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Accountability and Political Competition*

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Abstract

Is increasing political competition good for voters? We study this question in the political career concerns framework. Our results show that the relationship between political competition, viewed as the cost of challenging incumbent politicians, and the politicians' incentive to behave in the voters' interest is undetermined. The same holds for the relationship between political competition and voter welfare, where selection of politicians into office also matters. In particular, voter welfare need not be maximized when challenging incumbent politicians is costless. So, unlike in economy markets, where increased competition is beneficial, in political markets increased competition can have adverse effects. We tie our results to a contractual incompleteness that is typical of political markets, namely, that the set of instruments available to discipline politicians' behavior is limited.

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1 Introduction

"An ideal political democracy is defined as: an institutional arrangement for arriving at political decisions in which individuals endeavor to acquire political office through perfectly free competition for the votes of a broadly based electorate." (Becker 1958, p.106).

By and large, economists view competition in a positive light. In economic markets, the argument goes that competition among firms for consumers and the resulting risk of loss of market share ensures that firms behave in the consumers' best interest and only the best firms survive. Similarly to firms, which need not behave in the consumers' best interest, politicians need not behave in the voters' best interest. Moreover, also similarly to firms, politicians differ in quality; some politicians are better than others at producing beneficial outcomes for voters. Is it then the case that competition among politicians for votes and the implied risk of political takeover ensures that politicians in power act in the best interest of voters and only the best politicians survive?

In this paper, we investigate whether competition among politicians indeed benefits voters. We do so in the political career concerns framework.[1] In this framework, an office-motivated incumbent politician privately chooses how much effort to exert. The incumbent's performance depends positively on his or her effort and unknown ability. A (representative) voter observes the incumbent's performance, which provides information about the incumbent's ability, and then decides whether to retain or replace the incumbent. So, what motivates the politician to exert effort and benefit the voter is his or her desire to influence the voter's assessment of his or her ability, thus increasing the likelihood of retention. The political career concerns framework captures in a parsimonious way the presence of principal-agent and informational frictions in political "markets."[2]

We introduce political competition in the career concerns framework in the form of a citizen who decides whether or not to run for office against the incumbent. The citizen's ability is also unknown and the voter observes a signal about this ability before making a retention decision.

[1] Political career concerns models adapt the career concerns model of Holmström (1999) to a political economy setting. They are widely used to study how a politician's concern for his or her future career affects his or her behavior.

[2] The analogy with economic markets is useful in our context. In the same way that agents trade goods in economic markets, in an election voters trade votes for public-policy outcomes. The rules for determining electoral outcomes constitute the market-clearing mechanism: they determine which "goods" are traded in equilibrium.

Running for office is costly, though. Similarly to entry barriers in economic markets, this cost affects the amount of political competition faced by the incumbent. A low cost of entry implies that mounting a challenge to the incumbent's position is easy. On the other hand, a high cost of entry implies that even highly a qualified citizen, i.e., a citizen with a strong signal about his or her ability, may be discouraged from running for office.[3] Other than the citizen's costly entry decision, our model is a standard political career concerns model.

We first study the relationship between political competition and effective accountability, i.e., the incumbent's incentive to exert effort in office. We show that, typically, an increase in political competition can be associated with both an increase and a decrease in effective accountability. Thus, a priori, there is no unequivocal relationship between effective accountability and political competition. Furthermore, we show that effective accountability need not be maximized when the cost of entry is zero.

In order to understand the above results, notice that the incumbent's incentive to exert effort is tied to his or her likelihood of retention given his or her performance in office. By making it easier for the citizen to run for office against the incumbent, an increase in political competition has an ambiguous impact on effective accountability. On the one hand, it reduces the likelihood that an incumbent with a good performance is retained, which is bad for incentives. On the other hand, it reduces the likelihood that an incumbent with a poor performance is retained, which is good for incentives. We show that both the situation in which the positive effect dominates the negative effect and the situation in which the negative effect dominates the positive effect are compatible with equilibrium behavior.

We then study the relationship between political competition and voter welfare. Voter welfare depends not only on the incumbent's effort but also on electoral selection. A change in political competition directly affects electoral selection by changing the incumbent's decision to run for office. *Ceteris paribus*, a reduction in the cost of entry increases voter welfare by making it more likely that the citizen runs for office, thus increasing the set of politicians available for the voter

[3]Our measure of political competition captures the ex-ante electoral advantage of incumbents. Another measure of such advantage is the margin of victory. There are other notions of political competition used in political economy and political science (e.g., inter-jurisdiction and inter-party competition). See Bardhan and Yang (2004) for a discussion of the different notions of political competition used in the literature.

to choose. By changing the incumbent's effort, a change in political competition also indirectly affects electoral selection. Indeed, a change in the incumbent's effort affects the informational content of the incumbent's performance in office, and so the assessment of the incumbent's ability conditional on his or her performance.[4] This, in turn, affects both the voter's retention decision and the citizen's entry decision, as the latter also depends on the assessment of the incumbent's ability—the citizen is less likely to run for office if he or she believes the incumbent is strong.

In our analysis of voter welfare, we show that there are natural conditions under which voter welfare and effective accountability move together. Thus, our results about the impact of political competition on effective accountability extend to voter welfare. In particular, voter welfare need not be maximized when the cost of entry is zero. So, unlike in economic markets, where free competition among firms for consumers benefits consumers, in political markets free competition for votes among politicians may not always work to produce the most favorable outcome for voters.

Our results about the relationship between political competition and voter welfare beg the question of what is the feature of political markets that is responsible for these results. An important feature of such markets is that retention is the only incentive device available to motivate politicians to behave in the voters' interest. We show in the paper that this contractual incompleteness is at the heart of our results. If the voter in our setting had a sufficiently rich set of instruments allowing a politician's payoff to depend on his or her performance in office, then an increase in political competition would always be beneficial to the voter.[5]

The rest of the paper is organized as follows. We discuss the related literature in the rest of this section. In Section 2, we introduce the model. In Section 3, we define and characterize equilibria. In Section 4, we study the impact of changes in political competition on effective accountability and voter welfare. In Section 5, show that our results about the impact of political competition on voter welfare are tied to the fact that retention is the only incentive device available to motivate politicians. In Section 6, we establish the robustness of our results to some of our modeling assumptions. Section 7 concludes. The Appendix contains omitted proofs and details.

[4]See Ashworth, Bueno de Mesquita, and Friedenberg (2017) for a discussion of this.

[5]Our results thus provide a rejoinder to Stigler (1972) and Wittman (1989, 1995), who argue that political competition, in much the same way as economic competition, produces (constrained) efficient outcomes.

Related Literature Several papers study either or both theoretically and empirically the relationship between political competition and different types of economic and political outcomes, often obtaining conflicting results. Iaryczower and Mattozzi (2008), De Paola and Scoppa (2011), and Galasso and Nannicini (2011) find a positive relationship between political competition and politician quality, while Dal Bó and Finan (2018) finds that this relationship may go in either direction. Polo (1998) finds that political competition may increase political rents, while Svaleryd and Vlachos (2009) finds the opposite result. Afridi, Dhillon, and Solan (2019) finds that political competition may increase corruption, while Ashworth, Geys, Heyndels, and Wille (2014) finds that political competition increases the efficiency of municipal administration.

In the context of redistributive politics, Myerson (1993) and Lizzeri and Persico (2005) show, respectively, that a higher number of candidates is associated with more unequal redistribution and greater distortion in the provision of public goods. On the other hand, Arvate (2013) documents that a higher number of candidates increases the supply of local public goods, while Besley and Preston (2007) documents that electoral bias in favor of one party leads mayors from that party to offer policies that suit the party's core constituency rather than swing voters.

Theoretical and empirical studies of the relation between political competition and economic growth and development reach mixed conclusions as well. Acemoglu and Robinson (2006) propose a model in which there is a U-shaped relationship between pre-existing political competition and economic development. Padovano and Ricciuti (2009) and Besley, Persson, and Sturm (2010) find a positive relationship between political competition and economic growth, while Alfano and Baraldi (2015, 2016) find an inverted U-shaped relationship.

Starting with Barro (1973) and Ferejohn (1986), a substantial literature has studied how reelection concerns motivate politicians to behave in the voters' interest.[6] The literature has recognized that elections play a dual role: to control politician behavior and to select politicians into office. The political career concerns framework captures this dual role of elections in a simple way.[7] To

[6]See Ashworth (2012) and Duggan and Martinelli (2017) for reviews of the literature, and Besley and Case (1995), Alt, Bueno de Mesquita, and Rose (2011), and Ferraz and Finan (2011) for evidence that electoral accountability affects politician behavior.

[7]For models outside of the career concerns framework in which elections play the dual role discussed above, see Banks and Sundaram (1998), Duggan (2000), Schwabe (2010), and Duggan and Martinelli (2016).

the best of our knowledge, our paper is the first to directly analyse the effect of political competition on accountability and selection in a political career concerns setting.

Within the political career concerns literature, the paper most closely related to ours is Ashworth et al. (2017), which analyzes the trade-off between accountability and selection. It departs from the assumption in Holmström (1999) that effort and ability are substitutes and shows that depending on how effort and ability interact in production, increasing effort can be associated with either more or less information about ability. This tension between accountability and selection can be so severe that voter welfare may decrease with effective accountability. For clarity of exposition, in our welfare analysis we abstract from this issue by restricting attention to a case in which there is no tension between a politician's effort and the informativeness of his or her output.[8]

The analysis in our paper also speaks to the literature on incumbency advantage.[9] The typical view in this literature is that higher retention probabilities due to office holding hurt voters by lowering effective accountability and reducing the voters' ability to replace low-quality incumbents. These detrimental effects are compounded by the so-called scare-off effect, according to which high-quality challengers are less likely to run in closed-seat elections. A corollary of this view on incumbency advantage is that increasing political competition is beneficial to voters. Our analysis shows that this conclusion is not warranted from a normative point of view.[10]

2 Model

We now present our model and make some remarks about it.

Agents There are three agents, an incumbent, a citizen who can run for office against the incumbent, and a representative voter. We refer to the incumbent and the citizen as the politicians. Politicians can be of two types: a low-ability type and a high-ability type. We denote a politician's type by τ, where $\tau = L$ if the politician is of low ability and $\tau = H$ otherwise. A politician's

[8]As we discuss in Section 4, our results hold more generally.

[9]See, e.g., Cox and Katz (1996), Levitt and Wolfram (1997), Gordon, Huber, and Landa (2007), Ashworth and Bueno de Mesquita (2008), Hall and Snyder (2015), and Stone, Maisel, and Maestas (2004).

[10]Our message is complementary to the message of Ashworth, Bueno de Mesquita, and Friendenberg (2019), which shows that incumbency advantage can arise solely due to the fact that office holding provides information to voters about an incumbent's type.

type is unknown and is independent of the other politician's type. The (ex-ante) probability that a politician is of high ability is $\pi_0 \in (0, 1)$. We refer to the probability that the other agents assign to a politician being of high ability as the politician's reputation.

Output A politician's output in office depends on his or her private choice of effort $a \in A = [0, \overline{a}]$ and type τ, and is either $y = h$, a 'success', or $y = \ell$, a 'failure'. The probability that a politician of type τ succeeds when he or she exerts effort a is $f(a, \tau) \in (0, 1]$, where f is a twice continuously differentiable, strictly increasing, and strictly concave function of a with $\partial f(0, \tau)/\partial a$ finite for each type τ. Moreover, $f(a, H) > f(a, L)$ for all $a \in A$, so that a high-ability politician is more likely to succeed than a low-ability politician regardless of effort. In an abuse of notation, we let $f(a, \pi) = \pi f(a, H) + (1 - \pi)f(a, L)$ be the expected probability that a politician of reputation π succeeds when he or she exerts effort a. The assumption that output is binary is done for expositional simplicity. In Section 5, we show that we can relax this assumption.

Learning The incumbent's performance in office is observable, and so can be used by all agents to update their beliefs about the incumbent's type.[11] Before deciding whether to run for office, the citizen observes a public signal about his or her ability.[12] For the citizen, observing the signal about his or her ability and observing his or her reputation are equivalent. So, in what follows we treat the citizen's signal and reputation as being the same. Let Ω be the c.d.f. in $[0, 1]$ describing the distribution of the citizen's reputation. It has full support and a continuous density ω.[13]

Preferences Politicians only care about holding office and effort is costly for them. The benefit of holding office is $B > 0$ and the cost of effort a is $c(a)$. The function c is twice continuously differentiable, increasing and convex, and satisfies the 'Inada' conditions $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. The voter's payoff from having a politician in office depends on the politician's output and on an additive politician-specific shock, ξ. Let $F = 0$ and $S > 0$ be the voter's payoff from a failure and a success, respectively. The voter's payoff is then equal

[11] Since types are independent, observing the incumbent's performance provides no information about the citizen.

[12] Our results remain the same if the voter only observes the citizen's signal in case the citizen runs for office. The assumption that the voter observes the signal about the citizen's ability implies that there is no asymmetry of information between the citizen and the voter at the election stage, so that the citizen's decision to run for office has no signalling content. Allowing for signalling would introduce a further source of inefficiency in our environment.

[13] Our results easily extend to the case in which Ω has support $[\underline{\pi}, \overline{\pi}] \subseteq [0, 1]$. We gain no insights by doing so.

to $S + \xi$ if the politician succeeds and equal to ξ if the politician fails. The politician-specific shocks represent a dimension of horizontal differentiation among politicians that co-exists with the dimension of vertical differentiation at the center of our analysis. These shocks are independent across politicians and distributed according to a c.d.f. Λ in $\mathbb{R}$ with support $[-\eta, \eta]$, where $\eta > 0$. The c.d.f. Λ has a bounded density λ that is strictly positive in $[-\eta, \eta]$ and continuous in $(-\eta, \eta)$.

Entry The citizen pays a cost $\kappa \in [0, \overline{\kappa}]$, with $\overline{\kappa} < B$, if he or she runs for office. This cost measures the extent of political competition faced by the incumbent: an increase in κ decreases political competition by making a challenge to the incumbent more difficult. In what follows, we use a change in the entry cost and a change in political competition interchangeably when referring to change in κ.

Timing Action takes place in two periods. In the first period, the incumbent privately chooses his or her effort, output is realized, and all agents update their beliefs about the incumbent's type. In the second period, the citizen observes his or her reputation and decides whether to run for office. Following that, the voter observes the realization of the preference shocks for the incumbent and the citizen (if the citizen runs for office) and chooses which politician to put in office. Finally, the politician in office in the second period chooses his or her effort.

Remarks As in the canonical political career concerns model, the incumbent has an incentive to exert effort so as to manipulate the voter's assessment of his or her ability and increase the likelihood of being retained in office. We depart from the canonical career concerns model by endogenizing the set of agents who run for office against the incumbent in the second period. Two features of our model are important in this regard, namely, the presence of horizontal differentiation and the assumption that the citizen's reputation is random. We discuss these two features next.

Horizontal differentiation introduces uncertainty in the voter's retention decision in period two. Without this uncertainty, the citizen's entry decision does not respond to changes in the entry cost: either the citizen always enters or the citizen never enters. On the other hand, small changes in the entry cost do not change the incumbent's probability of retention conditional on his or her output if the citizen's reputation is not random. Indeed, the citizen who is indifferent between entering or not is not marginal with respect to the voter's retention decision except in knife-edge cases.

3 Equilibria

We begin our analysis by defining and characterizing equilibria. The main results in this section are a characterization of the incumbent's equilibrium choice of effort in the first period and a proof that any interior choice of effort is as an equilibrium choice of effort for the incumbent in the first period for a suitably chosen cost function. Both results play a key role in our subsequent analysis.

3.1 Strategies and Equilibria

A strategy profile specifies the incumbent's effort choice in the first period, the citizen's 'entry' decision in the second period, i.e., the citizen's decision of whether to run for office or not, the voter's appointment decision in the second period, and the effort choice of the politician in office in the second period. Clearly, politicians have no incentive to exert effort in office in the second period. So, in what follows we take the politicians' choice of effort in the second period as given and omit it from our description of equilibria. Moreover, we refer to the incumbent's choice of effort in the first period simply as the incumbent's effort.

We consider pure-strategy Perfect Bayesian Equilibria. Notice that both the citizen's entry decision in the second period and the voter's appointment decision in the same period depend on their conjectures about the incumbent's effort, as this choice affects how the citizen and the voter update their beliefs about the incumbent's type; we discuss this below. In equilibrium, the citizen and the voter are correct about the incumbent's effort. We proceed by backward induction.

3.2 Appointment Decision

The voter reappoints the incumbent if the citizen does not run for office. Suppose now that the citizen runs for office and let π_I and π_C be, respectively, the incumbent's and the citizen's reputation in the second period. Moreover, let ξ_I and ξ_C be, respectively, the realization of the voter's preference shock for the incumbent and the citizen. Since politicians exert no effort in office in the second period, the voter replaces the incumbent if, and only if

$$Sf(0, \pi_I) + \xi_I \leq Sf(0, \pi_C) + \xi_C;$$

we assume, without loss, that the voter replaces the incumbent when indifferent between the incumbent and the citizen. Let $\theta = 2\eta/S[f(0,H) - f(0,L)] > 0$. The next result follows from straightforward algebra.

Lemma 1. *Suppose the citizen runs for office. The voter replaces the incumbent if, and only if,* $\xi_I \leq \xi_C + (\pi_C - \pi_I)2\eta/\theta$.

We now use Lemma 1 to determine the probability that the incumbent is replaced as a function of his or her reputation and the citizen's reputation. Let $G : \mathbb{R} \to [0,1]$ be such that

$$G(x) = \int_{-\infty}^{+\infty} \Lambda\left(\xi + 2x\eta/\theta\right) \lambda(\xi) d\xi.$$

By Lemma 1, the voter replaces the incumbent with probability $G(\pi_C - \pi_I)$. The function G is nondecreasing, with $G(-\theta) = 0$ and $G(\theta) = 1$, and is strictly increasing and continuously differentiable in $(-\theta, \theta)$; see the Appendix for a proof of these properties of G. Thus, the probability that the incumbent is replaced is zero if the reputation difference $\pi_C - \pi_I$ is sufficiently negative, increases with this difference, and becomes one when this difference is sufficiently positive.

The ratio θ measures the importance of preference shocks relative to vertical differentiation between politicians. When θ is small, and vertical differentiation is important, the probability $G(\pi_C - \pi_I)$ is responsive even to small changes in the reputation difference $\pi_C - \pi_I$. On the other hand, when θ is large, it follows that $G(-1) > 0$ and $G(1) < 1$. In this second case, the voter can find it optimal to replace a high-ability incumbent even if the citizen is of low ability and keep a low-ability incumbent even if the citizen is of high ability. We focus our analysis in the case in which θ is small and discuss the large-θ case in Section 5.

Assumption 1. $\theta < \min\{\pi_0, 1 - \pi_0\}$.

3.3 Entry Decision

The citizen's expected payoff from running for office when his or her reputation is π_C and the incumbent's reputation is π_I is

$$BG(\pi_C - \pi_I) - \kappa.$$

We assume the citizen does not run for office when indifferent between doing so and not doing so. This assumption is without loss when the entry cost is positive, as the ex-ante probability that the citizen is indifferent between the two decisions is zero in this case. This assumption constrains the incumbent's behavior when the entry cost is zero, though. Indeed, when $\kappa = 0$, it is (weakly) optimal for the citizen to run for office regardless of his or her reputation. In particular, it is optimal for the citizen to run for office even if his or her reputation is low enough that the probability of being selected for office is zero. We view the assumption that the citizen enters only if it is strictly optimal to do so as a reasonable equilibrium refinement. It rules out situations in which the citizen's equilibrium behavior when $\kappa = 0$ cannot be approximated by the citizen's equilibrium behavior when $\kappa > 0$ regardless of how small the entry cost is.

Let $H : [0, 1] \to [-\theta, \theta]$ be the inverse of G when we restrict the domain of G to $[-\theta, \theta]$. The function H is continuous, strictly increasing, and continuously differentiable in $(0, 1)$, with $H(0) = -\theta$ and $H(1) = \theta$. Given that $\kappa/B \in [0, 1]$ for all $\kappa \in [0, \overline{\kappa}]$, the indifference condition $G(\pi_C - \pi_I) = \kappa/B$ is equivalent to $\pi_C = \pi_I + H(\kappa/B)$. Now let

$$\pi_C(\pi_I, \kappa) = \pi_I + H\Big(\frac{\kappa}{B}\Big). \tag{1}$$

The next result follows immediately.

Lemma 2. *The citizen enters if, and only if,* $\pi_C > \pi_C(\pi_I, \kappa)$.

By Lemma 2, $\pi_C(\pi_I, \kappa)$ is the cutoff reputation of entry for the citizen when the incumbent's reputation is π_I and the cost of entry is κ. The properties of H imply that $\pi_C(\pi_I, \kappa)$ is strictly increasing in π_I and κ. Notice that $\pi_C(\pi_I, \kappa)$ need not be in $[0, 1]$, though. When $\pi_C(\pi_I, \kappa) < 0$, the citizen always enters. On the other hand, when $\pi_C(\pi_I, \kappa) \geq 1$, the citizen never enters.[14] It follows from Lemma 2 that the citizen runs for office with an interior probability if, and only if, $\pi_C(\pi_I, \kappa) \in (0, 1)$, in which case the citizen's probability of entry is responsive to changes in political competition.[15] Notice that $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$ by Assumption 1.

[14]So, technically, $\pi_C(\pi_I, \kappa)$ is not a belief. We still refer to $\pi_C(\pi_I, \kappa)$ as the citizen's cutoff reputation of entry as it is clear how he or she behaves when $\pi_C(\pi_I, \kappa) \notin [0, 1]$.

[15]When there is no horizontal differentiation among politicians, the citizen's entry decision does not respond to changes in κ. Indeed, if $\eta = 0$, then the citizen enters if, and only if, $\pi_C \geq \pi_I$.

3.4 Incumbent's Effort

To conclude our equilibrium characterization, we consider the incumbent's choice of effort in the first period. We begin by determining the probability that the incumbent is retained as a function of his or her reputation in the second period. We then discuss how the incumbent's second-period reputation depends on his or her effort and performance in office in the first period. Together, these two pieces of information allow us to the determine the incumbent's payoff given his or her effort. This, in turn, allows us to characterize the incumbent's equilibrium effort.

Retention The incumbent is retained either when $\pi_C \leq \pi_C(\pi_I, \kappa)$, and the citizen does not run for office, or when $\pi_C > \pi_C(\pi_I, \kappa)$ but $\xi_I > \xi_C + (\pi_C - \pi_I)2\eta/\theta$, and the citizen runs for office but is not chosen by the voter. The probability $Q(\pi_I, \kappa)$ that the incumbent is retained when his reputation is π_I and the entry cost is κ is then equal to

$$Q(\pi_I, \kappa) = \Omega(\pi_C(\pi_I, \kappa)) + \int_{\max\{0, \pi_C(\pi_I, \kappa)\}}^{1} [1 - G(\pi - \pi_I)]\omega(\pi)d\pi,$$

where we adopt the convention that the above integral is zero if $\pi_C(\pi_I, \kappa) > 1$. The next result establishes some properties of $Q(\pi_I, \kappa)$; see the Appendix for a proof.

Lemma 3. *The retention probability $Q(\pi_I, \kappa)$ is continuous in π_I for all $\kappa \in [0, \overline{\kappa}]$, nondecreasing in π_I and κ, and strictly increasing in π_I and κ if $\pi_C(\pi_I, \kappa) \in (0, 1)$. Moreover, $Q(\pi_I, \kappa)$ is continuously differentiable in the set $\{(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa}) : \pi_C(\pi_I, \kappa) \neq 0, 1\}$, with*

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B^2}H'\left(\frac{\kappa}{B}\right) \tag{2}$$

for all $(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa})$ such that $\pi_C(\pi_I, \kappa) \in (0, 1)$.

The intuition behind (2) is simple. When the cutoff reputation of entry is interior, the marginal increase in the probability that the incumbent is retained following a marginal increase in the cost of entry is proportional to: (*i*) the probability that the citizen is on the margin between entering or not entering; and (*ii*) the marginal increase $\partial\pi_C(\pi_I, \kappa)/\partial\kappa$ in the cutoff reputation of entry. It follows from (2) and the fact that $\omega(\pi) > 0$ for all $\pi \in (0, 1)$ that the incumbent's probability of retention is responsive to changes in the entry cost if the citizen's probability of entry is interior.[16]

[16] When the citizen's reputation is deterministic, $Q(\pi_I, \kappa) = 1$ if $\pi_C(\pi_I, \kappa) \geq \pi_C$, and $Q(\pi_I, \kappa) = 1 - G(\pi_C - \pi_I)$ otherwise. So, unless κ is such that $\pi_C(\pi_I, \kappa) = \pi_C$, the probability $Q(\pi_I, \kappa)$ does not respond to small changes in κ.

Belief Updating Let $\pi^+(y|a)$ be the incumbent's reputation in the second period when his or her output in the first period is y and the other agents believe that his or her effort is a. Then

$$\pi^+(h|a) = \frac{f(a,H)\pi_0}{f(a,H)\pi_0 + f(a,L)(1-\pi_0)}$$

and

$$\pi^+(\ell|a) = \frac{(1-f(a,H))\pi_0}{(1-f(a,H))\pi_0 + (1-f(a,L))(1-\pi_0)}.$$

Since $f(a,H) > f(a,L)$ for all $a \in A$, it follows that $\pi^+(\ell|a) < \pi_0 < \pi^+(h|a)$ for all $a \in A$. So, the incumbent's performance in office is informative regardless of his or her effort.

Payoffs and Effort We now compute the incumbent's payoff as a function of his or her effort, and from this determine the incumbent's equilibrium effort. Notice that the incumbent's payoff also depends on the other agents' conjecture about the incumbent's effort, a^e, as this conjecture determines how the incumbent's reputation responds to his or her performance. The incumbent's payoff given a and a^e is

$$U(a,a^e) = B\Big[f(a,\pi_0)Q(\pi^+(h|a^e),\kappa) + (1-f(a,\pi_0))Q(\pi^+(\ell|a^e),\kappa)\Big] - c(a)$$

when the entry cost is κ. Given that $Q(\pi^+(h|a^e),\kappa) \geq Q(\pi^+(\ell|a^e),\kappa)$ regardless of a^e and κ by Lemma 3, it follows that $U(a,a^e)$ is concave in a for all $a^e \in A$ no matter the entry cost.

The effort a^* is an equilibrium choice of effort for the incumbent if, and only if, a^* maximizes $U(a,a^e)$ when $a^e = a^*$. The next result provides a necessary and sufficient condition for $a^* \in A$ to be an equilibrium choice of effort for the incumbent. We use this condition when we analyze the impact of changes in political competition on effective accountability and voter welfare.

Lemma 4. *The effort a^* is an equilibrium choice of effort for the incumbent when the entry cost is κ if, and only if,*

$$B\frac{\partial f}{\partial a}(a^*,\pi_0)\left[Q(\pi^+(h|a^*),\kappa) - Q(\pi^+(\ell|a^*),\kappa)\right] = c'(a^*). \tag{3}$$

The interpretation of condition (3) is straightforward. The left-hand side of (3) is the marginal benefit of effort to the incumbent when he or she exerts effort a^* and the citizen and the voter correctly anticipate the incumbent's behavior. The right-hand side of (3) is the marginal cost of effort to the incumbent when he or she exerts effort a^*. The concavity of $U(a, a^e)$ in a for all $a^e \in A$ implies that (3) is a sufficient condition for a^* to maximize $U(a, a^*)$. In the Appendix, we show that the Inada conditions on the cost function c imply that (3) is also necessary for a^* to maximize $U(a, a^*)$.

It follows from (3) and the Inada condition $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ that $a^* \in A$ can be an equilibrium choice of effort for the incumbent only if $a^* < \overline{a}$. It also follows from (3) and the Inada condition $c'(0) = 0$ that $a^* = 0$ is an equilibrium choice of effort for the incumbent only if the incumbent's probability of retention is the same regardless of his or her output when $a^e = 0$. We show in the Appendix that is not possible since $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$, and so the incumbent's probability of retention responds to his or her performance in office regardless of the conjecture about his or her effort. We have thus established the following corollary to Lemma 4.

Corollary 1. *The incumbent's equilibrium choice of effort is always interior.*

3.5 Rationalization

We conclude this section by showing that any interior choice of effort for the incumbent is consistent with equilibrium behavior. We start with some notation and terminology. For each $a \in A$ and $\kappa \in [0, \overline{\kappa}]$, let

$$\Delta(a, \kappa) = B\frac{\partial f}{\partial a}(a, \pi_0)\left[Q(\pi^+(h|a), \kappa) - Q(\pi^+(\ell|a), \kappa)\right] - c'(a).$$

Suppose $a^* \in (0, \overline{a})$ is the incumbent's equilibrium choice of effort when the entry cost is κ. We say the equilibrium is *stable* if $\partial\Delta(a^*, \kappa)/\partial a$ exists and is negative; this concept is important in the next section.[17] Moreover, we say that a cost function c is *admissible* if it is twice continuously differentiable, increasing, convex, and satisfies the Inada conditions given in Section 2.

We now show that regardless of κ, every $a \in (0, \overline{a})$ can be an equilibrium choice of effort for the incumbent for a suitably chosen cost function.[18] Moreover, we can choose this cost function in such a way that a is the incumbent's unique equilibrium choice of effort and, in case $\partial\Delta(a, \kappa)/\partial a$

[17] Stability of equilibria ensures that we can do local comparative statics using the implicit function theorem and obtain meaningful comparative statics results; see the discussion in the next section.

[18] A straightforward application of the intermediate value theorem shows that (3) has a solution for all $\kappa \in [0, \overline{\kappa}]$ regardless of the cost function (and the other primitives of the model). Thus, an equilibrium always exists.

exists, the equilibrium is stable. Recall that besides the cost function, the primitives of the model are the probability π_0 that politicians are of high type, the production function f, the distribution of the citizen's reputation Ω, the benefit of holding office B, the cost of entry κ, the voter's payoff from a success S and the distribution of the politician-specific preference shocks Λ.

Lemma 5. *Fix all the model's primitives but the cost function. For each $a^* \in (0, \overline{a})$, there exists an admissible cost function c such that a^* is the unique equilibrium choice of effort for the incumbent when the cost function is c. Moreover, for all $\kappa \in [0, \overline{\kappa}]$ such that $\partial\Delta(a^*, \kappa)/\partial a$ exists, we can choose c such that the equilibrium is stable.*

Lemma 5 is not surprising. Indeed, fix all the model's primitives but the cost function c and let

$$MB(a, \kappa) = B\frac{\partial f}{\partial a}(a, \pi_0)\left[Q(\pi^+(h|a), \kappa) - Q(\pi^+(\ell|a), \kappa)\right]$$

be the incumbent's marginal benefit of exerting effort a when the entry cost is κ and the other agents correctly anticipate the incumbent's behavior. Now let $a^* \in (0, \overline{a})$. We know from the proof of Corollary 1 that $MB(a^*, \kappa)$ is positive. Since $MB(a^*, \kappa)$ is bounded above by $B\partial f(0, \pi_0)/\partial a$, it is easy to see that there exists an admissible cost function c with $c'(a^*) = MB(a^*, \kappa)$. In the Appendix, we show that we can take c to be such that $c'(a) \neq MB(a, \kappa)$ for all $a \neq a^*$ and, in case $\partial MB(a^*, \kappa)/\partial a$ exists, $\partial MB(a^*, \kappa)/\partial a < c''(a^*)$.

4 Competition, Accountability, and Voter Welfare

In this section, we study how effective accountability and voter welfare respond to changes in political competition. We begin by discussing our approach to comparative statics. We then present our results about the impact of changes in political competition on effective accountability, followed by our results about the impact of changes in political competition on voter welfare.

4.1 Comparative Statics

It follows from the analysis of the previous section that the incumbent's incentive to exert effort when the other agents conjecture that his or her choice of effort is a is proportional to the increase

$$\delta(a, \kappa) = Q(\pi^+(h|a), \kappa) - Q(\pi^+(\ell|a), \kappa)$$

in the incumbent's probability of retention following a success. By affecting the citizen's entry decision, a change in political competition affects the incumbent's probability of retention conditional on his or her reputation, thus changing $\delta(a,\kappa)$ for all $a \in A$. This, in turn, alters the incumbent's incentive to exert effort, changing the incumbent's equilibrium choice of effort.

When the incumbent's equilibrium choice of effort is unique for every entry cost, his or her response to a change in political competition is unambiguous: if the incumbent's equilibrium choice of effort is a^*, then a change in κ that increases $\delta(a^*,\kappa)$ leads to higher effort and a change in κ that decreases $\delta(a^*,\kappa)$ leads to lower effort. Indeed, suppose that $\Delta(a,\kappa) = 0$ has a unique solution in $(0,\overline{a})$ for all $\kappa \in [0,\overline{\kappa}]$. Let $a_1^* \in (0,\overline{a})$ be the unique equilibrium choice of effort for the incumbent when $\kappa = \kappa_1 \in (0,\overline{\kappa})$. Now let $\kappa_2 \in (0,\kappa_1)$ and suppose, by contradiction, that $\delta(a_1^*,\kappa_2) > \delta(a_1^*,\kappa_1)$ but the unique equilibrium choice of effort for the incumbent when $\kappa = \kappa_2$ is $a_2^* \in (0,a_1^*]$.[19] Thus, $\Delta(a_1^*,\kappa_2) > \Delta(a_1^*,\kappa_1) = 0 = \Delta(a_2^*,\kappa_2)$. Since $c'(\overline{a}) > B\partial f(0,\pi_0)/\partial a$ implies that $\lim_{a\to\overline{a}} \Delta(a,\kappa_2) < 0$, it then follows that $\Delta(a_3^*,\kappa_2) = 0$ for some $a_3^* \in (a_1^*,\overline{a})$, contradicting equilibrium uniqueness when the entry cost is κ_2.

The incumbent's equilibrium choice of effort is globally unique only in some special cases, though.[20] Indeed, we show in the Appendix that if there exist $0 < a_1 < a_2 < \overline{a}$ and $\kappa \in [0,\overline{\kappa}]$ such that $MB(a_1,\kappa)/a_1 < MB(a_2,\kappa)/a_2$, then there exists an admissible cost function c such that equilibria with different effort choices for the incumbent exist when the cost function is c and the entry cost is κ.

In the absence of global uniqueness of the incumbent's equilibrium choice of effort, we rely on local comparative statics analysis using the implicit function theorem. The equation $\Delta(a^*,\kappa) = 0$ defines the incumbent's equilibrium choice of effort implicitly as a function of the entry cost. When $\Delta(a,\kappa)$ is continuously differentiable in a neighborhood of (a^*,κ), we can apply the implicit function theorem to study how the incumbent's equilibrium choice of effort in a given equilibrium responds to small, i.e., local, changes in the entry cost; we know from Lemma 3 that if (a^*,κ) is such that $\pi_C(\pi^+(y|a),\kappa) \neq 0,1$ for $y \in \{\ell,h\}$, then $\Delta(a,\kappa)$ is continuously differentiable in

[19] The proof that a decrease in the entry cost that lowers δ^* leads to less effort is similar, and thus omitted.

[20] One such case is when $\pi^+(h|a)$ is strictly decreasing in a and $\pi^+(\ell|a)$ is strictly increasing in a, so that an increase in the incumbent's effort decreases the dispersion in the distribution of his or her second-period reputation.

a neighborhood of (a^*, κ) when $\kappa > 0$.[21] This requires us to restrict attention to equilibria in which the incumbent's effort a^* is such that $\partial\Delta(a^*, \kappa)/\partial a \neq 0$, otherwise one cannot ensure local uniqueness of the incumbent's equilibrium choice of effort, a necessary condition for a meaningful comparative statics exercise.

On the other hand, as is well known, with multiple equilibria, local comparative statics analysis using the implicit function theorem can lead to ambiguous results in the sense that a local change in the incumbent's incentive to exert effort can lead to opposite changes in the incumbent's effort depending on the equilibrium under play. Hence, as is standard when using the implicit function theorem for local comparative statics analysis, we restrict attention to stable equilibria. This disciplines our analysis by ensuring that a local change in political competition leads to an unambiguous and economically meaningful response in the incumbent's effort: if the incumbent's equilibrium choice of effort is a^*, then a local change in the cost of entry that increases $\delta(a^*, \kappa)$ leads to higher effort, while a local change in the cost of entry that decreases $\delta(a^*, \kappa)$ leads to lower effort.[22]

4.2 Political Competition and Effective Accountability

We now use our equilibrium characterization to study how changes in political competition affect effective accountability in stable equilibria. Given that we rely on local comparative statics analysis using the implicit function theorem, from now on we understand changes in political competition as local changes. Moreover, given our focus on stable equilibria, in the remainder of this section an equilibrium always means a stable equilibrium.

We know from Lemma 3 that regardless of his or her effort and output, the incumbent's probability of retention is nondecreasing in the entry cost and is strictly increasing in the entry cost as long as the citizen's probability of entry is interior. So, for any $a \in (0, \overline{a})$ and $\kappa \in (0, \overline{\kappa})$, the impact of a reduction in κ on $\delta(a, \kappa)$ is ambiguous: it depends on whether the reduction in the citizen's probability of retention after a success is greater or smaller than the reduction in the citizen's probability of retention after a failure. Thus, a priori, the impact of an increase in political competition on effective accountability can be either positive or negative.

[21] Additional care is needed when $\kappa = 0$, as the implicit function theorem is not immediately applicable in this case.

[22] This follows from a straightforward application of the implicit function theorem.

As it turns out, there are conditions under which $\delta(a, \kappa)$ responds unambiguously to a reduction in κ regardless of a. For instance, suppose there exists $\kappa \in (0, \overline{\kappa}]$ such that

$$0 < \pi_C(\pi^+(\ell|a), \kappa) < 1 < \pi_C(\pi^+(h|a), \kappa) \text{ for all } a \in (0, \overline{a}). \tag{4}$$

Then, when the entry cost is κ, the citizen does not enter if the incumbent succeeds and enters with interior probability if the incumbent fails regardless of the incumbent's effort. In this case,

$$\delta(a, \kappa) = 1 - Q(\pi^+(\ell|a), \kappa)$$

and an increase in political competition increases $\delta(a, \kappa)$ for all $a \in (0, \overline{a})$. Consequently, an increase in political competition increases effective accountability when the entry cost is such that (4) holds. A similar argument shows that an increase in political competition decreases effective accountability when the entry cost is such that

$$\pi_C(\pi^+(\ell|a), \kappa) < 0 < \pi_C(\pi^+(h|a), \kappa) < 1 \text{ for all } a \in (0, \overline{a}). \tag{5}$$

Conditions (4) and (5) are rather stringent, though. In particular, they are never satisfied if horizontal differentiation is sufficiently small. Indeed, let $\overline{\pi}^+(\ell) = \sup_{a \in (0,\overline{a})} \pi^+(\ell|a) \in (0, \pi_0)$ and $\underline{\pi}^+(h) = \inf_{a \in (0,\overline{a})} \pi^+(h|a) \in (\pi_0, 1)$. A necessary condition for (4) is that $\underline{\pi}^+(h) + H(\kappa/B) \geq 1$. Given that $H(\kappa/B)$ is bounded above by θ, we then have that (4) holds only if $\theta \geq 1 - \underline{\pi}^+(h)$. Likewise, a necessary condition for (5) is that $\overline{\pi}^+(\ell) + H(\kappa/B) \leq 0$. Since $H(\kappa/B)$ is bounded below by $-\theta$, we then have that (5) holds only if $\theta \geq \overline{\pi}^+(\ell)$. So, (4) and (5) are violated for all $\kappa \in (0, \overline{\kappa}]$ and $a \in (0, \overline{a})$ when $\theta < \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$.[23]

The next result we establish is that conditions (4) and (5) are essentiality necessary for an unambiguous response of effective accountability to an increase in political competition: if there exist $\kappa \in (0, \overline{\kappa}]$ and $a \in (0, \overline{a})$ such that the probability of entry for the citizen is interior regardless of the incumbent's performance when the incumbent's effort is a and the entry cost is κ, then the response of effective accountability to an increase in political competition is undetermined. In what follows, we say that a cumulative distribution function Ω in $[0, 1]$ is an *admissible* distribution of the citizen's reputation if it has a continuous density ω.

[23]Notice that $\theta > \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$ is sufficient for either (4) or (5) to hold for some $\kappa \in (0, B]$. Indeed, since $\min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\} < \min\{\pi_0, 1 - \pi_0\}$, then either $\theta \in (\overline{\pi}^+(\ell), \pi_0)$ or $\theta \in (1 - \underline{\pi}^+(h), 1 - \pi_0)$ when $\theta > \min\{\overline{\pi}^+(\ell), 1 - \underline{\pi}^+(h)\}$. In the first case, (5) holds for κ small. In the second case, (4) holds for κ close to B.

Proposition 1. *Suppose the entry cost is $\kappa \in (0, \overline{\kappa})$ and fix all other primitives of the model but the cost function c and the distribution of the citizen's reputation Ω. If there exists $a^* \in (0, \overline{a})$ with*

$$0 < \pi_C(\pi^+(\ell|a^*), \kappa) < \pi_C(\pi^+(h|a^*), \kappa) < 1, \tag{6}$$

then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c, \Omega) = (c_1, \Omega_1)$ and when $(c, \Omega) = (c_2, \Omega_2)$; and (ii) an increase in political competition increases effective accountability in one case and decreases it in the other.

Proof. Suppose (6) holds for $a^* \in (0, \overline{a})$. Let Ω_1 and Ω_2 be admissible distributions of the citizen's reputation with densities ω_1 and ω_2 such that $\omega_1(\pi_C(\pi^+(h|a^*), \kappa)) < \omega_1(\pi_C(\pi^+(\ell|a^*), \kappa))$ and $\omega_2(\pi_C(\pi^+(h|a^*), \kappa)) > \omega_2(\pi_C(\pi^+(\ell|a^*), \kappa))$. By Lemma 5, for each Ω_i there exists an admissible cost function c_i such that a^* is the unique equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c_i and the distribution of the citizen's reputation is Ω_i. Now let $Q_i(\pi_I, \kappa)$ be the incumbent's probability of retention given his or her second-period reputation and the entry cost when the distribution of the citizen's reputation is Ω_i, and let $\delta_i(a, \kappa) = Q_i(\pi^+(h|a), \kappa) - Q_i(\pi^+(\ell|a), \kappa)$. The desired result follows since, by Lemma 3,

$$\frac{\partial \delta_i}{\partial \kappa}(a^*, \kappa) \propto \omega_i(\pi_C(\pi^+(h|a^*), \kappa)) - \omega_i(\pi_C(\pi^+(\ell|a^*), \kappa)). \qquad \square$$

The idea behind Proposition 1 is simple. The response of the incumbent's effort to a change in political competition depends on how sensitive to changes in competition are the 'supplies' of challengers to the incumbent after a success and a failure. An increase in political competition increases effort if the supply of challengers after a failure is more responsive to a change in the cost of entry then the supply of challengers after a success, and decreases otherwise. Lemma 5 shows that both situations are compatible with the *same* equilibrium choice of effort for the incumbent.

A corollary to Proposition 1 is that without additional assumptions on the distribution of the citizen's reputation, one cannot predict how effective accountability responds to a change in political competition when the cost of entry is such that (6) holds for some effort level a. For instance, if the distribution Ω of the citizen's reputation is strictly concave, then $\omega(\pi_C(\pi^+(h|a^*), \kappa)) < \omega(\pi_C(\pi^+(\ell|a^*), \kappa))$ for all $(a^*, \kappa) \in (0, \overline{a}) \times (0, \overline{\kappa})$ such that (6) holds. On the other hand, if Ω

is strictly convex, then $\omega(\pi_C(\pi^+(h|a^*), \kappa)) > \omega(\pi_C(\pi^+(\ell|a^*), \kappa))$ for all $(a^*, \kappa) \in (0, \overline{a}) \times (0, \overline{\kappa})$ such that (6) holds. Since $\theta < \overline{\pi}^+(\ell)$ implies that there exist $a \in (0, \overline{a})$ and $\kappa' \in (0, \overline{\kappa}]$ such that (6) holds for the effort level a when the entry cost is $\kappa \in (0, \kappa')$, one then cannot predict how effective accountability responds to a change in political competition when horizontal differentiation is sufficiently small if the entry cost is also sufficiently small.

Proposition 1 does not concern how the set of equilibrium effort choices for the incumbent depends on the cost of entry. The next result shows that effective accountability need *not* be maximized when the cost of entry is zero if horizontal differentiation is small enough.[24]

Proposition 2. *Fix all the model's primitives but the cost function, the distribution of the citizen's reputation, and the entry cost. If $\theta < \overline{\pi}^+(\ell)$, then there exist a cost function and a distribution of the citizen's reputation for which effective accountability is not maximized when entry is costless.*

Proof. Suppose $\theta < \overline{\pi}^+(\ell)$. Then $\theta < \pi^+(\ell|a^*)$ for some $a^* \in (0, \overline{a})$. Let Ω be an admissible distribution of the citizen's reputation with density ω satisfying $\omega(\pi^+(\ell|a^*) - \theta) < \omega(\pi^+(h|a^*) - \theta)$. Now let c be an admissible cost function such that a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable when the entry cost is zero, the cost function is c, and the distribution of the citizen's reputation is Ω; notice that $\partial\Delta(a^*, 0)/\partial a$ exists by construction. Take the cost function and the distribution of the citizen's reputation to be c and Ω, respectively.

We show in the Appendix that the uniqueness of the incumbent's equilibrium choice of effort and the stability of the equilibrium when $\kappa = 0$ together imply that there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique and differentiable in κ for all $\kappa \in (0, \underline{\kappa})$, and converges to a^* when κ decreases to zero. Moreover, if $a^*(\kappa)$ is the incumbent's unique equilibrium choice of effort when the entry cost is $\kappa \in (0, \underline{\kappa})$, then $\partial\Delta(a^*(\kappa), \kappa)/\partial a$ exists and is negative for all $\kappa \in (0, \underline{\kappa})$. Implicitly differentiating the equilibrium condition $\Delta(a^*(\kappa), \kappa) = 0$, we have that

$$\frac{\partial\Delta}{\partial a}(a^*(\kappa), \kappa)\frac{da^*}{d\kappa}(\kappa) + \frac{\partial\Delta}{\partial\kappa}(a^*(\kappa), \kappa) = 0$$

for all $\kappa \in (0, \underline{\kappa})$. Now observe from Lemma 3 that

$$\frac{\partial\Delta}{\partial\kappa}(a^*(\kappa), \kappa) \propto \omega\Big(\pi^+(h|a^*(\kappa)) + H\left(\frac{\kappa}{B}\right)\Big) - \omega\Big(\pi^+(\ell|a^*(\kappa)) + H\left(\frac{\kappa}{B}\right)\Big)$$

[24] It is also possible to show that for any positive cost of entry there are cases in which multiple stable equilibria exist and effective accountability responds differently to an increase in political competition in these equilibria.

for all $\kappa \in (0, \underline{\kappa})$. Since $\lim_{\kappa \to 0} \pi^+(y|a^*(\kappa)) + H(\kappa/B) = \pi^+(y|a^*) - \theta$ for each $y \in \{\ell, h\}$ and the density ω is continuous, it follows that $\partial \Delta(a^*(\kappa), \kappa)/\partial \kappa > 0$ if κ is sufficiently close to zero. Reducing $\underline{\kappa}$ even further, we can then conclude that $da^*(\kappa)/d\kappa > 0$ for all $\kappa \in (0, \underline{\kappa})$. Thus, effective accountability is not maximized when $\kappa = 0$. □

4.3 Political Competition and Voter Welfare

We now study the impact of changes in political competition on voter welfare. A change in political competition directly affects voter welfare by changing the citizen's cutoff reputation of entry, thus affecting the pool of politicians available for the voter to choose in the second period. A change in political competition also indirectly affects voter welfare by changing the incumbent's effort. This change in the incumbent's effort not only affects the voter's payoff in the first period, but also affects the voter's payoff in the second period. First, it changes the informational content of the incumbent's performance, affecting the selection of politicians for office. Second, it further affects the citizen's entry decision by changing the incumbent's reputation after each possible output.

Ashworth et al. (2017) show that depending on how effort affects the incumbent's performance, there is a tension between accountability and selection, as an increase in effort may reduce the informational content of the incumbent's performance. In order to have a cleaner analysis, we rule this out by assuming that the production function f is such that $\pi^+(h|a)$ is strictly increasing in a and $\pi^+(\ell|a)$ is strictly decreasing in a. So, an increase in the incumbent's effort increases the dispersion in the distribution of his or her second-period reputation, making the incumbent's performance more informative about ability. As we discuss below, we can relax this assumption.

The remainder of this part is organized as follows. First, we compute voter welfare. Then, we discuss how voter welfare responds to a change in political competition. We show that: (*i*) if horizontal differentiation is small enough, then one cannot predict how voter welfare responds to changes in political competition when the entry cost is also small enough; and (*ii*) voter welfare need not be maximized when the entry cost is zero.[25] Thus, our conclusions about the impact of changes in political competition on effective accountability extend to voter welfare.

[25] We ignore the politicians' payoffs in our welfare analysis. This assumption is justified on the grounds that the set of politicians is small compared to the set of voters.

Computing Voter Welfare Voter welfare is the sum of the first-period welfare and the second-period welfare. The first-period welfare is $W_1(a) = Sf(a, \pi_0)$, the voter's expected payoff when the incumbent's effort is a. In order to determine the second-period welfare, let $\mu(y|a)$ be the probability that the incumbent's output is y when his or her effort is a and let z be the random variable such that $z = \xi_I - \xi_C$. Notice that $\mathbb{E}_z[z] = 0$ since ξ_I and ξ_C have zero mean. Suppose the incumbent's equilibrium effort is a and his or her output is y. Two events are possible. Either the citizen enters and the voter's expected payoff is $\mathbb{E}_z\big[\max\{Sf(0, \pi^+(y|a), \kappa) + z, Sf(0, \pi_C)\}\big]$ or the citizen does not enter and the voter's expected payoff is $Sf(0, \pi^+(y|a))$. Given that the citizen enters if, and only if, his or her reputation is greater than the cutoff reputation of entry $\pi_C(\pi^+(y|a), \kappa)$, it follows that the second-period welfare is

$$W_2(a,\kappa) = \sum_{y \in \{\ell, h\}} \mu(y|a) \Bigg\{ \int_{\pi_C(\pi^+(y|a),\kappa)}^{1} \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a)) + z, Sf(0,\pi)\}\big]\omega(\pi)d\pi$$
$$+\Omega(\pi_C(\pi^+(y|a),\kappa))\mathbb{E}_z\big[Sf(0,\pi^+(y|a)) + z\big]\Bigg\}; \quad (7)$$

we adopt the convention that the above integral is zero if $\pi_C(\pi^+(y|a), \kappa) \geq 1$ and the citizen does not enter regardless of his or her reputation.

Let Γ denote the cumulative distribution function of the random variable z. We show in the Appendix that $W_2(a, \kappa) = W_2^+(a) - W_2^-(a, \kappa)$, where

$$W_2^+(a) = \sum_{y \in \{\ell, h\}} \mu(y|a) \int_0^1 \mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a)) + z, Sf(0,\pi)\}\big]\omega(\pi)d\pi$$

and

$$W_2^-(a,\kappa) = \sum_{y \in \{\ell, h\}} \mu(y|a) \int_0^{\pi_C(\pi^+(y|a),\kappa)} \underbrace{\left(\int_{-2\eta}^{(\pi - \pi^+(y|a))2\eta/\theta} \Gamma(z)dz \right)}_{I(\pi)} \omega(\pi)d\pi.$$

The term $W_2^+(a)$ is the voter's second-period welfare when the citizen always enters, while the term $W_2^-(a, \kappa)$ is the voter's welfare loss in the second period due to the cost of entry. When the cost of entry is positive, the citizen may choose not to enter even if there is a positive probability that he or she is better for the voter than the incumbent, which lowers voter welfare. Since the integrand $I(\pi)$ is strictly increasing in π, it follows that

$$\int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \Gamma(z)dz \right) \omega(\pi)d\pi \leq I(\pi_C) = \int_{-2\eta}^{H(\kappa/B)2\eta/\theta} \Gamma(z)dz;$$

recall that $\pi_C(\pi_I, \kappa) = \pi_I + H(\kappa/B)$. Given that $\lim_{\kappa \to 0} H(\kappa/B)2\eta/\theta = -2\eta$, it then follows that $W_2^-(a, \kappa)$ converges to zero as κ decreases to zero regardless of the effort a.

Welfare Changes We now discuss how a change in political competition affects voter welfare. Suppose the entry cost is $\kappa_0 \in (0, \overline{\kappa})$ and let $a^*(\kappa)$ be the incumbent's equilibrium choice of effort as a function of the entry cost when this cost is in a neighborhood of κ_0; our restriction to stable, and thus regular, equilibria ensures that $a^*(\kappa)$ is well-defined and continuously differentiable in κ. Now let $W^*(\kappa) = W(a^*(\kappa), \kappa)$ be the voter's welfare when κ is in a neighborhood of κ_0. The (first-order) effect of a change in the entry cost on voter welfare is[26]

$$\begin{aligned}\frac{dW^*}{d\kappa}(\kappa_0) = & \left[\frac{dW_1}{da}(a^*(\kappa_0)) + \frac{dW_2^+}{da}(a^*(\kappa_0)) - \frac{\partial W_2^-}{\partial a}(a^*(\kappa_0), \kappa_0)\right]\frac{da^*}{d\kappa}(\kappa_0) \\ & - \frac{\partial W_2^-}{\partial \kappa}(a^*(\kappa_0), \kappa_0).\end{aligned}$$

The direct effect of a change in the entry cost on voter welfare is

$$\Delta W^*_{\text{direct}} = -\frac{\partial W_2^-}{\partial \kappa}(a^*(\kappa_0), \kappa_0)$$

The term $\partial W_2^-(a, \kappa)/\partial \kappa$ describes the effect on the voter's second-period welfare of the resulting change in the citizen's cutoff reputation of entry. In the Appendix, we compute $\partial W_2^-(a, \kappa)/\partial \kappa$ and show that this term is positive. Intuitively, a higher cost of entry increases the cutoff reputation of entry, which decreases voter welfare by reducing the pool of politicians available for the voter to choose in the second period. Thus, $\Delta W^*_{\text{direct}}$ is negative.

The indirect effect of a change in the entry cost on voter welfare is

$$\Delta W^*_{\text{indirect}} = \left[\frac{dW_1}{da}(a^*(\kappa_0)) + \frac{dW_2^+}{da}(a^*(\kappa_0)) - \frac{\partial W_2^-}{\partial a}(a^*(\kappa_0), \kappa_0)\right]\frac{da^*}{d\kappa}(\kappa_0)$$

The term $dW_1(a)/da = S\partial f(a, \pi_0)/\partial a$ describes the effect of a change in the incumbent's effort on the voter's first-period welfare, while the terms $dW_2^+(a)/da$ and $\partial W_2^-(a, \kappa)/\partial a$ describe the effect of a change in the incumbent's effort on the voter's second-period welfare due to the change in the informational content of the incumbent's performance and the change in the citizen's entry decision conditional on the incumbent's performance, respectively.

[26]The differentiability of $W_1(a)$ and $W_2^-(a, \kappa)$ is straightforward to establish. The differentiability of $W_2^+(a)$ follows from standard results in measure theory; see the Appendix for a proof.

Clearly, $dW_1(a)/da$ is positive. Since $\pi^+(h|a)$ is strictly increasing in a and $\pi^+(\ell|a)$ is strictly decreasing in a, the convexity of $\mathbb{E}_z[\max\{Sf(0,\pi^+)+z, Sf(0,\pi)\}]$ in π^+ implies that $dW_2^+(a)/da$ is also positive. Intuitively, if effort increases the dispersion in the incumbent's second-period reputation, then more effort improves the selection of politicians for office, increasing voter welfare.[27] In the Appendix, we compute $\partial W_2^-(a,\kappa)/\partial a$ and show that it converges to zero in the limit as κ decreases to zero uniformly in a. So, the indirect effect of a change in political competition on voter welfare due to the resulting change in the incumbent's effort has the same sign at the change in the incumbent's effort if the entry cost is sufficiently small.

In general, a comparison of the direct and indirect effects on voter welfare of a change in political competition is not possible. However, the next result shows that when horizontal differentiation is sufficiently small, there are values of the model's primitives for which a stable equilibrium exists in which the sign of $dW^*(\kappa_0)/d\kappa$ is the same as the sign of $\Delta W^*_{\text{indirect}}$ as long as the entry cost is also small enough. The proof of Lemma 6 is in the Appendix; recall from the proof of Proposition 2 that if $\theta < \overline{\pi}^+(\ell)$, then there exists $a^* \in (0, \overline{a})$ with $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$.

Lemma 6. *Fix all the primitives of the model but the cost function, the distribution of the citizen's reputation, and the entry cost, and suppose that $\theta < \overline{\pi}^+(\ell)$. For all $a^* \in (0, \overline{a})$ with the property that $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$, there exist $\kappa' \in (0, \overline{\kappa})$ such that if the entry cost is $\kappa_0 \in (0, \kappa')$, then: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium for a suitably chosen cost function and a suitably chosen distribution of the citizen's reputation; and (ii) $dW^*(\kappa_0)/d\kappa$ has the same sign as $\Delta W^*_{\text{indirect}}$ in this equilibrium.*

An immediate corollary of Lemma 6 and the discussion preceding it is that when horizontal differentiation is small enough, there are choices of the model's primitives for which a stable equilibrium exists in which $dW^*(\kappa_0)/d\kappa$ and $da^*(\kappa_0)/d\kappa$ have the same sign if the entry cost is also small enough. The next result now follows immediately from Propositions 1 and 2.[28]

[27] More precisely, the assumptions on $\pi^+(h|a)$ and $\pi^+(\ell|a)$ imply that the distribution of the incumbent's second-period reputation decreases in the second-order stochastic sense with an increase in a.

[28] While Proposition 3 shows that voter welfare need not be maximized when entry is costless, clearly it is never optimal for the voter to have the entry cost so high that the citizen never runs for office.

Proposition 3. *Fix all the primitives of the model but the cost function c, the distribution of the citizen's reputation Ω, and the entry cost κ. Suppose that $\theta < \overline{\pi}^{+}(\ell)$. The following facts hold.*

1. *Let $a^* \in (0, \overline{a})$ satisfy $0 < \pi^{+}(\ell|a^*) - \theta < \pi^{+}(h|a^*) - \theta < 1$. There exists $\kappa' \in (0, \overline{\kappa})$ such that if $\kappa \in (0, \kappa')$, then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c, \Omega) = (c_1, \Omega_1)$ and when $(c, \Omega) = (c_2, \Omega_2)$; and (ii) an increase in political competition increases voter welfare in one case and decreases it in the other.*

2. *Voter welfare is not maximized when the entry cost is zero for a suitably chosen cost function and a suitably chosen distribution of the citizen's reputation.*

We can dispense with the assumption that effort increases the informativeness of output in our welfare analysis; see the Appendix for a proof. The key observation is that we can take the production function to be such that while the incumbent's performance is sensitive to his or her effort, the informational content of the incumbent's performance is small. In this case, the sign of $dW_1(a)/da + dW_2^{+}(a)/da$ is positive even if effort does not increase the informativeness of output.

5 Contractual Incompleteness

An important feature of our environment is that the only mechanism the voter has to incentivize a politician is retention. In this section, we discuss what happens when the voter has a richer set of instruments that he or she can use to provide incentives for a politician and show that the contractual incompleteness in our environment is at the heart of our results about the relationship between voter welfare and political competition.[29] For simplicity, we assume that the incumbent is risk neutral and that providing additional incentives to the incumbent is costless. It is straightforward to drop the assumption of risk neutrality. We return to the second issue at the end of this section.

Suppose the voter can provide the incumbent with an additional nonnegative payoff $v(y)$ in case his or her output is y; so, there is limited liability in the sense that the voter cannot impose

[29] Anesi and Buisseret (2019) show that in the presence of moral hazard and adverse selection, repeated elections as an instrument to both discipline and select politicians can approach efficiency in a long-run relationship between a voter and a set of politicians, playing the role of output-contingent transfers. An interesting question is whether repeated elections can play a similar role in a political career concerns environment.

punishments on the incumbent.[30] It will be clear from the analysis that follows that our results would remain the same if the payoffs $v(\ell)$ and $v(h)$ were contingent on retention, which would amount to making the benefit of holding office conditional on the incumbent's performance. The incumbent's payoff as a function of his or her effort and the other agents' conjecture about his or her effort is now given by

$$U(a, a^e) = f(a, \pi_0)\big[BQ(\pi^+(h|a), \kappa) + v(h)\big] + (1 - f(a, \pi_0))\big[BQ(\pi^+(\ell|a), \kappa) + v(\ell)\big] - c(a).$$

A straightforward modification of the argument leading to Lemma 4 in Section 3 shows that a^* is an equilibrium choice of effort for the incumbent if, and only if,

$$\frac{\partial f}{\partial a}(a^*, \pi_0)\Big\{B\big[Q(\pi^+(h|a^*), \kappa) - Q(\pi^+(\ell|a^*), \kappa)\big] + v(h) - v(\ell)\Big\} = c'(a^*). \tag{8}$$

So, by adjusting the difference $v(h) - v(\ell)$, the voter can affect the incumbent's effort.

In our analysis, we assume that $c'(a)$ is finite for all $a \in A$. Thus, by adjusting the reward difference $v(h) - v(\ell)$ appropriately, the voter can induce the incumbent to exert any effort in A. The assumption that every choice of effort by the incumbent is incentive feasible once the voter can provide the incumbent with additional nonnegative output-contingent rewards is reasonable. It implies that what can prevent the voter from inducing the incumbent to behave in the voter's best interest is the fact that retention is the only incentive device available to the voter.

Let $W(a, \kappa)$ be the voter's welfare when the incumbent's effort is a and the entry cost is κ and $a_{\max}(\kappa)$ be the choice of effort that maximizes $W(a, \kappa)$.[31] Typically, $a_{\max}(\kappa)$ is not an equilibrium choice of effort for the incumbent when the entry cost is κ if retention is the only incentive device available to the voter. Indeed, if $a_{\max}(\kappa)$ is a solution to (8) when $v(\ell) = v(h) = 0$ and the entry cost is κ, then any perturbation in the benefit B of holding office or the marginal cost $c'(a_{\max}(\kappa))$ of exerting effort $a_{\max}(\kappa)$ changes the incumbent's equilibrium choice of effort without changing voter welfare, and so $a_{\max}(\kappa)$. On the other hand, for any $\kappa \in [0, \overline{\kappa}]$, there exist choices of $v(\ell)$ and

[30] We assume that the voter cannot provide the politician in office in period two with any incentive to exert effort. When output-contingent contracts are possible in period two, the voter can ensure that the politician in office in period two chooses the static first-best effort. It is straightforward to extend the analysis to this case.

[31] Notice that if effort increases the informativeness of the incumbent's performance, then $a_{\max}(\kappa) = \overline{a}$ when entry is costless. Maximizing effort need not maximize voter welfare when entry is costly, though, as doing so discourages entry after a success, which might hurt welfare.

$v(h)$ for which $a_{\max}(\kappa)$ is a solution to (8), and so an equilibrium choice of effort the incumbent, when the entry cost is κ. Now let $W^{**}(\kappa) = W(a_{\max}(\kappa), \kappa)$. By the envelope theorem[32]

$$\frac{dW^{**}}{d\kappa}(\kappa) = -\frac{\partial W_2^-}{\partial \kappa}(a_{\max}(\kappa), \kappa) < 0.$$

Therefore, when the set of contracts that the voter can offer to incumbent is sufficiently rich, an increase in political competition is always beneficial to the voter.

As discussed above, we abstract from the cost of incentive provision in our analysis. This assumption is justified on the grounds that the set of politicians is small relative to the set of voters, and so the per-capita cost of incentive provision is small. Nevertheless, our analysis would go unchanged if it was costly to provide incentives to politicians. Loosely speaking, incorporating this cost in the voter's welfare would mean that for any entry cost κ, the voter's maximization problem would now consist of choosing a triple $(a, v(\ell), v(h))$ that maximizes the voter's welfare subject to the constraint that the effort a is incentive-feasible given the payoff vector $(v(\ell), v(h))$ when the entry cost is κ. An envelope-theorem argument would still show that the impact of a change in κ on voter welfare is equal to the direct impact of a change in κ on voter welfare when welfare is evaluated at the welfare-maximizing triple $(a, v(\ell), v(h))$. The latter term is negative for the same reason as in the previous section: an increase in the cost of entry increases the cutoff reputation of entry for the citizen, thus reducing the pool of candidates available for the voter to choose in the second period.

An implicit assumption in our analysis in this section is that if (8) has more than one solution, then the incumbent's equilibrium choice of effort is the one most desired by the voter. It is important to emphasize that this assumption is consistent with the core of the analysis in the previous section, as Propositions 1 and 2 about the ambiguous effect of political competition on effective accountability and Proposition 3 about the ambiguous effect of political competition on voter welfare were derived under conditions ensuring equilibrium uniqueness. Moreover, as we show in the next section in our analysis of the finite-output extension of our model, with output-contingent rewards the voter can uniquely pin down the incumbent's behavior when there are more than two output levels by suitably designing these rewards (under some technical assumptions).

[32] The result is immediate if $\partial W(a_{\max}(\kappa), \kappa)/\partial a = 0$. If, instead, $\partial W(a_{\max}(\kappa), \kappa)/\partial a \neq 0$, then $a_{\max}(\kappa)$ is not interior and a local change in κ does not affect $a_{\max}(\kappa)$, in which case the result also holds.

6 Robustness

We assume that output is binary and restrict attention to the case in which horizontal differentiation is small. In this section, we show that our results hold when we relax these two assumptions. We start with the analysis of the finite-output case.

6.1 Finite-Output Case

Suppose a politician's output is now $y \in Y = \{y_0, \ldots, y_N\}$, with $y_0 < \ldots < y_N$ and $N \geq 1$, and keep all else in the model the same. For each $y \in Y$, let $f(y|a,\tau) \in (0,1]$ be the probability that a politician of type $\tau \in \{L, H\}$ produces output y when he or she exerts effort a. For each $y \in Y$ and $\tau \in \{L, H\}$, the function $f(y|a,\tau)$ is twice continuously differentiable in a with bounded derivatives $\partial f(y|a,\tau)/\partial a$. Moreover, we make the following two assumptions:

1. The ratio $f(y|a,H)/f(y|a,L)$ is strictly increasing in y for all $a \in A$.

2. For all $a' > a$ and $\tau \in \{L, H\}$, the ratio $f(y|a',\tau)/f(y|a,\tau)$ is strictly increasing in y.

Assumptions 1 and 2 are standard monotone likelihood-ratio assumptions. Assumption 1 implies that $\pi^+(y|a)$ is strictly increasing in y for all $a \in A$. Let $F(y|a,\tau) = \sum_{y' \leq y} f(y|a,\tau)$ be the probability that a politician of type τ produces output y or less when he or she exerts effort a. Assumption 2 implies that for each $\tau \in \{L, H\}$, the probability $F(y|a,\tau)$ is strictly decreasing in a for all $y < y_N$, i.e., an increase in effort increases the distribution of the politician's output strictly in the first-order stochastic sense for both types of politician.

Let $f(y|a,\pi_0) = \pi_0 f(y|a,H) + (1-\pi_0) f(y|a,L)$ be the ex-ante probability that the incumbent produces output y when he or she exerts effort a. Then

$$U(a, a^e) = B \sum_{i=0}^{N} f(y_i|a,\pi_0) Q(\pi^+(y|a^e), \kappa) - c(a)$$

is the incumbent's payoff when he or she exerts effort a and the other agents believe that his or her effort is a^e. As before, $Q(\pi_I, \kappa)$ is the probability that the incumbent is retained if his or her reputation in the second period is π_I and the entry cost is κ. Now let $F(y|a,\pi_0) = \pi_0 F(y|a,H) + (1-\pi_0) F(y|a,L)$ be the ex-ante probability that the incumbent produces output y or less when he or she exerts effort a. Straightforward algebra shows that

$$U(a, a^e) = B\left\{Q(\pi^+(y_N|a^e), \kappa) + \sum_{i=0}^{N-1} F(y_i|a, \pi_0)\left[Q(\pi^+(y_i|a^e), \kappa) - Q(\pi^+(y_{i+1}|a^e), \kappa)\right]\right\} - c(a)$$

In light of the above expression for $U(a, a^e)$, we make the following additional assumption:

3. $F(y|a, \tau)$ is convex in a for each $y \in Y$ and $\tau \in \{L, H\}$.

Assumption 3 implies that $U(a, a^e)$ is strictly concave in a for all $a^e \in A$, in which case the Inada conditions on the cost function c imply that the first-order condition $\partial U(a, a^e)/\partial a = 0$ is necessary and sufficient for a to maximize the incumbent's payoff given a^e.[33] From this it follows that

$$B\left\{\sum_{i=1}^{N-1} \frac{\partial F}{\partial a}(y_i|a^*, \pi_0)\left[Q(\pi^+(y_i|a^*), \kappa) - Q(\pi^+(y_{i+1}|a^*), \kappa)\right]\right\} = c'(a^*) \tag{9}$$

is necessary and sufficient for a^* to be an equilibrium choice of effort for the incumbent. The above equation extends condition (3) in the binary-output case to the finite-output case.

For ease of exposition, we restrict the distribution of the citizen's reputation to be such that $\omega(0) = \omega(1) = 0$. We show in the Appendix that this implies that the retention probability $Q(\pi_I, \kappa)$ is continuously differentiable in $(0, 1) \times (0, \overline{\kappa})$, which in turn implies that the function

$$\Delta(a, \kappa) = B\left\{\sum_{i=1}^{N-1} \frac{\partial F}{\partial a}(y_i|a, \pi_0)\left[Q(\pi^+(y_i|a), \kappa) - Q(\pi^+(y_{i+1}|a), \kappa)\right]\right\} - c'(a)$$

is continuously differentiable in $(0, \overline{a}) \times (0, \overline{\kappa})$. We can relax the restriction that $\omega(0) = \omega(1) = 0$ at the cost of more cumbersome statements for our results.

Since $\pi^+(y|a)$ is strictly increasing in y for all $a \in A$, the martingale property of beliefs implies that for each $a \in A$, there exists $j \in \{1, \ldots, N-1\}$ such that $\pi^+(y_j|a) < \pi_0 < \pi^+(y_{j+1}|a)$. So, given that $\pi_C(\pi_0, \kappa)$ is interior for all $\kappa \in [0, \overline{\kappa}]$ by Assumption 1, the left-hand side of (9) is positive for all $a^* \in A$; as before, $\pi_C(\pi_I, \kappa) = \pi_I + H(\kappa/B)$ is the cutoff reputation of entry for a citizen with reputation π_I when the entry cost is κ. Therefore, the incumbent's effort is positive in any equilibrium. The analysis now proceeds almost exactly as in Sections 3 and 4.

[33] Assumption 3 is the counterpart in our environment of the assumption that the distribution of an agent's output is convex in effort used in standard moral problems to ensure the validity of the first-order approach.

First, notice that we can immediately extend Lemma 5 to the finite-output case. We can then use this result to establish that if there exist $j \in \{0, \ldots, N-1\}$, $a^* \in (0, \overline{a})$, and $\kappa \in (0, \overline{\kappa})$ with

$$0 < \pi_C(\pi^+(y_j|a^*), \kappa) < \pi_C(\pi^+(y_{j+1}|a^*), \kappa) < 1,$$

then the following holds when the entry cost is κ: (*i*) there exist choices of the cost function and distribution of the citizen's reputation for which a^* is the incumbent's equilibrium choice of effort and a^* increases with an increase in political competition; and (*ii*) there exist choices of the cost function and distribution of the citizen's reputation for which a^* is the incumbent's equilibrium choice of effort and a^* decreases with an increase in political competition. Moreover, if we let $\overline{\pi}^+(y) = \sup_{a \in A} \pi^+(y|a)$ be the highest reputation possible for the incumbent when his or her output is $y \in Y$, then we can also use the extension of Lemma 5 to show that if $\theta < \overline{\pi}^+(y_{N-1})$, then effective accountability need not be maximized when entry is costless. For completeness, we include formal statements and proofs of these results in the Appendix.

Finally, welfare analysis in the finite-output case is taken almost verbatim from the previous section. The only difference is that now summations in y are over the set Y and the probability $\mu(y|a)$ that the incumbent's output is y when his or her effort is a is equal to $f(y|a, \pi_0)$.

To conclude the analysis of the finite-output case, we discuss what happens when the voter can provide the incumbent with an additional non-negative payoff $v(y)$ in case the incumbent's output is y. It is straightforward to show that given a vector $v = (v(y_0), \ldots, v(y_N))$ of output-contingent rewards, a^* is an equilibrium choice of effort for the incumbent if, and only if,

$$\sum_{i=0}^{N-1} \frac{\partial F}{\partial a}(y_i|a^*, \pi_0)\Big\{B\big[Q(\pi^+(y_i|a^*), \kappa) - Q(\pi^+(y_{i+1}|a^*), \kappa)\big] + v(y_i) - v(y_{i+1})\Big\} = c'(a^*). \quad (10)$$

As in the binary-output case, by adjusting the vector of rewards, v, the voter can ensure that the welfare maximizing choice of effort, $a_{\max}(\kappa)$, is a solution to (10). From this, an envelope-theorem argument shows that welfare is strictly increasing in political competition.

We now show that under a technical assumption, the voter can choose v so that $a_{\max}(\kappa)$ is the unique solution to (10). For each $a \in A$, let $w(a) = (\partial F(y_0|a, \pi_0)/\partial a, \ldots, \partial F(y_{N-1}|a, \pi_0)/\partial a)$. Suppose the following holds:

4. For all $a, a' \in A$ such that $a \neq a'$, the vectors $w(a)$ and $w(a')$ are not colinear.

Notice that Assumption 4 can only be satisfied if $N \geq 2$. Now let v^* be a vector of rewards for which there exists $a' \neq a_{\max}(\kappa)$ such that $a_{\max}(\kappa)$ and a' are solutions to (10). For each vector of rewards v, define Δv to be such that $\Delta v = (v(y_0) - v(y_1), \ldots, v(y_{N-1}) - v(y_N))$ and consider an alternative vector of rewards $\widehat{v}$ such that $\Delta\widehat{v} = \Delta v^* + \varepsilon$, where ε is orthogonal to $w(a^*)$. By construction, $a_{\max}(\kappa)$ is still a solution to (10) when the vetor rewards is $\widehat{v}$. On the other hand, since Assumption 4 implies that ε is not orthogonal to $w(a')$, the effort a' is not. Thus, by suitably adjusting rewards, the voter can ensure that $a_{\max}(\kappa)$ is the only solution to (10).

The economic intuition behind the above result is straightforward. The voter can ensure that a given effort level a is the unique equilibrium choice of effort for the incumbent if he or she can adjust rewards in such a way that any effort $a' \neq a$ is not a best response to the conjecture $a^e = a'$. This, however, is only possible if there are at least three output levels. Indeed, with two output levels, (explicit) incentives are completely determined by the reward difference $v(y_1) - v(y_0)$, and any change in rewards that makes $a' \neq a$ suboptimal given the conjecture $a^e = a'$ also makes a suboptimal given the conjecture $a^e = a$. On the other hand, with three or more output levels, incentives are determined by the interaction between the payoff gains $v(y_{i+1}) - v(y_i)$ and the marginal increases in output $\partial F(y_i|a, \pi_0)/\partial a$, with $i \in \{0, \ldots, N-1\}$. Assumption 4 ensures that these extra degrees of freedom for the voter can be used to pin down uniquely the incumbent's equilibrium behavior.

6.2 Large Horizontal Differentiation

Suppose $\theta > 1$, so that $G(-1) > 0$ and $G(1) < 1$, and the voter can find it optimal to replace a high-ability incumbent even if the citizen is of low ability and keep a low-ability incumbent even if the citizen is of high ability. First notice that since $H(\kappa/B) < -1$ if κ is sufficiently close to zero and $H(\kappa/B) > 1$ if κ is sufficiently close to B, then there exist $0 < \kappa' < \kappa'' < B$ such that $\pi_C(\pi_I, \kappa) \leq 0$ regardless of π_I if $\kappa \leq \kappa'$ and $\pi_C(\pi_I, \kappa) \geq 1$ regardless of π_I if $\kappa \geq \kappa''$. So, when the entry cost is small, the citizen always enters and a local change in the entry cost has no effect on effective accountability. Likewise, when the entry cost is close to B, the citizen never enters and a local change in the entry cost also has no effect on effective accountability. Thus, unlike the case with small horizontal differentiation, the impact of a change in political competition on effective accountability can be ambiguous only for intermediate values of the entry cost.

Now, for each $\pi_I \in [0,1]$, let $\kappa_1 = \kappa_1(\pi_I)$ be such that $\pi_C(\pi_I, \kappa_1) = 0$ and $\kappa_2 = \kappa_2(\pi_I) > \kappa_1$ be such that $\pi_C(\pi_I, \kappa_2) = 1$. Both κ_1 and κ_2 are well-defined and strictly decreasing in π_I since H is strictly increasing with $H(0) = -\theta < -1$ and $H(1) = \theta > 1$. By construction, $\pi_C(\pi_0, \kappa) \in (0,1)$ if $\kappa \in (\kappa_1(\pi_0), \kappa_2(\pi_0))$, in which case the incumbent's probability of retention is sensitive to his or her performance in office. Clearly, Proposition 1 holds when the entry cost is in this intermediate range.[34] In what follows, we show that both effective accountability and voter welfare need not be maximized when entry is costless.

Fix all the model's primitives but the entry cost κ and let $a_0 \in (0, \overline{a})$ the incumbent's optimal choice of effort when the citizen enters with probability one regardless the incumbent's performance. Then a_0 is the incumbent's unique equilibrium choice of effort when $\kappa \leq \kappa_1(\pi^+(h|a_0))$. Now consider the impact on the incumbent's behavior of a local increase in the entry cost when $\kappa = \kappa_1(\pi^+(h|a_0))$. Since the probability that the citizen enters if the incumbent succeeds drops below one after this change in the entry cost but the probability that the citizen enters if the incumbent fails remains equal to one, the marginal benefit of effort to the incumbent when he or she exerts effort a_0 and the other agents correctly anticipate his or her behavior increases. This, in turn, leads to a higher equilibrium effort for the incumbent. Moreover, given that there is full entry up to the point where the entry cost becomes equal to $\kappa_1(\pi^+(h|a_0))$, a local increase in the entry cost when κ assumes this value has a second-order effect on voter welfare. Thus, voter welfare is also not maximized when entry is costless if effort increases the informativeness of output.

7 Concluding Remarks

Political markets are markets in which there is a wedge between the private and social returns of effort. This creates the need of institutional arrangements to provide politicians with an incentive to behave in the voters' interest. In democracies, these incentives are provided by the possibility of reelection. For this to work, however, voters need to credibly commit to reward a good per-

[34]The condition $0 < \pi_C(\pi^+(\ell|a), \kappa) < \pi_C(\pi^+(h|a), \kappa) < 1$ is equivalent to $\kappa_1(\pi^+(\ell|a)) < \kappa < \kappa_2(\pi^+(h|a))$. Notice that the interval $(\kappa_1(\pi^+(\ell|a)), \kappa_2(\pi^+(h|a)))$ might be empty if $\pi^+(h|a)$ and $\pi^+(\ell|a)$ are too far apart.

formance and punish a bad one. Political competition helps voters punish a bad performance by providing them with viable alternatives to an incumbent. Political competition makes the promise of rewarding a good performance through retention less credible, though. We show that this tension between the good and the bad aspects of political competition implies that the relationship between political competition and voter welfare is undetermined. In particular, voter welfare need not be maximized when entry of politicians is free. We also show that if voters have additional instruments to reward politicians for their performance in office, then an increase in political competition is always beneficial to voters as long as these instruments are sufficiently rich.

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8 Appendix: Omitted Proofs and Details

Here, we provide all proofs and details that were omitted from the main text.

Properties of the Function G

Recall that $G : \mathbb{R} \to [0,1]$ is such that

$$G(x) = \int_{-\infty}^{+\infty} \Lambda(\xi + 2x\eta/\theta)\lambda(\xi)d\xi.$$

Clearly, Λ nondecreasing implies that G is nondecreasing. Since $\xi + 2x\eta/\theta \in (-\eta, \eta)$ for some $\xi \in (-\eta, \eta)$ if, and only if, $x \in (-\theta, \theta)$, it follows that $G(-\theta) = 0$ and $G(\theta) = 1$. Standard results in measure theory, see, e.g., Corollary 5.9 in Bartle (1966), show that G is differentiable with

$$G'(x) = (2\eta/\theta)\int_{-\infty}^{+\infty} \lambda(\xi + 2x\eta/\theta)\lambda(\xi)d\xi.$$

Since $\lambda(\xi) > 0$ for all $\xi \in [-\eta, \eta]$, it follows that G is strictly increasing in $(-\theta, \theta)$. The continuity G' in $(-\theta, \theta)$ follows from the dominated convergence theorem and the continuity of λ in $(-\eta, \eta)$.

Proof of Lemma 3

Notice that $Q(\pi_I, \kappa) = 1$ if $\pi_C(\pi_I, \kappa) \geq 1$ and that

$$Q(\pi_I, \kappa) = 1 - \int_{\max\{0, \pi_C(\pi_I, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi \tag{11}$$

otherwise. Given that $G(\pi - \pi_I)$ is nonincreasing in π_I for all $\pi \in [0,1]$ and $\pi_C(\pi_I, \kappa)$ is strictly increasing in π_I and κ, it follows that $Q(\pi_I, \kappa)$ is nondecreasing in π_I and κ and strictly increasing in π_I and κ if $\pi_C(\pi_I, \kappa) \in (0,1)$. The continuity of $Q(\pi_I, \kappa)$ in π_I follows from the fact that the map $(x, \pi_I) \mapsto \int_x^1 G(\pi - \pi_I)\omega(\pi)d\pi$ is jointly continuous by dominated convergence theorem.[35]

We now compute the derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ and show that they are continuous in (π_I, κ) when $\pi_C(\pi_I, \kappa) \neq 0, 1$. Notice that $\partial Q/\partial \pi_I(\pi_I, \kappa) = \partial Q/\partial \kappa(\pi_I, \kappa) = 0$ if $\pi_C(\pi_I, \kappa) > 1$ and that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \int_0^1 G'(\pi - \pi_I)\omega(\pi)d\pi$$

[35] Just note that $\int_x^1 G(\pi - \pi_I)\omega(\pi)d\pi = \int_0^1 F(x, \pi_I, \pi)d\pi$, where $F(x, \pi_I, \pi) = \mathbb{I}_{[x,1]}(\pi)G(\pi - \pi_I)\omega(\pi)$ and $\mathbb{I}_{[x,1]}$ is the indicator function of the interval $[x, 1]$.

and $\partial Q(\pi_I, \kappa)/\partial\kappa = 0$ if $\pi_C(\pi_I, \kappa) < 0$. Suppose then that $\pi_C(\pi_I, \kappa) \in (0, 1)$. Since $\pi_C(\pi_I, \kappa)$ is differentiable in π_I and κ for all $(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa})$ and $G(\pi_C(\pi_I, \kappa) - \pi_I) = \kappa/B$,

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B} + \int_{\pi_C(\pi_I,\kappa)}^{1} G'(\pi - \pi_I)\omega(\pi)d\pi$$

and

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\pi_C(\pi_I, \kappa))\frac{\kappa}{B^2}H'\left(\frac{\kappa}{B}\right)$$

by the fundamental theorem of calculus. Clearly, $\partial Q/\partial\kappa(\pi_I, \kappa)$ is continuous if $\pi_C(\pi_I, \kappa) \neq 0, 1$. The continuity of $\partial Q/\partial\pi_I(\pi_I, \kappa)$ in the same set follows from the dominated convergence theorem.

Proof of Lemma 4

The sufficiency of (3) was established in the main text. Suppose now that a^* maximizes $U(a, a^*)$. Then $a^* < \overline{a}$, as the Inada condition $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ implies that $\partial U(\overline{a}, a^e)/\partial a < 0$ for all $a^e \in A$. A necessary condition for $a^* < \overline{a}$ to maximize $U(a, a^*)$ is $\partial U(a^*, a^*)/\partial a \leq 0$ and $\partial U(a^*, a^*)/\partial a = 0$ if $a^* > 0$. Since $c'(0) = 0$ implies that $\partial U(0, a^e)/\partial a \geq 0$ for all $a^e \in A$, it also follows that $\partial U(a^*, a^*)/\partial a = 0$ when $a^* = 0$. Thus, condition (3) is necessary as well.

Proof of Corollary 1

Since $\pi^+(\ell|a^e) < \pi_0 < \pi^+(h|a^e)$ for all $a^e \in A$ and $\pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$, it follows that $\pi_C(\pi^+(\ell|a^e), \kappa) < 1$ and $\pi_C(\pi^+(h|a^e), \kappa) > 0$ for all $a^e \in A$ and $\kappa \in [0, \overline{\kappa}]$. So, $Q(\pi^+(h|a^e), \kappa) > Q(\pi^+(\ell|a^e), \kappa)$ regardless of a^e and κ. This, in turn, implies that the left-hand side of (3) is positive for all $a^* \in [0, \overline{a})$, and so $a^* = 0$ cannot be a solution to (3); notice that $\partial f(a, \tau)/\partial a > 0$ for all $a \in [0, \overline{a})$ since $f(a, \tau)$ is strictly increasing and strictly concave in a.

Proof of Lemma 5

Fix all the model's primitives but the cost function and define the function $\widehat{\lambda} : A \to \mathbb{R}$ to be such that $\widehat{\lambda}(a) = \max\{0, MB(a^*, \kappa) + \xi(a - a^*)\}$, where

$$\xi = \frac{1}{\overline{a} - a^*}\left[B\frac{\partial f}{\partial a}(0, \pi_0) - MB(a^*, \kappa)\right] + \max\left\{\frac{MB(a^*, \kappa)}{a^*}, \sup_{a \in A\setminus\{a^*\}} \frac{MB(a, \kappa) - MB(a^*, \kappa)}{a - a^*}\right\}.$$

One can show that $MB(a,\kappa)$ is locally Lipschitz in a for all $\kappa \in [0,\overline{\kappa}]$, so that ξ is well-defined; we delay the proof of this technical fact to the end of the proof of Lemma 5 for ease of exposition. Notice that $\widehat{\lambda}(a)$ is increasing in a, with $\widehat{\lambda}(\overline{a}) > B\partial f(0,\pi_0)/\partial a$. Given that $\xi > MB(a^*,\kappa)/a^*$, there exists $a_0 \in (0,a^*)$ such that $\widehat{\lambda}(a) = 0$ if $a \leq a_0$ and $\widehat{\lambda}(a) > 0$ otherwise. Moreover, since $\widehat{\lambda}(a^*) = MB(a^*,\kappa)$ and $\xi > \big(MB(a,\kappa) - MB(a^*,\kappa)\big)/(a-a^*)$ for all $a \in A \setminus \{a^*\}$, it also follows that $\widehat{\lambda}(a) < MB(a,\kappa)$ for all $a < a^*$ and $\widehat{\lambda}(a) > MB(a,\kappa)$ for all $a > a^*$.

Now fix $0 < \varepsilon < \min\{a_0, a^* - a_0\}$ and let $\lambda : A \to \mathbb{R}$ be such that $\lambda(a) = \widehat{\lambda}(a)$ if $|a - a_0| \geq \varepsilon$ and $\lambda(a) = \alpha(a - a_0 + \varepsilon)^n$ if $|a - a_0| < \varepsilon$, where $\alpha(2\varepsilon)^n = MB(a^*,\kappa) + \xi(a_0 + \varepsilon - a^*)$ and $\alpha n(2\varepsilon)^{n-1} = \xi$.[36] Notice that $\lambda(a_0 - \varepsilon) = \widehat{\lambda}(a_0 - \varepsilon) = 0$ and $\lambda(a_0 + \varepsilon) = \widehat{\lambda}(a_0 + \varepsilon) = \alpha(2\varepsilon)^n$. Moreover, $\lambda'(a_0 - \varepsilon) = 0$ and $\lambda'(a_0 + \varepsilon) = \xi$. So, $\lambda(a)$ is increasing and continuously differentiable in a. On the other hand, by letting n be sufficiently large and ε be sufficiently small, we can ensure that $\lambda(a) < MB(a,\kappa)$ for all $a \in A$ such that $|a - a_0| < \varepsilon$.

To conclude the argument, let $c(a) = \int_0^a \lambda(s)ds$. By construction, c is twice continuously differentiable, increasing, convex, and such that $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0,\pi_0)/\partial a$. So, c is admissible. Moreover, $c'(a^*) = MB(a^*,\kappa)$ and $c'(a) \neq MB(a,\kappa)$ for all $a \neq a^*$. Thus, a^* is the unique equilibrium choice of effort for the incumbent when the cost function is c. Finally, if $\partial MB(a^*,\kappa)/\partial a$ exists, then $c''(a^*) = \xi > \partial MB(a^*,\kappa)/\partial a$ by the definition of ξ, and so the equilibrium is stable.

We conclude the proof of Lemma 5 by showing that $MB(a,\kappa)$ is locally Lipschitz in a for all $\kappa \in [0,\overline{\kappa}]$. Since the composition, product, and sum of locally Lipschitz functions is locally Lipschitz and a differentiable function is locally Lipschitz, we obtain the desired result if we show that $Q(\pi_I,\kappa)$ is Lipschitz in π_I for all $\kappa \in [0,\overline{\kappa}]$. Fix $\kappa \in [0,\overline{\kappa}]$ and let $0 \leq \pi_I < \pi_I' \leq 1$. Then

$$Q(\pi_I',\kappa) - Q(\pi_I,\kappa) = \int_{\max\{0,\pi_C(\pi_I,\kappa)\}}^{\max\{0,\pi_C(\pi_I',\kappa)\}} G(\pi - \pi_I)\omega(\pi)d\pi + \int_{\max\{0,\pi_C(\pi_I',\kappa)\}}^{1} \big[G(\pi - \pi_I) - G(\pi - \pi_I')\big]\omega(\pi)d\pi.$$

Now let $\overline{\omega} = \sup_{\pi\in[0,1]} \omega(\pi) < \infty$ and $\overline{\lambda} = \sup_{x\in[-\eta,\eta]} \lambda(x) < \infty$; $\overline{\omega}$ and $\overline{\lambda}$ are finite given the assumptions on ω and λ. Given that $G(x) \in [0,1]$ and $|G'(x)| \leq (2\eta/\theta)\overline{\lambda}$ regardless of x, it

[36] We solve for the system of equations defining α and n as follows. First, substitute ξ by $\alpha n(2\varepsilon)^{n-1}$ in the first equation to obtain α; note that $\alpha > 0$ since $a^* > a_0 + \varepsilon$. Then obtain n residually by the second equation.

follows from the last equation and the mean value theorem that

$$|Q(\pi_I', \kappa) - Q(\pi_I, \kappa)| \leq \big(\overline{\omega} + (2\eta/\theta)\overline{\lambda}\big)|\pi_I' - \pi_I|.$$

This establishes the desired result.

Sufficient Conditions for Equilibrium Multiplicity

The following lemma establishes sufficient conditions for multiple equilibria to exist.

Lemma 7. *Fix all the model's primitives but the cost function and let κ be the entry cost. Suppose there exist $0 < a_1 < a_2 < \overline{a}$ with $MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$. There exists an admissible cost function c such that if the cost function is c, then a_1 and a_2 are equilibrium effort choices for the incumbent.*

Proof. Let $\widehat{\lambda}_2 : A \to \mathbb{R}$ be such that $\widehat{\lambda}_2(a) = \max\{0, MB(a_2, \kappa) + \xi_2(a - a_2)\}$, where

$$\xi_2 = \frac{1}{\overline{a} - a_2}\left[B\frac{\partial f}{\partial a}(0, \pi_0) - MB(a_2, \kappa)\right] + \frac{MB(a_2, \kappa) - MB(a_1, \kappa)}{a_2 - a_1}.$$

Moreover, let $\widehat{\lambda}_1 : A \to \mathbb{R}$ be such that $\widehat{\lambda}_1(a) = \max\{0, MB(a_1, \kappa) + \xi_1(a - a_1)\}$, where

$$\xi_1 = \frac{\widehat{\lambda}_2(a_2 - \eta) - MB(a_1, \kappa)}{a_2 - a_1 - \eta}$$

and $0 < \eta < a_2 - a_1$. Since $\lim_{\eta \to 0} \widehat{\lambda}_2(a_2 - \eta) = MB(a_2, \kappa)$ and

$$MB(a_1, \kappa) - \left(\frac{MB(a_2, \kappa) - MB(a_1, \kappa)}{a_2 - a_1}\right)a_1 \propto MB(a_1, \kappa)a_2 - MB(a_2, \kappa)a_1 < 0$$

by assumption, there exists $\overline{\eta} > 0$ such that $\xi_1 > MB(a_1, \kappa)/a_1$ if $\eta \in (0, \overline{\eta})$. Thus, $\widehat{\lambda}_1(a) = 0$ for a sufficiently close to zero if $\eta \in (0, \overline{\eta})$.

Fix $\eta < \overline{\eta}$ and define $\widehat{\lambda} : A \to \mathbb{R}$ to be such that $\widehat{\lambda}(a) = \max\{\widehat{\lambda}_1(a), \widehat{\lambda}_2(a)\}$. By construction, $\widehat{\lambda}_1(a_2 - \eta) = \widehat{\lambda}_2(a_2 - \eta)$. Moreover,

$$\xi_2 > \frac{MB(a_2, \kappa) - MB(a_1, \kappa)}{a_2 - a_1} > \frac{\widehat{\lambda}_2(a_2 - \eta) - MB(a_1, \kappa)}{a_2 - a_1 - \eta} = \xi_1,$$

where the second inequality follows from the fact that the second ratio is strictly decreasing in η. Thus, $\widehat{\lambda}(a) = \widehat{\lambda}_1(a)$ if, and only if, $a \leq a_2 - \eta$. A straightforward modification of the argument in the proof of Lemma 5 now shows that there exists an admissible cost function c with $c'(a_1) = MB(a_1, \kappa)$ and $c'(a_2) = MB(a_2, \kappa)$. This concludes the proof. □

Proof of Proposition 2

We complete the proof of Proposition 2 by showing that under the assumptions in its statement, there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique and differentiable in κ for all $\kappa \in (0, \underline{\kappa})$.

We first show that there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique for all $\kappa \in (0, \underline{\kappa})$. Suppose not. Then there exists a sequence $\{\kappa_n\}$ in $[0, \overline{\kappa}]$ converging to zero and two sequences $\{a_n\}$ and $\{a'_n\}$ in $(0, \overline{a})$ such that $a_n \neq a'_n$ and $\Delta(a_n, \kappa_n) = \Delta(a'_n, \kappa_n) = 0$ for all $n \in \mathbb{N}$. Moving to subsequences if necessary, we can assume that $\{a_n\}$ and $\{a'_n\}$ are both convergent. Let a and a' be the limits of $\{a_n\}$ and $\{a'_n\}$, respectively. Given that $\Delta(a, \kappa)$ is jointly continuous, it follows that $\Delta(a, 0) = \Delta(a', 0) = 0$. So, $a = a' = a^*$, as a^* is the unique equilibrium effort choice for the incumbent when $\kappa = 0$. Now observe that the assumption on a^* and θ implies that there exists $0 < \varepsilon < a^*$ and $0 < \delta < \overline{\kappa}$ such that $\Delta(a, \kappa)$ is continuously differentiable in $(a^* - \varepsilon, a^* + \varepsilon) \times (0, \overline{\kappa} - \delta)$. So, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, we can find $a''_n \in [\min\{a_n, a'_n\}, \max\{a_n, a'_n\}]$ with $\partial\Delta(a''_n, \kappa_n)/\partial a = 0$. Moreover, since $\lim_n a_n = \lim_n a'_n = a^*$, it also follows that $\{a''_n\}$ converges to a^*. Thus, $\partial\Delta(a^*, 0)/\partial a = \lim_n \partial\Delta(a''_n, \kappa_n)/\partial a = 0$. This, however, contradicts the stability of the equilibrium when the entry cost is zero.

For each $\kappa \in (0, \underline{\kappa})$, let $a^*(\kappa)$ be the incumbent's unique equilibrium choice of effort when the entry cost is κ. A standard argument shows that since $\Delta(a, \kappa)$ is jointly continuous, $\lim_{\kappa \to 0} a^*(\kappa) = a^*$. By reducing $\underline{\kappa}$ if necessary, we then have that $a^*(\kappa) \in (a^* - \varepsilon, a^* + \varepsilon)$ and $\kappa \in (0, \overline{\kappa} - \delta)$ for all $\kappa \in (0, \underline{\kappa})$. We now show that $a^*(\kappa)$ is differentiable in κ for all $\kappa \in (0, \underline{\kappa})$, reducing $\underline{\kappa}$ even further if necessary. Since $\Delta(a, \kappa)$ is continuously differentiable in $(a^* - \varepsilon, a + \varepsilon) \times (0, \overline{\kappa} - \delta)$ and $\partial\Delta(a^*, 0)/\partial a < 0$, it then follows that $\partial\Delta(a^*(\kappa), \kappa)/\partial a < 0$ for $\kappa \in (0, \underline{\kappa})$, reducing $\underline{\kappa}$ if necessary. The desired result now follows from the implicit function theorem.

Decomposition of Second-Period Welfare

Here, we establish the decomposition of $W_2(a, \kappa)$ in terms $W_2^+(a)$ and $W_2^-(a, \kappa)$. Let γ be the density of Γ. Since $Sf(0, \pi^+) + z \geq Sf(0, \pi)$ if, and only if, $z \geq (\pi - \pi^+)2\eta/\theta$,

$$\max\{Sf(0, \pi^+) + z, Sf(0, \pi)\} = Sf(0, \pi^+) + z + \max\left\{0, (\pi - \pi^+)2\eta/\theta - z\right\},$$

and the random variable z has zero mean and support $[-2\eta, 2\eta]$, it follows that

$$\begin{aligned}
&\Omega(\pi_C(\pi^+(y|a),\kappa))\mathbb{E}_z\big[Sf(0,\pi^+(y|a))+z\big]\\
&\quad=\int_0^{\pi_C(\pi^+(y|a),\kappa)}\bigg(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\big[Sf(0,\pi^+(y|a))+z\big]\gamma(z)dz\bigg)\omega(\pi)d\pi\\
&\qquad+\int_0^{\pi_C(\pi^+(y|a),\kappa)}\bigg(\int_{(\pi-\pi^+(y|a))2\eta/\theta}^{2\eta}\max\{Sf(0,\pi^+(y|a))+z,Sf(0,\pi)\}\gamma(z)dz\bigg)\omega(\pi)d\pi\\
&\quad=\int_0^{\pi_C(\pi^+(y|a),\kappa)}\mathbb{E}_z\big[\max\{Sf(0,\pi^+(y|a))+z,Sf(0,\pi)\}\big]\omega(\pi)d\pi\\
&\qquad-\int_0^{\pi_C(\pi^+(y|a),\kappa)}\left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta-z\big\}\gamma(z)dz\right)\omega(\pi)d\pi.
\end{aligned}$$

The desired result is a consequence of (7) and the definitions of $W_2^+(a)$ and $W_2^-(a,\kappa)$ together with the fact that integration by parts implies that

$$\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\max\big\{0,(\pi-\pi^+(y|a))2\eta/\theta-z\big\}\gamma(z)dz=\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta}\Gamma(z)dz.$$

Differentiability of W_2^+

Since

$$W_2^+(a)=\sum_{y\in\{\ell,h\}}\mu(y|a)\mathbb{E}_z\left[\int_0^1\max\{Sf(0,\pi^+(y|a)+z,Sf(0,\pi)\}\omega(\pi)d\pi\right],$$

the differentiability of W_2^+ follows from Corollary 5.9 in Bartle (1966) and the differentiability in a of $\mu(y|a)$ and $\pi^+(y|a)$ if

$$G(\pi^+,z)=\int_0^1\max\{Sf(0,\pi^+)+z,Sf(0,\pi)\}\omega(\pi)d\pi \tag{12}$$

is differentiable in π^+ for all $z\in\mathbb{R}$. We establish the latter fact in what follows.

First notice that

$$G(\pi^+,z)=\big[Sf(0,\pi^+)+z\big]\,\Omega(\pi^++z/\alpha)+\int_{\pi^++z/\alpha}^1 Sf(0,\pi)\omega(\pi)d\pi,$$

where $\alpha=S[f(0,H)-f(0,L)]$ and we used the fact that $Sf(0,\pi)\geq Sf(0,\pi^+)+z$ if, and only if, $\pi\geq\pi^++z/\alpha$; we adopt the convention that the last integral is zero if $\pi^++z/\alpha>1$. Clearly, for each $z \in \mathbb{R}$, $G(\pi^+, z)$ is differentiable in π^+ for all π^+ such that $\pi^+ + z/\alpha \neq 1$. Let then π^+ be such that $\pi^+ + z/\alpha = 1$. It is immediate to see that

$$\lim_{h\downarrow 0} \frac{G(\pi^+ + h, z) - G(\pi^+, z)}{h} = S\frac{\partial f}{\partial \pi}(0, \pi^+).$$

Since, by L'Hospital's rule,

$$\lim_{h\uparrow 0} \frac{\big[Sf(0, \pi^+ + h) + z\big]\Omega(\pi^+ + h + z/\alpha) - \big[Sf(0, \pi^+) + z\big]}{h} = S\frac{\partial f}{\partial \pi}(0, \pi^+) + Sf(0, \pi^+)\omega(1),$$

and, by the fundamental theorem of calculus,

$$\lim_{h\downarrow 0} \frac{1}{h} \int_{1-h}^{1} Sf(0, \pi)\omega(\pi)d\pi = -Sf(0, 1)\omega(1),$$

we than have that

$$\lim_{h\downarrow 0} \frac{G(\pi^+ - h, z) - G(\pi^+, z)}{h} = S\frac{\partial f}{\partial \pi}(0, \pi^+)$$

Thus, $G(\pi^+, z)$ is also differentiable in π^+ when $\pi^+ + z/\alpha = 1$. This establishes the desired result.

Welfare Changes

We now compute the derivatives $\partial W_2^-(a, \kappa)/\partial \kappa$ and $\partial W_2^-(a, \kappa)/\partial a$ and show that these terms have the properties described in the main text. First notice that the fundamental theorem of calculus implies that

$$\frac{\partial W_2^-}{\partial \kappa}(a, \kappa) = \frac{1}{B} H'\left(\frac{\kappa}{B}\right) \sum_{y \in \{\ell, h\}} \mu(y|a)\omega(\pi_C(\pi^+(y|a), \kappa)) \int_{-2\eta}^{H(\kappa/B)2\eta/\theta} \Gamma(z)dz.$$

Clearly, the above derivative is positive.

Now notice, also from the fundamental theorem of calculus, that

$$\begin{aligned} \frac{\partial W_2^-}{\partial a}(a, \kappa) \;=\; & \sum_{y \in \{\ell, h\}} \frac{\partial \mu}{\partial a}(y|a) \int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi - \pi^+(y|a))2\eta/\theta} \Gamma(z)dz \right) \omega(\pi)d\pi \\ & + \sum_{y \in \{\ell, h\}} \mu(y|a) \frac{\partial \pi^+}{\partial a}(y|a)\omega(\pi_C(\pi^+(y|a), \kappa)) \int_{-2\eta}^{H(\kappa/B)2\eta/\theta} \Gamma(z)dz \\ & - \sum_{y \in \{\ell, h\}} \mu(y|a) \frac{2\eta}{\theta} \frac{\partial \pi^+}{\partial a}(y|a) \int_0^{\pi_C(\pi^+(y|a),\kappa))} \Gamma\bigg(\Big(\pi - \pi^+(y|a)\Big)\frac{2\eta}{\theta} \bigg)\omega(\pi)d\pi. \end{aligned}$$

We claim that $\lim_{\kappa\to 0} \partial W_2^-(a,\kappa)/\partial a = 0$ uniformly in a. Given that

$$\int_0^{\pi_C(\pi^+(y|a),\kappa)} \left(\int_{-2\eta}^{(\pi-\pi^+(y|a))2\eta/\theta} \Gamma(z)dz \right) \omega(\pi)d\pi \leq \int_{-2\eta}^{H(\kappa/B)2\eta/\theta} \Gamma(z)dz$$

and

$$\int_0^{\pi_C(\pi^+(y|a),\kappa))} \Gamma\left(\left(\pi - \pi^+(y|a)\right) \frac{2\eta}{\theta} \right) \omega(\pi)d\pi \leq \Gamma\left(H\left(\frac{\kappa}{B}\right) \frac{2\eta}{\theta} \right),$$

the desired result follows from the fact that $\lim_{\kappa\to 0} H(\kappa/B)2\eta/\theta = -2\eta$ if $\omega(\pi_C(\pi^+(y|a),\kappa))$, $\partial\mu(y|a)/\partial a$, and $\partial\pi^+(y|a)/\partial a$ are uniformly bounded in a for every y. Since ω is continuous, $\omega(\pi_C(\pi^+(y|a),\kappa))$ is uniformly bounded in a regardless of y. Straightforward algebra shows that the same holds for $\partial\mu(y|a)/\partial a$ and $\partial\pi^+(y|a)/\partial a$ since $\partial f(0,L)/\partial a$ and $\partial f(0,H)/\partial a$ are finite.

Proof of Lemma 6

We begin by establishing the following auxiliary result.

Claim 1. $\lim\limits_{\kappa\to 0} \dfrac{1}{\kappa} \displaystyle\int_{-\eta}^{H(\kappa/B)2\eta/\theta} \Gamma(z)dz = 0.$

Proof. Let $\lambda(\eta) = \lim_{x\uparrow\eta} \lambda(x)$ and $\lambda(-\eta) = \lim_{x\downarrow -\eta} \lambda(x)$. Both limits exist and are positive given the assumptions on λ. We first show that

$$\lim_{\kappa\to 0} \frac{\int_{-\infty}^{+\infty} \lambda\left(\xi + H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta} \right) \lambda(\xi)d\xi}{\lambda(\eta)\lambda(-\eta)2\eta\left[1 + H\left(\frac{\kappa}{B}\right)\frac{1}{\theta}\right]} = 1. \tag{13}$$

We know there exists $\kappa' > 0$ such that $H(\kappa/B) < 0$ if $\kappa \in [0,\kappa')$. Suppose that $\kappa \in (0,\kappa')$. Since $\xi + H(\kappa/B)2\eta/\theta$ belongs to the interval $[-\eta,\eta]$ if, and only if,

$$-\eta\left(1 + H\left(\frac{\kappa}{B}\right)\frac{2}{\theta}\right) \leq \xi \leq \eta\left(1 - H\left(\frac{\kappa}{B}\right)\frac{2}{\theta}\right)$$

and $\eta\big(1 - H(\kappa/\theta)(2/\theta)\big) > \eta$ if $H(\kappa/B) < 0$, it follows that

$$\int_{-\infty}^{+\infty} \lambda\left(\xi + H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta} \right)\lambda(\xi)d\xi = \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \lambda\left(\xi + H\left(\frac{\kappa}{B}\right)\frac{2\eta}{\theta} \right)\lambda(\xi)d\xi.$$

Now observe that

$$\begin{aligned}
\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \lambda\bigg(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\bigg)\lambda(\xi)d\xi & \\
= & \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \lambda(-\eta)\lambda(\eta)d\xi \\
& + \lambda(-\eta)\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \left[\lambda(\xi)-\lambda(\eta)\right]d\xi \\
& + \int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \bigg[\lambda\bigg(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\bigg) - \lambda(-\eta)\bigg]\lambda(\xi)d\xi.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\int_{-\infty}^{+\infty} \lambda\bigg(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\bigg)\lambda(\xi)d\xi}{\lambda(\eta)\lambda(-\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]} &= 1 \\
&+ \frac{1}{\lambda(\eta)\lambda(-\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \bigg[\lambda\bigg(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\bigg) - \lambda(-\eta)\bigg]d\xi \\
&+ \frac{1}{\lambda(\eta)2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \left[\lambda(\xi)-\lambda(\eta)\right]d\xi \qquad (14)
\end{aligned}$$

Equation (13) holds if the last two terms in the right-hand side of (14) converge to zero as κ converges to zero. We establish this fact in what follows.

First notice, making the change of variables $\xi \mapsto \xi - H(\kappa/B)2\eta/\theta$, that

$$\begin{aligned}
\frac{1}{2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \bigg[\lambda\bigg(\xi + H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\bigg) - \lambda(-\eta)\bigg]d\xi & \\
= \frac{1}{h}\int_{-\eta}^{-\eta+h}[\lambda(\xi)-\lambda(-\eta)]d\xi = \frac{1}{h}\int_{-\eta}^{-\eta+h}\lambda(\xi)d\xi - \lambda(-\eta),
\end{aligned}$$

where $h = 2\eta(1+H(\kappa/B)/\theta)$. Since $\lim_{\kappa\to 0} h = 0$, the fundamental theorem of calculus implies that the right-hand side of the above equation converges to zero as κ converges to zero. Equation (13) follows from the fact the fundamental theorem of calculus also implies that

$$\lim_{\kappa\to 0}\frac{1}{2\eta\left[1+H\Big(\frac{\kappa}{B}\Big)\frac{1}{\theta}\right]}\int_{-\eta\left(1+H(\frac{\kappa}{B})\frac{2}{\theta}\right)}^{\eta} \left[\lambda(\xi)-\lambda(\eta)\right]d\xi = \frac{1}{h}\int_{\eta-h}^{\eta}\lambda(\xi)d\xi - \lambda(\eta) = 0.$$

We now establish the claim. For this, notice that

$$\begin{aligned}
\lim_{\kappa\to 0}\frac{1}{\kappa}\int_{-\eta}^{H(\kappa/B)2\eta/\theta}\Gamma(z)dz &= \lim_{\kappa\to 0}\Gamma\left(H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)H'\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\frac{1}{B}\\
&= \lim_{\kappa\to 0}\frac{\Gamma\left(H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)}{B\int_{-\infty}^{+\infty}\lambda\left(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)\lambda(\xi)d\xi}\\
&= \lim_{\kappa\to 0}\frac{\Gamma\left(H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)}{B\lambda(\eta)\lambda(-\eta)\left[2\eta+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right]}\\
&= \lim_{\kappa\to 0}\frac{\frac{2\eta}{\theta}\gamma\left(H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)}{B\lambda(\eta)\lambda(-\eta)}.
\end{aligned}$$

The first equality follows from L'Hospital's rule. The second equality follows from the fact that $H'(x) = 1/G'(H(x))$ and $G'(y) = (2\eta/\theta)\int_{-\infty}^{+\infty}\lambda(\xi + 2y\eta/\theta)\lambda(\xi)d\xi$.[37] The third equality follows from (13). The last equality follows from a second application of L'Hospital's rule, where γ is the density of Γ. On the other hand, since $\lim_{\kappa\to 0}\lambda(\xi + H(\kappa/B)2\eta/\theta) = \lambda(\xi - 2\eta) = 0$ for all $\xi < \eta$ and $\Gamma(z) = \int_{-\eta}^{+\eta}\Lambda(\xi+z)\lambda(\xi)d\xi$, it follows that

$$\lim_{\kappa\to 0}\gamma\left(H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right) = \lim_{\kappa\to 0}\int_{-\eta}^{+\eta}\lambda\left(\xi+H\Big(\frac{\kappa}{B}\Big)\frac{2\eta}{\theta}\right)\lambda(\xi)d\xi = 0$$

by the dominated convergence theorem. This concludes the proof of the claim. □

We can now establish Lemma 6. Fix all the primitives of the model except the cost function, the distribution of the citizen's reputation, and the entry cost, and assume that $\theta < \overline{\pi}^+(\ell)$. Let $a^* \in (0, \overline{a})$ be such that $0 < \pi^+(\ell|a^*) - \theta < \pi^+(h|a^*) - \theta < 1$ and take the distribution of the citizen's reputation to satisfy $\omega(\pi^+(h|a^*)-\theta) - \omega(\pi^+(\ell|a^*)-\theta) \neq 0$. By construction, there exists $\hat{\kappa} \in (0, \overline{\kappa})$ such that $\pi^+(\ell|a^*) + H(\kappa/B) > 0$ and $\pi^+(h|a^*) + H(\kappa/B) < 1$ for all $\kappa \in (0, \hat{\kappa})$. For each $\kappa \in (0, \hat{\kappa})$, let c_κ be an admissible cost function such that if the entry cost is κ, then a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable if the cost function is c_κ. Finally, let $\kappa_0 \in (0, \hat{\kappa})$ be the entry cost and take the cost function to be $c = c_{\kappa_0}$.

[37] Corollary 5.9 in Bartle (1966) shows that we can exchange the order of integration and derivation when computing the derivative of G.

First notice that

$$\frac{da^*}{d\kappa}(\kappa_0) = \frac{\frac{\partial f}{\partial a}(a^*, \pi_0)\left[\omega(\pi^+(h|a^*) + H(\kappa_0/B)) - \omega(\pi^+(\ell|a^*) + H(\kappa_0/B))\right]}{c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)} \cdot \frac{\kappa_0}{B} H'\left(\frac{\kappa_0}{B}\right)$$

and recall that

$$\frac{\partial W_2^-}{\partial \kappa}(a, \kappa_0) = \frac{1}{B} H'\left(\frac{\kappa_0}{B}\right) \sum_{y\in\{\ell,h\}} \mu(y|a)\omega(\pi_C(\pi^+(y|a), \kappa_0)) \int_{-2\eta}^{H(\kappa_0/B)2\eta/\theta} \Gamma(z)dz,$$

Since $c''(a^*) > \partial MB(a^*, \kappa_0)/\partial a$, as the equilibrium is stable, it follows that

$$\frac{dW^*}{\partial \kappa}(\kappa_0) \propto \frac{\kappa_0}{B} H'\left(\frac{\kappa_0}{B}\right) \left\{ \left[S\frac{\partial f}{\partial a}(a^*, \pi_0) + \frac{dW_2^+}{da}(a^*) - \frac{\partial W_2^-}{\partial a}(a^*, \kappa_0)\right] \frac{\partial f}{\partial a}(a^*, \pi_0) D(\kappa_0) \right.$$
$$\left. - \frac{B}{\kappa_0}\left[c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)\right] \sum_{y\in\{\ell,h\}} \mu(y|a)\omega\big(\pi^+(y|a^*) + H(\kappa_0/B)\big) \int_{-2\eta}^{H(\kappa_0/B)2\eta/\theta} \Gamma(z)dz \right\}$$

and that

$$\Delta W^*_{\text{indirect}} \propto \frac{\kappa_0}{B} H'\left(\frac{\kappa_0}{B}\right) \left[S\frac{\partial f}{\partial a}(a^*, \pi_0) + \frac{dW_2^+}{da}(a^*) - \frac{\partial W_2^-(a_*, \kappa_0)}{\partial a}\right] \frac{\partial f}{\partial a}(a^*, \pi_0) D(\kappa_0),$$

where $D(\kappa_0) = \omega(\pi^+(h|a^*) + H(\kappa_0/B)) - \omega(\pi^+(\ell|a^*) + H(\kappa_0/B))$.

Given that

$$\lim_{\kappa_0\to 0} \frac{1}{\kappa_0}\left[c''(a^*) - \frac{\partial MB}{\partial a}(a^*, \kappa_0)\right] \omega\big(\pi^+(y|a^*) + H(\kappa_0/B)\big) \int_{-2\eta}^{H(\kappa_0/B)2\eta/\theta} \Gamma(z)dz = 0$$

for all $y \in \{\ell, h\}$ by Claim 1 and $\lim_{\kappa_0\to 0} D(\kappa_0) = \omega(\pi^+(h|a^*) - \theta) - \omega(\pi^+(\ell|a^*) - \theta) \neq 0$, we can then conclude that $dW^*(\kappa_0)/d\kappa$ and $\Delta W^*_{\text{indirect}}$ have the same sign as long as κ_0 is small enough and $\lim_{\kappa_0\to 0} \Delta W^*_{\text{indirect}} \neq 0$. The latter fact holds since $dW_1(a)/da + dW_2^+(a)/\partial a > 0$ for all a and $\lim_{\kappa\to 0} \partial W_2^-(a, \kappa)/\partial a = 0$ uniformly in a. This concludes the proof of the Lemma.

Relaxing the Assumption that Effort the Increases Informativeness of Output

Here, we show that we can relax the assumption that effort increases the informativeness of output in our welfare analysis. Fix $\eta > 0$ and let $g : [0, \bar{a} + \eta] \to (0, 1]$ be twice continuously differentiable, strictly increasing and strictly concave, and such that $g'(0)$ is finite and $g'(a)/[1 - g(a)]$ is strictly decreasing. Now let $\varepsilon \in (0, \eta)$ and set $f(a, L) = g(a)$ and $f(a, H) = g(a + \varepsilon)$. Then

$$\pi^+(y|a) = \frac{\pi_0}{\pi_0 + \mathcal{L}_y(a, \varepsilon)(1 - \pi_0)},$$

where $\mathcal{L}_h(a,\varepsilon) = g(a)/g(a+\varepsilon)$ and $\mathcal{L}_\ell(a,\varepsilon) = (1-g(a))/(1-g(a+\varepsilon))$.

First notice that

$$0 < \frac{\partial \mathcal{L}_h}{\partial a}(a,\varepsilon) = \frac{g'(a)g(a+\varepsilon) - g'(a+\varepsilon)g(a)}{g(a+\varepsilon)^2} < \frac{g'(a)g(a+\varepsilon) - g'(a+\varepsilon)g(a)}{g(0)^2}; \tag{15}$$

the first inequality in (15) follows from the fact that g is strictly increasing. Moreover,

$$\frac{g'(a+\varepsilon)}{1-g(a+\varepsilon)} - \frac{g'(a)}{1-g(a+\varepsilon)} < \frac{\partial \mathcal{L}_\ell}{\partial a}(a,\varepsilon) = \frac{g'(a+\varepsilon)[1-g(a)] - g'(a)[1-g(a+\varepsilon)]}{[1-g(a+\varepsilon)]^2} < 0; \tag{16}$$

the first inequality in (16) follows the fact that g is strictly increasing in a while the second inequality in (16) follows from the assumption that $g'(a)/[1-g(a)]$ is strictly decreasing. In particular, an increase in the incumbent's effort decreases the dispersion in his or her second-period reputation, making the incumbent's performance in office *less* informative of his or her ability.

Now observe that the numerator in the rightmost term in (15) is continuous in a and converges pointwise and monotonically to zero as ε decreases to zero. Moreover, the leftmost term in (16) is also continuous in a and converges pointwise and monotonically to zero as ε decreases to zero. Thus, by Dini's theorem, both $\partial\mathcal{L}_h(a,\varepsilon)/\partial a$ and $\partial\mathcal{L}_\ell(a,\varepsilon)/\partial a$ converge uniformly to zero as ε decreases to zero. From this, it easily follows that both $\mathcal{L}_h(a,\varepsilon)$ and $\mathcal{L}_\ell(a,\varepsilon)$ converge uniformly to one as ε decreases to zero.[38] Consequently, given that

$$\frac{\partial \pi^+}{\partial a}(y|a) = \pi^+(y|a)(1-\pi^+(y|a))\frac{1}{\mathcal{L}_y(a,\varepsilon)}\frac{\partial \mathcal{L}_y}{\partial a}(a,\varepsilon),$$

we can then conclude that $\partial\pi^+(h|a)/\partial a$ and $\partial\pi^+(\ell|a)/\partial a$ converge uniformly to zero as ε decreases to zero, and so $\pi^+(h|a)$ and $\pi^+(\ell|a)$ converge uniformly to zero as ε decreases to zero.

To finish, it follows from the discussion about the differentiability of W_2^+ that the function $G(\pi^+, z)$ given by (12) is differentiable in π^+ for all $z \in \mathbb{R}$. So, by Corollary 5.9 in Bartle (1966), we have that

$$\begin{aligned} \frac{dW_2^+}{da}(a) \;=\; & \sum_{y\in\{\ell,h\}} \frac{\partial \mu}{\partial a}(y|a)\mathbb{E}_z\left[G(\pi^+(y|a),z)\right] \\ & + \sum_{y\in\{\ell,h\}} \mu(y|a)\frac{\partial \pi^+}{\partial a}(y|a)\mathbb{E}_z\left[\frac{\partial G}{\partial \pi^+}(\pi^+(y|a),z)\right]. \end{aligned}$$

[38] Notice that $|\mathcal{L}_y(a,\varepsilon)-1| \le |\mathcal{L}_y(0,\varepsilon)-1| + \int_0^a |\partial\mathcal{L}_y(s,\varepsilon)/\partial a| ds \le |\mathcal{L}_y(0,\varepsilon)-1| + \int_0^{\overline{a}} |\partial\mathcal{L}_y(s,\varepsilon)/\partial a| ds$.

Now notice that $\mathbb{E}_z[G(\pi^+(y|a), z)]$ converges to $\mathbb{E}_z[G(\pi_0, z)]$ uniformly in a. Moreover, given that $Sf(0, \pi^+ + z/\alpha) = Sf(0, \pi^+) + z$ for $\alpha = S[f(0, H) - f(0, L)]$, it also follows from the discussion about the differentiability of $W_2^+(a)$ and the fundamental theorem of calculus that

$$\frac{\partial G}{\partial \pi^+}(\pi^+, z) = \left[S\frac{\partial f}{\partial \pi}(0, \pi^+) + z\right] \Omega\left(\pi^+ + \frac{z}{\alpha}\right) \leq Sg'(0) + z.$$

So, $dW_2^+(a)/da$ converges uniformly to 0 as ε decreases to zero. Since $\partial f(0, \pi_0)/\partial a \geq g'(a + \varepsilon)$, we then have that there exists $\overline{\varepsilon} \in (0, \eta)$ with the property that if $\varepsilon \in (0, \overline{\varepsilon})$, then

$$\frac{dW_1}{da}(a) + \frac{dW_2^+}{da}(a) = S\frac{\partial f}{\partial a}(0, \pi_0) + \frac{dW_2^+}{da}(a)$$

is positive and bounded away from zero as long as $g'(a)$ is bounded below by a positive constant. This shows that we can drop the assumption that effort increases the informativeness of output in our welfare analysis.

Continuous Differentiability of $Q(\pi_I, \kappa)$

Here, we prove that $Q(\pi_I, \kappa)$ is continuously differentiable in $(0, 1) \times (0, \overline{\kappa})$ if $\omega(0) = \omega(1) = 0$. We know from Lemma 3 that $Q(\pi_I, \kappa)$ is continuously differentiable in $\{(\pi_I, \kappa) \in (0, 1) \times (0, \overline{\kappa}) : \pi_C(\pi_I, \kappa) \neq 0, 1\}$. Hence, we only need to prove that $Q(\pi_I, \kappa)$ is differentiable when $\pi_C(\pi_I, \kappa) \in \{0, 1\}$ and establish the continuity of the partial derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ in the latter set. We restrict attention to the case in which $\pi_C(\pi_I, \kappa) = 1$ since the proof in the case in which $\pi_C(\pi, \kappa) = 0$ is similar.

Suppose that $\pi_C(\pi_I, \kappa) = 1$ and let $\{h_n\}$ be such that $h_n \to 0$. Then

$$\frac{Q(\pi_I + h_n, \kappa) - Q(\pi_I, \kappa)}{h_n} = -\frac{1}{h_n}\int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi$$

and

$$\frac{Q(\pi_I, \kappa + h_n) - Q(\pi_I, \kappa)}{h_n} = -\frac{1}{h_n}\int_{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi$$

Since $\pi_C(\pi_I + h_n, \kappa) = h_n + \pi_C(\pi_I, \kappa) = 1 + h_n$, it follows that

$$\left|\frac{1}{h_n}\int_{\max\{0, \pi_C(\pi_I + h_n, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi\right| \leq A\left|\frac{\Omega(1) - \Omega(1 + h_n)}{h_n}\right|,$$

where $A = \sup_{\pi \in [0,1]} |G(\pi - \pi_I)|$. Given that Ω is differentiable and $\Omega'(1) = \omega(1) = 0$, it then follows that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \lim_{n \to \infty} \frac{Q(\pi_I + h_n, \kappa) - Q(\pi_I, \kappa)}{h_n} = 0.$$

Now note that $\pi_C(\pi_I, \kappa + h_n) = \pi_C(\pi_I, \kappa) + \alpha(h_n)$, where $\alpha(h) = H'(\kappa/B)(h/B) + o(h/B)$. So,

$$\left| \frac{1}{h_n} \int_{\max\{0, \pi_C(\pi_I, \kappa + h_n)\}}^{1} G(\pi - \pi_I)\omega(\pi) d\pi \right| \leq A \frac{|\Omega(1) - \Omega(1 + \alpha(h_n))|}{|h_n|}.$$

Since $\Omega(1 + \alpha(h)) = \Omega(1) + o(\alpha(h))$ and $o(\alpha(h_n))/h_n \to 0$, as $\lim_{h \to 0} \alpha(h) = 0$ and $\alpha(h)/h$ is bounded when h is small, it then follows that

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \lim_{n \to \infty} \frac{Q(\pi_I, \kappa + h_n) - Q(\pi_I, \kappa)}{h_n} = 0.$$

The continuity of the partial derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ when $\pi_C(\pi_I, \kappa) = 1$ follows from the proof of Lemma 3 together with the fact that $\omega(\pi_C(\pi_I, \kappa))$ converges to zero as $\pi_C(\pi_I, \kappa)$ converges to one from below.

Propositions 1 and 2 in the Finite-Output Case

Here, we state and prove the counterparts of Propositions 1 and 2 in the finite-output case.

Proposition 4. *Suppose the entry cost is $\kappa \in (0, \overline{\kappa})$ and fix all other primitives of the model but the cost function c and the distribution of the citizen's reputation Ω. Now suppose there exists $a^* \in (0, \overline{a})$ and $j \in \{0, \ldots, N-1\}$ with*

$$0 < \pi_C(\pi^+(y_j|a^*), \kappa) < \pi_C(\pi^+(y_{j+1}|a^*), \kappa) < 1. \quad (17)$$

Then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when $(c, \Omega) = (c_1, \Omega_1)$ and $(c, \Omega) = (c_2, \Omega_2)$; and (ii) an increase in political competition increases effective accountability in one case and decreases it in the other.

Proof. Suppose $a^* \in (0, \overline{a})$ satisfies (17). Let $k = \inf\{j \in \{1, \ldots, N-1\} : \pi_C(\pi^+(y_j|a^*), \kappa) > 0\}$ and $l = \sup\{j \in \{1, \ldots, N-1\} : \pi_C(\pi^+(y_j|a^*), \kappa) < 1\}$. Both k and l are well-defined and $k < l$. Now let Ω_1 and Ω_2 be admissible distributions of the citizen's reputation with densities ω_1 and ω_2 such that $\omega_1(\pi_C(\pi^+(y_{j+1}|a^*),\kappa)) < \omega_1(\pi_C(\pi^+(y_j|a^*),\kappa))$ for all $j \in \{k,\ldots,l\}$ and $\omega_2(\pi_C(\pi^+(y_{j+1}|a^*),\kappa)) > \omega_2(\pi_C(\pi^+(y_j|a^*),\kappa))$ for all $j \in \{k,\ldots,l\}$. By the extension of Lemma 5 to the finite-output case, for each Ω_i there exists an admissible cost function c_i such that a^* is an equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c_i and the distribution of the citizen's reputation is Ω_i.

Now let

$$\Delta_i(a,\kappa) = B\sum_{j=1}^{N}\frac{\partial F}{\partial a}(y_j|a^*,\pi_0)\big[Q_i(\pi^+(y_j|a),\kappa) - Q_i(\pi^+(y_{j+1}|a),\kappa)\big] - c_i(a), \tag{18}$$

where $Q_i(\pi_I,\kappa)$ is the incumbent's probability of retention as a function of his or her second-period reputation and the entry cost when the distribution of the citizen's reputation is Ω_i. From (9), the equation $\Delta_i(a,\kappa) = 0$ defines the incumbent's equilibrium choice of effort implicitly as a function of κ when the cost function is c_i and the distribution of the citizen's reputation is Ω_i. The desired result follows from the fact $\partial\Delta_i(a^*,\kappa)/\partial\kappa < 0$ for $i = 1,2$ from equilibrium stability and the fact that

$$\frac{\partial\Delta_i}{\partial\kappa}(a^*,\kappa) \propto B\sum_{j=1}^{N}\frac{\partial F}{\partial a}(y_j|a^*,\pi_0)\big[\omega_i(\pi^+(y_j|a),\kappa) - \omega_i(\pi^+(y_{j+1}|a),\kappa)\big]$$

from Lemma 3. □

Proposition 5. *Fix all the model's primitives but the cost function, the distribution of the citizen's reputation, and the entry cost. If $\theta < \overline{\pi}^+(y_{N-1})$, then there exist a cost function and a distribution of the citizen's reputation for which effective accountability is not maximized when entry is costless.*

Proof. Suppose $\theta < \overline{\pi}^+(y_{N-1})$. Then there exists $a^* \in (0,\overline{a})$ such that $\theta < \pi^+(y_{N-1}|a^*)$. Let $k = \inf\{j \in \{1,\ldots,N-1\} : \pi^+(y_j|a^*) \geq \theta\}$ and choose Ω to be an admissible distribution of the citizen's reputation with density ω such that $\omega(\pi^+(y_j|a^*) - \theta) < \omega(\pi^+(y_{j+1}|a^*) - \theta)$ for all $j \in \{k,\ldots,N-1\}$. Now let c be an admissible cost function such that a^* is the unique equilibrium effort choice for the incumbent and the equilibrium is stable when the entry cost is zero, the cost function is c, and the distribution of the citizen's reputation is Ω. Take the cost function and the distribution of the citizen's reputation to be c and Ω, respectively. The same argument as in the proof of Proposition 2 establishes the desired result, where now $\Delta(a,\kappa)$ is given by (18). □