omega(\pi_C(\pi^+(h|a_1), \kappa))$ and $\omega(\pi_C(\pi^+(\ell|a_2), \kappa)) < \omega(\pi_C(\pi^+(h|a_2), \kappa))$ if $\mu > \mu_2$.

It follows from the previous paragraph that $\lim_{\mu\to1} \delta(a_1, \kappa) = 0$ and $\lim_{\mu\to1} \delta(a_2, \kappa) > 0$. So,

$$\lim_{\mu\to1} \frac{MB(a_1, \kappa)}{a_1} = \lim_{\mu\to1} \frac{g'(a_1)}{a_1} B[\pi_0 + \mu(1-\pi_0)]\delta(a_1, \kappa) = 0$$

and, given that $\lim_{\mu\to1} a_2 = \overline{a}$,

$$\lim_{\mu\to1} \frac{MB(a_2, \kappa)}{a_2} = \lim_{\mu\to1} \frac{g'(a_2)}{a_2} B[\pi_0 + \mu(1-\pi_0)]\delta(a_2, \kappa) > 0.$$

Thus, there exists $\mu_3 \in (0, 1)$ such that $\mu > \mu_3$ implies that $MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$. Now observe that since $\pi^+(h|a)$ is constant in a, we have that

$$\begin{aligned}\frac{\partial MB}{\partial a}(a_1, \kappa) &= B\frac{\partial^2 f}{\partial a^2}(a_1, \pi_0)\delta(a_1, \kappa) + B\frac{\partial f}{\partial a}(a_1, \pi_0)\frac{\partial \delta}{\partial a}(a_1, \kappa) \\ &\propto g''(a_1)\delta(a_1, \kappa) - g'(a_1)\frac{\partial Q}{\partial \pi_I}(\pi^+(\ell|a_1), \kappa)\frac{\partial}{\partial a}\pi^+(\ell|a)\bigg|_{a=a_1}.\end{aligned}$$

Given that $\lim_{\mu \to 1} \partial \pi^+(\ell|a)/\partial a = 0$ for all $a \in (0, \overline{a})$ from straightforward algebra, it then follows that $\lim_{\mu \to 1} \partial MB(a_1, \kappa)/\partial a = 0$. Therefore, there exists $\mu_4 \in (0,1)$ such that $\partial MB(a_1, \kappa)/\partial a < MB(a_2, \kappa)/a_2$ if $\mu > \mu_4$.

To conclude, let $\mu^* = \max\{\mu_1, \mu_2, \mu_3, \mu_4\} \in (0,1)$ and suppose $\mu > \mu^*$. By Corollary 2, there exists an admissible cost function c such that if the cost function is c, then a_1 and a_2 are equilibrium effort choices for the incumbent in stable equilibria. Take the cost function to be c. Given that

$$\frac{\partial \delta}{\partial \kappa}(a_i, \kappa) \propto \omega(\pi_C(\pi^+(h|a_i), \kappa)) - \omega(\pi_C(\pi^+(\ell|a_i), \kappa),$$

we then have that from the above construction that $\partial \delta(a_1, \kappa)/\partial \kappa$ and $\partial \delta(a_2, \kappa)/\partial \kappa$ have opposite signs. This concludes the proof.

Proof of Proposition 5 (omitted details)

Here we complete the proof of Proposition 5 by showing that under the assumptions in the statement there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique and differentiable in κ for all $\kappa \in (0, \underline{\kappa})$.

We first show that there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique for all $\kappa \in (0, \underline{\kappa})$. Suppose not. Then there exists a sequence $\{\kappa_n\}$ in $[0, \overline{\kappa}]$ converging to zero and two sequences $\{a_n\}$ and $\{a'_n\}$ in $(0, \overline{a})$ such that $a_n \neq a'_n$ and $\Delta(a_n, \kappa_n) = \Delta(a'_n, \kappa'_n) = 0$ for all $n \in \mathbb{N}$. Moving to subsequences if necessary, we can assume that $\{a_n\}$ and $\{a'_n\}$ are both convergent. Let a and a' be the limits of $\{a_n\}$ and $\{a'_n\}$, respectively. Given that $\Delta(a, \kappa)$ is jointly continuous, it follows that $\Delta(a, 0) = \Delta(a', 0) = 0$. So, $a = a' = a^*$, as a^* is the unique equilibrium effort choice for the incumbent when $\kappa = 0$. Now observe that for all $n \in \mathbb{N}$, there exists $a''_n \in [\min\{a_n, a'_n\}, \max\{a_n, a'_n\}]$ such that $\partial \Delta(a''_n, \kappa_n)/\partial a = 0$. Since $\lim_n a_n = \lim_n a''_n = a^*$, it also follows that $\{a''_n\}$ converges to a^*. Since $\Delta(a, \kappa)$ is continuously differentiable in $(0, \overline{a}) \times (0, \overline{\kappa})$, we can then conclude that $\lim_n \partial \Delta(a''_n, \kappa_n)/\partial a = \partial \Delta(a^*, 0)/\partial a = 0$. This, however, contradicts the stability of the equilibrium when the entry cost is zero.

For each $\kappa \in (0, \underline{\kappa})$, let $a^*(\kappa)$ be the incumbent's unique equilibrium choice of effort when the entry cost is κ. We now show that $a^*(\kappa)$ is differentiable in κ for all $\kappa \in (0, \underline{\kappa})$, reducing $\underline{\kappa}$ if necessary. A standard argument shows that since $\Delta(a, \kappa)$ is jointly continuous, $\lim_{\kappa \to 0} a^*(\kappa) = a^*$.

Given that $\Delta(a,\kappa)$ is continuously differentiable in $(0,\overline{a}) \times (0,\overline{\kappa})$ and $\partial\Delta(a^*,0)/\partial a < 0$, it then follows that $\partial\Delta(a^*(\kappa),\kappa)/\partial a < 0$ if κ is sufficiently close to zero. Reducing $\underline{\kappa}$ if necessary, we can assume that $\partial\Delta(a^*(\kappa),\kappa)/\partial a < 0$ for $\kappa \in (0,\underline{\kappa})$. By the implicit function theorem, we can then conclude that $a^*(\kappa)$ is differentiable in κ for all $\kappa \in (0,\underline{\kappa})$.

Proof of Lemma 6

In order to prove Lemma 6, we first establish the following result.

Claim 1. *The following fact holds:*

$$\lim_{\kappa\to 0}\frac{1}{\kappa}\int_{-\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz = 0.$$

Proof. We first show that

$$\lim_{\kappa\to 0}\frac{\frac{2\eta}{\theta}\int_{-\infty}^{+\infty}\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)\lambda(\xi)d\xi}{\lambda(\eta)\lambda(-\eta)2\eta\left[1 + H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]} = 1. \tag{11}$$

We know that there exists $\kappa' > 0$ such that $H(\kappa/B) < 0$ if $\kappa \in [0,\kappa')$. Suppose that $\kappa \in (0,\kappa')$. Since $\xi + H(\kappa/B)2\eta/\theta$ belongs to the interval $[-\eta,\eta]$ if, and only if,

$$-\eta\left(1 + H\Big(\frac{\kappa}{B}\Big)\frac{2}{\theta}\right) \le \xi \le \eta\left(1 - H\Big(\frac{\kappa}{B}\Big)\frac{2}{\theta}\right)$$

and $\eta\big(1 - 2H(\kappa/\theta)\big) > \eta$ if $H(\kappa/B) < 0$, it follows that

$$\int_{-\infty}^{+\infty}\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)\lambda(\xi)d\xi = \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)\lambda(\xi)d\xi.$$

Now observe that

$$\begin{aligned}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)\lambda(\xi)d\xi &= \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\lambda(-\eta)\lambda(\eta)d\xi \\ &+ \lambda(\eta)\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big) - \lambda(-\eta)\right]\lambda(\xi)d\xi \\ &+ \lambda(-\eta)\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda(\xi) - \lambda(\eta)\right]d\xi \\ &+ \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big) - \lambda(-\eta)\right]\left[\lambda(\xi) - \lambda(\eta)\right]d\xi.\end{aligned}$$

Hence,

$$\frac{\frac{2\eta}{\theta}\int_{-\infty}^{+\infty}\lambda\Big(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)\lambda(\xi)d\xi}{\lambda(\eta)\lambda(-\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}=1$$

$$+\frac{1}{\lambda(-\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)-\lambda(-\eta)\right]d\xi$$

$$+\frac{1}{\lambda(\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda(\xi)-\lambda(\eta)\right]d\xi$$

$$+\frac{1}{\lambda(\eta)\lambda(-\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)-\lambda(-\eta)\right][\lambda(\xi)-\lambda(\eta)]d\xi. \tag{12}$$

The limit in (11) holds if the last three terms in the right-hand side of (12) converge to zero as κ converges to zero. We establish this fact in what follows.

First notice, making the change of variables $\xi\mapsto\xi-H(\kappa/B)2\eta/\theta$, that

$$\frac{1}{2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)-\lambda(-\eta)\right]d\xi$$
$$=\frac{1}{h}\int_{-\eta}^{-\eta+h}[\lambda(\xi)-\lambda(-\eta)]d\xi,$$

where $h=2\eta(1+H(\kappa/B)/\theta)$. Since $\lim_{\kappa\to0}h=0$, the fundamental theorem of calculus implies that the right-hand side of the above equation converges to zero as κ converges to zero. Given that $-\eta\left(1+H(\kappa/B)2/\theta\right)=\eta-h$, the fundamental theorem of calculus also implies that

$$\lim_{\kappa\to0}\frac{1}{2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda(\xi)-\lambda(\eta)\right]d\xi=0.$$

Finally, notice that

$$\lim_{\kappa\to0}\frac{1}{2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta}\left[\lambda\Big(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\Big)-\lambda(-\eta)\right]\left[\lambda(\xi)-\lambda(\eta)\right]d\xi$$
$$=\lim_{h\to0}\frac{1}{h}\int_{\eta-h}^{\eta}\left[\lambda\left(\xi-2\eta+h\right)-\lambda(-\eta)\right]\left[\lambda(\xi)-\lambda(\eta)\right]d\xi, \tag{13}$$

where h is the same as above. Since

$$\left|\frac{1}{h}\int_{\eta-h}^{\eta}\left[\lambda\left(\xi-2\eta+h\right)-\lambda(-\eta)\right]\left[\lambda(\xi)-\lambda(\eta)\right]d\xi\right|\leq\frac{D(h)}{h}\int_{\eta-h}^{h}|\lambda(\xi)-\lambda(\eta)|d\xi,$$

where $D(h)=\sup_{\xi\in[\eta-h,\eta]}|\lambda\left(\xi-2\eta+h\right)-\lambda(-\eta)|$ and $\lim_{h\to 0}D(h)=0$ by the continuity of λ, a straightforward argument shows that the limit in the right-hand side of (13) is zero as well. From this we can conclude that (11) holds.

We now establish the desired limit. For this, notice that

$$\begin{aligned}\lim_{\kappa\to 0}\frac{1}{\kappa}\int_{-\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz &= \lim_{\kappa\to 0}\Gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)H'\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\frac{1}{B}\\ &= \lim_{\kappa\to 0}\frac{\Gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)}{B\int_{-\infty}^{+\infty}\lambda\left(\xi+H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)\lambda(\xi)d\xi}\\ &= \lim_{\kappa\to 0}\frac{\Gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)}{B\theta\lambda(\eta)\lambda(-\eta)\left[1+H\left(\frac{\kappa}{B}\right)\frac{1}{\theta}\right]}\\ &= \lim_{\kappa\to 0}\frac{2\eta\gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)}{B\theta\lambda(\eta)\lambda(-\eta)}.\end{aligned}$$

where: (*i*) the first equality follows from L'Hospital's rule; (*ii*) the second equality follows from the fact that $H'(x)=1/G'(H(x))$ and $G'(x)=(2\eta/\theta)\int_{-\infty}^{+\infty}\Lambda(\xi+2x\eta/\theta)\lambda(\xi)d\xi$; (*iii*) the third equality follows from (11); and (*iv*) the last equality follows from a second application of L'Hospital's rule. On the other hand, given that $\lim_{\kappa\to 0}\lambda(\xi+H(\kappa/B)2\eta/\theta)=\lambda(\xi-2\eta)=0$ for all $\xi<\eta$, it follows that

$$\lim_{\kappa\to 0}\gamma\left(H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)=\lim_{\kappa\to 0}\int_{-\eta}^{+\eta}\lambda\left(\xi+H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta}\right)\lambda(\xi)d\xi=0$$

by the dominated convergence theorem.[34] This concludes the proof of the claim. □

We can now establish Lemma 6. Fix all the primitives of the model except the cost function, the distribution of the citizen's reputation, and the entry cost and assume that $\theta<\overline{\pi}^{+}(\ell)$. Let

[34] Notice that $\Gamma(z)=\int_{-\eta}^{+\eta}\Lambda(\xi+z)\lambda(\xi)d\xi$.

$a^* \in (0, \overline{a})$ be such that $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$ and take the distribution of the citizen's reputation to be such that $\omega(\pi^+(h|a^*) - \theta) - \omega(\pi^+(\ell|a^*) - \theta) \neq 0$. Moreover, for each $\kappa \in (0, \overline{\kappa})$, let c_κ be an admissible cost function such that if the entry cost is κ, then a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable if the cost function is c_κ. Now let $\kappa_0 \in (0, \overline{\kappa})$ be the entry cost and take the cost function to be c_{κ_0}. Since

$$\frac{da^*}{d\kappa}(\kappa_0) = \frac{\frac{\partial f}{\partial a}(a^*, \pi_0)\left[\omega(\pi^+(h|a^*) + H(\kappa_0/B)) - \omega(\pi^+(\ell|a^*) + H(\kappa_0/B))\right]}{c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)} \cdot \frac{\kappa_0}{B} H'\left(\frac{\kappa_0}{B}\right).$$

and $c''(a^*) > \partial MB(a^*, \kappa_0)/\partial a$, as the equilibrium is stable, it follows that

$$\frac{dW^*}{\partial \kappa}(\kappa_0) \propto \frac{\kappa_0}{B} H'\left(\frac{\kappa_0}{B}\right)\Bigg\{\left[S\frac{\partial f}{\partial a}(a^*, \pi_0) + \frac{dW_2^+}{da}(a^*) - \frac{\partial W_2^-}{\partial a}(a^*, \kappa_0)\right]\frac{\partial f}{\partial a}(a^*, \pi_0) D(\kappa_0)$$
$$- \frac{1}{\kappa_0}\left[c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)\right] \sum_{y \in \{\ell, h\}} \mu(y|a)\omega\big(\pi^+(y|a^*) + H(\kappa_0/B)\big) \int_{-2\eta}^{H(\kappa_0/B)2\eta/\theta} \Gamma(z)dz\Bigg\},$$

where $D(\kappa_0) = \omega(\pi^+(h|a^*) + H(\kappa_0/B)) - \omega(\pi^+(\ell|a^*) + H(\kappa_0/B))$. Given that

$$\lim_{\kappa_0 \to 0} \frac{1}{\kappa_0}\left[c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)\right]\omega\big(\pi^+(y|a^*) + H(\kappa_0/B)\big)\int_{-2\eta}^{H(\kappa_0/B)2\eta/\theta} \Gamma(z)dz = 0$$

for all $y \in \{\ell, h\}$ by Claim 1 and $\lim_{\kappa_0 \to 0} D(\kappa_0) = \omega(\pi^+(h|a^*) - \theta) - \omega(\pi^+(\ell|a^*) - \theta) \neq 0$, we then have that $dW^*(\kappa_0)/d\kappa$ and $\Delta W^*_{\text{indirect}}$ have the same sign as long as κ_0 is small enough.

Appendix C: Further Omitted Details (Not for Publication)

This appendix provides some further details that were omitted from the main text. We first establish the properties of the function G defined in Section 3. We then derive comparative statics results under the assumption that the incumbent's equilibrium choice of effort is globally unique. Following that, we establish the differentiability of $W_2^+(a)$ and show that we can drop the assumption that effort increases the informativeness of output in Proposition 6. Finally, we state and prove the counterparts of Propositions 3 and 5 in the finite-output case.

Properties of the Function G

Recall that $G : \mathbb{R} \to [0, 1]$ is such that

$$G(x) = \int_{-\infty}^{+\infty} \Lambda(\xi + 2x\eta/\theta)\lambda(\xi)d\xi.$$

Clearly, Λ nondecreasing implies that G is nondecreasing. Since $\xi + 2x\eta/\theta \in (-\eta, \eta)$ for some $\xi \in (-\eta, \eta)$ if, and only if, $x \in (-\theta, \theta)$, it follows that $G(-\theta) = 0$ and $G(\theta) = 1$. Standard results in measure theory, see, e.g., Corollary 5.9 in Bartle (1966), show that G is differentiable with

$$G'(x) = (2\eta/\theta) \int_{-\infty}^{\infty} \lambda(\xi + 2x\eta/\theta)\lambda(\xi)d\xi.$$

Given that λ is strictly positive in $[-\eta, \eta]$, it follows that G is strictly increasing in $(-\theta, \theta)$. To conclude, the continuity $G'(x)$ for all $x \in (-\theta, \theta)$ follows from the dominated convergence theorem and the continuity of λ in $(-\eta, \eta)$.

Comparative Statics under Equilibrium Uniqueness

Here we prove that an increase in political competition that increases δ^* leads to higher effort when the incumbent's equilibrium choice of effort is unique for every entry cost. The proof that an increase in political competition that lowers δ^* leads to lower effort when the incumbent's equilibrium choice of effort is unique for every entry cost is similar, and thus omitted.

Suppose that $\Delta(a, \kappa) = 0$ has a unique solution in $(0, \overline{a})$ for all $\kappa \in [0, \overline{\kappa}]$. Now suppose, by contradiction, that there exist $0 \leq \kappa_1 < \kappa_2 \leq \overline{\kappa}$ and $a_1^*, a_2^* \in (0, \overline{a})$ with $\Delta(a_i^*, \kappa_i) = 0$ for $i = 1, 2$

and $\delta(a_2^*, \kappa_1) > \delta(a_2^*, \kappa_2)$ but $a_1^* \geq a_2^*$. Hence, $\Delta(a_2^*, \kappa_1) > \Delta(a_2^*, \kappa_2) = 0 = \Delta(a_1^*, \kappa_1)$. Since the Inada condition $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ implies that $\lim_{a \to \overline{a}} \Delta(a, \kappa_1) < 0$, it then follows that $\Delta(a_3^*, \kappa_1) = 0$ for some $a_3^* \in (a_2^*, \overline{a})$. This, however, contradicts equilibrium uniqueness when the entry cost κ_1. Thus, an increase in political that increases δ^* leads to higher effort in equilibrium.

Differentiability of $W_2^+(a)$

Here we establish the differentiability of $W_2^+(a)$. First notice that

$$W_2^+(a) = \sum_{y \in \{\ell, h\}} \mu(y|a) \mathbb{E}_z \underbrace{\left[\int_0^1 \max\{Sf(0, \pi^+(y|a) + z, Sf(0, \pi)\} \omega(\pi) d\pi \right]}_{G(\pi^+(y|a), z)}.$$

Now observe that

$$\begin{aligned} G(\pi^+, z) &= \int_0^1 \max\{Sf(0, \pi^+) + z, Sf(0, \pi)\} \omega(\pi) d\pi \\ &= \left[Sf(0, \pi^+) + z\right] \Omega(\pi^+ + z/\alpha) + \int_{\pi^+ + z/\alpha}^1 Sf(0, \pi) \omega(\pi) d\pi \end{aligned}$$

where $\alpha = S[f(0, H) - f(0, L)]$ and we used the fact that $Sf(0, \pi) \geq Sf(0, \pi^+) + z$ if, and only if, $\pi \geq \pi^+ + z/\alpha$; we adopt the convention that the last integral is zero if $\pi^+ + z/\alpha > 1$. The desired result follows from Corollary 5.9 in Bartle (1966) and the differentiability in a of $\mu(y|a)$ and $\pi^+(y|a)$ if $G(\pi^+, z)$ is differentiable in π^+ for all $z \in \mathbb{R}$.

Clearly, for each $z \in \mathbb{R}$, $G(\pi^+, z)$ is differentiable in π^+ for all π^+ such that $\pi^+ + z/\alpha \neq 1$. Let then π^+ be such that $\pi^+ + z/\alpha = 1$. It is immediate to see that

$$\lim_{h \downarrow 0} \frac{G(\pi^+ + h, z) - G(\pi^+, z)}{h} = S \frac{\partial f}{\partial \pi}(0, \pi^+).$$

Now observe that

$$\lim_{h \downarrow 0} \frac{G(\pi^+ - h, z) - G(\pi^+, z)}{h} = S \frac{\partial f}{\partial \pi}(0, \pi^+) + \frac{1}{h} \int_{1-h}^1 Sf(0, \pi) \omega(\pi) d\pi,$$

where we used the fact that $\omega(1) = 0$. Since

$$\lim_{h \downarrow 0} \frac{1}{h} \int_{1-h}^1 Sf(0, \pi) \omega(\pi) d\pi = -Sf(0, 1) \omega(1) = 0$$

by the fundamental theorem of calculus, we then have that $G(\pi^+, z)$ is also differentiable in π^+ when $\pi^+ + z/\alpha = 1$. This establishes the differentiability of $W_2^+(a)$.

Relaxing Assumption that Effort Increases Informativeness of Output in Proposition 6

Here we show that we can relax the assumption that effort increases the informativeness of output in Proposition 6. Fix $\eta > 0$ and let $g : [0, \overline{a} + \eta] \to (0, 1]$ be twice continuously differentiable, strictly increasing and strictly concave. Moreover, supose that $g'(0)$ finite and $g'(a)/[1 - g(a)]$ is strictly decreasing. Now let $\varepsilon \in (0, \eta)$ and set $f(a, L) = g(a)$ and $f(a, H) = g(a + \varepsilon)$. Then

$$\pi^+(y|a) = \frac{\pi_0}{\pi_0 + Q_y(a, \varepsilon)(1 - \pi_0)},$$

where $Q_h(a, \varepsilon) = g(a)/g(a+\varepsilon)$ and $Q_\ell(a, \varepsilon) = (1-g(a))/(1-g(a+\varepsilon))$; $Q_\ell(a, \varepsilon)$ is well defined since $g(a + \varepsilon) < 1$ for all $a \in [0, \overline{a}]$ and $\varepsilon < \eta$.

First notice that

$$0 < \frac{\partial Q_h}{\partial a}(a, \varepsilon) = \frac{g'(a)g(a + \varepsilon) - g'(a + \varepsilon)g(a)}{g(a + \varepsilon)^2} < \frac{g'(a)g(a + \varepsilon) - g'(a + \varepsilon)g(a)}{g(0)^2}; \tag{14}$$

the first inequality in (14) follows from the fact that g is strictly increasing and strictly concave. Moreover, notice that

$$\frac{g'(a + \varepsilon)}{1 - g(a + \varepsilon)} - \frac{g'(a)}{1 - g(a + \varepsilon)} < \frac{\partial \hat{Q}_\ell}{\partial a}(a, \varepsilon) = \frac{g'(a + \varepsilon)[1 - g(a)] - g'(a)[1 - g(a + \varepsilon)]}{[1 - g(a + \varepsilon)]^2} < 0; \tag{15}$$

the first inequality in (15) follows the fact that g is strictly increasing in a while the second inequality in (15) follows from the assumption that $g'(a)/[1 - g(a)]$ is strictly decreasing. In particular, an increase in the incumbent's effort decreases the dispersion in his period-two reputation, making his performance in office *less* informative of his ability.

Now observe that the numerator in (14) is continuous in a and converges pointwise and monotonically to zero as ε decreases to zero. Moreover, the term in left-hand side (15) is also continuous in a and converges pointwise and monotonically to zero as ε decreases to zero. Thus, by Dini's theorem, both $\partial Q_h(a, \varepsilon)/\partial a$ and $\partial Q_\ell(a, \varepsilon)/\partial a$ converge uniformly to zero as ε decreases to zero. From this, it easily follows that both $Q_h(a, \varepsilon)$ and $Q_\ell(a, \varepsilon)$ converge uniformly to one as ε decreases to zero.[35] Consequently, given that

$$\frac{\partial \pi^+}{\partial a}(y|a) = \pi^+(y|a)(1 - \pi^+(y|a))\frac{1}{Q_y(a, \varepsilon)}\frac{\partial Q_y}{\partial a}(a, \varepsilon),$$

[35] Notice that $|Q_y(a, \varepsilon) - 1| \leq |Q_y(0, \varepsilon) - 1| + \int_0^a |\partial Q_y(s, \varepsilon)/\partial a| ds \leq |Q(0, \varepsilon) - 1| + \int_0^{\overline{a}} |\partial Q(s, \varepsilon)/\partial a| ds$

we can then conclude that $\partial\pi^+(h|a)/\partial a$ and $\partial\pi^+(\ell|a)/\partial a$ converge uniformly to zero as ε decreases to zero, and so $\pi^+(h|a)$ and $\pi^+(\ell|a)$ converge uniformly to zero as ε decreases to zero.

To finish, if we let

$$G(\pi^+, z) = \int_0^1 \max\{Sf(0,\pi^+) + z, Sf(0,\pi)\}\omega(\pi)d\pi,$$

then it follows from the discussion about the differentiability of $W_2^+(a)$ that $G(\pi^+, z)$ is differentiable in π^+ for all $z \in \mathbb{R}$. So, by Corollary 5.9 in Bartle (1966), we have that

$$\begin{aligned}\frac{dW_2^+}{da}(a) &= \sum_{y\in\{\ell,h\}} \frac{\partial\mu}{\partial a}(y|a)\mathbb{E}_z\left[G(\pi^+(\pi^+(y|a), z)\right] \\ &\quad + \sum_{y\in\{\ell,h\}} \mu(y|a)\frac{\partial\pi^+}{\partial a}(y|a)\mathbb{E}_z\left[\frac{\partial G}{\partial\pi^+}(\pi^+(y|a), z)\right].\end{aligned}$$

Given that $Sf(0,\pi^+ + z/\alpha) = Sf(0,\pi^+) + z$ for $\alpha = S[f(0,H) - f(0,L)]$, it also follows from the discussion about the differentiability of $W_2^+(a)$ and the fundamental theorem of calculus that

$$\frac{\partial G}{\partial\pi^+}(\pi^+, z) = \left[S\frac{\partial f}{\partial\pi}(0,\pi^+) + z\right]\Omega\left(\pi^+ + \frac{z}{\alpha}\right).$$

So, $dW_2^+(a)/da$ converges uniformly to 0 as ε decreases to zero. Since $\partial f(0,\pi_0)/\partial a \geq g'(a+\varepsilon)$, we then have that there exists $\overline{\varepsilon} \in (0,\eta)$ with the property that if $\varepsilon \in (0,\overline{\varepsilon})$, then

$$\frac{dW_1}{da}(a) + \frac{dW_2^+}{da}(a) = S\frac{\partial f}{\partial a}(0,\pi_0) + \frac{dW_2^+}{da}(a)$$

is positive and bounded away from zero as long as $g'(a)$ is bounded below by a positive constant. This shows that we can drop the assumption that effort increases the informativeness of output from Proposition 6.

Propositions 3 and 5 in the Finite-Output Case

We first state and prove the counterpart of Proposition 3 in the finite-output case of Section 5.

Proposition 7. *Suppose the entry cost is $\kappa \in (0,\overline{\kappa})$ and fix all other primitives of the model but the cost function c and the distribution of the citizen's reputation Ω. Now suppose there exists $a^* \in (0,\overline{a})$ and $j \in \{0,\ldots,N-1\}$ with*

$$0 < \pi_C(\pi^+(y_j|a^*),\kappa) < \pi_C(\pi^+(y_{j+1}|a^*),\kappa) < 1. \tag{16}$$

Then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c,\Omega) = (c_1,\Omega_1)$ and $(c,\Omega) = (c_2,\Omega_2)$; and (ii) an increase in political competition increases effective accountability in one case and decreases it in the other.

Proof. Suppose $a^* \in (0,\overline{a})$ satisfies (16). Let $k = \inf\{j \in \{1,\dots,N-1\} : \pi_C(\pi^+(y_j|a^*),\kappa) > 0\}$ and $l = \sup\{j \in \{1,\dots,N-1\} : \pi_C(\pi^+(y_j|a^*),\kappa) < 1\}$. Notice that both k and l are well-defined and $k < l$. Now let Ω_1 and Ω_2 be admissible distributions of the citizen's reputation with densities ω_1 and ω_2 such that $\omega_1(\pi_C(\pi^+(y_{j+1}|a^*),\kappa)) < \omega_1(\pi_C(\pi^+(y_j|a^*),\kappa))$ for all $j \in \{k,\dots,l\}$ and $\omega_2(\pi_C(\pi^+(y_{j+1}|a^*),\kappa)) > \omega_2(\pi_C(\pi^+(y_j|a^*),\kappa))$ for all $j \in \{k,\dots,l\}$. By the extension of Lemma 5 to the finite-output case, for each Ω_i there exists an admissible cost function c_i such that a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c_i and the distribution of the citizen's reputation is Ω_i.

Now let

$$\Delta_i(a,\kappa) = B\sum_{j=1}^{N}\frac{\partial F}{\partial a}(y_j|a^*,\pi_0)\big[Q_i(\pi^+(y_j|a),\kappa) - Q_i(\pi^+(y_{j+1}|a),\kappa)\big] - c_i(a),$$

where $Q_i(\pi_I,\kappa)$ is the incumbent's probability of retention as a function of his period-two reputation and the entry cost when the distribution of the citizen's reputation is Ω_i. From (8), the equation $\Delta_i(a,\kappa) = 0$ defines the incumbent's equilibrium choice of effort implicitly as a function of κ when the cost function is c_i and the distribution of the citizen's reputation is Ω_i. The desired result follows from the fact $\partial\Delta_i(a^*,\kappa)/\partial\kappa < 0$ for $i = 1, 2$ from equilibrium stability and

$$\frac{\partial\Delta_i}{\partial\kappa}(a^*,\kappa) \propto B\sum_{j=1}^{N}\frac{\partial F}{\partial a}(y_j|a^*,\pi_0)\big[\omega_i(\pi^+(y_j|a),\kappa) - \omega_i(\pi^+(y_{j+1}|a),\kappa)\big]$$

from Lemma 3. □

Proposition 8. *Fix all the model's primitives but the cost function, the distribution of the citizen's reputation, and the entry cost. If $\theta < \overline{\pi}^+(y_{N-1})$, then there exist a cost function and a distribution of the citizen's reputation for which effective accountability is not maximized when entry is costless.*

Proof. Suppose $\theta < \overline{\pi}^+(y_{N-1})$. Then there exists $a^* \in (0,\overline{a})$ such that $\theta < \pi^+(y_{N-1}|a^*)$. Let $k = \inf\{j \in \{1,\dots,N-1\} : \pi^+(y_j|a^*) \geq \theta\} \leq N-1$ and choose Ω to be an admissible distribution of the citizen's reputation with density ω such that $\omega(\pi^+(y_j|a^*) - \theta) < \omega(\pi^+(y_{j+1}|a^*) - \theta)$ for all $j \in \{k, \ldots, N-1\}$. Now let c be an admissible cost function such that a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable when the entry cost is zero, the cost function is c, and the distribution of the citizen's reputation is Ω. Take the cost function and the distribution of the citizen's reputation to be c and Ω, respectively. The same argument as in the proof of Proposition 5 establishes the desired result, where now $\Delta(a, \kappa)$ is given by the expression in the previous proof. $\square$