ESG RECHERCHE

DOCUMENT DE TRAVAIL / WORKING PAPER

No. 2018-09

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Janvier 2018

ESG UQÀM

Département des sciences économiques
École des sciences de la gestion
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Working Time Regulation, Unequal Lifetimes and Fairness*

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December 1, 2017

Abstract

We examine the redistributive impact of working time regulations in an economy with unequal lifetimes. We first compare the laissez-faire equilibrium with the *ex post* egalitarian optimum, where the realized lifetime well-being of the worst off (usually the short-lived) is maximized, and show that, unlike the laissez-faire, this social optimum involves an increasing working time age profile and equalizes the realized lifetime well-being of the short-lived and the long-lived. We then examine whether working time regulations can compensate the short-lived. It is shown that uniform working time regulations cannot improve the situation of the short-lived with respect to the laissez-faire, and can only reduce well-being inequalities at the cost of making the short-lived worse off. However, age-specific regulations involving lower working time for the young and higher working time for the old make the short-lived better off, even though such regulations may not fully eradicate well-being inequalities.

Keywords: Longevity, premature deaths, labor supply, working time regulations, compensation.

JEL codes: J10, J18, J22.

*We would like to thank K. Behrens, V. Barham, P. De Donder, L. Larue, J. Martin, F. Moizeau, P. Pestieau, M. Yazbeck and D. Zwarthoed for their comments on this paper. We are also grateful to two anonymous referees for their remarks and suggestions on this manuscript.

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1 Introduction

Limiting the number of hours an individual can work in a day is an old idea, which dates back to the first attempts to characterize the contours of an ideal society. In his *Utopia*, Thomas More (1516) described an island where individuals would work 6 hours a day, the rest of time being dedicated to education. In Tommaso Campanella's *City of the Sun* (1602), each adult would work only 4 hours a day. The main motivation behind these caps on working time was the anti-materialistic idea that goods should serve humans, as opposed to humans becoming instruments of goods.

Since the early 19th century, workers' movements have called for working time reductions. Following those social movements, governments introduced working time regulations specifying a maximum number of hours worked every week. The effects of working time reductions have been studied in details by economists who estimated their impact on unemployment (Crépon and Kramartz 2002, Chemin and Wasmer 2009), firm productivity (Crépon et al 2004), actual hours worked (Hunt 1999), and female labor supply (Goux et al 2011).[1]

Working time regulations also have significant redistributive effects. Working time reductions that are compensated (i.e., carried out at constant total labor earnings) imply a rise in hourly wage, which affects the distribution of income among factors of production.[2] But beyond the impact of working time reductions on the wage share, little attention has been paid so far to the redistributive effects of working time regulations from a *lifecycle* perspective.

In a world with equal longevity, working time regulations would be neutral from a lifecycle perspective. Everyone would go through all stages of life and face the same constraints imposed by working time regulations. However, under the more realistic assumption of unequal lifetimes, working time regulations are no longer neutral. By affecting working time and budget constraints in distinct ways across long-lived and short-lived workers, these regulations influence the distribution of well-being across workers with different longevity.

The goal of this paper is to examine the redistributive consequences of working time regulations in an economy where individuals have unequal lifetimes. We study the conditions under which working time regulations can be used in order to provide some compensation to the worst off, who is in general the short-lived. By "compensating the short-lived", we mean making the short-lived better off with respect to the laissez-faire equilibrium (i.e. in the absence of regulation) and possibly - but not necessarily - eradicating inequalities in lifetime well-being between the long-lived and the short-lived.

The study of the compensation of the short-lived is motivated by the Principle of Compensation (Fleurbaey and Maniquet 2004, Fleurbaey 2008), according to which well-being inequalities due to circumstances should be abolished. This principle applies to well-being inequalities due to unequal lifetime, since a large

[1] See Askenazy (2013) for a survey of these studies.

[2] Holmlund and Pencavel (1988) and Friesen (2000) show that a reduction in the working time is associated with a significant rise in the hourly wage in countries such as Sweden, the Netherlands and Canada.

part of longevity inequalities are independent of individual choices.[3] Hence, individuals cannot be held responsible for those inequalities. According to the Principle of Compensation, governments should therefore intervene so as to abolish those well-being inequalities due to circumstances.

Compensating an individual for a premature death raises several specific difficulties. As stressed in Fleurbaey et al (2014), a major difficulty with the compensation of short-lived individuals resides in the fact that *ex ante* (i.e., before the duration of life is revealed), short-lived individuals cannot be identified, whereas, *ex post* (i.e., after the duration of life is revealed), it is too late to affect the well-being of the short-lived. Another, related difficulty is that, whereas Fleurbaey and Maniquet's (2004) defence of the Principle of Compensation focuses on situations where the worst off agrees with being compensated, this may not be the case here, because individuals do not know *ex ante* that they will be short-lived (and thus the worst off). Thus there may be a conflict between, on the one hand, what young individuals (who include those who will turn out to be short-lived) choose *ex ante* in terms of consumption and working time profiles, and, on the other hand, the consumption and working time profiles that maximize the well-being of the short-lived *ex post*.

This paper examines the difficulties raised by the compensation for a premature death, and pays particular attention to the conditions under which imposing working time regulations can help compensating the short-lived. Our analysis, which relies on a lifecycle model with risky lifetime, proceeds in three steps.[4]

We first characterize the laissez-faire equilibrium, where individuals choose their working time freely, and compare it with the *ex post* egalitarian social optimum, where the realized lifetime well-being of the short-lived is maximized subject to the resource constraint and the egalitarian constraint (i.e. the long-lived should not be worse off than the short-lived). We show that, in affluent economies, this social optimum involves, unlike the laissez-faire equilibrium, an increasing age profile in terms of worked hours, as well as a decreasing age profile in terms of consumption. By transferring leisure time and consumption to the young (who include those who will turn out to be short-lived), the *ex post* egalitarian optimum improves the situation of the short-lived, and allows for an equalization of the realized lifetime well-being of short-lived and long-lived individuals, despite the non-identification *ex ante* of the premature dead. That social optimum can be decentralized by means of lump sum transfers and a tax on saving, which induce individuals to opt for a decreasing consumption age profile and an increasing working time age profile.

In a second stage, we consider the problem of a government who behaves like a Stackelberg leader and selects a uniform working time regulation that maximizes the realized lifetime well-being of the short-lived (subject to the resource

[3] Christensen et al (2006) emphasize that about 1/4 to 1/3 of longevity inequalities are due to the genetic background, over which individuals have little control.

[4] For the sake of presentation, our results are here given only for the case of affluent economies (where productivity is sufficiently high so that the long-lived is better off than the short-lived). See the manuscript for the results concerning poor economies (where productivity is so low that the long-lived is necessarily worse off than the short-lived).

constraint and the egalitarian constraint), while anticipating the behavioral response of individuals in terms of saving. We obtain negative results: uniform working time regulations cannot improve the situation of the short-lived with respect to the laissez-faire, and those regulations could only reduce well-being inequalities at the cost of making the short-lived worse off.[5]

Finally, we study the problem of a government who selects a non-uniform, age-specific working time regulation that maximizes the realized lifetime well-being of the short-lived (subject to the same constraints as above), while anticipating, here again, the response of individuals in terms of saving. We show that age-specific working time regulations involving lower working time for the young can make the short-lived better off than at the laissez-faire. Indeed, although individuals still smooth their consumption, it is possible, by forcing the young to work less and the old to work more, to make the short-lived better off, thanks to more consumption and less labor at the young age. Note, however, that those regulations may not suffice to fully eradicate well-being inequalities.[6]

Those analytical findings are illustrated by numerical simulations, which show that the *ex post* egalitarian optimum would involve, at the young age, a strong working time reduction, close to the standards of Campanella's *City of the Sun* (20 hours a week), whereas old-age working time would be increased with respect to current regulations. Our simulations also show that age-specific working time regulations can significantly improve the situation of the short-lived, and reduce inequalities in realized lifetime well-being. Hence, our calculations provide a second-best argument supporting, on the grounds of fairness with respect to the unlucky short-lived, age-specific working time regulations involving an increasing working time age profile.

This paper complements the large literature on the effects of working time regulations by studying their redistributive consequences in an economy peopled of short-lived and long-lived workers. This paper also complements recent studies on how the government could reduce inequalities in lifetime well-being due to unequal lifetimes, as in Fleurbaey and Ponthiere (2013) and Fleurbaey et al (2016). The former paper examines prevention choices without considering production. The latter paper focuses only on the choice of a retirement age without considering intensive margins in labor choices. We complement those papers by considering the impact of working time regulations on inequalities in lifetime well-being between short-lived and long-lived individuals.

The paper is organized as follows. Section 2 presents the model. The laissez-faire equilibrium is studied in Section 3. Section 4 characterizes the *ex post* egalitarian optimum and studies its decentralization. Section 5 examines the redistributive consequences of imposing uniform working time regulations. Age-specific working time regulations are studied in Section 6. Finally, Section 7 illustrates our findings numerically. Section 8 concludes.

[5] The reason why imposing a uniform working time age profile cannot increase consumption or well-being at the young age in comparison to the laissez-faire is that the intratemporal arbitrage between labor and consumption is already solved optimally at the laissez-faire.

[6] The capacity of age-specific working time regulations to bring well-being equality is here limited by the upper bound on the labor supply of the surviving old.

2 The model

We consider a 2-period economy. Each period has a duration normalized to 1. The population is a continuum of individuals of size 1.

Period 1 is young adulthood, during which individuals work $\ell_y \in [0,1]$ units of time, save and consume.[7] Period 2 is old adulthood, during which individuals work $\ell_o \in [0,1]$ units of time and consume.[8] Lifetime is risky: only a fraction $\pi \in [0,1]$ of young adults reach the old age. Thus, although all individuals are identical *ex ante* (i.e., before the duration of individual life is revealed), they will differ *ex post*, since some of them (in proportion π) will turn out to be long-lived, whereas others (in proportion $1-\pi$) will turn out to be short-lived.

Individual preferences on lotteries of life satisfy the expected utility hypothesis, and lifetime well-being is supposed to be time-additive. Normalizing the utility of being dead to zero, preferences are represented by:

$$u(c) - v(\ell_y) + \pi \left[u(d) - v(\ell_o)\right], \tag{1}$$

where c denotes consumption at young adulthood, d denotes consumption at mature adulthood, and $u(\cdot)$ satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. We assume that there exists a consumption level $\bar{c} \geq 0$ such that $u(\bar{c}) = 0$. The disutility of labor is supposed to be increasing and convex: $v'(\cdot) > 0$ and $v''(\cdot) > 0$. We also have $v(0) = 0$ and $v'(0) = 0$. Note that the utility function exhibits no pure time preferences. However, the probability to survive to the old age π can be interpreted as a biological discount factor.

The labor market is assumed to be perfectly competitive, with an hourly wage equal to $w > 0$.[9]

There exists a perfect annuity market, with actuarially fair return:

$$\hat{R} = \frac{R}{\pi}, \tag{2}$$

where R equals one plus the interest rate. For the sake of simplicity, we assume that $R = 1$.

The first-period budget constraint is:

$$c + s = \ell_y w, \tag{3}$$

where s denotes saving. The second-period budget constraint is:

$$d = \frac{s}{\pi} + \ell_o w. \tag{4}$$

The intertemporal resource constraint is thus:

$$c + \pi d = \ell_y w + \pi \ell_o w. \tag{5}$$

[7] We abstract here from the childhood period, which is not relevant for the issue at stake.

[8] We abstract from the retirement decision. See Fleurbaey et al (2016) on the selection of fair retirement age under fixed working time.

[9] Throughout this paper, we assume, without loss of generality, that $w > \bar{c}$. This assumption allows us to avoid extreme cases that are irrelevant for the issue at stake.

3 The laissez-faire equilibrium

As a starting point, let us consider an economy without any working time regulation, that is, an economy where individuals can freely choose how many hours they work at each point in their life (i.e. the intensive margins of labor).

At the laissez-faire equilibrium, individuals choose their consumptions c and d as well as the quantities of hours worked ℓ_y and ℓ_o in such a way as to maximize their expected lifetime well-being subject to their resource constraint:

$$\begin{aligned}
&\max_{c,d,\ell_y,\ell_o} u\left(c\right)-v\left(\ell_y\right)+\pi\left[u\left(d\right)-v\left(\ell_o\right)\right]\\
&\text{s.t. } \ell_y w+\pi\ell_o w\geq c+\pi d\\
&\text{s.t. } \ell_y\geq 0 \text{ and } 1-\ell_y\geq 0\\
&\text{s.t. } \ell_o\geq 0 \text{ and } 1-\ell_o\geq 0
\end{aligned}$$

The last four constraints guarantee that $0\leq \ell_y,\ell_o\leq 1$.

Substituting for the resource constraint, the Lagrangian can be written as:

$$\max_{d,\ell_y,\ell_o} u\left(\ell_y w+\pi\ell_o w-\pi d\right)-v\left(\ell_y\right)+\pi\left[u\left(d\right)-v\left(\ell_o\right)\right]+\varphi_y\ell_y+\rho_y\left(1-\ell_y\right)+\varphi_o\ell_o+\rho_o(1-\ell_o)$$

where φ_y (resp. φ_o) is the Lagrange multiplier associated to the constraint $\ell_y\geq 0$ (resp. $\ell_o\geq 0$) and ρ_y (resp. ρ_o) is the Lagrange multiplier associated to the constraint $\ell_y\leq 1$ (resp. $\ell_o\leq 1$). First-order conditions (FOCs) yield:

$$\begin{aligned}
\pi u'(c) &= \pi u'(d), && (6)\\
wu'(c) &= v'(\ell_y)+\rho_y-\varphi_y, && (7)\\
wu'(c) &= v'(\ell_o)+\frac{\rho_o}{\pi}-\frac{\varphi_o}{\pi}, && (8)
\end{aligned}$$

as well as

$$\begin{aligned}
\varphi_y &\geq 0,\ell_y\geq 0,\\
\varphi_o &\geq 0,\ell_o\geq 0,\\
\rho_y &\geq 0,1-\ell_y\geq 0,\\
\rho_o &\geq 0,1-\ell_o\geq 0,
\end{aligned}$$

with complementary slackness.

The first FOC implies that, at the laissez-faire equilibrium, consumption is smoothed along the life cycle: $c=d$. The reason lies in the existence of a perfect annuity market with actuarially fair returns, combined with the absence of pure time preferences. The FOCs for labor supply correspond to what Jevons (1871) calls the "final equivalence of labor and utility". At the interior optimum, working time should equalize the utility gain from additional consumption obtained from one additional hour of work (i.e., $wu'(c)$) to the disutility from that additional hour (i.e., $v'\left(\ell_y\right)$ at the young age and $v'(\ell_o)$ at the old age). Given that consumption is smoothed, we also have that labor is smoothed: $\ell_y=\ell_o=\ell^{LF}$. Proposition 1 summarizes our results.

Proposition 1
- *If $wu'(w) < v'(1)$, there exists a unique laissez-faire equilibrium with an interior working time at each period. At that laissez-faire equilibrium, we have: $c^{LF} = d^{LF} = \ell^{LF} w$ and $\ell_y = \ell_o = \ell^{LF} \in]0, 1[$.*
- *If $wu'(w) \geq v'(1)$, there exists a unique laissez-faire equilibrium with corner working time at each period. At that laissez-faire equilibrium, we have: $c^{LF} = d^{LF} = \ell^{LF} w$ and $\ell_y = \ell_o = \ell^{LF} = 1$.*

Proof. See the Appendix. ■

Thus, in the absence of working time regulation, individuals perfectly smooth consumption and labor over time (i.e., $c^{LF} = d^{LF}$ and $\ell_y = \ell_o = \ell^{LF}$). As a consequence of the absence of retirement, individuals do not save at all, but consume at each period the product of their labor, that is, $w\ell^{LF}$. An interesting feature of the laissez-faire is that the consumption and working time profiles are independent from the survival probability π. This is a direct consequence of assuming a perfect annuity market.[10]

Note that Proposition 1 describes not only the laissez-faire equilibrium, but also the utilitarian social optimum. Actually, a utilitarian social planner maximizing the average realized utility within the population (or alternatively the expected utility of a representative agent) would choose consumption and labor profiles that are exactly the same as the ones prevailing at the laissez-faire.

Let us now examine the prevalence of inequalities at the laissez-faire. Within our model, all individuals are *ex ante* identical and make the same decisions concerning consumption and labor profiles. All individuals thus enjoy the same expected lifetime well-being. However, once the durations of life are revealed (i.e., *ex post*), the population becomes composed of a fraction π of long-lived individuals and of a fraction $1 - \pi$ of short-lived individuals. Those two groups enjoy different levels of realized lifetime well-being. Short-lived individuals have a realized lifetime well-being equal to $u\left(\ell^{LF} w\right) - v\left(\ell^{LF}\right)$, whereas for long-lived individuals it is equal to $2\left[u\left(\ell^{LF} w\right) - v\left(\ell^{LF}\right)\right]$. Hence short-lived individuals are worse off than long-lived individuals if and only if:

$$U^{LL} - U^{SL} = u\left(\ell^{LF} w\right) - v\left(\ell^{LF}\right) > 0.$$

where U^{LL} (resp. U^{SL}) denote the lifecycle utility of a long-lived (resp. short-lived) individual.

Whether the above inequality is valid or not depends on the level of the wage and on the functional forms for $u(\cdot)$ and $v(\cdot)$. Regarding the effect of the wage, one can show that a rise in the wage increases the differential $U^{LL} - U^{SL}$. Indeed, differentiating $U^{LL} - U^{SL}$ with respect to w yields:

$$u'(c^{LF})\frac{dc^{LF}}{dw} - v'(\ell^{LF})\frac{d\ell^{LF}}{dw} = u'(c^{LF})\ell^{LF} > 0,$$

[10] Indeed, substituting for constant consumption and labor profiles into the intertemporal budget constraint yields:

$$c^{LF}(1+\pi) = \ell^{LF} w(1+\pi).$$

Thus the life expectancy $1 + \pi$ can be simplified from both sides, making the laissez-faire consumption independent from π. But given the "final equivalence of labor and utility", if consumption is independent from π, labor supply is also independent from π.

where we made use of the FOC on ℓ^{LF}.

In affluent economies, where productivity is high, w is large, so that the long-lived are better off than the short-lived. On the contrary, in poor economies, where productivity is low, the opposite case arises and the long-lived are worse off than the short-lived: $U^{LL} - U^{SL} < 0$.[11] This situation is unusual as it means that the wage is so low that individuals would prefer to die early rather than surviving to the old age. Given that this case is quite extreme, we will, throughout this manuscript, focus mainly on affluent economies and discuss this other possibility only for the sake of completeness.

4 The ex post egalitarian optimum

Inequalities in realized lifetime well-being between the long-lived and the short-lived can be questioned from an ethical perspective. Actually, in our model, all individuals are perfectly identical *ex ante*, that is, before the duration of life is revealed. No one can do anything to influence his or her survival chances. Longevity inequalities are thus arbitrary and can be regarded as circumstances on which individuals have no influence. Hence, if one adheres to the Principle of Compensation (Fleurbaey and Maniquet, 2004, Fleurbaey 2008), according to which well-being inequalities due to circumstances should be abolished, it follows that the government should intervene to reduce inequalities in realized lifetime well-being due to unequal exogenous lifetimes.

Applying the Principle of Compensation in the context of unequal lifetimes raises several difficulties. A first major problem concerns the identification of those who will turn out to be the worst-off. *Ex ante* (i.e. before the duration of each life is known), the government can hardly identify individuals who will turn out to be the worst-off (i.e., in affluent economies, the short-lived). Indeed, governments have access to large statistical information on factors affecting survival, but have no information at the individual level, which is the level relevant for compensation. *Ex post*, that is, once the duration of each life is known, short-lived persons are identified, but it is too late to compensate them.

Another related problem that arises in the context of affluent economies is that, since individuals do not know *ex ante* whether they will be short-lived or not, there may disagree with government policies aimed at compensating the short-lived. In other words, young individuals (who include the short-lived), when planning their life cycle, may disagree with being compensated, since they do not know at that time that they will turn out to be short-lived.

In the following, we study the compensation for unequal lifetimes by considering the *ex post* egalitarian social welfare criterion, which takes as objective the maximization of the realized lifetime well-being of the worst-off in the society.[12] As discussed in Fleurbaey et al (2014) in a model with fixed labor supply,

[11] In the Appendix we derive, under standard functional forms for $u(\cdot)$ and $v(\cdot)$, the threshold for the wage rate w beyond which the long-lived are better off than the short-lived.

[12] Note that, in order to do justice to the idea of compensating the worst-off, one cannot, in the present context, rely on a utilitarian social welfare function, since, as we showed in

this social welfare objective leads to selecting the allocation that maximizes the realized lifetime well-being of the worst-off, that is, his well-being *ex post*, i.e. once the durations of life revealed. As such, this social criterion deliberately abstracts from preferences on non-degenerate lotteries of life, in order to focus only on preferences over degenerate lotteries (i.e., either short life or long life).

This focus on the distribution of realized lifetime well-being rather than on the distribution of expected lifetime well-being can, at first glance, be criticized as a form of extreme paternalism. However, focusing on realized lifetime well-being can be justified on the grounds that, from the perspective of individual lifetime well-being, only achievements matter, and not what individuals expected to achieve given their beliefs and anticipations. In our context, having a lower or higher life expectancy is irrelevant to assess individual lifetime well-being: what really matters is how long individuals actually live.[13] Moreover, one can also justify the *ex post* egalitarian social welfare objective on the grounds that, in affluent economies, individuals who turn out to be the worst off (i.e. the short-lived) would, in the hypothetical case where they would still be alive, prefer the *ex post* egalitarian allocation over any other allocation (including the laissez-faire) that does not make the long-lived worse off than the short-lived.

4.1 The social planner's problem

Under the *ex post* egalitarian social welfare criterion, and taking into account the impossibility to identify the short-lived *ex ante*, the social planner's problem can be written as the maximization of the minimum level of realized lifetime well-being, subject to the resource constraint of the economy:

$$\begin{aligned}
&\max_{c,d,\ell_y,\ell_o} \min \left\{ u\left(c\right) - v\left(\ell_y\right), u\left(c\right) - v\left(\ell_y\right) + u\left(d\right) - v\left(\ell_o\right) \right\} \\
&\text{s.t. } \ell_y w + \pi \ell_o w \geq c + \pi d \\
&\text{s.t. } \ell_y \geq 0 \text{ and } 1 - \ell_y \geq 0 \\
&\text{s.t. } \ell_o \geq 0 \text{ and } 1 - \ell_o \geq 0
\end{aligned}$$

where the last four constraints guarantee that $0 \leq \ell_y, \ell_o \leq 1$.

Note that, because of the identification constraint, the social planner cannot distinguish between the short-lived and the long-lived. As a consequence, the realized lifetime well-being of the short-lived, i.e., $u\left(c\right) - v\left(\ell_y\right)$, must be identical to the realized well-being of the long-lived when being young, i.e., $u\left(c\right) - v\left(\ell_y\right)$.

The above objective function is not differentiable. But we can rewrite that problem as the maximization of first-period utility subject to the constraint that the long-lived is not worse off than the short-lived. The Lagrangian of that

Section 3, this social criterion legitimates the laissez-faire and the resulting inequalities in realized lifetime well-being between short-lived and long-lived individuals.

[13] In particular, the fact that a prematurely dead person expected to have a long life does not seem to be a relevant piece of information to assess his lifetime well-being.

problem can be written as:

$$\max_{d,\ell_y,\ell_o} \quad u(\ell_y w + \pi \ell_o w - \pi d) - v(\ell_y) + \mu \left(u(d) - v(\ell_o)\right)$$
$$+\gamma_y \ell_y + \gamma_o \ell_o + \kappa_y \left(1 - \ell_y\right) + \kappa_o \left(1 - \ell_o\right)$$

where μ is the Lagrange multiplier associated to the egalitarian constraint $u(d) - v(\ell_o) \geq 0$, γ_y (resp. γ_o) is the Lagrange multiplier associated to the constraint $\ell_y \geq 0$ (resp. $\ell_o \geq 0$) and κ_y (resp. κ_o) is the Lagrange multiplier associated to the constraint $\ell_y \leq 1$ (resp. $\ell_o \leq 1$). FOCs are:

$$u'(c) = \frac{\mu}{\pi} u'(d), \tag{9}$$
$$wu'(c) = v'(\ell_y) + \kappa_y - \gamma_y, \tag{10}$$
$$wu'(c) = \frac{\mu}{\pi} v'(\ell_o) + \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}, \tag{11}$$

as well as the conditions

$$\begin{aligned} \mu &\geq 0, u(d) - v(\ell_o) \geq 0, \\ \gamma_y &\geq 0, \ell_y \geq 0, \\ \gamma_o &\geq 0, \ell_o \geq 0, \\ \kappa_y &\geq 0, 1 - \ell_y \geq 0, \\ \kappa_o &\geq 0, 1 - \ell_o \geq 0, \end{aligned}$$

with complementary slackness. Proposition 2 summarizes our results.

Proposition 2

- *At the ex post egalitarian optimum, the egalitarian constraint holds necessarily:* $u(d^*) = v(\ell_o^*) \implies U^{SL} = U^{LL}$.
- *Regarding consumption and labor profiles, two cases can arise:*
  - *In the affluent economy* ($\frac{\mu}{\pi} < 1$)*, we have:* $\bar{c} < d^* < c^*$ *and three cases are possible: either* $0 < \ell_y^* < \ell_o^* < 1$ *or* $0 < \ell_y^* < 1 = \ell_o^*$ *or* $\ell_y^* = \ell_o^* = 1$.
  - *In the poor economy* ($\frac{\mu}{\pi} > 1$)*, we have* $c^* < d^*$, $\bar{c} \lesseqgtr c^*$ *and* $\bar{c} < d^*$*, and three cases are possible: either* $0 < \ell_o^* < \ell_y^* < 1$ *or* $0 < \ell_o^* < 1 = \ell_y^*$ *or* $\ell_y^* = \ell_o^* = 1$.

Proof. See the Appendix. ■

The *ex post* egalitarian optimum differs from the laissez-faire equilibrium (and the utilitarian social optimum), since it does not involve flat consumption profiles and flat labor profiles. When the shadow price of relaxing the egalitarian constraint is low ($\mu < \pi$), the egalitarian optimum involves a decreasing consumption profile with the age, as well as an increasing labor profile with the age. Hence, according to that social criterion, old workers should work more than younger ones. On the contrary, when the shadow price of relaxing the egalitarian constraint is high ($\mu > \pi$), the egalitarian optimum involves a consumption profile increasing with the age and decreasing labor supply. That second case corresponds to a situation of extreme poverty where resources are so scarce that living longer decreases lifetime well-being.

Focusing on the first, more plausible case, we obtain that the *ex post* egalitarian optimum involves a decreasing consumption age-profile and an increasing labor age-profile. The intuition behind that result goes as follows. Given that short-lived individuals only enjoy the young age, the social planner can only make the short-lived better off by transferring consumption and leisure towards the young age, while reducing old-age consumption and leisure. Forcing older workers to work more hours and to consume less implies that more resources (goods and leisure time) can be transferred to the young age. This increases the realized lifetime well-being of individuals who will turn out to be short-lived.

Although the social planner does not know *ex ante* who will be short-lived or long-lived, imposing decreasing consumption profiles and increasing labor profiles for *all* individuals suffices to improve the situation of the unlucky individuals who will turn out to be short-lived (in comparison with the laissez-faire).[14] Thus, the *ex post* egalitarian social optimum overcomes the identification problem mentioned previously. Moreover, if individuals who turned out to die prematurely could rank consumption and labor profiles, they would prefer the ones prevailing at the *ex post* egalitarian optimum to those prevailing at the laissez-faire.

Note also that we have, at the *ex post* egalitarian optimum, a full equalization of the realized lifetime well-being of short-lived and long-lived individuals (i.e., $U^{LL} = U^{SL}$), since the egalitarian constraint is necessarily binding at that optimum, even when the optimal working time of the old is a corner solution. The underlying intuition is that, if the egalitarian constraint were not satisfied when $\ell_o^* = 1$, it could be possible to reduce d^* and increase c^* until we obtain $u(d^*) = v(1)$, and, hence, until lifetime well-being is equalized between the long-lived and the short-lived. A corner solution for ℓ_o^* does not prevent here the equalization of lifetime well-being.

Thus, although the fraction π of the population enjoys a longer lifetime than the fraction $1 - \pi$, this does not prevent equality in realized lifetime well-being. Longevity inequality due to Nature has here no effect on realized lifetime well-being, since old-age consumption is fixed to a level such that the utility of old-age consumption is equal to the disutility of old-age labor, that is, such that the surviving old are indifferent between dying prematurely and surviving. The realized lifetime well-being levels associated to a long life and a short life are, by construction of consumption and labor profiles, exactly equal in the two cases.

[14] The underlying intuition goes as follows. At the *ex post* egalitarian optimum, consumption and working time profiles are chosen so as to maximize the well-being of the short-lived, whereas at the laissez-faire, consumption and working time profiles are chosen in such a way as to maximize a weighted sum of the well-being of the short-lived and the well-being at the old age. Given that the resource constraint is the same in the two optimization problems, the short-lived must be better off at the *ex post* egalitarian optimum than at the laissez-faire.

4.2 Decentralization

As shown above, the *ex post* egalitarian optimum maximizes the realized lifetime well-being of the worst off, and implies a full equalization of realized lifetime well-being across long-lived and short-lived individuals. That social optimum leads thus to fully neutralize the impact of unequal lifetimes on realized lifetime well-being. More importantly, that optimum is, from an *ex post* view point, strictly preferred by the worst off over any other feasible allocation.

Note, however, that individuals do not know *ex ante* whether they will be long-lived or short-lived, and that their decisions in terms of consumption profile and working time profile (studied in Section 3) differ from the profiles under the *ex post* egalitarian optimum. How could then a government manage to induce individuals to opt for the socially optimal profiles in terms of consumption and working time? To answer that question, this subsection studies the tax-schedule that would allow to decentralize the *ex post* egalitarian optimum.

Comparing the FOCs in Section 3 with those in Section 4.1, it can be shown that the decentralization of the *ex post* egalitarian social optimum requires, in affluent economies, a tax on the return of saving τ, so as to induce the optimal decreasing consumption pattern, as well as lump sum transfers so as to satisfy the egalitarian constraint. To see this, let us reconsider the choice of consumption profile by an individual. Under a tax on saving τ, the FOC for saving obtained from the individual's problem becomes $u'(c) = (1-\tau)u'(d)$ while the socially optimal profile satisfies $u'(c^*) = \frac{\mu}{\pi}u'(d^*)$. Hence, in order to induce the socially optimal consumption profile, the government needs to set the tax on saving returns equal to:

$$\tau^* = 1 - \frac{\mu}{\pi} = 1 - \frac{u'(c^*)}{u'(d^*)}.$$

In affluent economies (i.e. $\mu < \pi$), individuals therefore face a tax on saving which induces them to have the optimal decreasing consumption profile. The government also needs to impose lump sum transfers $T^* = d^* - d^{LF}$ so as to ensure that the egalitarian constraint, $u(d^*) = v(\ell_o^*)$ is satisfied at the decentralized optimum. Under those two instruments (τ^*, T^*), the FOCs for labor will lead to the optimal levels ℓ_y^* and ℓ_o^* when consumptions take their optimal levels c^* and d^*.

In the case of poor economies (i.e. $\mu > \pi$), individuals face a subsidy on saving, so as to induce an increasing consumption profile. One needs also lump sum transfers. Proposition 3 summarizes our results.

Proposition 3 *In an affluent economy (resp. poor economy), the ex post egalitarian optimum can be decentralized by means of a tax (resp. a subsidy) on saving returns equal to:* $\tau^* = 1 - wu'(c^*)/v'(\ell_o^*)$*, as well as lump sum transfers* $T^* = d^* - d^{LF}$ *so as to satisfy the egalitarian constraint.*

Proof. See above. ■

Thus, although unequal longevity leads to large welfare inequalities at the laissez-faire equilibrium, Proposition 3 shows that it is possible for governments, by imposing particular taxes on saving and lump sum transfers, to decentralize the *ex post* egalitarian optimum, at which the realized lifetime well-being of the short-lived is maximized, and at which realized lifetime well-being is equalized across all individuals. As shown by the tax formula, the tax on saving returns should be positive in an affluent economy, so as to induce the optimal decreasing consumption profiles and increasing working time profiles. On the contrary, in a poor economy, saving should be subsidized, so as to induce the optimal increasing consumption profiles and decreasing working time profiles.

Note that the decentralization of the *ex post* egalitarian social optimum does not require any working time regulation. Actually, the two instruments (τ^*, T^*) are sufficient to decentralize the social optimum. The underlying intuition goes as follows. By taxing saving, the government will, on the basis of Jevons's (1871) "final equivalence of labor and utility", increase the marginal utility gain from consumption at the old age, which will induce old individuals to work more, in line with the social optimum. Thus working time regulations are not needed in a first-best world.

While Proposition 3 shows that a tax on saving and lump sum transfers suffice to decentralize the *ex post* egalitarian optimum, one may wonder whether such a decentralization is realistic. Actually, in the real world, there exist strong political constraints, and those policy instruments may not be available. This motivates us to depart from this first-best problem and to consider other policy instruments that exist in real economies. Given that this paper focuses on working time regulations, a natural question that arises is whether existing uniform working time regulations can decentralize the *ex post* egalitarian optimum. That issue is examined in the next section.

5 Uniform working time regulations

Let us now examine whether imposing uniform working time regulations allows for an improvement of the situation of the unlucky short-lived with respect to the laissez-faire equilibrium, and, also, whether this can contribute to reduce inequalities between the long-lived and the short-lived.[15] The underlying motivation is that governments impose, since the 19th century, uniform working time regulations specifying a number of working hours per week. One can thus wonder whether imposing such regulations could be set so as to improve the situation of the short-lived with respect to the long-lived.

In order to examine that issue, let us consider the problem of a social planner who chooses a uniform working time $\bar{\ell}$ imposed on young and old workers, in such a way as to maximize the well-being of the short-lived, under the constraint that the long-lived is not worse off than the short-lived.[16]

[15] By "uniform" working time regulations, we mean regulations that consist in imposing flat working time profiles, i.e., a constant quantity of worked hours along the life cycle (i.e., $\bar{\ell}_y = \bar{\ell}_o = \bar{\ell}$). Non-uniform (age-specific) working time regulations are studied in Section 6.

[16] It should be stressed here that $\bar{\ell}$ does not denote an upper bound for the number of hours worked, but the exact number of hours worked. This assumption simplifies the presentation

In that modified social planning problem, the social planner does not directly control for individual consumption profiles, unlike in the previous section. The social planner acts here as a Stackelberg leader, who imposes a working time $\bar{\ell}$ while anticipating the reaction of individuals in terms of saving behavior.[17]

Regarding that behavioral reaction, we know from Section 3 that, given a particular working time profile, individuals smooth their consumption profile, so that $c = d$ which together with the individual's resource constraint implies $c = d = \bar{\ell}w$. Thus when choosing the optimal working time profile $\bar{\ell}$, the social planner anticipates that individual consumption behavior is such that $c = d = \bar{\ell}w$.

In the light of this, the problem of the social planner can be written as:

$$\begin{aligned}
&\max_{\bar{\ell}} \min \left\{ u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right), u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right) + u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right) \right\} \\
&\text{s.t. } \bar{\ell} \geq 0 \text{ and } 1 - \bar{\ell} \geq 0
\end{aligned}$$

As above, we can rewrite that problem as the maximization of first-period utility subject to the constraint that the long-lived is not worse off than the short-lived. The Lagrangian associated to the problem is:

$$\max_{\bar{\ell}} u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right) + \mu\left[u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right)\right] + \gamma\bar{\ell} + \kappa\left(1 - \bar{\ell}\right)$$

where μ is the Lagrange multiplier associated to the egalitarian constraint, while γ and κ are the Lagrange multipliers associated to the constraints $\bar{\ell} \geq 0$ and $\bar{\ell} \leq 1$.

The FOC is:

$$\left[wu'(\bar{\ell}w) - v'(\bar{\ell})\right](1 + \mu) = \kappa - \gamma, \tag{12}$$

as well as

$$\begin{aligned}
\mu &\geq 0, u\left(\bar{\ell}w\right) - v\left(\bar{\ell}\right) \geq 0, \\
\gamma &\geq 0, \bar{\ell} \geq 0, \\
\kappa &\geq 0, 1 - \bar{\ell} \geq 0,
\end{aligned}$$

with complementary slackness.

In the Appendix, we study those conditions, and we show that the solution of that social planning problem consists in imposing a working time level $\bar{\ell}$ exactly equal to the laissez-faire level ℓ^{LF}.

Proposition 4 *Consider a government imposing a uniform working time regulation $\bar{\ell}$ while anticipating the reaction of individuals in terms of saving.*

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of results, and is also close to real economies where individuals generally work full time, i.e. at the level of the uniform regulation.

[17] Given that the policy instrument considered here (a uniform working time $\bar{\ell}$) is extremely basic, one can regard this social planning problem as a kind of "third-best problem" (since this involves strong restrictions on available policy instruments).

- *The regulation $\bar{\ell}$ that maximizes the realized lifetime well-being of the short-lived coincides with the working time chosen at laissez-faire ℓ^{LF}, and, hence, cannot make the short-lived better off than at the laissez-faire. Inequalities remain thus the same as at the laissez-faire.*

- *If the government imposes another regulation $\tilde{\ell} \neq \ell^{LF}$, inequalities in realized lifetime well-being between the long-lived and the short-lived are reduced, but at the cost of making the short-lived worse off than at the laissez-faire.*

Proof. See the Appendix. ■

Proposition 4 states a negative result regarding the possibility of uniform working time regulations to improve the situation of the unlucky short-lived. Actually, it shows that the optimal uniform working time regulation coincides with the working time chosen by individuals at the laissez-faire. This makes the uniform working time regulation useless as a policy instrument aimed at improving the situation of the short-lived.

Moreover, Proposition 4 states that any other uniform working time regulation $\tilde{\ell} \neq \ell^{LF}$ would reduce inequalities in realized lifetime well-being between the long-lived and the short-lived, but at the cost of making the short-lived even worse off than at the laissez-faire. This is due to the fact that the utility obtained from this additional second period of life is exactly equal to the utility of a short-lived individual, i.e. $U^{LL} - U^{SL} = u(w\bar{\ell})$, when the regulation of labour supply is uniform. Hence, the only way to reduce well-being inequalities would here consists in deteriorating further the situation of the worst-off, which is clearly undesirable.

In the light of those negative results, uniform working time regulations can hardly help compensating the short-lived. This motivates us to consider, in the next section, another class of working time regulations, which are not uniform but age-specific.

6 Age-specific working time regulations

This section considers the impact of introducing age-specific working time regulations, which, unlike in Section 5, consists of constraints on working time that differ along the life cycle. The underlying intuition for introducing age-specific working time regulations is that, at the *ex post* egalitarian optimum, we have, in general, an increasing labor profile with the age (i.e., $\ell_y^* < \ell_o^*$). Hence it makes sense to examine the extent to which imposing age-specific labor standards - instead of uniform labor standards - can help making the short-lived better off in comparison to the laissez-faire, and can also help reducing well-being inequalities between the long-lived and the short-lived.

To examine this issue, let us consider the problem of a social planner who chooses age-specific working time regulations $\left(\bar{\ell}_y, \bar{\ell}_o\right)$ in such a way as to maximize the realized lifetime well-being of the short-lived, under the egalitarian constraint (i.e. the long-lived is not worse off than the short-lived). As in Section 5, the social planner does not control directly for the consumption profile, but, rather, behaves as a Stackelberg leader that imposes working time regulations while anticipating the reaction of individuals in terms of saving.[18] Since, in our economy, individuals smooth their consumption across periods, the budget constraint implies that, under age-specific working time regulations $\left(\bar{\ell}_y, \bar{\ell}_o\right)$, consumption in each period is equal to:

$$c = d = w\frac{\bar{\ell}_y + \pi\bar{\ell}_o}{1+\pi}. \tag{13}$$

Hence the social planner's problem can be written as:

$$\begin{aligned} &\max_{\bar{\ell}_y,\bar{\ell}_o}\min\left\{ \begin{array}{c} u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v\left(\bar{\ell}_y\right), \\ u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v\left(\bar{\ell}_y\right) + u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v\left(\bar{\ell}_o\right) \end{array} \right\} \\ &\text{s.t. } \bar{\ell}_y \geq 0 \text{ and } 1-\bar{\ell}_y \geq 0 \\ &\text{s.t. } \bar{\ell}_o \geq 0 \text{ and } 1-\bar{\ell}_o \geq 0 \end{aligned}$$

As above, the social planner's problem can be rewritten as the maximization of the realized lifetime well-being of the short-lived under the egalitarian constraint. The associated Lagrangian is:

$$\begin{aligned} \max_{\bar{\ell}_y,\bar{\ell}_o} \quad & u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v\left(\bar{\ell}_y\right) + \mu\left[u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v(\bar{\ell}_o)\right] \\ & +\gamma_y\bar{\ell}_y + \gamma_o\bar{\ell}_o + \kappa_y\left(1-\bar{\ell}_y\right) + \kappa_o\left(1-\bar{\ell}_o\right) \end{aligned}$$

where μ is the Lagrange multiplier associated to the egalitarian constraint, γ_y (resp. γ_o) is the Lagrange multiplier associated to the constraint $\bar{\ell}_y \geq 0$ (resp. $\bar{\ell}_o \geq 0$) and κ_y (resp. κ_o) is the Lagrange multiplier associated to the constraint $\bar{\ell}_y \leq 1$ (resp. $\bar{\ell}_o \leq 1$). The FOCs are:

$$wu'(c)\frac{1+\mu}{1+\pi} = v'(\bar{\ell}_y) + \kappa_y - \gamma_y, \tag{14}$$

$$wu'(c)\frac{1+\mu}{1+\pi} = \frac{\mu}{\pi}v'(\bar{\ell}_o) + \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}, \tag{15}$$

as well as

$$\begin{aligned} \mu &\geq 0, u\left(w\frac{\bar{\ell}_y+\pi\bar{\ell}_o}{1+\pi}\right) - v(\bar{\ell}_o) \geq 0, \\ \gamma_y &\geq 0, \bar{\ell}_y \geq 0 \text{ and } \kappa_y \geq 0, 1-\bar{\ell}_y \geq 0, \\ \gamma_o &\geq 0, \bar{\ell}_o \geq 0 \text{ and } \kappa_o \geq 0, 1-\bar{\ell}_o \geq 0, \end{aligned}$$

with complementary slackness. Proposition 5 summarizes our results.

[18] One can regard the social planning problem of Section 6 as a "second-best problem" since the set of policy instruments $\left(\bar{\ell}_y, \bar{\ell}_o\right)$ is here less constrained that in the problem of Section 5.

Proposition 5 *Consider a government imposing age-specific working time regulation* $(\bar{\ell}_y, \bar{\ell}_o)$ *while anticipating the reaction of individuals in terms of consumption and saving. Under the regulation* $(\bar{\ell}_y, \bar{\ell}_o)$ *that maximizes the realized lifetime well-being of the short-lived, the egalitarian constraint may be satisfied or not (unlike at the first-best, where it is always satisfied):*

- *If the egalitarian constraint holds, we have, in the affluent economy* ($\frac{\mu}{\pi} < 1$)*, either* $0 < \bar{\ell}_y < \bar{\ell}_o < 1$ *or* $0 < \bar{\ell}_y < \bar{\ell}_o = 1$ *or* $\bar{\ell}_y = \bar{\ell}_o = 1$*. In the poor economy* ($\frac{\mu}{\pi} > 1$)*, we have either* $0 < \bar{\ell}_o < \bar{\ell}_y < 1$ *or* $0 < \bar{\ell}_o < \bar{\ell}_y = 1$ *or* $\bar{\ell}_y = \bar{\ell}_o = 1$*.*

- *If the egalitarian constraint does not hold, we have, in the affluent economy* ($\frac{\mu}{\pi} < 1$)*, either* $0 < \bar{\ell}_y < \bar{\ell}_o = 1$ *or* $\bar{\ell}_y = \bar{\ell}_o = 1$*. In the poor economy* ($\frac{\mu}{\pi} > 1$)*, we have either* $0 < \bar{\ell}_o < \bar{\ell}_y = 1$ *or* $\bar{\ell}_y = \bar{\ell}_o = 1$*.*

Proof. See the Appendix. ■

In an affluent economy, the age-specific working time regulation that maximizes the realized lifetime well-being of the worst off involves an *increasing* working time age profile, whereas, in a poor economy, it involves a *decreasing* working time age profile. Indeed, in an affluent economy, the short-lived are worse off than the long-lived. However, it is possible, by reducing the working time of the young and by increasing the working time of the old, to increase temporal well-being at the young age, and, hence, to make the unlucky short-lived better off. The opposite argument holds in the poor economy.

It should be stressed that Proposition 5 rules out two extreme cases: on the one hand, the standard pension system where the young works and the old does not work (i.e. $\bar{\ell}_y > 0, \bar{\ell}_o = 0$), and, on the other hand, the inverted or reversed pension system, where the young would not work and the old would work (i.e. $\bar{\ell}_y = 0, \bar{\ell}_o > 0$). Proposition 5 shows that, from the perspective of the worst-off, it is always better that everyone - including the worst-off - works some amount of time (because of our assumption $v'(0) = 0$). Note that, if one allowed for $v'(0) > 0$, our results would be modified and we could find some support for an inverted pension system ($\bar{\ell}_y = 0, \bar{\ell}_o > 0$) in an affluent economy.

Note that, unlike what happens at the egalitarian first-best optimum, where there is always an equalization of realized lifetime well-being between the long-lived and the short-lived, this equalization does not necessarily hold at the second-best optimum. The reason why the egalitarian constraint may not hold lies in the fact that per period working time is bounded upwards (it cannot exceed 1). True, this limitation was also present in the first-best setting, but it should be reminded that, in the first-best, the social planner could directly control for the shape of the consumption profile, so that the fact that working time was bounded at 1 did not prevent to reach equality of lifetime well-being by reducing old-age consumption and increasing young-age consumption. That adjustment is no longer possible here, since the government only imposes working time age profiles, but leaves individuals choose their consumption age profile. This crucial difference explains why the age-specific working time regulation does not necessarily lead to equalizing realized lifetime well-being across individuals.

7 Numerical illustration

Let us now put some numbers into the model, in order to see the extent to which the *ex post* egalitarian optimum differs from the laissez-faire, and to examine the impact of age-specific working time regulations on the distribution of lifetime well-being across short-lived and long-lived individuals.

Throughout this section, we will assume that (period) utility of consumption and (period) disutility of work are represented by the functions:

$$u(c) = T_1 \frac{c^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \alpha \text{ and } v(\ell) = T_2 \frac{\beta \ell^2}{2},$$

where T_1 and T_2 are parameters capturing the fact that only a fraction of the period is worked, whereas consumption takes place during the whole period. For simplicity, we set $T_1 = 52$ and $T_2 = 47$, which amounts to assume that a fraction $\frac{47}{52}$ of a period is worked (i.e., 5 weeks holidays per year).

Regarding the calibration of preference parameters α, β and γ, we proceed as follows. Following Becker et al. (2005), we assume that $\alpha = -16.2$ and that $\gamma = 1.250$. Regarding the calibration of β, which captures the intensity of the disutility of labor, a major difficulty lies in the fact that the working time that would be chosen at the laissez-faire equilibrium is not observable, since in contemporary economies the working time is subject to regulations. Thus we can hardly take the observed working time as resulting from a free optimization problem of the agent. In order to study the robustness of our results to the disutility of labor, we will consider a large interval of values for $\beta \in]0, 27]$.

Regarding the calibration of π, we proceed as follows. Considering a young adult of age 20, who can work two periods (young adulthood and old adulthood) of 30 years each, the maximal longevity is 80 years, and life expectancy is equal to $50 + 30\pi$ in our model. Hence we set $\pi = 0.9$, which yields a life expectancy of 77 years (close to OECD average). Finally, the hourly wage is set to $w = \$10$.

Figure 1 compares, for various values of the preference parameter β, the working time at the young and old ages prevailing at the laissez-faire equilibrium, at the first-best *ex post* egalitarian optimum, and at the second-best (i.e. optimal age-specific working time regulations).[19] In each case, the working time is, as expected, decreasing with the intensity of the disutility of labor (i.e. parameter β).

[19] The working time is expressed in hours per week. Note that the maximum working time equals 80 hours per week, which corresponds to 5 days of 16 hours of daily work (leaving only 8 hours for sleep and daily activities).

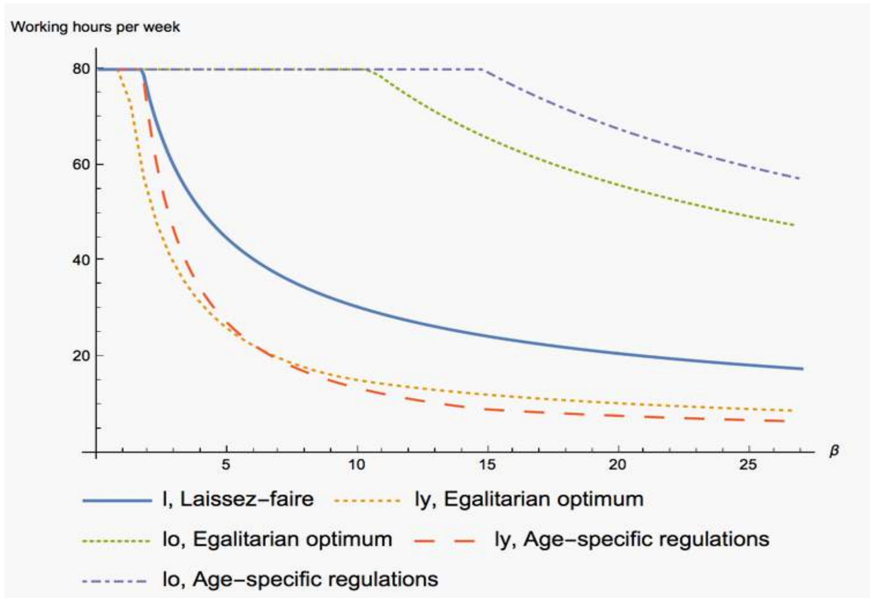

Fig. 1. Working time for the young and the old, at the laissez-faire, the *ex post* egalitarian optimum and the optimal age-specific regulations.

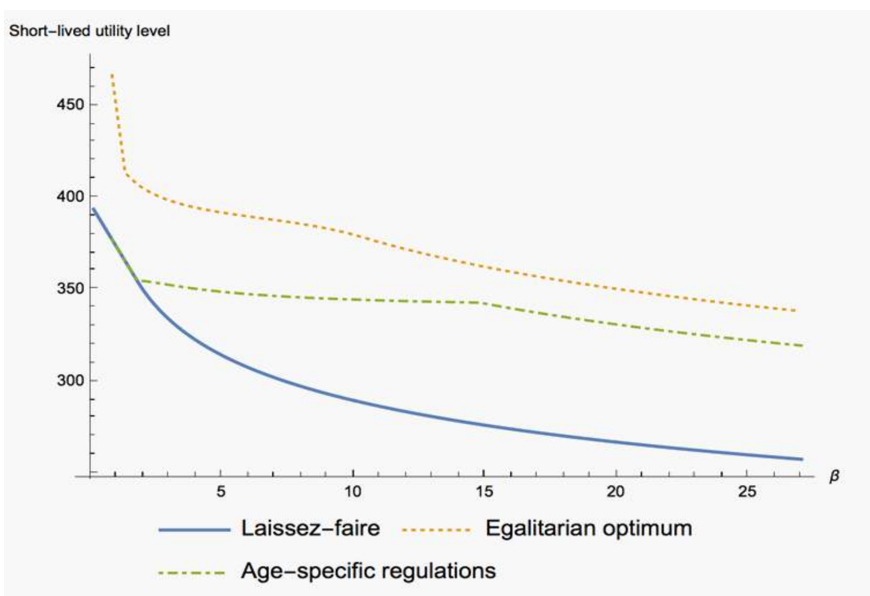

Fig. 2. Well-being of the short-lived at the laissez-faire, the *ex post* egalitarian optimum and the optimal age-specific regulations.

In comparison to the laissez-faire (where the working time of the young and the old are equal), the *ex post* egalitarian optimum implies a lower working time for the young and a higher working time for the old, to an extent that varies with the disutility of labor. If, for instance, the working time at the laissez-faire equals 40 hours per week, then the optimal working time of the young equals, at the *ex post* egalitarian optimum, about 25 hours per week, whereas the optimal working time of the old would be 80 hours per week.[20] When there is a larger disutility of labor, both the working time for the young and for the old at the *ex post* egalitarian optimum are reduced, and the ratio old working time / young working time tends to fall as β takes higher levels, but it is still the case that, at the *ex post* egalitarian optimum, the young works much less than the old.

The *ex post* egalitarian optimum involves thus a strongly increasing labor profile with the age, which enables to increase the well-being of the short-lived and to neutralize well-being inequalities with respect to the laissez-faire. For young adults, the number of hours worked is, for the largest interval of values for β, close to or slightly below 20 hours per week under the *ex post* egalitarian optimum. This is quite close to the 4 hours a day proposed by Campanella (1602). However, for old workers, the optimal number of hours worked (between 50 and 80 hours per week) is much larger. The intuition behind that differentiated treatment of young and old workers is that, contrary to Campanella (1602) where limiting working time is a way to avoid alienation, working time is used here as a way to compensate the unlucky short-lived. That point is illustrated by Figure 2, which shows that the realized lifetime well-being of the short-lived is larger at the *ex post* egalitarian optimum than at the laissez-faire.

Figures 1 and 2 allow us also to compare the laissez-faire and the *ex post* egalitarian optimum with what prevails under optimal age-specific working time regulations, which are the solutions of the second-best social planning problem studied in Section 6. As shown on Figure 1, under (optimal) age-specific working time regulations, the quantity of hours worked by the young is lower than at the laissez-faire and is higher (resp. lower) than at the *ex post* egalitarian optimum for low (resp. high) levels of β. Regarding the working time of the old under (optimal) age-specific regulations, it is higher than at the laissez-faire and either equal or higher than at the *ex post* egalitarian optimum. This result can be interpreted as follows. If the government cannot use a tax on saving to improve the situation of the short-lived, it has to increase the quantity of labor of the surviving old so as to raise the level of the flat consumption profile, in order to make the short-lived better off. As shown on Figure 2, imposing lower working time for the young and higher working time to the old (with respect to the laissez-faire) allows to improve the situation of the unlucky short-lived.

Finally, our simulations allow us also to illustrate the impact of imposing working time regulations on the size of inequalities in realized lifetime well-being between the long-lived and the short-lived. As shown on Figure 3, those inequalities, which are substantial at the laissez-faire, are reduced to zero at the

[20] This coincides with the corner solution $\ell_o^* = 1$ in Section 4. As shown on Figure 1, this corner solution arises for β lower than 15. For $\beta > 15$, there is an interior solution, corresponding to $\ell_o^* < 1$ in the model.

ex post egalitarian optimum. The *ex post* egalitarian optimum does not only improve the situation of the unlucky short-lived with respect to the laissez-faire; it also brings a full compensation of the short-lived, in the sense that enjoying a short or a long life has no impact on individual lifetime well-being.

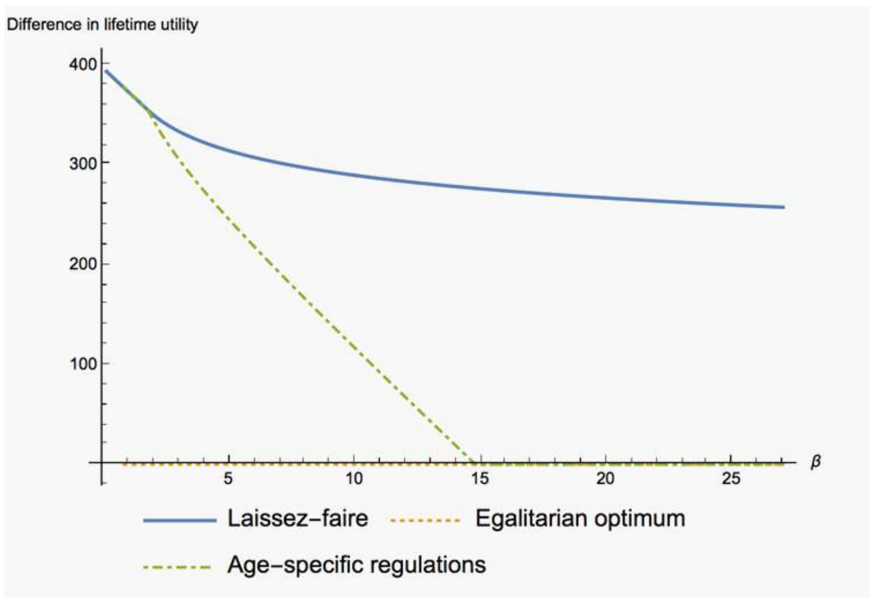

Fig. 3. Well-being inequalities at the laissez-faire, the *ex post* egalitarian optimum and the optimal age-specific regulations.

Figure 3 illustrates also the impact of imposing working time regulations on inequalities in realized lifetime well-being between the long-lived and the short-lived. Remind first that, as shown in Section 4, imposing uniform working time regulations yields, at best, the laissez-faire equilibrium, so that, if one does not want to make the short-lived worse off than at the laissez-faire, the imposed uniform working time regulations coincide with the working time prevailing at the laissez-faire. Thus uniform working time regulations lead to the same inequalities as those shown for the laissez-faire on Figure 3. However, imposing age-specific working time regulations leads to reduce inequalities in realized lifetime well-being. As shown on Figure 3, this reduction is not total for low values of β, so that some inequalities remain. The underlying intuition is that, for a low disutility of labor, the old-age working time is a corner solution coinciding with maximal working time (see Figure 1). It is then impossible to make the old work more, which prevents a full equalization of lifetime well-being between short-lived and long-lived individuals. To the contrary, for sufficiently large values of β, the optimal age-specific working time regulations involve an interior working time, and imposing optimal age-specific working time regulations allows to reduce inequalities in lifetime well-being to zero.

8 Conclusion

Whereas a large literature quantifies the consequences of working time regulations in terms of employment and productivity, this paper proposes to cast another light on working time regulations by examining their redistributive effects. In particular, this paper explores, from a lifecycle perspective, the redistributive impact of working time regulations in an economy where individuals have unequal longevity.

We showed that, in the absence of any regulation, unequal lifetimes generate inequalities in realized lifetime well-being between the long-lived and the short-lived that are increasing with labor productivity. However, despite the difficulty to identify *ex ante* who will be short-lived, those inequalities in well-being due to Nature can be fully neutralized at the *ex post* egalitarian social optimum. The intuition behind that somewhat surprising result is that the *ex post* egalitarian optimum involves a strongly decreasing consumption age profile, as well as a strongly increasing working time age profile, which, when combined, lead to transfer resources (goods and leisure time) towards the young. This allows to fully compensate unlucky individuals who turn out to be short-lived.

While the decentralization of that social optimum can be achieved by a tax on saving and lump sum transfers, those policy instruments are not easily available in real economies. This motivated us to consider the capacity of existing uniform working time regulations to improve the situation of the short-lived, and to reduce inequalities in lifetime well-being. Our analysis led us here to a negative result: imposing uniform working time regulations cannot improve the situation of the short-lived with respect to the laissez-faire. Moreover, uniform working time regulations can only reduce inequalities in realized lifetime well-being at the cost of making the short-lived worse off.

However, our analyses highlighted a positive result concerning age-specific working time regulations: we showed that, in a second-best world, imposing working time regulations involving a strongly increasing working time profile with the age cannot only make the short-lived better off than at the laissez-faire, but can also reduce inequalities in lifetime well-being (but not completely when there is a low disutility of labor, unlike at the first-best). This paper provides thus a second-best argument supporting age-specific working time regulations on the grounds of fairness with respect to the unlucky short-lived.

To conclude, it is useful to come back to the starting point of this paper: the key role played by working time regulations in the ideal societies described by More (1516) and Campanella (1602). Undoubtedly, those authors use working time regulations in order to achieve various goals, such as the will to free workers from being instruments of goods. Our study suggests that, in a second-best world where policy instruments are restricted, age-specific working time regulations can help overcoming unfair inequalities due to Nature, and can lead to more justice between the short-lived and the long-lived. Working time regulations remain, in the 21st century, a key policy instrument for a better society.

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10 Appendix

10.1 Proof of Proposition 1

The FOCs of the individual's problem are:

$$\begin{aligned} \pi u'(c) &= \pi u'(d) \\ wu'(c) &= v'(\ell_y) + \rho_y - \varphi_y \\ wu'(c) &= v'(\ell_o) + \frac{\rho_o}{\pi} - \frac{\varphi_o}{\pi} \end{aligned}$$

as well as conditions $\rho_y \geq 0, 1 - \ell_y \geq 0$; $\rho_o \geq 0, 1 - \ell_o \geq 0$; $\varphi_y \geq 0, \ell_y \geq 0$ and $\varphi_o \geq 0, \ell_o \geq 0$ with complementary slackness.

We obviously have, from the first FOC, $c = d$. Regarding the working time profile, several cases can arise.

- If $0 < \ell_y, \ell_o < 1$, we have $\rho_y = \rho_o = \varphi_y = \varphi_o = 0$. The FOCs become $wu'(c) = v'(\ell_y)$ and $wu'(c) = v'(\ell_o)$. Hence we have $0 < \ell_y = \ell_o < 1$.

- If $\ell_y = 0$, we have $\rho_y = 0$ and $\varphi_y \geq 0$. The second FOC becomes: $wu'(c) = v'(0) - \varphi_y \iff wu'(c) = -\varphi_y$. Given that $u'(c) > 0$, that case is not possible.

- If $\ell_o = 0$, we have $\rho_o = 0$ and $\varphi_o \geq 0$. The third FOC becomes: $wu'(c) = v'(0) - \frac{\varphi_o}{\pi} \iff wu'(c) = -\frac{\varphi_o}{\pi}$. Again that case is not possible.

- If $\ell_y = 1$, we have $\rho_y \geq 0$ and $\varphi_y = 0$. The second FOC becomes: $wu'(c) = v'(1) + \rho_y$.

  - If $0 < \ell_o < 1$, so that $\rho_o = \varphi_o = 0$, we then have $wu'(c) = v'(1) + \rho_y$ and $wu'(c) = v'(\ell_o)$, which implies $v'(1) + \rho_y = v'(\ell_o)$. That case is not possible.
  - If $\ell_o = 1$, so that $\rho_o \geq 0$ and $\varphi_o = 0$, we then have: $wu'(c) = v'(1) + \rho_y$ and $wu'(c) = v'(1) + \rho_o$. That case is possible.

- If $\ell_o = 1$, we have $\rho_o \geq 0$ and $\varphi_o = 0$. The third FOC becomes: $wu'(c) = v'(1) + \frac{\rho_o}{\pi}$. If $0 < \ell_y < 1$, so that $\rho_y = \varphi_y = 0$, we then have $wu'(c) = v'(\ell_y)$ and $wu'(c) = v'(1) + \frac{\rho_o}{\pi} \iff v'(1) + \frac{\rho_o}{\pi} = v'(\ell_y)$. That case is not possible.

In sum, only two cases can arise: either $0 < \ell_y = \ell_o < 1$ or $\ell_y = \ell_o = 1$. Given that consumption is smoothed, we have $c = \ell w + \pi \ell w - \pi c$, implying $c = d = w\ell$. Hence whether one case arises or the other depends on whether $wu'(w) \gtreqless v'(1)$. If $wu'(w) < v'(1)$, the laissez-faire equilibrium is interior and we have $0 < \ell_y = \ell_o < 1$. On the contrary, when $wu'(w) \geq v'(1)$, we have the corner solution $\ell_y = \ell_o = 1$.

10.2 Analytical example: the threshold for w

In order to discuss whether the long-lived is better off than the short-lived at the laissez-faire, let us take the following functional forms for $u(\cdot)$ and $v(\cdot)$:

$$u(c) = \frac{(w\ell)^{1-\sigma}}{1-\sigma} - \alpha \text{ and } v(\ell) = \frac{\beta\ell^2}{2}.$$

The condition for optimal working time $wu'(w\ell) = v'(\ell)$ and the egalitarian condition $u(w\ell) = v(\ell)$ can be rewritten as, respectively:

$$w(w\ell)^{-\sigma} = \beta\ell \iff \ell = \left(\frac{w^{1-\sigma}}{\beta}\right)^{\frac{1}{1+\sigma}} \text{ and } \frac{(w\ell)^{1-\sigma}}{1-\sigma} - \alpha = \frac{\beta\ell^2}{2}.$$

Hence substituting for the first equation in the second, we obtain:

$$U^{LL} \gtreqless U^{SL} \iff w \gtreqless \left[\frac{\alpha}{\beta^{\frac{\sigma-1}{1+\sigma}}\left[\frac{1}{1-\sigma} - \frac{1}{2}\right]}\right]^{\frac{1+\sigma}{2(1-\sigma)}}.$$

The RHS consists of the threshold for the wage beyond which the long-lived is better off than the short-lived. The threshold depends on α, β and σ. The higher the wage is, the more likely it is that the LHS exceeds the RHS, implying that $U^{LL} > U^{SL}$.

10.3 Proof of Proposition 2

The FOCs of the social planner's problem are:

$$\begin{aligned} \pi u'(c) &= \mu u'(d) \\ wu'(c) &= v'(\ell_y) + \kappa_y - \gamma_y \\ \pi wu'(c) &= \mu v'(\ell_o) + \kappa_o - \gamma_o \end{aligned}$$

as well as the conditions $\mu \geq 0, u(d) - v(\ell_o) \geq 0$; $\kappa_y \geq 0, 1 - \ell_y \geq 0$; $\kappa_o \geq 0, 1 - \ell_o \geq 0$; $\gamma_y \geq 0, \ell_y \geq 0$ and $\gamma_o \geq 0, \ell_o \geq 0$ with complementary slackness.

1. Let us suppose that the egalitarian constraint is satisfied, so that $\mu \geq 0$ and $u(d) - v(\ell_o) = 0$.

   (a) In the interior case ($0 < \ell_o, \ell_y < 1$), we have $\kappa_y = \kappa_o = \gamma_y = \gamma_o = 0$, so that: $v'(\ell_y) = \frac{\mu}{\pi} v'(\ell_o)$. We thus have either an increasing or a decreasing consumption profile, depending on whether $\frac{\mu}{\pi} \lesseqgtr 1$.

   (b) If now $\kappa_y \geq 0$ so that $\ell_y = 1$, and $\kappa_o = 0$ so that $\ell_o < 1$. In that situation, we have $\gamma_y = 0$ (since $\ell_y = 1$). We then have: $wu'(c) = v'(1) + \kappa_y$ and $wu'(c) = \frac{\mu}{\pi} v'(\ell_o) - \gamma_o \implies v'(1) + \kappa_y = \frac{\mu}{\pi} v'(\ell_o) - \frac{\gamma_o}{\pi}$.

- If $\gamma_o = 0$, i.e. $\ell_o > 0$, two cases can arise depending on whether $\frac{\mu}{\pi} \gtrless 1$. 1)If $\frac{\mu}{\pi} < 1$, we have a contradiction since the above equality would imply $v'(\ell_o) > v'(1) + \kappa_y$ and thus $\ell_o > 1$. That corner case $(\ell_y = 1, \ell_o < 1)$ is not possible in affluent economies. 2) When $\frac{\mu}{\pi} > 1$, $(\ell_y = 1, \ell_o < 1)$ is possible.
- If $\gamma_o \geq 0$, $\ell_o = 0$, we have $v'(1) + \kappa_y = \frac{\mu}{\pi} v'(0) - \gamma_o$ which is possible only if $\mu/\pi > 1$ (i.e. poor economy). But the case $(\ell_y = 1, \ell_o = 0)$ is not possible when $\frac{\mu}{\pi} < 1$ (affluent economy).

(c) If $\kappa_y = 0$ so that $\ell_y < 1$, and $\kappa_o \geq 0$ and $\ell_o = 1$. We then have that $\gamma_o = 0$ and $wu'(c) = \frac{\mu}{\pi} v'(1) + \frac{\kappa}{\pi} = v'(\ell_y) - \gamma_y$.

- If $\gamma_y = 0$ implying $\ell_y > 0$, when $\frac{\mu}{\pi} > 1$, we have a contradiction. That case $(\ell_y < 1, \ell_o = 1)$ is thus not possible. But when $\frac{\mu}{\pi} < 1$ (affluent economies) that case $(\ell_y < 1, \ell_o = 1)$ is possible.
- If $\gamma_y \geq 0$ implying $\ell_y = 0$, we have $\frac{\mu}{\pi} v'(1) + \frac{\kappa_o}{\pi} = v'(0) - \gamma_y$, which is impossible.

(d) Consider now the double corner case, where $\ell_o = \ell_y = 1$, so that $\kappa_o \geq 0$, $\kappa_y \geq 0$ and $\gamma_y = \gamma_o = 0$. We then have $wu'(c) = \frac{\mu}{\pi} v'(1) + \frac{\kappa_o}{\pi} = v'(1) + \kappa_y$. When $\frac{\mu}{\pi} > 1$, we have $\kappa_y > \frac{\kappa_o}{\pi}$. When $\frac{\mu}{\pi} < 1$, we have the opposite. That case is possible.

(e) Consider now the double corner case, where $\ell_o = \ell_y = 0$, so that $\kappa_o = \kappa_y = 0$ and $\gamma_y, \gamma_o > 0$. This implies $c = d = 0$. But the egalitarian constrain would then become $u(0) = v(0)$, which is impossible, since the LHS is negative and the RHS is zero. The case $(\ell_y = 0, \ell_o = 0)$ is not possible here.

(f) Consider now the case where $\ell_y = 0$ so that $\kappa_y = 0$ and $\gamma_y \geq 0$. We have: $wu'(c) = \frac{\mu}{\pi} v'(\ell_o) + \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi} = v'(0) - \gamma_y$. If $0 < \ell_o < 1$, $\kappa_o = \gamma_o = 0$, so that we have $\frac{\mu}{\pi} v'(\ell_o) = -\gamma_y$. This is not possible.

(g) Consider now the case where $\ell_o = 0$ so that $\kappa_o = 0$ and $\gamma_o \geq 0$. We have: $wu'(c) = -\frac{\gamma_o}{\pi} = v'(\ell_y) + \kappa_y - \gamma_y$. If $0 < \ell_y < 1$, $\kappa_y = \gamma_y = 0$, implying $-\frac{\gamma_o}{\pi} = v'(\ell_y)$. which is not possible either.

2. Suppose now that the egalitarian constraint does not hold so that $\mu = 0$. The FOC for d is now strictly negative and equal to $-\pi u'(c)$ which implies that $d = 0$. However, under the egalitarian constraint, we need to have $u(0) - v(\ell_o) > 0$ which is not possible since $u(0) < 0$. Hence, this case is not possible.

10.4 Proof of Proposition 4

The FOCs of the social planner's problem are:

$$\left[wu'(w\bar{\ell}) - v'(\bar{\ell})\right](1 + \mu) = \kappa - \gamma$$

as well as the conditions $\mu \geq 0, u\left(w\bar{\ell}\right) - v\left(\bar{\ell}\right) \geq 0; \gamma \geq 0, \bar{\ell} \geq 0$ and $\kappa \geq 0, 1 - \bar{\ell} \geq 0$ with complementary slackness.

1. Suppose the egalitarian constraint holds so that $\mu \geq 0$ and $u\left(w\bar{\ell}\right) = v\left(\bar{\ell}\right)$. The FOC becomes $\left[u'(c)w - v'(\bar{\ell})\right] = \frac{\kappa-\gamma}{1+\mu}$. For any interior value of $\bar{\ell}$ (i.e. $\gamma = \kappa = 0$), we would have $u'(w\bar{\ell})w = v'(\bar{\ell})$ which cannot hold at the same time as the egalitarian constraint $(u(w\bar{\ell}) = v(\bar{\ell}))$. The same reasoning applies when $\bar{\ell} = 0$ (i.e. $\gamma \geq 0$ and $\kappa = 0$) and when $\bar{\ell} = 1$ (i.e. $\gamma = 0$ and $\kappa \geq 0$). Hence, the egalitarian constraint can never be binding.

2. Suppose that the egalitarian constraint does not hold, so that we have $\mu = 0$ and $u\left(w\bar{\ell}\right) > v\left(\bar{\ell}\right)$. The FOC becomes: $\left[u'(w\bar{\ell})w - v'(\bar{\ell})\right] = \kappa - \gamma$. If $\bar{\ell}$ is interior, we have $\kappa = \gamma = 0$ and the solution is given by $u'(w\bar{\ell})w = v'(\bar{\ell})$ as in the laissez-faire equilibrium. If $\bar{\ell} = 0$, we have $\kappa = 0$ and $\gamma \geq 0$, so that $u'(c)w = -\gamma$ which is not possible since $u'(c) > 0$. If $\bar{\ell} = 1$, we have $\kappa \geq 0$ and $\gamma = 0$, so that $[u'(w)w - v'(1)] = \kappa$. That case is possible. Thus we have either $0 < \bar{\ell} < 1$ or $\bar{\ell} = 1$.

Inequalities in realized lifetime well-being are: $U^{LL} - U^{SL} = u(w\bar{\ell}) - v(\bar{\ell})$. To reduce $U^{LL} - U^{SL}$, the government must choose $\tilde{\ell} \neq \bar{\ell}$. We would then have: $u(w\tilde{\ell}) - v(\tilde{\ell}) < u(w\bar{\ell}) - v(\bar{\ell})$. Thus, there would be a reduction of $U^{LL} - U^{SL}$ with respect to the optimal regulation (and also with respect to the laissez-faire). But this reduction in inequalities would be achieved at the cost of making the short-lived worst-off than at the laissez-faire since his utility would now be $u(w\tilde{\ell}) - v(\tilde{\ell})$ instead of $u(w\bar{\ell}) - v(\bar{\ell})$.

10.5 Proof of Proposition 5

The FOCs of the social planner's problem are:

$$\begin{aligned} u'(c)\frac{w}{1+\pi}(1+\mu) &= v'(\bar{\ell}_y) + \kappa_y - \gamma_y \\ u'(c)\frac{w}{1+\pi}(1+\mu) &= \frac{\mu}{\pi}v'(\bar{\ell}_o) + \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi} \end{aligned}$$

as well as the conditions $\mu \geq 0, u\left(w\frac{\bar{\ell}_y + \pi\bar{\ell}_o}{1+\pi}\right) - v(\bar{\ell}_o) \geq 0$; $\gamma_y \geq 0, \bar{\ell}_y \geq 0$; $\kappa_y \geq 0, 1 - \bar{\ell}_y \geq 0$; $\gamma_o \geq 0, \bar{\ell}_o \geq 0$ and $\kappa_o \geq 0, 1 - \bar{\ell}_o \geq 0$ with complementary slackness.

1. Take the case where the egalitarian constraint is satisfied. We have $\mu \geq 0$ and $u\left(w\frac{\bar{\ell}_y + \pi\bar{\ell}_o}{1+\pi}\right) - v(\bar{\ell}_o) = 0$.

   (a) If $0 < \bar{\ell}_o, \bar{\ell}_y < 1$, we have $\kappa_y = \kappa_o = \gamma_y = \gamma_o = v'(\bar{\ell}_y)$ and $u'(c)\frac{w}{1+\pi}(1+\mu) = \frac{\mu}{\pi}v'(\bar{\ell}_o)$. When $\frac{\mu}{\pi} < 1$, we have thus $v'(\bar{\ell}_o) > v'(\bar{\ell}_y)$, implying $\bar{\ell}_o > \bar{\ell}_y$. When $\frac{\mu}{\pi} > 1$, the opposite holds.

   (b) If $\bar{\ell}_y = 0$, we have $\kappa_y = 0$ and $\gamma_y \geq 0$. We have $u'(c)\frac{w}{1+\pi}(1+\mu) = v'(0) - \gamma_y \frac{\mu}{\pi}v'(\bar{\ell}_o) + \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}$. If $\bar{\ell}_o = 0$, we have a contradiction: the egalitarian constraint cannot hold as $u(0) \neq v(0)$. If $0 < \bar{\ell}_o < 1$, we have $\kappa_o = 0$ and $\gamma_o = 0$, so that $-\gamma_y = \frac{\mu}{\pi}v'(\bar{\ell}_o)$ which is not possible. If $\bar{\ell}_o = 1$, we have $\kappa_o \geq 0$ and $\gamma_o = 0$, so that we have $-\gamma_y = \frac{\mu}{\pi}v'(1) + \frac{\kappa_o}{\pi}$. That case is not possible either.

(c) Using the same reasoning as in (b) we can show that $\bar{\ell}_o = 0$ is never possible.

(d) If $\bar{\ell}_y = 1$ and $\bar{\ell}_o < 1$, we have $\gamma_y = \gamma_o = 0$, $\kappa_y \geq 0$ and $\kappa_o = 0$,. FOCs become $u'(c)\frac{w}{1+\pi} = \frac{\kappa_y + v'(1)}{1+\mu} = \frac{\frac{\mu}{\pi}v'(\bar{\ell}_o)}{1+\mu}$. If $\frac{\mu}{\pi} < 1$, a contradiction is reached. If $\frac{\mu}{\pi} > 1$, that case is possible.

(e) If $\bar{\ell}_y < 1$ and $\bar{\ell}_o = 1$, we have $\gamma_y = \gamma_o = 0$, $\kappa_y = 0$, and $\kappa_o \geq 0$. FOCs become $u'(c)\frac{w}{1+\pi} = \frac{v'(\bar{\ell}_y)}{1+\mu} = \frac{\kappa_o + \mu v'(1)}{\pi(1+\mu)}$. If $\frac{\mu}{\pi} > 1$, that case is not possible. If $\frac{\mu}{\pi} < 1$, that case is possible.

(f) If $\bar{\ell}_y = \bar{\ell}_o = 1$, we have $\gamma_y = \gamma_o = 0$, $\kappa_y, \kappa_o \geq 0$. FOCs become: $u'(c)(1+\mu)\frac{w}{1+\pi} = \kappa_y + v'(1) = \frac{\kappa_o}{\pi} + \frac{\mu}{\pi}v'(1)$. That case is possible.

2. Take now the case where the egalitarian constraint does not hold. We have $\mu = 0$ and $u\left(w\frac{\bar{\ell}_y + \pi\bar{\ell}_o}{1+\pi}\right) - v(\bar{\ell}_o) > 0$.

   In that case, the FOCs become: $u'(c)\frac{w}{1+\pi} - v'(\bar{\ell}_y) = \kappa_y - \gamma_y$ and $u'(c)\frac{w\pi}{1+\pi} = \kappa_o - \gamma_o$. Hence $u'(c)\frac{w}{1+\pi} = v'(\bar{\ell}_y) + \kappa_y - \gamma_y = \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}$.

   (a) Take the case where both $\bar{\ell}_y$ and $\bar{\ell}_o$ are interior. We then have $v'(\bar{\ell}_y) = 0$ leading to $\bar{\ell}_y = 0$. A contradiction.

   (b) If $\bar{\ell}_y = 0$, we have $\kappa_y = 0$ and $\gamma_y \geq 0$. This leads to $-\gamma_y = \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}$. The case where $\bar{\ell}_y = \bar{\ell}_o = 0$ is not possible as it would not satisfy the egalitarian constraint ($u(0) - v(0) < 0$ in that case). If $0 < \bar{\ell}_o < 1$, we have $\kappa_o = \gamma_o = 0$. This leads to $-\gamma_y = 0$. That case is impossible. If $\bar{\ell}_o = 1$, we have $\kappa_o \geq 0$ and $\gamma_o = 0$. This leads to $-\gamma_y = \frac{\kappa_o}{\pi}$. That case is impossible.

   (c) If $\bar{\ell}_y = 1$, we have $\kappa_y \geq 0$ and $\gamma_y = 0$. This leads to $v'(1) + \kappa_y = \frac{\kappa_o}{\pi} - \frac{\gamma_o}{\pi}$. If $\bar{\ell}_o = 0$, we have $\kappa_o = 0$ and $\gamma_o \geq 0$. This leads to $v'(1) + \kappa_y = -\frac{\gamma_o}{\pi}$. That case is impossible. If $0 < \bar{\ell}_o < 1$, we have $\kappa_o = \gamma_o = 0$. This leads to $v'(1) + \kappa_y = 0$. That case is impossible. If $\bar{\ell}_o = 1$, we have $\kappa_o \geq 0$ and $\gamma_o = 0$. This leads to $v'(1) + \kappa_y = \frac{\kappa_o}{\pi}$. That case is possible.

   (d) If $\bar{\ell}_o = 0$, we have $\kappa_o = 0$ and $\gamma_o \geq 0$. We have $v'(\bar{\ell}_y) + \kappa_y - \gamma_y = -\frac{\gamma_o}{\pi}$. From (b), we know that $\bar{\ell}_y = \bar{\ell}_o = 0$ is not possible. If $0 < \bar{\ell}_y < 1$, we have $\kappa_y = \gamma_y = 0$. We have $v'(\bar{\ell}_y) = -\frac{\gamma_o}{\pi}$. That case is impossible. If $\bar{\ell}_y = 1$, we have $\kappa_y \geq 0$ and $\gamma_y = 0$. We have $v'(1) + \kappa_y = -\frac{\gamma_o}{\pi}$. That case is impossible.

   (e) If $\bar{\ell}_o = 1$, we have $\kappa_o \geq 0$ and $\gamma_o = 0$. We have $v'(\bar{\ell}_y) + \kappa_y - \gamma_y = \frac{\kappa_o}{\pi}$. From (b), we know that $\bar{\ell}_y = 0$ is impossible. If $0 < \bar{\ell}_y < 1$, we have $\kappa_y = \gamma_y = 0$. We have $v'(\bar{\ell}_y) = \frac{\kappa_o}{\pi}$. That case is possible. If $\bar{\ell}_y = 1$, we have $\kappa_y \geq 0$ and $\gamma_y = 0$. We have $v'(1) + \kappa_y = \frac{\kappa_o}{\pi}$. That case is possible.