Financial Development and Capital Flows: Appraisal of the "Allocation Puzzle" by the Schumpeterian Growth

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Abstract

We explore the role of financial development to explain the negative correlation between capital inflows and productivity catch-up. As observed in the data, countries with higher productivity growth rates export capital while countries with lower productivity growth rates receive positive capital inflows. This is contradictory to the predictions of the standard neoclassical growth model. Gourinchas and Jeanne (2013) called this paradox the "allocation puzzle". Under perfect credit market, our calibrated Schumpeterian growth model also predicts a positive correlation between capital inflows and productivity catch-up. We show that the "allocation puzzle" is more prevalent than previously thought. It can be actually generalized to a larger sample by covering more countries and a longer time period. We then introduce credit constraints in a calibrated Schumpeterian growth model to address this paradox. Our main result indicates that, when the level of financial development prevents countries from catching up relative to the world technological frontier, countries import capital to compensate for their insufficient level of domestic savings.

KEYWORDS: Capital Flows, Financial Development, Productivity, Schumpeterian Growth

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1 Introduction

According to the neoclassical growth theory, capital must flow into countries where the marginal product is higher, in contrast to what we observe with data.[1] Starting with Lucas (1990), who showed that capital flows from rich to poor countries in small amounts, some articles emphasized the role of financial frictions to explain the discrepancy between theoretical predictions and observed data. By using a calibrated neoclassical growth model, Gourinchas and Jeanne (2013) showed that the "allocation puzzle" may be explained by a wedge affecting savings decisions; the "allocation puzzle" is related to the downstream capital flows from high-growth to low-growth countries while the "Lucas puzzle"allocation puzzle" is related to the downstream capital flows from high-income to low-income countries.

In this paper, we introduce an endogenous imperfect creditor protection *à la* Aghion, Howitt and Mayer-Foulkes (2005) in a calibrated Schumpeterian growth model to address the "allocation puzzle". Our model shows that countries above some threshold level of financial development will catch-up relative to the world technological frontier while the others will fall behind. Then, following Gourinchas and Jeanne (2013), we decompose theoretical net capital inflows and focus on the contributions of investment and savings on the motion of external debt. We find an interesting prediction for countries which fall below the world technological frontier because of their level of financial development: predicted net capital inflows going toward domestic saving is strongly and negatively correlated with productivity catch-up. Because entrepreneurs have a limited access to credit in countries with low level of financial development, they have to self-finance new projects with a greater fraction of their own wealth. Therefore, the credit constraint tends to reduce capital income in these countries and increases current consumption relative to future consumption. The representative domestic consumer borrows from abroad to finance this increase of consumption. On the other hand, a country with a low level of financial development fails to innovate because investments in new projects are insufficient to reach the productivity level of the world technological frontier. Thus, a country with a low level of financial development falls below the technological frontier and also has a positive capital inflows because of the limited access to credit in the domestic financial market. Our model is able to replicate the direction of capital inflows observed in the data, for groups of countries with low levels of financial development. Our choice of a domestic credit constraint rather than a friction in the international credit market is primarily motivated by two reasons. First, as mentioned by Gourinchas and Jeannes (2013), international financial frictions can just mute capital flows by increasing the cost of external finance relative to domestic finance, but these frictions cannot reverse the direction of the capital flows. Therefore, most of the articles using a financial constraint to address the "allocation puzzle" are more focused on domestic distortions. According to Gertler and Rogoff (1990), domestic financial frictions, determined endogenously and depending on the country's wealth, can mute and possibly reverse capital flow direction from rich countries to poor ones. A country's level of financial development determines the capacity of borrowing to private agents who would like to invest in a risky project. Second, as we show in this

[1] Prasad, Rajan and Subramanian (2007) showed that capital tends to flow from high-growth to low-growth non-industrialized countries.

paper[2], financial development measured by the size of credit to the private sector provided by domestic financial institutions is positively correlated with productivity catch-up[3], as suggested by Gourinchas and Jeanne (2013), yet also negatively correlated with capital inflows. The more a country is financially developed, the more likely it is to catch up relative to the technological frontier and import less capital, than less financially developed countries.

We show in this paper that the "allocation puzzle" can also be generalized to worldwide economies no matter their level of development, and also that the Schumpeterian growth model predicts a positive relationship between capital inflows and productivity catch-up when entrepreneurs have unlimited access to credit. With an extended sample including OECD and non-OECD countries, we show that observed net capital inflows are negatively correlated with productivity growth as in most of the works in the literature used primarily developing countries in their samples (Prasad *et al.*, 2007, Aguiar and Amador, 2011). As we do not perform a decomposition between public and private debts in observed net capital inflows, as in most of empirical works, we broaden our sample to developed countries [4]. We are only interested in the general pattern of total net capital inflows, in other words, the negative correlation of observed capital flows and productivity growth, which we use to assess the prediction of our model. Alfaro *et al.* (2014) argued that the sign of correlation between net capital flows and productivity changes depend on the selected sample; with a sample dominated by Asian and African countries, they find a robust negative correlation, while this correlation is weakly positive with a larger sample. Gourinchas and Jeanne (2013) used a sample of developing countries and found that total net capital inflows are negatively correlated with productivity growth. We generalize their results to the entire world. Our results can withstand changes resulting from limiting the sample to developing countries, and only using data from the period of 1980 to 2000 instead of 1980 to 2010. We also use two different measures for total net capital inflows, but the sign of the correlation between capital flows and productivity growth remains the same.

Intuitively, we can interpret the predictions of our model as follows. When agents have unlimited access to credit, the country will catch-up relative to the world technological frontier since they anticipate higher future income, will then increase their consumption. Given that their current income is unchanged and that they have access to international financial market, they will borrow from abroad. When agents have limited access to credit and the level of financial development is sufficient, the same pattern is observed, but the predicted capital inflows going toward saving is higher. This is attributable to credit constraints, which reduce capital income and increases current consumption relative to future consumption. In contrast, a financially underdeveloped country will likely fall below the world technological frontier. Agents will borrow on the international financial market to finance the increase of their consumption and their debt will also increase due to the reduction of their wealth created by the expenditure on new projects.

This article is related to several strands in the literature. First, this paper is linked to the literature on financial development and economic growth. Since Goldsmith (1969), the literature

[2] See figures **??**, **??**, **??** and **??**.

[3] See Aghion *et al.* (2005) for theoretical support and empirical evidence.

[4] The World Bank data do not report details on foreign debt for countries classified as "developed". Therefore, empirical articles that perform decomposition of net debt can only focus on developing countries.

in development economics has established evidence of the strong positive relationship between financial development and economic growth. We show in this paper that, depending on their level of financial development, the productivity of countries grows at a higher or lower rate than the technological frontier productivity growth rate. Our findings corroborate the conclusions reached by Aghion *et al*. (2005) who show that countries converge at the technological frontier growth rate only if their level of financial development is above a critical level.

Second, this paper is related to the literature on the determinants of capital flows. Many commentators have focused on determinants of the relationship between savings, investment, and growth to explain why the standard neoclassical growth model fails to predict the negative correlation between productivity growth and net capital inflows. It is well known in the literature that saving is strongly positively correlated with growth across countries (Modigliani, 1970, Carroll and Weil, 1994, Carroll and Weil, 2000), as well as with investment (Feldstein and Horioka, 1980, Attanasio *et al*., 2000). Among determinants affecting these relationships, friction on the domestic financial market seems to be a potential candidate. A financial friction can reduce the agents' ability to borrow against future income (Caballero, Farhi and Gourincha, 2008). Morever with a lack of social insurance, this could also increase precautionary savings (Mendoza *et al*., 2009, Carroll and Jeanne, 2009). Aghion *et al*. (2009) showed that distortions in the domestic financial market prevent domestic savings from substituting perfectly for foreign savings. Therefore, agents in poor countries have to increase their saving to be able to invest in a new project, while saving matters for growth in these countries. Some articles in the literature are also focused on distortions affecting investment in physical capital to explain the puzzle (Buera, 2016, Caselli and Feyrer, 2007). We propose a model with an endogenous credit constraint that affects the consumption-saving behavior of agents. We show that the credit constraint measured by the level of financial development tends to reduce the total wealth of agents. These effects are more severe in countries that are likely to fall below the technological frontier and, in particular, will have to rely more on external debt.

Third, this article is linked to the wide literature documenting the negative correlation between capital flows and growth. Our analysis is close to that of Gourinchas and Jeanne (2013). We show that the Schumpeterian growth model under perfect financial markets (Howitt, 2000, Aghion and Howitt, 1998a, b) also yields the same predictions as the standard neoclassical growth model. In addition to Gourinchas and Jeanne (2013), our model endogenizes the productivity catch-up. We also show that this negative correlation holds for developing countries and is strong when extended to the rest of the world.

The rest of the paper is organized as follows. In Section 2 we present the Schumpeterian growth model under perfect financial markets and we propose the theoretical ratio of cumulated net capital inflows to initial output. In section 3, we introduce imperfect creditor protection in our model and we derive its implications. In section 4, we discuss the data and calibration, and we compare the model's predictions with the observed data. Section 5 concludes the article.

2 The Schumpeterian Growth Framework

We use the Schumpeterian growth model including physical capital accumulation developed by Howitt (2000) and Aghion and Howitt (1998a, b). Time is discrete and there is a continuum of individuals in each country. There are J small open countries, indexed by $j = 1, ..., J$, which exchanges goods and factors, and are technologically interdependent in the sense that they use technological ideas developed elsewhere in the world. Each country can borrow and lend at an exogenously given world real interest rate $r^{\star}$.

2.1 Household

The economy consists of a set of identical households (whose size is normalized to 1), but where the number of infinite lifetime individuals in each household grows at the exogenous rate n, so that $L_j(t) = (1 + n_j)^t L_j(0)$. Each individual supplies inelastically one unit of labor. The representative household maximizes the following constant relative risk aversion (CRRA) utility function:

$$\max_{\{c_j(t)\}_{t=0,1,...}} U_j(0) = L_j(0) \sum_{t=0}^{\infty} \beta^t \frac{c_j(t)^{1-\gamma} - 1}{1 - \gamma}, \tag{2.1}$$

where $c_j(t) = C_j(t)/L_j(t)$ is the per-worker consumption in country j at date t, $\beta \equiv \dfrac{1 + n_j}{1 + \rho}$ is the effective discount rate and ρ is the subjective discount rate, with $\rho > n_j$, and $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution. Denoting $\mathcal{A}_j(t)$ as the asset holding of the representative household at time t, the law of motion of total assets is given by:

$$\mathcal{A}_j(t + 1) = w_j(t)L_j(t) + (1 + r^{\star})\mathcal{A}_j(t) - C_j(t), \tag{2.2}$$

and we assume that the following no-Ponzi condition holds:

$$\lim_{t\to\infty} \left(\frac{1}{1 + r^{\star}}\right)^t \mathcal{A}_j(t + 1) \geq 0. \tag{2.3}$$

The Euler condition for this small open economy is given by:

$$c_j(t)^{-\gamma} = \beta(1 + r^{\star})c_j(t + 1)^{-\gamma}, \tag{2.4}$$

so that, we follow Gourinchas and Jeanne (2013) in assuming that the world interest rate is:

$$1 + r^{\star} = \frac{(1 + g)^{\gamma}}{\beta}, \tag{2.5}$$

where we implicitly assume that the rest of the world is composed of advanced countries at their steady state level, and sharing the same preference parameters of the J small countries under consideration here.

2.2 Production

Production of final good. Let us assume each country produces a single good. The final good is produced under perfect competition by labor and a continuum of intermediate products, according to the production function:

$$Y_j(t) = \left(\frac{L_j(t)}{Q_j(t)}\right)^{1-\alpha} \int_0^{Q_j(t)} A_j(\nu,t)^{1-\alpha} x_j(\nu,t)^{\alpha} d\nu, \tag{2.6}$$

where $Y_j(t)$ is the country's j gross output at date t, $L_j(t)$ is the flow of raw labor used in production, $Q_j(t)$ measures the number of different intermediate products produced and used in the country j at date t, $x_j(\nu,t)$ is the flow output of intermediate product $\nu \in [0, Q_j(t)]$ used at date t, and $A_j(\nu,t)$ is a productivity parameter attached to the latest version of intermediate product ν.

We assume that labor supply and population size are identical. They both grow exogenously at the fixed proportional rate n_j. The form of the production function, that is, the presence of the term $Q_j(t)$ dividing the labor, ensures that growth in product variety does not affect aggregate productivity. Therefore, we suppose, as Aghion and Howitt (1998) and Howitt (2000) do, that the number of products grows as a result of serendipitous imitation, not deliberate innovation. Imitation is limited to domestic intermediate products; thus, each new product will have the same productivity parameter as a randomly chosen existing product within the country. Each agent has the same propensity to imitate $\xi > 0$, which we assume to be identical for each country j. Moreover, we assume that the exogenous fraction ψ of existing intermediate products disappears each period. Thus, the aggregate flow of new products is: $Q_j(t+1) - Q_j(t) = \xi L_j(t) - \psi Q_j(t)$, so that the number of workers per product $l_j(t) \equiv L_j(t)/Q_j(t)$ converges monotonically to the constant:

$$l = \psi/\xi. \tag{2.7}$$

Assuming that this convergence has already occurred, so that: $L_j(t) = lQ_j(t)$ for all t. The form of the production function (**??**) ensures that growth in product variety does not affect aggregate productivity. This and the fact that population growth induces product proliferation guarantees that the model does not exhibit the sort of scale effect that Jones (1995) argues is contradicted by postwar trends in research and development (R&D) spending and productivity. Without loss of generality, we set l=1 in the rest of the paper.

The final good is used for consumption, as an input into entrepreneurial innovation or invested to create new units of physical capital. Producers of the final good act as perfect competitors in all markets, so that the inverse demands for intermediate goods and labor are given by:

$$\text{(FOC)} \quad \begin{cases} p_j(\nu,t) = \alpha \left(\dfrac{A_j(\nu,t)L_j(t)}{Q_j(t)}\right)^{1-\alpha} x_j(\nu,t)^{\alpha-1} \quad \text{for all sectors } \nu \in [0, Q_j(t)] \\ , w_j(t) = (1-\alpha)\dfrac{Y_j(t)}{L_j(t)}. \end{cases} \tag{2.8}$$

Production of intermediate goods. Each intermediate good is produced with physical capital using a one-to-one technology such as:

$$x_j(\nu,t) = K_j(\nu,t),$$

where $K_j(\nu,t)$ is the physical capital used in sector ν at date t in country j to produce $x_j(\nu,t)$ units of intermediate goods. For each intermediate good ν, there is an innovator who enjoys a monopoly power in the production of this intermediate good and maximizes profits according to:

$$\max_{\{x_j(\nu,t)\}} \pi_j(\nu,t) = p_j(\nu,t)x_j(\nu,t) - (r^\star+\delta)x_j(\nu,t) = \alpha\left(\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}\right)^{1-\alpha} x_j(\nu,t)^\alpha - (r^\star+\delta)x_j(\nu,t),$$

where δ is the depreciation rate of physical capital. The equilibrium quantity of intermediate good ν is given by:

$$x_j(\nu,t) = \alpha^{\frac{2}{1-\alpha}}(r^\star+\delta)^{-\frac{1}{1-\alpha}}\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}.$$

Replacing in the inverse demand, we obtain the equilibrium price as: $p_j(\nu,t) = \alpha^{-1}(r^\star+\delta)$.

Aggregate stock of capital. The aggregate stock of physical capital demanded is:

$$K_j(t) = \int_0^{Q_j(t)} K_j(\nu,t)d\nu = \int_0^{Q_j(t)} x_j(\nu,t)d\nu = \alpha^{\frac{2}{1-\alpha}}(r^\star+\delta)^{-\frac{1}{1-\alpha}}A_j(t)L_j(t),$$

where $A_j(t) \equiv \frac{1}{Q_j(t)}\int_0^{Q_j(t)} A_j(\nu,t)d\nu$ is the productivity average in country j, so that the capital stock per-efficient unit of labor, denoted by $\widehat{k}$, is constant and given by:

$$\widehat{k} = \left(\frac{\alpha^2}{r^\star+\delta}\right)^{\frac{1}{1-\alpha}}.$$

Aggregate profits. Substituting the equilibrium quantity of intermediate good and the equilibrium price leads to the equilibrium profit of the monopoly in the sector ν of country j at date t, as:

$$\pi_j(\nu,t) = \frac{1-\alpha}{\alpha}(r^\star+\delta)x_j(\nu,t) = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(r^\star+\delta)^{-\frac{\alpha}{1-\alpha}}\frac{A_j(\nu,t)L_j(t)}{Q_j(t)},$$

which we can rewrite in function of the per-efficient unit of labor physical capital, $\widehat{k}$, as: $\pi(\hat{k})\frac{A_j(\nu,t)L_j(t)}{Q_j(t)}$ where $\pi(\widehat{k}) \equiv \alpha(1-\alpha)\widehat{k}^\alpha$. The aggregate profits are therefore given by:

$$\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu = \pi(\widehat{k})A_j(t)L_j(t).$$

Equilibrium wage and output. Introducing equilibrium quantity of intermediate product in each sector ν in the production function of final good sector leads to the equilibrium quantity of per-worker final good: $y_j(t) = A_j(t)\widehat{k}^{\alpha}$, where $y_j(t) \equiv Y_j(t)/L_j(t)$ is the per-worker GDP. The equilibrium wages are $w_j(t) = (1-\alpha)A_j(t)\widehat{k}^{\alpha} = \omega(\widehat{k})A_j(t)$ where $\omega(\widehat{k}) \equiv (1-\alpha)\widehat{k}^{\alpha}$.

Per worker GDP is given by the sum of incomes (wages, profits of monopolists and rent of capital) in the economy as $y_j(t) = \pi(\widehat{k})A_j(t) + \omega(\widehat{k})A_j(t) + (r^{\star}+\delta)\widehat{k}A_j(t) = A_j(t)\widehat{k}^{\alpha}$ so that the growth rate of the economy is therefore given by the growth rate of the average productivity.[5]

2.3 Innovation and dynamics of aggregate productivity

Assume that each period and in each sector ν, there exists a large number of innovators who invest $Z_j(\nu,t)$ units of final goods in R&D. When successful with a probability $\mu_j(\nu,t)$, an innovator replaces the incumbent monopolist next period and reach the worldwide technological frontier denoted by $\overline{A}(t)$, and when there is no innovation in sector ν, the level of productivity remains at its previous level of $A_j(\nu,t)$. Therefore, the law of motion of productivity in each sector ν is given by:

$$A_j(\nu,t+1) = \begin{cases} \overline{A}(t+1) & \text{with probability} \quad \mu_j(\nu,t), \\ A_j(\nu,t) & \text{with probability} \quad (1-\mu_j(\nu,t)). \end{cases} \tag{2.9}$$

The probability of innovation is linear and given by:

$$\mu_j(\nu,t) = \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)},$$

where $\lambda > 0$ is the productivity of R&D, and where we deflate R&D expenditures in each sector by $\overline{A}(t)$ in order to recognize the force of increasing complexity; as technology advances, the resource cost of further advances increases proportionally. The leading-edge technological is the worldwide technology frontier denoted as $\overline{A}(t)$ and its growth rate is g, so that $\overline{A}(t) = (1+g)^t\overline{A}(0)$.

Moreover, an incumbent monopolists that innovated at date t and are still producing at date $t+1$, with a probability of $(1-\mu_j(\nu,t))$, have the following firm value written in recursive form:

$$V_j(\nu,t) = \frac{1}{1+r^{\star}}\left(\pi_j(\nu,t) + (1-\mu_j(\nu,t))V_j(\nu,t+1)\right), \tag{2.10}$$

so that the problem of the innovator is given by:

$$\max_{\{Z_j(\nu,t)\}} \mu_j(\nu,t)V_j(\nu,t+1) - Z_j(\nu,t). \tag{2.11}$$

Therefore, the innovator invests $Z_j(\nu,t)$ units of final good in R&D and obtains the value $V_j(\nu,t+$

[5] $y_j(t)$ is indeed the per-worker GDP of the country j, since the sum of value added in all sectors is given by: $\left(y_j(t) - \int_0^{Q_j(t)} p_j(\nu,t)x_j(\nu,t)\right) + \left(\int_0^{Q_j(t)} p_j(\nu,t)x_j(\nu,t) - 0\right) = y_j(t)$.

1) with a probability $\mu_j(\nu,t)$. The FOC of the innovator's problem gives the Schumpeterian non-arbitrage condition, which is: $\lambda v_j(\nu,t+1) = \frac{1}{1+g}$, where $v_j(\nu,t+1) \equiv \dfrac{V_j(\nu,t+1)}{\overline{A}(t+1)}$ is the value of a firm in sector ν in efficient units, so that the equilibrium probability to innovate is given by:

$$\mu_j^\star = \lambda\pi(\widehat{k}) - \frac{r^\star - g}{1+g}. \tag{2.12}$$

Finally, total R&D expenditures are given by:

$$Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu = \int_0^{Q_j(t)} \left[\frac{\overline{A}(t)}{\lambda}\mu_j^\star\right] d\nu = \frac{\overline{A}(t)}{\lambda}\mu_j^\star Q_j(t) = z(\widehat{k})\overline{A}(t)L_j(t),$$

where $z(\widehat{k}) = \left(\pi(\widehat{k}) - \dfrac{r^\star - g}{\lambda(1+g)}\right)$.

Denoting the proximity to the technological frontier by $a_j(t) = A_j(t)/\overline{A}(t)$, using equation (**??**), it evolves according to:

$$a_j(t+1) = \mu_j^\star + \frac{1-\mu_j^\star}{1+g}a_j(t) \equiv F_1(a_j(t)), \tag{2.13}$$

and the steady-state equilibrium of the proximity to the frontier is given by:

$$a_j^\star = \frac{1+g}{g+\mu_j^\star}\mu_j^\star,$$

where $\mu_j^\star$ is given by equation (**??**).

2.4 Net Capital Inflows

In our Schumpeterian growth model, net capital inflows can be decomposed, as in Gourinchas and Jeanne (2013), in terms of convergence, trend, investment and saving. We therefore write the volume of capital inflows in terms of the exogenous parameters of the model to be able to compare the prediction of the model to the observed data.

Market clearing implies that the assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is the country's j external debt, and $V_j(t) = \int_0^{Q_j(t)} V_j(\nu,t)d\nu$ is the total value of corporate assets. Therefore, the resources constraint can be rewritten as:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t)).$$

Given the recursive form of the value of firms and the free entry condition in the R&D sector

$\mu_j(\nu,t)V_j(\nu,t+1) = Z_j(\nu,t)$, we have:

$$\int_0^{Q_j(t)} \left(r^\star V_j(\nu,t) - \Delta V_j(\nu,t)\right) d\nu = \int_0^{Q_j(t)} \left(\pi_j(\nu,t) - Z_j(\nu,t)\right) d\nu, = \Pi_j(t) - Z_j(t),$$

where $\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu$ is the aggregate profits and $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu$ is the aggregate R&D expenditures. Therefore, the representative household's budget constraint in country j at date t is written as:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + \Pi_j(t) - Z_j(t).$$

We can now rewrite the budget constraint in terms of per-efficient worker variables as:

$$\widehat{c}_j(t)+(1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1) - \widehat{d}_j(t+1)\right) = (1+r^\star)\left(\widehat{k}_j(t) - \widehat{d}_j(t)\right)+\omega(\widehat{k})+\pi(\widehat{k})-\frac{z(\widehat{k})}{a_j(t)},$$

where, $\widehat{x} = x/A$ denotes the per-worker variables in efficiency units and $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1) \equiv \dfrac{A_j(t+1) - A_j(t)}{A_j(t)}$.

As in Gourinchas and Jeanne (2013) state, we assume that the economy reaches its steady-state at a finite date $T < \infty$. The economy steady growth path is $g_j(t+1) = g$, $a_j(T) = a_j^\star$, $\widehat{k}_j(t+1) = \widehat{k}_j(t) = \widehat{k}$ and $\widehat{d}_j(t+1) = \widehat{d}_j(t) = \widehat{d}_j(T)$, so that the steady-state debt value is given by:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - \frac{z(\widehat{k})}{a_j(T)} - \widehat{c}_j(T)}{r^\star - G_j}, \tag{2.14}$$

where $1 + G_j \equiv (1+g)(1+n_j)$. Steady-state consumption in terms of the proximity to the technological frontier is given as:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)}{a_j(T)/a_j(0)}, \tag{2.15}$$

and the initial consumption per-efficient worker is given by:

$$\widehat{c}_j(0) = \frac{r^\star - G_j}{a_j(0)(1+r^\star)} \sum_{t=0}^{\infty} \left(\frac{1+G_j}{1+r^\star}\right)^t \left(\omega(\widehat{k}) + \pi(\widehat{k})\right) a_j(t) - \frac{z(\widehat{k})}{a_j(0)} + (r^\star - G_j)\left(\widehat{k}_j(0) - \widehat{d}_j(0)\right). \tag{2.16}$$

Finally, we follow Gourinchas and Jeanne (2013) and use the change in external debt between dates 0 and T normalized by initial GDP as the measure of capital inflows:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(T) - D_j(0)}{Y_j(0)}. \tag{2.17}$$

Given equations (**??**), (**??**), (**??**) and (**??**), we obtain the volume of capital inflows in terms of the exogenous parameters of the model as follows:[6]

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\frac{\widehat{k} - \widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$\underbrace{+ \left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T \left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t (1 - f(t))\right]}_{\Delta D^s/Y_0}, \tag{2.18}$$

where $f(t) \leq 1$ and $f(t) = 1$ for $t \geq T$.

The decomposition of the capital inflows in equation (??) leads to the following terms, similar to Gourinchas and Jeanne (2013): the convergence term ($\Delta D^c/Y_0$), which represents the initial level of capital scarcity, the trend term ($\Delta D^t/Y_0$), which is the impact of initial debt on capital inflows, the investment term ($\Delta D^i/Y_0$), and the saving term ($\Delta D^s/Y_0$), which both represent the effect of the productivity catch up. The investment term reflects the amount of external debt dedicated to domestic investment while the saving term is the impact of domestic saving on external debt.

It is worth noting that the Schumpeterian framework developed in our paper allows us to endogenize the productivity catch-up parameter which depends on the proximity to the technological frontier, unlike the one used by Gourinchas and Jeanne (2013). Because the evolution of the proximity to the technological frontier, $a_j(T)$ is endogenous according to equation (??), we obtain an endogenous productivity catch-up and also a function $f(t)$, which is given by: $f(t) = 1 - \left(\frac{1-\mu^\star}{1+g}\right)^t$ and therefore depends explicitly on probability to innovate given by equation (??).

3 The Imperfect Credit Market Model

With perfect credit markets, as it was implicitly assumed in the previous section, innovators have unlimited access to credit and all countries converge on the technological frontier growth rate. We now introduce a asymmetric information as in Aghion et *al*. (2005). This constraint will affect the total amount potential innovators could invest in R&D, the equilibrium probability to innovate and the evolution of the proximity to the technological frontier. As we will show, countries fall in three different groups depending on their own level of financial development: the non-credit constrained group, the credit constrained with convergence group and the credit-constrained divergence group. Each group leads to different predicted net capital inflow.

3.1 Innovation Under Credit Constraints

Each period, a potential innovator with current total wealth $\mathcal{A}_j(t)$ decides to invest $Z_j(\nu, t)$ units of final goods in R&D in each sector ν. Assuming she invests the amount $\mathcal{A}_j(\nu, t)$, defined by some constant and exogenous fraction of her total wealth, in each sector, she needs to borrow

[6] See Appendix A.

the amount $Z_j(\nu,t) - \mathcal{A}_j(\nu,t)$ for each project. We also assume as in Aghion et *al* (2005) and Bernanke and Gertler (1989) that she can pay a cost $H_j Z_j(\nu,t)$ to hide her successful result and defraud her creditor. To ensure she pays back the loan, we assume that the cost of defraud is greater than the repayment of the loan. As shown in appendix B, the innovator could only invest up to a finite multiple of her total wealth in equilibrium:

$$Z_j(\nu,t) \leq \phi_j \mathcal{A}_j(\nu,t), \tag{3.1}$$

where $\phi_j = \frac{1+r^\star}{1+r^\star - H_j}$, $\phi_j \in [1,\infty)$ is the credit multiplier and H_j is the parameter of the hiding cost.

The innovator now chooses $Z_j(\nu,t)$ to maximize her expected net profit of being an incumbent in date t+1 with probability $\mu_j(\nu,t) = \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}$, namely:

$$\begin{aligned} \max_{\{Z_j(\nu,t)\}} \quad & \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)} V_j(\nu,t+1) - Z_j(\nu,t) \\ \text{subject to} \quad & Z_j(\nu,t) \leq \phi_j \mathcal{A}_j(\nu,t). \end{aligned} \tag{3.2}$$

The first order conditions of (**??**) give the Schumpeterian non-arbitrage condition and the expenditure in R&D for each sector ν:

$$\lambda V_j(\nu,t+1) = (1+\Gamma_j(\nu,t))\overline{A}(\nu,t) \quad \text{and} \quad Z_j(\nu,t) = \phi_j \mathcal{A}_j(\nu,t),$$

where $\Gamma_j(\nu,t) > 0$ is the Lagrange multiplier associated with the credit constraint[7]. Since the probability of innovation is the same in all sectors at the equilibrium, the potential innovator will self-finance the same amount $\mathcal{A}_j(\nu,t)$ in each sector and the total expenditure in R&D in the economy is given therefore by $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu = \phi_j q_j(t)\mathcal{A}_j(t),$, where $q_j(t) < 1$ represents the fraction of the households' total wealth devoted to R&D. The entrepreneur will now innovate with probability $\mu(a_j(t)) < \mu^\star$ given by:

$$\mu(a_j(t)) = \lambda\phi\hat{\mathcal{F}}_j(t)a_j(t), \tag{3.3}$$

where $\hat{\mathcal{F}}_j(t) \equiv q(t)\hat{\mathcal{A}}_j(t)$ is self-financing per-efficient worker. The evolution of the proximity to the technological frontier is now given by:

$$a_j(t+1) = \mu(a_j(t)) + \frac{1-\mu(a_j(t))}{1+g} a_j(t) \equiv F_2(a_j(t)), \tag{3.4}$$

which is an increasing concave function and $F_2(0) = 0$.

When a country is credit constrained but has sufficient level of financial development ($\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) > 1$), function $F_2(a_j(t))$ has a slope greater than one and converges to the limit proximity $\widehat{a}_j$. This proximity is lower than the one without credit constraint ($\widehat{a}_j < a^\star$). Hence, the higher is the level of financial development, the higher is the size of investment in

[7] $\Gamma_j(\nu,t) = 0$ corresponds to the case where the entrepreneur is not financially constrained. We do not consider the case where $\Gamma_j(\nu,t) = 0$ and $Z_j(\nu,t) = \phi\mathcal{A}_j(\nu,t)$).

R&D and the higher is the probability to innovate and reach the productivity growth rate of the world technological frontier at the equilibrium. Otherwise $\phi_j < g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) < 1)$, the slope of $F_2(a_j(t))$ is lower than one and and it converges to 0. The economy fails to innovate and grows at a lower rate.

3.2 Household

From the evolution of the representative household's assets and the evolution of the value of the firms in equilibrium, given by $r^\star V_j(t) - \Delta V_j(t) = \Pi_j(t) - (1+\Gamma_j(t))\phi_j\mathcal{F}_j(t)$, we can write the new budget constraint as:

$$C_j(t)+K_j(t+1)-D_j(t+1) = w_j(t)L_j(t)+(1+r^\star)(1-\tau_j(t))(K_j(t)-D_j(t))+\Pi_j(t)-(1+\Gamma_j(t))q_j(t)\phi_jV_j(t), \tag{3.5}$$

where $\tau_j(t) = \frac{(1+\Gamma_j(t))q_j(t)}{1+r^\star-H_j}$ is the saving wedge. This wedge will act as a "tax" on household saving and will be spent in R&D. Saving wedge increases with the level of financial development H_j.

In countries where the financial institutions are developed, to defraud creditors is expensive. Lenders are more induce to pay back their loans than to defraud. Hence, the higher is the hiding cost that will have to be paid by the investor who defaults on his creditor, the more the impact of asymmetric information will be reduced. Thus, the size of credit granted by financial institutions to the investor may be higher and the probability to innovate may increase.

The representative household will now maximize the utility function given by equation (**??**), subject to budget constraints (**??**). The Euler condition for the small open economy is now given by:

$$c_j(t)^{-\gamma} = \beta(1+r^\star)(1-\tau_j(t))c_j(t+1)^{-\gamma}, \tag{3.6}$$

and finally

$$c_j(t) = c_j(0)(1+g)^t\Phi_j(t)^{min(t,T)}. \tag{3.7}$$

The saving wedge will affect consumption growth. Indeed, consumption will now grow by the factor $(1+g)\Phi_j(t)$ in every period $t < T$ and by the factor $(1+g)$ afterwards, with $\Phi_j(t) = (1-\tau_j(t))^{1/\gamma}$.

3.3 Net Capital Inflows Under Credit Constraint

Once again, we need to write the volume of capital inflows in terms of the exogenous parameters. From the country's aggregate resource constraint, we write the steady-state debt in terms of efficiency units as:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j(T) - \widehat{c}_j(T)}{r^\star - G_j}. \tag{3.8}$$

Because of the Euler equation presented in (**??**), steady-state consumption in terms of the proximity to the technological frontier becomes:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)\Phi_j^T}{a_j(T)/a_j(0)}, \tag{3.9}$$

and the initial consumption per-efficiency worker will be:

$$\begin{aligned}\widehat{c}_j(0) = \; & (r^\star - g)\Theta\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)\right)\\ & +(r^\star - g)\Theta\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right),\end{aligned} \tag{3.10}$$

where $\Theta_j = \dfrac{(1+r^\star)-(1+G_j)\Phi_j}{r^\star - G_j + \left(\dfrac{1+G_j}{1+r^\star}\right)^T \Phi_j^T(1+G_j)(1-\Phi_j)}$.

Finally, using equations (**??**), (**??**), (**??**) and the definition of the change in external debt given by (**??**), we can write the volume of net capital inflows under credit constraint as:

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)} = & \overbrace{\left(\frac{\widehat{k}-\Theta_j\Phi_j^T\widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T-1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)}-1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0} \\ & \underbrace{+\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\Theta_j\Phi_j^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{a_j(T)\Phi_j^{(t-T)}-a_j(t)}{a_j(T)-a_j(0)}\right)\right]}_{\Delta D^s/Y_0}.\end{aligned} \tag{3.11}$$

Our model with credit constraint leads to equation (**??**), which gives a general form of the volume and direction of the capital inflows. We can thus identify again the four terms as above: the convergence term $\Delta D^c/Y_0$, representing the part of the international borrowing going toward investment to reach the steady state, the trend term $\Delta D^t/Y_0$, representing the part of the initial debt on capital inflows, the investment term $\Delta D^i/Y_0$ and the saving term $\Delta D^s/Y_0$, representing the effect of productivity growth on capital inflows. Imperfection in the domestic financial market will affect all the terms except the investment term; capital scarcity is higher in the case of credit constraint, $\left(\widehat{k}-\Theta_j\Phi_j^T\widehat{k}_j(0)\right) > \left(\widehat{k}-\widehat{k}_j(0)\right) \; \forall \; \Theta_j\Phi_j^T > 1$ because a fraction of the initial capital is used in R&D; the gap of capital needed to be financed by external debt is therefore higher. The impact of initial debt on external debt is reduced because a fraction of the initial debt goes toward R&D. The trend term will therefore be lower in an imperfect financial market. The investment term does not change as shown in equation (**??**) because the domestic friction only affects saving. The country could always borrow on the international financial market to finance domestic investment as its productivity grows.

In the next section, we discuss on the particular forms of equation (**??**) according to the level of financial development.

3.4 Theoretical predictions

From the above, we can separate economies into three different groups according to the level of financial development, and predict the direction and the volume of capital inflows with respect to the productivity evolution between 0 and T.

3.4.1 Capital Inflows in an Perfect Credit Market

Looking at the condition $\phi_j \in [1, \infty)$ where $\phi_j = \infty$ corresponds to the case without financial friction, in which that entrepreneurs have unlimited access to credit, it follows that countries with a high financial development level, $H_j \geq (1 + r^\star)$, are assumed to be financially unconstrained. Therefore, one shows that $\Phi_j = \Theta_j = 1$, $q_j = 0$, and thus equation (**??**) become similar to equation (**??**). The proximity of the country to the technological frontier will evolve according to $F_1(a_j(t))$ and will converge to:

$$a_j^\star = \frac{1+g}{g+\mu_j^\star}\mu^\star.$$

The growth rate of productivity will be the same as the technological frontier productivity growth rate $g = \left(\frac{1+r^\star}{\beta}\right)^{1/\gamma} - 1$ for $t \geq T$. For this group of countries, the model predictions are similar to Gourinchas and Jeanne (2013) as follows:

Proposition 1. Without capital scarcity and initial debt, a country will have a positive net capital inflow only if it converges.

$$\Delta D_j/Y_j(0) > 0 \quad \text{only if} \quad a_j(T) > a_j(0)$$

Proposition 2. For two identical countries i and j, except for their productivity catch-up, country j will receive more capital inflows than i only if j catches up relative to the technological frontier faster than i.

$$\Delta D_j/Y_j(0) > \Delta D_i/Y_i(0) \quad \text{if and only if} \quad \left(\frac{a_j(T)}{a_j(0)} - 1\right) > \left(\frac{a_i(T)}{a_i(0)} - 1\right).$$

Interpretation: The positive slope of $\Delta D_j/Y_j(0)$ indicates that countries that catch-up relative to the technological frontier will import capital. As the country catches up and reaches the frontier, productivity grows at a higher rate and households, who anticipate higher future incomes, increase their consumption. Saving decreases and external debt has to increase since current income $(\omega(\widehat{k}) + \pi(\widehat{k}))$ does not change. Also, external debt has to increase with the productivity catch-up to finance domestic investment. Both effects of the productivity catch-up on saving and investment lead to the positive link between net capital inflows and productivity catch-up.

3.4.2 Capital inflows under imperfect credit market with convergence

In countries where the level of financial development is low $(H_j < (1 + r^\star))$, entrepreneurs have limited access to credit and thus cannot invest more than some percentage of their total wealth in

R&D. They will innovate with probability $\mu(a_j(t)) < \mu^\star$ and the proximity of the country to the technological frontier will evolve according to $F_2(a_j(t))$, that is an increasing concave function. With a sufficient level of financial development so that $\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) > 1$, the economy will converge to a limit $\widehat{a}_j < a^\star$ given by:[8]

$$\widehat{a}_j = (1+g) - \frac{g}{\lambda\phi_j\hat{\mathcal{F}}_j}.$$

The volume and direction of capital inflows between 0 and T (equation (**??**)) depend on the exogenous parameters of the model and is therefore given by:

$$\begin{aligned}\frac{\Delta D_j}{Y_j(0)} = & \overbrace{\left(\frac{\widehat{k} - \Theta_j\Phi_j^T\widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0} \\ & \underbrace{+\left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T\Theta_j\Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{\widehat{a}_j\Phi_j^{(t-T)} - a_j(0)\left(\frac{1+g_j}{1+g}\right)^t}{\widehat{a}_j - a_j(0)}\right)\right]}_{\Delta D^s/Y_0}.\end{aligned}$$

where $g_j = (1+g)\lambda\phi_j\hat{\mathcal{F}}_j$ and $g_j > g$ is the average growth rate of productivity, specific to the economy j for $t < T$, $g_j = g$ for $t \geq T$, and $a_j(0)$ is its initial proximity to the technological frontier. As the economy converges to $\widehat{a}_j > a_j(t) > a_j(0)$ and $\Phi_j < 1$, we can easily show the positive relationship between $\Delta D_j/Y_j(0)$ and $(a_j(T)/a_j(0) - 1)$.

For this group, the model also predicts a similar net capital inflows: without capital scarcity and initial debt, one observes positive capital inflows only if the country converges. Considering two identical countries except for their productivity catch-up, a country will receive more capital inflows than another one only if it catches up the technological frontier faster.

Interpretation. The behavior of the investment term is the same as that of the group of countries with perfect financial markets: external debt has to increase to finance domestic investment for the country to feature positive productivity growth. On the other hand, we observe two effects on the direction and the volume of the saving component when the economy experiences some imperfection on the domestic financial market, but has a sufficient level of financial development to catch-up to the world technological frontier.

The first effect is driven by the expenditure in R&D. Indeed, a positive fraction of wealth is used to self-finance part of a new project because of the credit constraint while it decreases the household's current income. The second effect is due to the permanent income hypothesis. As the country invests in R&D and expects a higher growth rate in its productivity, households anticipate higher future income, thus increasing their consumption and decreasing saving. The country will borrow on the international financial market to finance this increase of the consumption. Because of the decrease in current income induced by the first effect, the volume of

[8] We have $F_2(0) = 0$ and $F_2'(0) = \lambda\phi_j\hat{\mathcal{F}}_j + \frac{1}{1+g}$, so that countries with $\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$ converge to a positive proximity to the technological frontier, whereas others with $\phi_j < g/((1+g)\lambda\hat{\mathcal{F}}_j)$ diverge and belong to the third group presented below.

external debt going toward domestic saving is amplified.

Theoretically, both effects in combination with the effect of the investment component imply a positive relationship between capital inflows and productivity catch-up, as well as a higher volume of capital inflow compared to the group of countries with a perfect domestic credit market. To sum up, countries in this group catch-up relative to the technological frontier because of their level of financial development, and have positive net capital inflows. We observe a positive correlation between net capital inflows predicted by the saving component and productivity catch-up. A sufficient level of financial development amplifies the volume of the saving component, compared to an economy with a perfect credit market.

3.4.3 Capital inflows under imperfect credit market with divergence

This group is also characterized by countries with limited access to credit; the proximity to the technological frontier will evolve according to $F_2(a_j(t))$. Opposite to the previous group, countries in this group have an insufficient level of financial development so that $\phi_j < g/((1+g)\lambda\hat{\mathcal{F}}_j)$ and $F_2'(0) < 1$. The proximity of the country to the technological frontier will therefore converge to 0 at a lower growth rate, $g_j \in (0, g)$. By the l'Hôpital's rule, one shows that the productivity growth rate $g_j(t)$ will approach:

$$\lim_{t\to\infty} g_j(t) = (1+g)\lim_{t\to\infty}\left(\frac{a_j(t+1)}{a_j(t)}\right) - 1 = (1+g)\lim_{a\to 0} F_2'(a) = (1+g)\lambda\phi_j l\hat{\mathcal{F}}_j.$$

Equation (**??**), representing the volume and the direction of capital inflows between 0 and T, with respect to the exogenous parameters of the model, can be written as:

$$\frac{\Delta D_j}{Y_j(0)} = \overbrace{\left(\frac{\widehat{k} - \Theta_j\Phi_j^T\widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\left(\frac{a_j(T)}{a_j(0)} - 1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$\underbrace{+\left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T\Theta_j\Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{\left(\frac{1+g_j}{1+g}\right)^T\Phi_j^{(t-T)} - \left(\frac{1+g_j}{1+g}\right)^t}{\left(\frac{1+g_j}{1+g}\right)^T - 1}\right)\right]}_{\Delta D^s/Y_0}.$$

Given that the economy converges to $0 < a_j(t) < a_j(0)$ at a lower growth rate than the growth rate of the technological frontier and that $\Phi_j < 1$, we find a negative relationship between net capital inflows predicted by saving term and productivity catch-up[9]. The prediction is therefore different for this group of countries. Without capital scarcity and initial debt, we can observe the following directions and volume of capital inflows with respect to productivity catch-up:

Proposition 3. As a country diverges, the saving component of net capital inflows is positive and investment component of net capital inflows is negative:

$$\Delta D_j^s/Y_j(0) > 0 \quad \text{only if} \quad a_j(T) < a_j(0)$$

[9]For $g_j < g$ and $\Phi_j < 1$, we can show that $\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{\left(\frac{1+g_j}{1+g}\right)^T\Phi_j^{(t-T)} - \left(\frac{1+g_j}{1+g}\right)^t}{\left(\frac{1+g_j}{1+g}\right)^T - 1}\right) < 0.$

and

$$\Delta D^i_j/Y_j(0) < 0 \quad \text{only if} \quad a_j(T) < a_j(0).$$

Proposition 4. Consider two identical countries with insufficient level of financial development, except for their productivity catch-up, country i will have a higher saving component and a lower investment component of net capital inflows than j only if i falls behind the technological frontier faster than j.

$$\Delta D^s_j/Y_j(0) < \Delta D^s_i/Y_i(0) \quad \text{if and only if} \quad \left(\frac{a_j(T)}{a_j(0)} - 1\right) > \left(\frac{a_i(T)}{a_i(0)} - 1\right)$$

and

$$\Delta D^i_j/Y_j(0) > \Delta D^i_i/Y_i(0) \quad \text{if and only if} \quad \left(\frac{a_j(T)}{a_j(0)} - 1\right) > \left(\frac{a_i(T)}{a_i(0)} - 1\right).$$

Interpretation. As the credit constraint does not affect the share of international borrowing going toward domestic investment, we can observe the same pattern of the investment term as for the two groups above. As on the previous credit-constrained group, countries with an insufficient level of financial development experience the same effects from productivity growth and R&D expenditures on the direction and volume of the saving component of net capital inflows.

The total effect will be positive net capital inflows, denoting that the volume of net capital inflows will also be amplified by the extra external debt due to the decrease in current income. In contrast, countries in this group fail to innovate and will fall behind the technological frontier. Also, the more a country is financially underdeveloped, the more its wealth decreases because of the wedge, and it likely falls below the technological frontier. We therefore observe a negative correlation between net capital inflows predicted by the saving component and productivity catch-up; as shown previously, the credit constraint implies a higher volume of capital inflows predicted by the saving component than in an economy with a perfect credit market.

We have shown that the Schumpeterian growth model allows us to have several predictions of the relationship between productivity growth and net capital inflows, depending on the level of financial development. In the next section, we assess our model predictions with the observed data.

4 Empirical Assessment of the Model

4.1 Data

We follow the standard calibration adopted in the development accounting literature proposed by Caselli (2005). The depreciation rate of physical capital is set to δ=0.06 and the capital share to α=0.3. We set the annual discount factor to β=0.96 and we assume log preference (γ = 1) as in Gourinchas and Jeanne (2013). The growth rate of the world technological frontier is set to (1+g) = 1.017, which corresponds to the observed average growth rate of the USA's total factor productivity between 1980 and 2010. Therefore, the gross world interest rate is in accordance with equation (**??**) given by $1 + r^\star = 1.04$.

Regarding the measure of productivity, we use data from Version 9.0 of the Penn World Tables[10] (Feenstra *et al*, 2015), and from the World Bank's World Development Indicators for national accounts[11], population, GDP, price levels, income classification and investment. Productivity is obtained by using $A_j(t) = (y_j(t)/k_j(t)^\alpha)^{1/1-\alpha}$ and the level of capital stock per-efficient unit of labor $\widehat{k}_j(t)$ as $k_j(t)/A_j(t)$, where $y_j(t)$ and $k_j(t)$ are respectively the per-capita output and capital stock. Using the trend component of the productivity $A_j(t)^{hp}$ obtained with the Hodrick-Prescott filter, we then construct the proximity of each country to the technological frontier (that is defined as that of the U.S), as $a_j(t) = \left(A_j(t)^{hp}/\overline{A}(t)^{hp}\right)$.

We use the External Wealth of Nations Mark II database (EWN) of Lane and Milesi-Ferretti (2007) to calculate net capital inflows as the opposite of the ratio of the change in net foreign assets to initial GDP between 1980 and 2010. We normalize the series by GDP to control for the relative size of countries. We also use data on current accounts from the IMF's International Financial Statistics as an alternative measure of the net capital inflows. Since EWN and IFS data are in current US dollars, we use the PPP-adjustment method in Hsieh and Klenow (2007) to convert NFA from current US dollars to constant international dollars[12]. Since PWT 9.0 reports data on the price of investment $P_j(t)$, we use it as the price index. The PPP-adjustor is therefore computed as $\tilde{P}_j(t) = P_j(t)\left(\dfrac{CGDPO_j(t)}{RGDPO_j(t)}\right)$ where $CGDPO(RGDPO)$ is GDP at current (constant) US dollars (international dollars). As in Gourinchas and Jeanne (2013), the volume of capital inflows normalized by initial GDP between 1980 and 2010 for each country is constructed using:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(2010) - D_j(1980)}{Y_j(1980)}. \tag{4.1}$$

As we do not have a direct measure of the level of financial development, we follow what is standard in the literature by using the ratio of private credit to GDP as a proxy of the parameter H. Data on private credit come from Beck *et al* (2000) and represent the ratio of credit granted by financial intermediaries to the private sector, to GDP.[13]

The parameters regarding R&D for the countries with perfect credit markets are calibrated to match observed data in the United States; the probability of innovation $\mu^\star$ is set to 3.6%, which corresponds to the steady-state rate of creative destruction of firms in the U.S economy[14] and the productivity of R&D λ parameter is set by using equation (**??**). Table **??** summarizes the main parameters for the three groups of countries. ϕ_{conv} and ϕ_{div} are the average credit multipliers for

[10] http://www.aeaweb.org/articles?id=10.1257/aer.20130954

[11] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD

[12] According to Gourinchas and Jeanne (2013), this adjustment method does not affect the results and allows us to compare the capital flows measure to the capital accumulation or the output measures used in the development accounting literature.

[13] We also use data on private credit by deposit money banks and other financial institutions to GDP, and, alternatively data on private credit by deposit money banks to GDP, although this did not affect qualitatively our results.

[14] See Caballero and Jaffe (1993).

each group. The average productivity growth rate is computed using $g_{conv} = (1+g)\lambda\phi_{conv}\hat{\mathcal{F}}_j$ and $g_{div} = (1+g)\lambda\phi_{div}\hat{\mathcal{F}}_j$. We use the World Bank data on R&D expenditure and data on countries' total wealth to set q = 0.025 identical to each country with credit constraints, assuming that 2.5% of the total wealth in the economy is dedicated to R&D self-financing.

Table **??** around here

After excluding outlier countries, the final sample consists of 109 countries, including 90 non-OECD and 19 OECD countries. The start of the period is 1980 except for some countries (Angola, Burundi and Brunei based in 1985, Belize and Laos in 1984 and China in 1982), and the end of the period is 2010.

4.2 The "Allocation Puzzle": Evidence and Predictions

Figure **??** shows a quick illustration of the negative relationship between the average growth rate of total factor productivity (TFP) and the average ratio of net capital inflows measured by the negative of the ratio of the average current account to GDP for the whole sample over the period 1980-2010. In our sample, we include developed countries but the pattern of the capital inflows remains similar to the one illustrated by Gourinchas and Jeanne (2013): several countries with a negative average growth rate of TFP received positive capital inflows while others with a positive average growth rate of TFP exported capital. Also, one can observe that capital inflows decrease with productivity growth as shown by the regression line; the slope is negative (-0.91) and statistically significant at 1% with a standard error of (0.20). In figure **??**, we also observe that the volume of capital flows varies between -5% of GDP for countries exporting capital, and up to 15% of GDP for countries importing capital. This reveals the high volume of capital movement across countries. The standard neoclassical growth model predicts a lower capital should flow from rich to poor countries, but that also a positive correlation is expected between productivity growth and capital inflows. Figure **??** highlights that is observed both in the data, the opposite of the neoclassical growth model predictions. Therefore, the "allocation puzzle" (Gourinchas and Jeanne, 2013) and the "Lucas puzzle" (Lucas, 1990) are depicted in the same figure.

Figure **??** around here

To explore further the relationship between net capital inflows and productivity catch-up, table **??** groups countries according to their level of financial development. In group 1 we find countries with a high level of financial development, hence not financially constrained ($H_j \geq (1+r^\star)$). In group 2, we have countries with medium level of financial development ($H_j < (1+r^\star)$ and $\phi_j > g/((1+g)\lambda\hat{\mathcal{F}}_j)$). Finally, countries with a low level of financial development are in group 3 ($H_j < (1+r^\star)$ and $\phi_j \leq g/((1+g)\lambda\hat{\mathcal{F}}_j)$). On average, countries in our sample fall behind the technological frontier ($(a_j(T)/a_j(0)-1)=-0.19$) and receive positive capital inflows (63.80%), as measured by the ratio of change in external debt to initial output over the period 1980-2010. We observe the same pattern, on average, for non-OECD countries (productivity catch-up = -0.25 and net capital inflows = 70.64% of initial output). The finding for developing countries is similar to Gourinchas and Jeanne (2013), who found an average productivity catch up of -0.10 and an average net capital inflow of 31.49% with a sample of 68 developing countries. Compared to Non-OECD countries, on average, OECD countries are more likely to catch-up relative to the technological frontier (0.10) and to receive a positive, but lower, volume of capital inflows (31.41%). Among the three groups mentioned above, we observe a negative relationship between productivity catch-up and capital inflows only with the third group[15]. With a negative productivity catch-up (-0.27 on average), countries in this group borrow about 77.53% of their initial output. The pattern is different for the two other groups. With a positive productivity catch-up, countries with high (0.12) and sufficient (0.005) level of financial development borrow, respectively, on average, 42.02% and 18.08% of their initial output abroad. Our results are, without surprise, in line with the conclusion of Gourinchas and Jeanne (2013). Contrary to the prediction of the neoclassical growth model, net capital inflow are negatively correlated with productivity growth in the data.

Table **??** around here

In figure **??** we assess the predictions of the Schumpeterian growth model with a perfect financial market against the capital inflows observed in the data. Assuming that there is common population growth and no capital scarcity or initial debt,[16], we present only the predicted investment (D^i/Y_0) and saving terms (D^s/Y_0). On one hand, the observed negative relationship sharply appears in the graph: the slope of the regression line is negative and statistically significant (-1.19 significant at 5%).[17] One can observe that most countries that fall below the technological frontier (the countries in the left panel) received positive net capital inflows. According to our threshold of financial development, these are countries that likely diverge because of their low level of financial development. In this panel, we find African and Latin-American countries, which are characterized by lower long-run productivity growth rates and lower levels of financial development. Asian, and some European, countries are characterized by higher productivity growth rates and higher levels of financial development in the right panel.

Figure **??** around here

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[15]See table (**??**). By grouping countries with a low level of financial development according to their income, one can observe that external debt decreases with productivity catch-up

[16]Gourinchas and Jeanne (2013) show with multiple regressions and robustness checks that initial capital scarcity and population growth do not enter significantly in observed capital inflows. They also found that initial debt has a positive and significant coefficient, as predicted by their model.

[17]The slope is also negative (-1.12) and significant (5%) when we include only non-OECD countries.

On the other hand, we represent net capital inflows predicted by the Schumpeterian model in a perfect financial market. Under the assumption of absence of capital scarcity and initial debt, total capital inflow is the sum of the investment term and the saving term. We observe in the graph that both these terms have a positive slope for the reasons we invoked above. The volume of capital inflows predicted by the saving term is much greater than for the investment term. The slope of the saving term is 36.42, while that of the investment term is 2.49.[18] This is expected because of the assumption of infinite life of consumers in the model. According to Gourinchas and Jeanne (2013), households can perfectly smooth their consumption, so the saving term is more sensitive to the productivity growth. Gourinchas and Jeanne (2013) have predicted approximately the same magnitude for the investment term but a lower one for the saving term. Considering the positive correlation predicted by our model in a perfect financial market and the negative correlation observed in the data, we conclude that the Schumpeterian growth model also faces the "allocation puzzle" as in the neoclassical growth model. We now turn to the prediction of the Schumpeterian growth model with imperfection in the domestic financial market and compare it with the observations.

Figure **??** around here

Figures **??** and **??** show capital flows predicted by the saving and investment components against the observed data for the group of countries which are credit constrained. In Figure **??**, we group countries with a sufficient level of financial development. As we discussed previously, countries that belonging to this group catch-up with the technological frontier in the long run. As with the model without credit constraint, one can observe that net capital inflows predicted by saving and investment terms have positive slopes; because of the credit constraint, the saving component predicted by the model will be greater as we also explained in the previous section, but the predicted investment component remains unchanged. The saving term has a greater positive slope (164.73%). Regarding the observed data, productivity catch-up seems to have no significant effect on the capital inflows or, at best, a positive effect.[19] As we do not have a wider sample of convergent countries with credit constraint, the correlation between capital flows and productivity catch-up observed in the data is ambiguous. Therefore, we conclude that the "allocation puzzle" for this group of countries is related to the positive correlation predicted by the model and the positive (at most null) correlation observed in the data.

Figure **??** around here

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[18]Gourinchas and Jeanne (2013) predict a slope of the investment term=2.14 and a slope for the saving term=5.25. The slope of saving is higher in our model because we normalize the saving and investment terms by the initial income (as indicated in equation (**??**)) instead by normalizing by capital, as they did.

[19]Given that only a few countries belong to this group, we find that the regression of the net capital inflows on productivity catch-up gives is non-significant.

Our main finding is presented in figure ??. Countries in this graph are those which likely diverge because of a low level of financial development. When we look at the observed net external debt and productivity catch-up, most of the countries are located in the left panel. Few of them catch-up to the frontier despite their low level of financial development. The whole pattern is a decrease of net capital inflows with the productivity catch-up. We also draw the saving and investment terms predicted by the model with imperfection in the domestic financial market. The predicted investment term increases with the productivity catch-up as we argued above. An increase of one percentage point in productivity catch-up implies an increase of capital inflows predicted by investment term by 2.08% of initial output. This is identical to the model without credit constraint as well as for the model with a sufficient level of financial development. That is because the imperfection introduced in the model does not affect investment, but only the saving component of net capital inflows. Our model prediction about the investment component of capital flows are similar to the conclusion of Gourinchas and Jeanne (2013). Concerning the saving component, the low level of financial development, in addition to preventing the countries from catching up to the technological frontier, it also increases, generally, external borrowing going toward saving. The more a country is financially constrained, the more it falls below the technological frontier and the more its net capital inflows predicted by the saving component is higher. This leads to the negative correlation between productivity catch-up and net external debt predicted by the saving term.

Introducing a friction in the domestic financial market allows the Schumpeterian growth model to replicate the direction the saving part of net capital inflows as observed with the data for countries which fail to catch-up to the technological frontier because of their level of financial development. However, our model fails in replicating the volume of net capital inflows. Although the predicted investment component succeeds to replicate the magnitude of net capital inflows in absolute value as with the observed net external debt[20], our predicted saving component decreases by 202.94 % of initial output for an increase of one percentage point in the productivity catch-up for countries with an insufficient level of financial development; the predicted saving component increases by 164.73 % of initial output for an increase of one percentage point in the productivity catch-up for countries with a sufficient level of financial development. To be able to replicate perfectly the volume of observed net capital inflows, Gourinchas and Jeanne (2013) estimate an average saving wedge of about 1% of aggregate saving, which is relatively small. In Gourinchas and Jeanne (2013), the saving wedge τ_s for each country is computed such that the predicted net capital inflows perfectly match the observed net external debt. We propose in our model an endogenous wedge of about 5% of aggregate saving, that itself depends on the level of financial development. We assume that there is no initial debt or capital scarcity and use this wedge to predict separately the saving and the investment components. This is not the case in Gourinchas and Jeanne (2013), who also use an additional wedge for physical capital. The saving wedge is estimated in their model using the whole model (with initial debt and capital

[20]Net capital inflows predicted by the investment component increases by 2.08% and 1.10% of initial output for an increase of one percentage point in the productivity catch-up, respectively, for countries with insufficient and sufficient levels of financial development. According to the data, net capital inflows decrease by 1.19% of initial output for an increase of one per cent of productivity catch-up.

scarcity).

The Schumpeterian growth model gives the same prediction as the neoclassical growth model about capital inflows when there is no friction on the domestic financial market. We show that observed net capital inflows are negatively correlated with the average growth rate of productivity whereas the theoretical model predicts a positive correlation. In a perfect financial market, our model also faces the "allocation puzzle" and fails to explain the negative correlation between productivity growth and net capital inflows. However, by assuming a friction in the domestic financial market that reduces the capacity of entrepreneurs to borrow and invest in a new project, the model is able to replicate the negative relationship observed for countries which likely diverge.

5 Concluding Remarks

We addressed in this paper the "allocation puzzle" by looking at the movement of capital across countries according to their level of financial development. We introduced a credit constraint in a Schumpeterian growth model and showed that this constraint reduces a country's total wealth. When the level of financial development allows a country to catch-up relative to the world technological frontier, we find that net capital inflows increase with the productivity catch-up. In contrast, we find that the saving component of net capital inflows decreases with productivity catch-up when a country grows at a lower rate than the technological frontier. Since the "allocation puzzle" is more related to domestic saving (Gourinchas and Jeanne, 2013 and Alfaro *et al*, 2014), our model contributes to explaining the negative correlation between productivity catch-up and capital inflows. We also showed that the "allocation puzzle" holds in a Schumpeterian growth framework and, with more recent data, can be generalized to a larger sample that includes developed countries.

However, despite these interesting findings, our model is unable to replicate the volume of the external debt, and little capital flows from rich to poor countries (Lucas, 1990), according to the data, in contrast to the predicted capital flows. This model in shortcoming to replicate the volume of capital flows can be due to the fact that the saving wedge in our model is somewhat high.[21] In addition, our model does not take into account human capital, which is an important determinant of capital flows (Lucas, 1990) and saving (Aghion *et al*, 2009) in developing countries. We believe that these features can help to improve the predictions of our model. We will propose an approach in this direction in future research.

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A Appendix

A.1 *Ratio of cumulated net capital inflows to initial output under perfect financial market*

Ratio of the debt to initial GDP. We first write the ratio of the debt to initial GDP by expressing variables per-efficient worker.

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(T) - D_j(0)}{Y_j(0)} = \frac{\widehat{d}_j(T)\dfrac{A_j(T)L_j(T)}{A_j(0)L_j(0)} - \widehat{d}_j(0)}{\widehat{y}_j(0)},$$

where $\widehat{d}_j(t) \equiv \dfrac{D_j(t)}{A_j(t)L_j(t)}$ is the per-efficient worker debt and $\widehat{y}_j(t) \equiv \dfrac{Y_j(t)}{A_j(t)L_j(t)}$ is the per efficient worker GDP, for all $t \geq 0$. Using $a_j(t) \equiv \dfrac{A_j(t)}{\overline{A}(t)}$ the proximity to the frontier, $L_j(T) = L_j(0)(1+n_j)^T$ and $\overline{A}(T) = \overline{A}(0)(1+g)^T$, we can write the debt ratio as:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{\dfrac{a_j(T)}{a_j(0)}\widehat{d}_j(T)(1+G_j)^T - \widehat{d}_j(0)}{\widehat{y}_j(0)}, \tag{A.1}$$

where $1 + G_j \equiv (1+g)(1+n_j)$ without loss of generality.

Steady-state debt per-efficient worker. The law of motion for total assets is given by:

$$\mathcal{A}_j(t+1) = w_j(t)L_j(t) + (1+r^\star)\mathcal{A}_j(t) - C_j(t)$$

where market clearing implies that assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is country's j external debt, and $V_j(t) = \int_0^{Q_j(t)} V_j(\nu,t)d\nu$ is the total value of corporate assets. We have:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t)).$$

Given the recursive form of the firm value $V_j(\nu,t) = \frac{1}{1+r^\star}\left(\pi_j(\nu,t) + (1-\mu_j(\nu,t))V_j(\nu,t+1)\right)$ and the free entry condition in the R&D sector $\mu_j(\nu,t)V_j(\nu,t+1) = Z_j(\nu,t)$, we have:

$$\begin{aligned}\int_0^{Q_j(t)} \left(r^\star V_j(\nu,t) - \Delta V_j(\nu,t)\right)d\nu &= \int_0^{Q_j(t)} \left(\pi_j(\nu,t) - Z_j(\nu,t)\right)d\nu \\ &= \Pi_j(t) - Z_j(t),\end{aligned}$$

so that:

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + \Pi_j(t) - Z_j(t),$$

where $\Pi_j(t) = \int_0^{Q_j(t)} \pi_j(\nu,t)d\nu$ is aggregate profits and $Z_j(t) = \int_0^{Q_j(t)} Z_j(\nu,t)d\nu$ stands for aggregate R&D expenditures.

We can now write the budget constraint in terms of per-efficient worker variables as:

$$\widehat{c}_j(t)+(1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1)-\widehat{d}_j(t+1)\right) = (1+r^\star)\left(\widehat{k}_j(t)-\widehat{d}_j(t)\right)+\omega(\widehat{k})+\pi(\widehat{k})-\frac{z(\hat{k})}{a_j(t)},$$

where $z(\hat{k}) = \left(\pi(\hat{k}) - \frac{\xi}{\psi\lambda}\frac{r^\star - g}{1+g}\right)$ and $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1) \equiv \frac{A_j(t+1)-A_j(t)}{A_j(t)}$.

After time T, the economy steady growth path is $g_j(t+1) = g$, $\widehat{k}_j(t+1) = \widehat{k}_j(t) = \widehat{k}$ and $\widehat{d}_j(t+1) = \widehat{d}_j(t) = \widehat{d}_j(T)$, so that the steady-state debt value is given by:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - \frac{z(\widehat{k})}{a_j(T)} - \widehat{c}_j(T)}{r^\star - G_j}. \tag{A.2}$$

Steady-state consumption per-efficient worker. We now compute steady-state consumption in terms of the proximity to the technological frontier. Steady-state consumption per-effective worker is defined by:

$$\widehat{c}_j(T) = \frac{c_j(T)}{A_j(T)}.$$

We can therefore define the average productivity as $A_j(t) = a_j(t)\overline{A}(t) = \overline{A}(0)a_j(t)(1+g)^t$.

Using $c_j(T) = c_j(0)(1+g)^T$ and the definition for average productivity, we can write:

$$\widehat{c}_j(T) = \frac{c_j(0)(1+g)^T}{a_j(T)\overline{A}_j(0)(1+g)^T},$$

that becomes:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)}{a_j(T)/a_j(0)}. \tag{A.3}$$

Initial consumption per-efficient worker. The per-worker intertemporal budget constraint is:

$$\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t c_j(t) = (1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t w_j(t)+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t(\pi_j(t)-z_j(t)).$$

Using $c_j(t)=\widehat{c}_j(0)A_j(0)(1+g)^t$, $w_j(t)=\omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$, $\pi_j(t)=\pi(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$ and $z_j(t)=z(\widehat{k})\overline{A}_j(0)(1+g)^t$ we can write:

$$\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t\widehat{c}_j(0)A_j(0)(1+g)^t=(1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t\omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$$
$$+\sum_{t=0}^{\infty}\left(\frac{1+n_j}{1+r^\star}\right)^t\left(\pi(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t-z(\widehat{k})\overline{A}_j(0)(1+g)^t\right).$$

It follows that:

$$\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t\widehat{c}_j(0)A_j(0)=(1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t\omega(\widehat{k})a_j(t)\overline{A}_j(0)$$
$$+\sum_{t=0}^{\infty}\left(\frac{(1+g)(1+n_j)}{1+r^\star}\right)^t\left(\pi(\widehat{k})a_j(t)\overline{A}_j(0)-z(\widehat{k})\overline{A}_j(0)\right).$$

Since $(1+g)(1+n_j)=(1+G_j)$, without loss of generality and dividing both sides by $A_j(0)$, we can write:

$$\begin{aligned}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\widehat{c}_j(0) &= (1+r^\star)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)+\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\frac{a_j(t)}{a_j(0)}\omega(\widehat{k})\\ &+\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{a_j(t)}{a_j(0)}\pi(\widehat{k})-\frac{z(\widehat{k})}{a_j(0)}\right).\end{aligned}$$

Since :

$$\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t=\frac{1+r^\star}{r^*-G_j},$$

the previous equation implies that:

$$\widehat{c}_j(0)=\frac{r^\star-G_j}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)-\frac{z(\widehat{k})}{a_j(0)}+(r^\star-G_j)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right).\tag{A.4}$$

Ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$. Given equations (??),(??),(??) and (??), we can finally compute the volume of capital inflows in terms of the exogenous parameters of the model as follow: Using the initial consumption per-efficient worker equation, we can write steady-state consumption per effective worker as:

$$\widehat{c}_j(T)=\frac{r^\star-G_j}{a_j(T)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)-\frac{z(\widehat{k})}{a_j(T)}+\frac{a_j(0)}{a_j(T)}(r^\star-G_j)\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)).$$

We then introduce this last term into the steady-state debt per-worker equation. It follows that:

$$\widehat{d}_j(T)=\widehat{k}+\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{r^\star-G_j}\right)-\frac{a_j(0)}{a_j(T)}\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)-\frac{1}{a_j(T)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t).$$

Multiplying both side by $\dfrac{a_j(T)}{a_j(0)}(1+G_j)^T$, we have:

$$\frac{a_j(T)}{a_j(0)}(1+G_j)^T\widehat{d}_j(T) = \frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{r^\star - G_j}\right) - (1+G_j)^T\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)$$
$$+\frac{a_j(T)}{a_j(0)}(1+G_j)^T\widehat{k} - \frac{(1+G_j)^T}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t).$$

Introducing this last into equation (**??**), we finally obtain:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star - G_j)}\right) - (1+G_j)^T\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)}-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right) - \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}$$
$$+\frac{a_j(T)}{a_j(0)}(1+G_j)^T\frac{\widehat{k}}{\widehat{y}_j(0)} - \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t),$$

$$\frac{\Delta D_j}{Y_j(0)} = \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1) + \frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T + \frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star - G_j)}\right)$$
$$+\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T - \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t).$$

Let us denote:

$$\begin{aligned}
A &= \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(t)\right],\\
&= \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T)+a_j(T))\right],\\
&= \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T))\right]\\
&\quad+\frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)a_j(T)\right].
\end{aligned}$$

Using

$$\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t = \frac{1+r^\star}{r^* - G_j},$$

we can write:

$$
\begin{aligned}
A &= \frac{(1+G_j)^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})\right)(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right), \\
&= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right), \\
&= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))+\sum_{t=T}^{\infty}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right).
\end{aligned}
$$

Using $a_t(j)=a_j(T)\ \forall t\geq T$, then we can write A as

$$
\begin{aligned}
A &= \frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t(a_j(t)-a_j(T))\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right), \\
&= \left(1-\frac{a_j(T)}{a_j(0)}\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right] \\
&\quad +\frac{a_j(T)}{a_j(0)}(1+G_j)^T\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(r^\star-G_j)}\right).
\end{aligned}
$$

We then substitute this expression into the capital inflows equation, to obtain:

$$
\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} &= \frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T+\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)+\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T \\
&+\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right].
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} &= \overbrace{\frac{\widehat{k}-\widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0}+\overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T-1)}^{\Delta D^t/Y_0}+\overbrace{\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)}-1\right)(1+G_j)^T}^{\Delta D^i/Y_0} \\
&\underbrace{+\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(1-\left(\frac{a_j(t)-a_j(0)}{a_j(T)-a_j(0)}\right)\right)\right].}_{\Delta D^s/Y_0}
\end{aligned}
$$

In addition, we can use the evolving of the distance to the technological frontier to write:

$$\begin{aligned}
a_j(t+1) &= \mu_j(t) + \frac{1-\mu_j(t)}{1+g}a_j(t), \\
a_j(t+1) - a_j(T) &= \mu_j(t) - a_j(T) + \frac{1-\mu_j(t)}{1+g}a_j(t), \\
a_j(t+1) - a_j(T) &= \mu_j(t) - a_j(T) + \frac{1-\mu_j(t)}{1+g}(a_j(t) - a_j(T)) + \frac{1-\mu_j(t)}{1+g}a_j(T), \\
a_j(t+1) - a_j(T) &= \mu_j(t) - a_j(T) + \frac{1-\mu_j(t)}{1+g}a_j(T) + \frac{1-\mu_j(t)}{1+g}(a_j(t) - a_j(T)), \\
a_j(t+1) - a_j(T) &= \mu_j(t) - \frac{g+\mu_j(t)}{1+g}a_j(T) + \frac{1-\mu_j(t)}{1+g}(a_j(t) - a_j(T)), \\
a_j(t+1) - a_j(T) &= \mu_j(t) - \frac{g+\mu_j(t)}{1+g}\frac{1+g}{g+\mu_j(t)}\mu_j(t) + \frac{1-\mu_j(t)}{1+g}(a_j(t) - a_j(T)), \\
a_j(t+1) - a_j(T) &= \frac{1-\mu_j(t)}{1+g}\left(a_j(t) - a_j(T)\right).
\end{aligned}$$

By induction, it follows that:

$$\begin{aligned}
a_j(t) - a_j(T) &= \left(\frac{1-\mu_j(t)}{1+g}\right)^t \left(a_j(0) - a_j(T)\right), \\
\left(\frac{a_j(t) - a_j(T)}{a_j(0) - a_j(T)}\right) &= \left(\frac{a_j(T) - a_j(t)}{a_j(T) - a_j(0)}\right) = \left(\frac{1-\mu_j(t)}{1+g}\right)^t, \\
1 - \left(\frac{a_j(T) - a_j(t)}{a_j(T) - a_j(0)}\right) &= \left(\frac{a_j(t) - a_j(0)}{a_j(T) - a_j(0)}\right) = 1 - \left(\frac{1-\mu_j(t)}{1+g}\right)^t \equiv f(t).
\end{aligned}$$

where $f(t) \leq 1$ and $f(t) = 1$ for $t \geq T$.

Thus, the ratio of cumulated net capital inflows to initial output between $t = 0$ and $t = T$ becomes:

$$\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} = &\overbrace{\frac{\widehat{k} - \widehat{k}_j(0)}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^c/Y_0} + \overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G)^T - 1)}^{\Delta D^t/Y_0} + \overbrace{\frac{\widehat{k}}{\widehat{y}_j(0)}\left(\frac{a_j(T)}{a_j(0)} - 1\right)(1+G_j)^T}^{\Delta D^i/Y_0} \\
&\underbrace{+ \left(\frac{a_j(T)}{a_j(0)} - 1\right)\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k})}{\widehat{y}_j(0)(1+r^\star)}\right)(1+G_j)^T \left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t (1 - f(t))\right]}_{\Delta D^s/Y_0}. \qquad \text{(A.5)}
\end{aligned}$$

A.2 *Ratio of cumulated net capital inflows to initial output under imperfect financial market*

Ratio of the debt to initial GDP. We first write the ratio of the debt to initial GDP in terms of per-efficient worker variables.

$$\frac{\Delta D_j}{Y_j(0)} = \frac{D_j(T) - D_j(0)}{Y_j(0)} = \frac{\widehat{d}_j(T)\frac{A_j(T)L_j(T)}{A_j(0)L_j(0)} - \widehat{d}_j(0)}{\widehat{y}_j(0)},$$

where $\widehat{d}_j(t) \equiv \dfrac{D_j(t)}{A_j(t)L_j(t)}$ is the per-efficient worker debt and $\widehat{y}_j(t) \equiv \dfrac{Y_j(t)}{A_j(t)L_j(t)}$ is the per-efficient worker GDP, for all $t \geq 0$. Using $a_j(t) \equiv \dfrac{A_j(t)}{\overline{A}(t)}$ the proximity to the frontier, $L_j(T) = L_j(0)(1+n)^T$ and $\overline{A}(T) = \overline{A}(0)(1+g)^T$, we can rewrite the debt ratio as:

$$\frac{\Delta D_j}{Y_j(0)} = \frac{\frac{a_j(T)}{a_j(0)}\widehat{d}_j(T)(1+G)^T - \widehat{d}_j(0)}{\widehat{y}_j(0)}, \tag{A.6}$$

where $1 + G \equiv (1+g)(1+n)$ without loss of generality.

Steady-state debt per effective worker. The law of motion of total assets is given by:

$$\mathcal{A}_j(t+1) = w_j(t)L_j(t) + (1+r^\star)\mathcal{A}_j(t) - C_j(t),$$

where market clearing implies that the assets must be equal to: $\mathcal{A}_j(t) = K_j(t) - D_j(t) + V_j(t)$, where $K_j(t)$ is the stock of physical capital of country j, $D_j(t)$ is the country's j external debt, and $V_j(t) = \int_0^{Q_j(t)} V_j(\nu, t)d\nu$ is the total value of corporate assets. We have:

$$C_j(t) + K_j(t+1) + V_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) + V_j(t) - D_j(t)).$$

$$C_j(t) + K_j(t+1) - D_j(t+1) = w_j(t)L_j(t) + (1+r^\star)(K_j(t) - D_j(t)) + (r^\star V_j(t) - \Delta V_j(t)).$$

The evolution of the value of the firms in equilibrium is given by $r^\star V_j(t) - \Delta V_j(t) = \Pi_j(t) - (1+\Gamma_j(t))\phi_j\mathcal{F}_j(t)$. We can now write the budget constraint per-efficient worker variables as:

$$\widehat{c}_j(t) + (1+g_j(t+1))(1+n_j)\left(\widehat{k}_j(t+1) - \widehat{d}_j(t+1)\right) = (1+r^\star)\left(\widehat{k}_j(t) - \widehat{d}_j(t)\right) + \omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j(T),$$

where $g_j(t+1)$ is the growth rate of average productivity, i.e., $g_j(t+1) \equiv \dfrac{A_j(t+1) - A_j(t)}{A_j(t)}$.

After time T, the economy steady growth path is $g_j(t+1) = g$, $\widehat{k}_j(t+1) = \widehat{k}_j(t) = \widehat{k}$, $\widehat{v}_j(t+1) = \widehat{v}_j(t) = \widehat{v}_j(T)$ and $\widehat{d}_j(t+1) = \widehat{d}_j(t) = \widehat{d}_j(T)$, so that the steady-state debt value is given by:

$$\widehat{d}_j(T) = \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j(T) - \widehat{c}_j(T)}{r^\star - G_j}. \tag{A.7}$$

Steady-state consumption per-efficient worker. We now compute the steady-state consumption in terms of the proximity to the technological frontier.

Steady-state consumption per-efficient worker is defined by:

$$\widehat{c}_j(T) = \frac{c_j(T)}{A_j(T)}.$$

We can therefore define the average productivity $A_j(t) = a_j(t)\overline{A}(t) = \overline{A}(0)a_j(t)(1+g)^t$.

It follows from the Euler equation:

$$\left(\frac{c_j(t+1)}{c_j(t)}\right)^\gamma = \beta(1+r^\star)(1-\tau_j(t)),$$

where $\tau_j(t) = \frac{(1+\Gamma_j(t))q_j(t)}{1+r^\star-H_j}$. Then:

$$c_j(t) = c_j(0)(1+g)^t\Phi_j(t)^{min(t,T)},$$

where $\Phi_j(t) = (1-\tau_j(t))^{1/\gamma}$. Using $c_j(T) = c_j(0)(1+g)^T\Phi^T$ and the average productivity definition, we can write:

$$\widehat{c}_j(T) = \frac{c_j(0)\Phi^T(1+g)^T}{a_j(T)\overline{A}_j(0)(1+g)^T}.$$

And finally:

$$\widehat{c}_j(T) = \frac{\widehat{c}_j(0)\Phi^T}{a_j(T)/a_j(0)}. \tag{A.8}$$

Initial consumption per-efficient worker. The per worker intertemporal budget constraint is:

$$\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t c_j(t) = (1+r^\star)(k_j(0)-d_j(0))+\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t(w_j(t)+\pi_j(t)-(1+\Gamma_j)\phi_j\mathcal{F}_j(t)).$$

Using $c_j(t) = \widehat{c}_j(0)A_j(0)(1+g)^t\Phi^{min(t,T)}$, we show that the left hand side is given by:

$$\sum_{t=0}^{\infty}\left(\frac{1+n}{1+r^\star}\right)^t c_j(t) = \frac{A_j(0)\widehat{c}_j(0)}{\left(1-\dfrac{1+G}{1+r^\star}\right)\Theta},$$

where $\Theta = \dfrac{(1+r^\star)-(1+G)\Phi}{r^\star-G+\left(\dfrac{1+G}{1+r^\star}\right)^T\Phi^T(1+G)(1-\Phi)}$ and using $w_j(t) = \omega(\widehat{k})a_j(t)\overline{A}_j(0)(1+g)^t$, it follows that:

$$\widehat{c}_j(0) = (r^\star-g)\Theta\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)+\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\right). \tag{A.9}$$

Ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$. Given equation (**??**),(**??**),(**??**) and (**??**), we can finally compute the volume of capital inflows in terms of the exogenous parameters of the model as follow. First, using the initial consumption per-efficient worker equation, we can write steady-state consumption per-efficient worker as:

$$\widehat{c}_j(T) = \frac{a_j(0)}{a_j(T)}(r^\star-g)\Theta\Phi^T\left(\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)+\left(\widehat{k}_j(0)-\widehat{d}_j(0)\right)\right).$$

We then introduce this last into steady-state debt per-worker equation. It follows that:

$$\begin{aligned}
\widehat{d}_j(T) &= \widehat{k} + \frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G} - \frac{a_j(0)}{a_j(T)}\Theta\Phi^T\left(\widehat{k}_j(0) - \widehat{d}_j(0)\right) \\
&-\frac{a_j(0)}{a_j(T)}\Theta\Phi^T\left[\frac{1}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t)\right].
\end{aligned}$$

Multiplying both sides by $\frac{a_j(T)}{a_j(0)}(1+G)^T$, we have:

$$\begin{aligned}
\frac{a_j(T)}{a_j(0)}(1+G)^T\widehat{d}_j(T) = \frac{a_j(T)}{a_j(0)}(1+G)^T\widehat{k} + \frac{a_j(T)}{a_j(0)}(1+G)^T\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G}\right) \\
-(1+G)^T\Theta\Phi^T\left(\widehat{k}_j(0) - \widehat{d}_j(0)\right) - \frac{(1+G)^T\Theta\Phi^T}{a_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)\right)a_j(t).
\end{aligned}$$

Introducing in equation (**??**), we finally obtain:

$$\begin{aligned}
\frac{\Delta D_j}{Y_j(0)} = \frac{a_j(T)}{a_j(0)}(1+G)^T\frac{\widehat{k}}{\widehat{y}_j(0)} + \frac{a_j(T)}{a_j(0)}(1+G)^T\left(\frac{\omega(\widehat{k}) + \pi(\widehat{k}) - (1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j}{r^\star - G}\right) - \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)} \\
-(1+G)^T\Theta\Phi^T\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)} - \frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right) - \frac{(1+G)^T\Theta\Phi^T}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\omega(\widehat{k})a_j(t).
\end{aligned}$$

Let us write:

$$\begin{aligned}
B &= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t a_j(t), \\
&= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) + (a_j(T) - a_j(T))\,\Phi^{min(0,t-T)}\right), \\
&= \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{min(0,t-T)}\right) - \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t a_j(T)\Phi^{min(0,t-T)}, \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{(t-T)}\right) + \sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{(T-T)}\right) \\
&\quad + \sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t a_j(T)\Phi^{min(0,t-T)}.
\end{aligned}$$

Using $a_j(t) = a_j(T)\ \forall t \geq T$; then we have:

$$\begin{aligned}
B &= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{(t-T)}\right) + a_j(T)\sum_{t=0}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^{min(0,t-T)}, \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{(t-T)}\right) + a_j(T)\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^{t-T} + \sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^{T-T}\right), \\
&= \sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t) - a_j(T)\Phi^{(t-T)}\right) + a_j(T)\Phi^{-T}\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^t + \sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^T\right).
\end{aligned}$$

Finally, using:

$$\left(\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^t+\sum_{t=T}^{\infty}\left(\frac{1+G}{1+r^\star}\right)^t\Phi^T\right)=\frac{1+r^\star}{r^\star-G}\Theta^{-1},$$

we obtain:

$$B=\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right)+a_j(T)\Phi^{-T}\Theta^{-1}\frac{1+r^\star}{r^\star-G}.$$

We then reintroduce this expression into the capital inflows equation. That gives:

$$\frac{\Delta D_j}{Y_j(0)}=\frac{a_j(T)}{a_j(0)}(1+G)^T\frac{\widehat{k}}{\widehat{y}_j(0)}-(1+G)^T\Theta\Phi^T\left(\frac{\widehat{k}_j(0)}{\widehat{y}_j(0)}-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}\right)-\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}$$
$$-\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\widehat{\mathcal{F}}_j(T)}{a_j(0)\widehat{y}_j(0)(1+r^\star)}\right)(1+G)^T\Theta\Phi^T\sum_{t=0}^{T-1}\left(\frac{1+G}{1+r^\star}\right)^t\left(a_j(t)-a_j(T)\Phi^{(t-T)}\right).$$

Thus, the ratio of cumulated net capital inflows to initial output between $t=0$ and $t=T$ becomes:

$$\frac{\Delta D_j}{Y_j(0)}=\overbrace{\left(\frac{\widehat{k}-\Theta_j\Phi_j^T\widehat{k}_j(0)}{\widehat{y}_j(0)}\right)(1+G_j)^T}^{\Delta D^c/Y_0}+\overbrace{\frac{\widehat{d}_j(0)}{\widehat{y}_j(0)}((1+G_j)^T\Theta_j\Phi_j^T-1)}^{\Delta D^t/Y_0}+\overbrace{\left(\frac{a_j(T)}{a_j(0)}-1\right)\frac{\widehat{k}}{\widehat{y}_j(0)}(1+G_j)^T}^{\Delta D^i/Y_0}$$
$$\underbrace{+\left(\frac{a_j(T)}{a_j(0)}-1\right)\left(\frac{\omega(\widehat{k})+\pi(\widehat{k})-(1+\Gamma_j)\phi_j\hat{\mathcal{F}}_j}{\widehat{y}_j(0)(1+r^\star)}\right)((1+G_j)^T\Theta_j\Phi_j^T)\left[\sum_{t=0}^{T-1}\left(\frac{1+G_j}{1+r^\star}\right)^t\left(\frac{a_j(T)\Phi_j^{(t-T)}-a_j(t)}{a_j(T)-a_j(0)}\right)\right]}_{\Delta D^s/Y_0}.$$

A.3 *Problem of the entrepreneur under credit constraint*

Assume that an entrepreneur in country j wants to undertake a new project and have a limited access to credit. She can only borrow the amount $B_j(\nu,t)$ from a financial institution and self-finance a fraction $\mathcal{A}_j(\nu,t)$ of her total wealth with respectively returns $\eta(t)$ and $\upsilon(t)$ to invest in R&D. She pays back the loan and recovers her self-financed amount if and only if she succeeds with probability $\mu(\nu,t)$. The entrepreneur cannot invest more than her self-finance plus the amount borrowed from the financial institution. Therefore, we can write:

$$B_j(\nu,t)+\mathcal{A}_j(\nu,t)\geq Z(\nu,t). \tag{A.10}$$

We also assume that the expected return of the loan and the expected return of the self-financed are not greater than a risk-free return to ensure a non-arbitrage between the two returns. This is represented by these equations:

$$\lambda\frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t)\geq(1+r^\star)B_j(\nu,t), \tag{A.11}$$

and

$$\lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\upsilon(t))\mathcal{A}_j(\nu,t) \geq (1+r^\star)\mathcal{A}_j(\nu,t). \tag{A.12}$$

Finally, we assume that the entrepreneur can defraud the financial institution; she can pay a cost $H_j Z_j(\nu,t)$ to hide her successful result to the financial institution and if she does, she will not pay back his loan $\lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t)$. To ensure she will pay the loan, we assume that the cost of defraud is greater than the repayment of the loan:

$$H_j(t)Z_j(\nu,t) \geq \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)}(1+\eta(t))B_j(\nu,t). \tag{A.13}$$

The entrepreneur problem is to maximize the expected net profit of becoming the incumbent in the next period. It is given by the expected value of being the incumbent minus the discounted refund of the total investment in R&D, namely:

$$\max_{\{Z_j(\nu,t),B_j(\nu,t),\mathcal{A}_j(\nu,t),\rho(t),\upsilon(t)\}} \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)} \left(V_j(\nu,t+1) - \frac{1}{1+r^\star}\left((1+\eta(t))B_j(\nu,t) + (1+\upsilon(t))\mathcal{A}_j(\nu,t)\right) \right) \tag{A.14}$$

subject to **??**, **??**, **??** and **??**.

By combining constraints **??**, **??**, **??** and **??**, the innovator problem can be rewrite as:

$$\begin{aligned} \max_{\{Z_j(\nu,t)\}} \quad & \lambda \frac{Z_j(\nu,t)}{\overline{A}(t)} V_j(\nu,t+1) - Z_j(\nu,t) \\ \text{subject to} \quad & Z_j(\nu,t) \leq \phi \mathcal{A}_j(\nu,t), \end{aligned} \tag{A.15}$$

where $\phi_j = \frac{1+r^\star}{1+r^\star - H_j}$ and $\phi_j \in [1,\infty)$.

	Perfect credit market	Imperfect credit market	
		Convergence	Divergence
Probability of innovation	$\mu^{\star}$=3.6%	$\mu(\widehat{a})$= 1.3%	$\mu(\widehat{a})$= 0%
Lagrange multiplier	$\Gamma = 0$	$\Gamma_{conv} = 0.13$	$\Gamma_{div} = 0.86$
Credit multiplier	$\phi^{\star} = \infty$	$\phi_{conv} = 3.99$	$\phi_{div} = 1.28$
Productivity growth rate	g=0.017	$g_{conv} = 0.028$	$g_{div} = 0.0091$

Table 1: Parameter Calibration

	Productivity $\left(\frac{a_j(T)}{a_j(0)} - 1\right)$	Financial development H_j	Capital flows $\Delta D_j/Y_j(0)$	Obs
Total sample	-0.19	41.56	63.80	109
OECD countries	0.10	94.06	31.41	19
Non-OECD countries (Developing countries)	-0.25	30.48	70.64	90
By financial development level:				
High	0.12	135.39	42.02	7
Medium	0.005	79.44	18.08	21
Low	-0.27	23.63	77.53	81

Table 2: Productivity catch-up and Capital Inflows (1980-2010) by level of financial development.

	Productivity $\left(\frac{a_j(T)}{a_j(0)} - 1\right)$	Financial development H_j	Capital flows $\Delta D_j/Y_j(0)$	Obs
Group of countries with level of financial development	-0.27	23.63	77.53	81
Low income	-0.40	14.40	130.43	39
Lower middle income	-0.19	28.47	103.51	23
Upper middle income	-0.09	30.98	-7.98	14
High income	-0.12	52.83	-215.10	5

Table 3: Productivity catch-up and Capital Inflows (1980-2010), Low level of Financial Development.

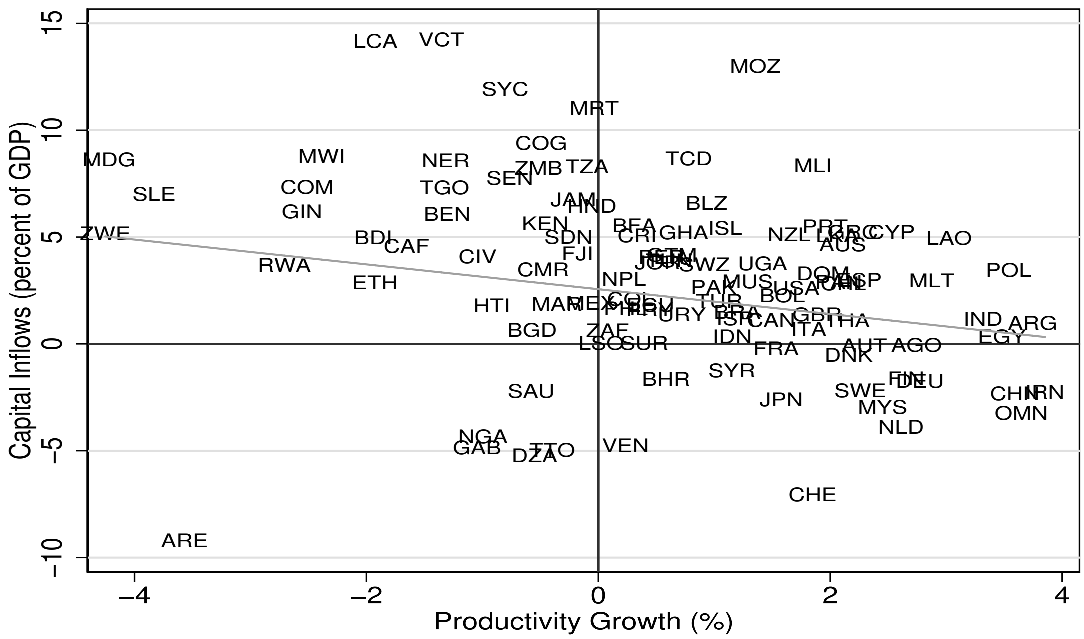

Figure 1: Productivity growth and average Capital Inflows (Negative of Current Account) between 1980 and 2010

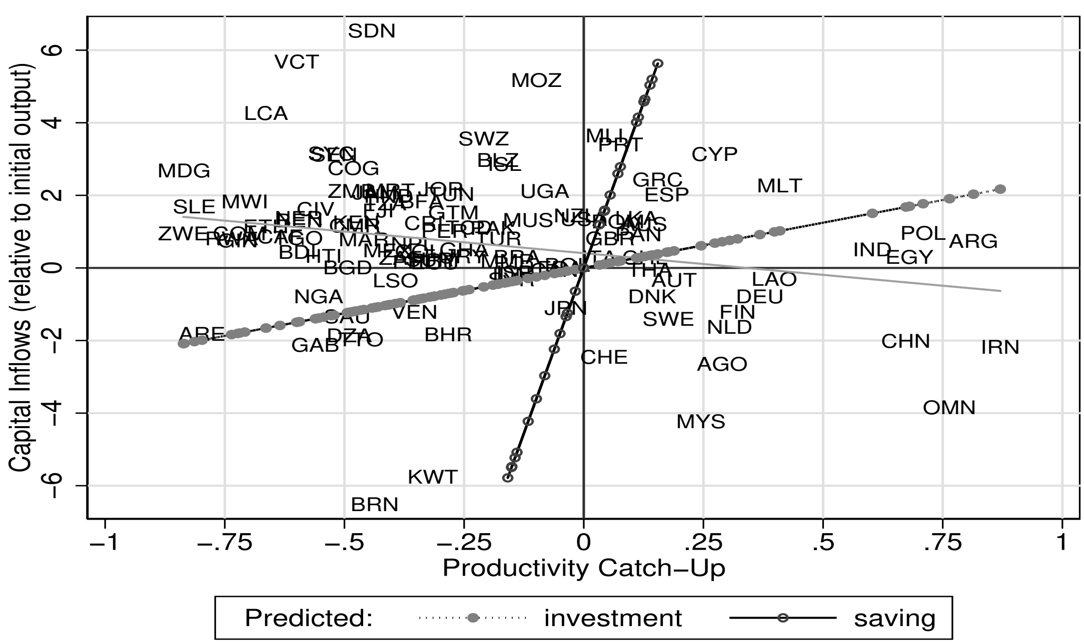

Figure 2: Productivity catch-up and average Capital Inflows (NFA) between 1980 and 2010

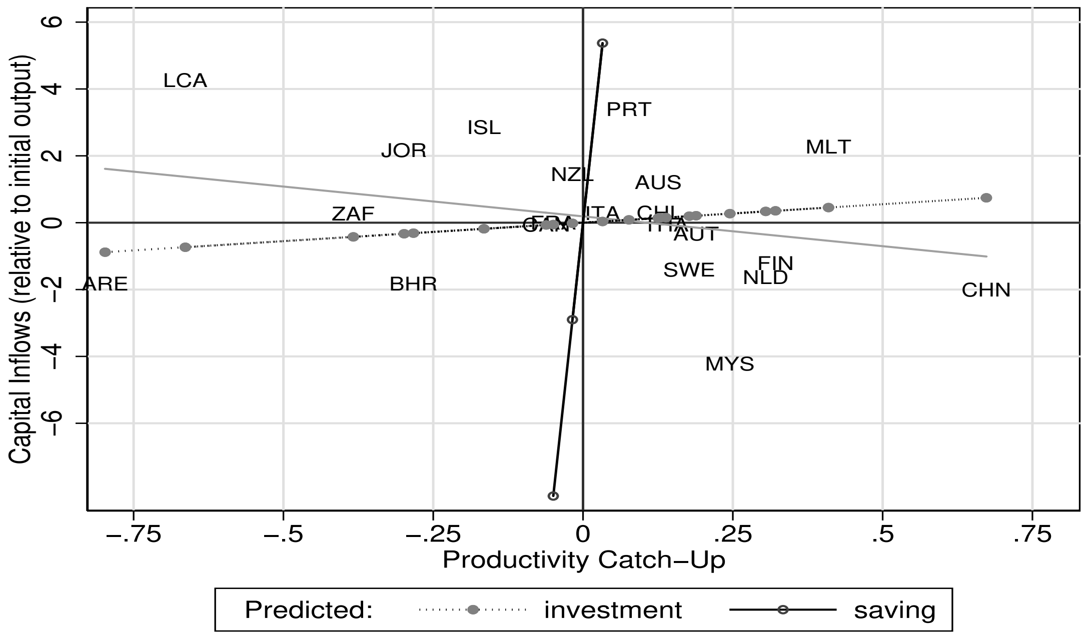

Figure 3: Productivity catch-up and average Capital Inflows between 1980 and 2010, Medium level of Financial Development

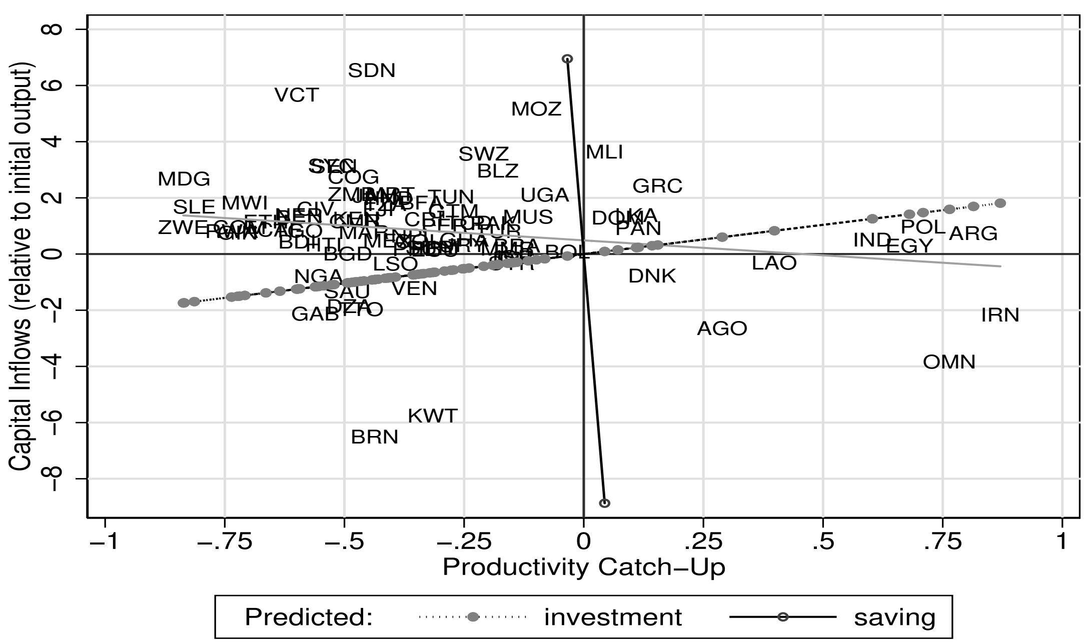

Figure 4: Productivity catch-up and average Capital Inflows between 1980 and 2010, Low level of Financial Development

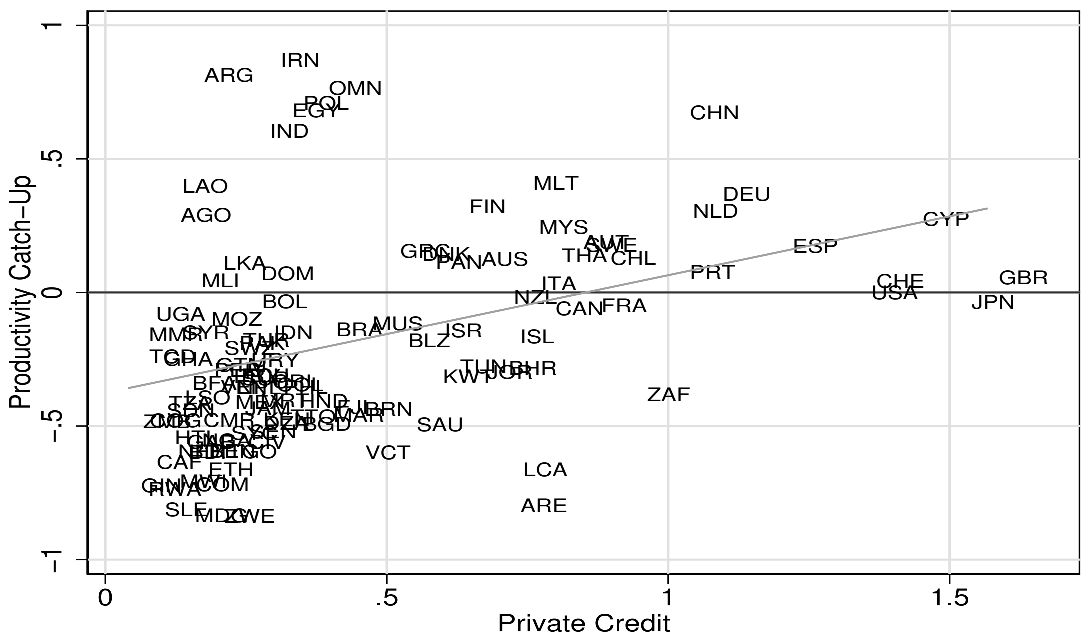

Figure 5: Average Private Credit and Productivity catch-up between 1980 and 2010

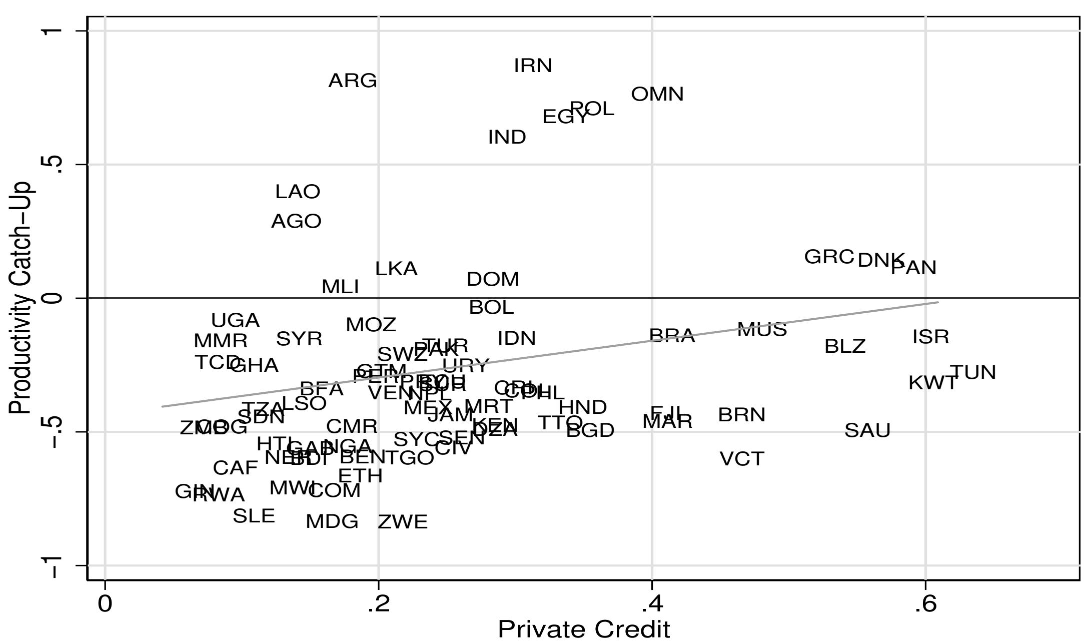

Figure 6: Average Private Credit and Productivity catch-up between 1980 and 2010, Low level of Financial Development

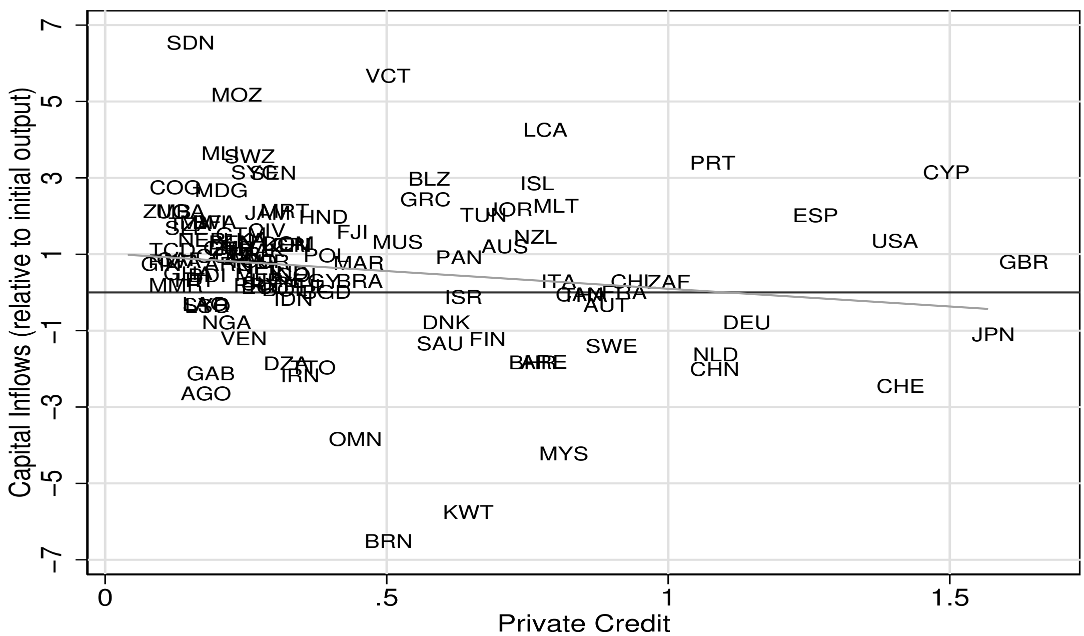

Figure 7: Average Private Credit and average Capital Inflows between 1980 and 2010

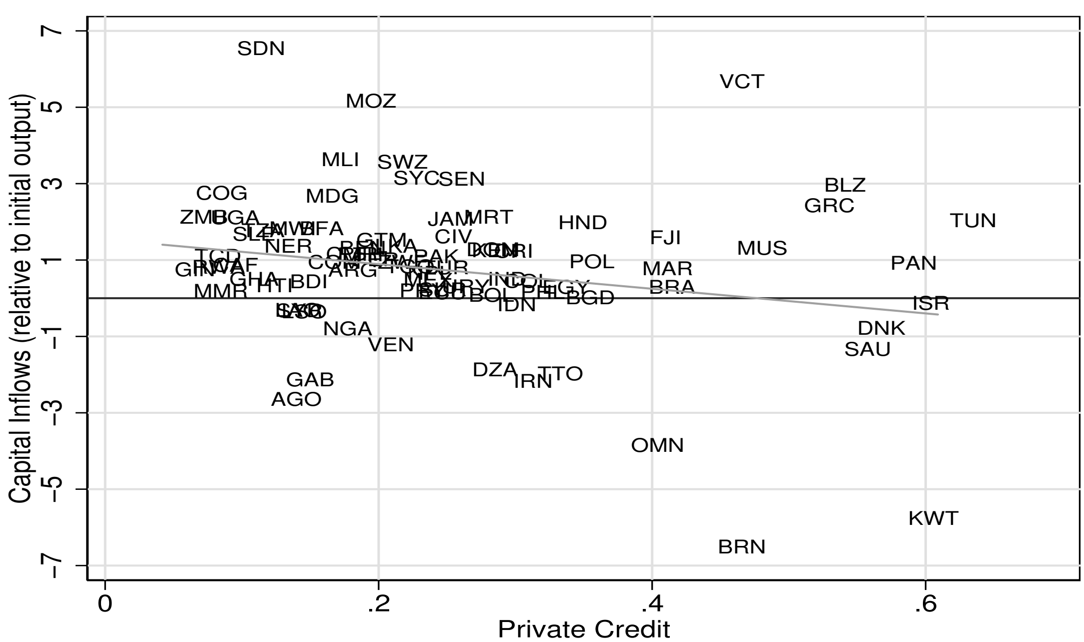

Figure 8: Average Private Credit and average Capital Inflows between 1980 and 2010, Low level of Financial Development