

# Valuing Life as an Asset, as a Statistic, and at Gunpoint<sup>1</sup>

Julien Hugonnier<sup>1,4,5</sup>, Florian Pelgrin<sup>2,6</sup> and Pascal St-Amour<sup>3,4,6</sup>

<sup>1</sup>École Polytechnique Fédérale de Lausanne

<sup>2</sup>EDHEC Business School

<sup>3</sup>HEC Lausanne, University of Lausanne

<sup>4</sup>Swiss Finance Institute

<sup>5</sup>CEPR

<sup>6</sup>CIRANO

February 23, 2021

<sup>1</sup>Financial support by the Swiss Finance Institute is gratefully acknowledged. We are grateful to Nezih Guner (Joint Managing Editor) and three anonymous referees for constructive remarks and suggestions. We have also benefited from very useful discussions with and comments from Georges Dionne, James K. Hammitt, Pierre-Carl Michaud, Michel Normandin, Frédéric Robert-Nicoud and Mathias Thoenig. The usual disclaimer applies.

## Abstract

The Human Capital (HK) and the Statistical (VSL) values differ sharply in their empirical pricing of a human life. Rationalizing these differences is complicated by the absence of common theoretical and empirical foundations. We contribute to the life valuation literature by providing the first *joint benchmark* estimates of the willingness to pay (WTP) to avoid increases in mortality risk, as well as of the values of life, in the context of a theoretically, and empirically integrated approach. The optimal investment to a flexible human capital problem with longevity risk is used to characterize the HK, whereas the indirect utility yields WTP. The marginal WTP solves for the VSL and the limiting WTP provides an alternative valuation calculated at Gunpoint (GPV). A structural estimation of the analytical solutions with 2017-PSID data confirms that the HK (300 K\$) and GPV (251 K\$) are close to one another and that the strong curvature of the WTP explains a much larger VSL (4.98 M\$).

**Keywords:** Value of Human Life, Human Capital, Value of Statistical Life, Gunpoint Value, Hicksian Willingness to Pay, Equivalent Variation, Mortality, Structural Estimation.

**JEL Classification:** J17, D15, G11.

# 1 Introduction

## 1.1 Motivation and overview

Computing the economic value of a human life is often required in policy, societal, as well as legal debates and has long generated a deep interest among researchers.<sup>1</sup> Indeed, life valuations are called upon in public health and safety issues, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. They are also important in long-run debates on quality versus quantity of life, such as whether to spend more resources on innovations that foster consumption growth or on those that prolong life expectancy.<sup>2</sup> Moreover, economic life values are resorted to in assessing the tolls of war, in wrongful death litigation, as well as in terminal care cost/benefit analysis.

An agent's willingness to pay (WTP) or to accept (WTA) compensation for changes in death risk exposure is a key ingredient for life valuation. Indeed, a shadow price of a life can be deduced through the individual marginal rate of substitution (MRS) between mortality and wealth. In the same vein, a collective MRS between life and wealth is relied upon by the Value of a Statistical Life (VSL) literature to calculate the societal WTP to save an unidentified (i.e. statistical) life. The VSL's domain of application relates to public health and safety decisions benefiting unidentified persons. In contrast, the Human Capital (HK) life value relies on asset pricing theory to compute the present value of an identified person's cash flows corresponding to his<sup>3</sup> labor income, net of the measurable investment expenses. HK values are used for valuing a given life, such as in wrongful death litigation,<sup>4</sup> or in measuring the economic costs of armed conflict.<sup>5</sup> Finally, a Gunpoint value of life (GPV) measures the maximal amount a person is willing to pay

---

<sup>1</sup>Landefeld and Seskin (1982); Kiker (1966) make reference to human-capital based evaluations of the value of life dating back to Petty (1691). See also Hofflander (1966) for historical perspectives on life valuation.

<sup>2</sup>See Jones (2016); Jones and Klenow (2016); Hall and Jones (2007); Murphy and Topel (2006); Becker et al. (2005) for quality vs quantity of life arbitrages.

<sup>3</sup>We henceforth refer to an agent using the 'he/his' pronouns not to distinguish gender, but solely to alleviate exposition.

<sup>4</sup>See Symmons (1938); Kiker (1966); Mishan (1971) for descriptions, historical perspectives and discussions of HK and VSL. See Viscusi (2000, 2007) for legal uses of HK and VSL life values. See also Posner and Sunstein (2005) for comparisons between administrative (e.g. VSL used by regulatory agencies) and legal (i.e. HK used in litigation) life value measurement.

<sup>5</sup>See Eden (1972) for a HK analysis of the value of enlisted men and officers' lives lost in Southeast Asian wars.

to avoid certain, instantaneous death. The GPV is theoretically relevant for end-of-life (e.g. terminal care) settings, yet, to the best of our knowledge, no empirical evaluation of the Gunpoint life value exists.<sup>6</sup>

In practice, both HK and VSL valuations of a human life yield strikingly different measures with VSL estimates being much larger than HK values. For example, Huggett and Kaplan (2016) identify HK values between 300 K–900 K\$, whereas the U.S. Transportation authority recommends using a VSL-type amount of 9.4 M\$ (U.S. Department of Transportation, 2016). Although it is well recognized that HK and VSL life values need not be equal,<sup>7</sup> rationalizing differences of such magnitude is complicated by the fact that HK and VSL evaluations neither share joint theoretical underpinnings, nor common database, nor encompassing identification strategy.

We contribute to life valuation by providing the first *joint benchmark* estimates of the WTP, HK, VSL and GPV, within the context of a theoretically, and empirically integrated approach. The proposed framework involves solving and structurally estimating a flexible life cycle problem which departs from standard approaches in two key dimensions. First, our model features endogenous financial and human capital accumulation for an agent exposed to financial and longevity risk. Human capital benefits income and can be interpreted either as skills (e.g. Ben-Porath, 1967; Heckman, 1976) or as health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), and is subject to stochastic depreciation shocks (e.g. unemployment, obsolescence or illness). Second, we rely on recursive utility which disentangles attitudes towards risk from those towards inter-temporal substitution. This specification is useful in guaranteeing strict preference for life over death, in allowing more flexible trade-offs between quantity (i.e. longevity) and quality of life (i.e. consumption), as well as in reconciling savings with other financial choices.<sup>8</sup>

Our first main contribution is theoretical and shows that the optimal rules and associated indirect utility function for that model are *sufficient* to fully integrate and characterize the four life valuation measures. This remarkable result can be traced back

---

<sup>6</sup>See Jones-Lee (1974); Cook and Graham (1977); Eeckhoudt and Hammitt (2004) for related definitions of the GPV, and Philipson et al. (2010); Round (2012); Hugonnier et al. (2020) for end-of-life discussions.

<sup>7</sup>See Conley (1976); Shepard and Zeckhauser (1984); Pratt and Zeckhauser (1996); Viscusi (2000, 2007) for discussions.

<sup>8</sup>See Hugonnier et al. (2013); Córdoba and Ripoll (2017) for discussion of preference for life and additional flexibility in longevity versus wealth trade-offs in recursive preferences. Epstein and Zin (1989, 1991); Duffie and Epstein (1992) discuss the role of separation in attitudes in reconciling consumption and financial decisions.

to two channels. First, the dynamics of capital along the optimal path determine the lifetime flow of income net of investment. This dividend can be capitalized using the stochastic discount factor consistent with the agent’s opportunity set to characterize the HK value. Second, the indirect utility calculated at the optimum can be combined with variational analysis (Hicks, 1946) to define the willingness to pay to prevent increases in mortality risk exposure. We show that the marginal WTP defines the VSL, whereas the limiting WTP identifies the Gunpoint value as the maximal willingness to pay that leaves an agent indifferent between living and dying. These four closed-form life valuations precisely pinpoint the contributions of fundamentals (i.e. preferences, risk distributions, or technology) and of state variables (i.e. wealth and capital levels) thereby allowing us to investigate how the WTP, HK, VSL and GPV are theoretically related to one another.

Our second main contribution is empirical. We structurally estimate the model’s distributional, technological and preferences parameters by associating human capital to health and by resorting to 2017 PSID data that correspond to the optimal consumption, portfolio, as well as health spending and insurance policies. A Revealed Preference perspective then allows us to combine the estimated deep parameters with observed wealth and health variables to estimate the analytical expressions for the willingness to pay, Human Capital, Statistical and Gunpoint values of life. Our encompassing approach thus ensures that the WTP and the three different life values are computed through a single-step estimation, using the same data set, and imposing strict compliance with common theoretical conditions thereby ensuring a shared identification strategy.

## 1.2 Contributions to the literature

The reliance on an integrated approach to life valuation provides answers to a number of open issues.<sup>9</sup> First, to what extent can the four different life valuation concepts be empirically revealed by observed financial and capital choices made by agents? We show that *all* measures are identifiable from the widely-used and representative PSID data set from which household consumption, portfolio, health investment and insurance decisions are explained with health and wealth covariates. Second, are the large differences between the HK and VSL due to disjoint theoretical and empirical frameworks? We show that it

---

<sup>9</sup>We provide a more comprehensive review of the relevant literature in Section A in the Online Appendix.

is not the case; our integrated estimates yields gaps between the two that are of the same order of magnitude as those identified by the segmented HK/VSL literature. Indeed, our PSID estimates for the HK value is 300 K\$, compared to our VSL estimate of 4.98 M\$. Third, how does the (previously un-quantified) Gunpoint value compare with these HK and VSL values? We show that our estimated GPV of 251 K\$ is both theoretically, and empirically close to the HK value.

Fourth, what are the theoretical reasons behind the much larger VSL estimates? We show that the estimated individual WTP is increasing, very concave and bounded above in the change in the death risk exposure. The VSL is the marginal willingness to pay (MWTP), whereas the GPV is the limiting WTP. The VSL is much larger than the GPV because a linear projection with slope equal to the MWTP necessarily over-estimates the upper bound of an increasing and concave WTP. Fifth, what role do technological and distributional assumptions play in these life values? We show that the human capital accumulation and risks parameters uniquely pin down the shadow price (i.e. Tobin's- $q$ ) of human capital. The latter can be combined with observed capital and wealth to obtain a net total wealth measure. Human and/or net total wealth condition all four life value measures. Finally, what role do the preferences play? We show that they are absent from the Human Capital value. Minimal consumption is a key driver for the VSL, WTP and GPV. Attitudes towards risk and time, especially the elasticity of inter-temporal substitution (EIS), determine the VSL and the WTP. However, since death is certain and instantaneous in a Gunpoint threat, risk aversion and the EIS play no role in the GPV.

In addition to the segmented research on HK, WTP, VSL and GPV, our paper contributes to the literature on encompassing and on theoretical models of life valuations. First, the links between the WTP, the VSL and a GPV equivalent have been explored by Jones-Lee (1974) in a static setup. In addition, Conley (1976); Shepard and Zeckhauser (1984); Rosen (1988) use life cycle models of human capital to relate HK and the VSL. However, none of these contributions link all four main valuations in an encompassing framework and none provide joint estimation of the HK, WTP, VSL and GPV measures as we do. Second, our paper is related to theoretical life valuation models. Córdoba and Ripoll (2017); Bommier et al. (2019); Hugonnier et al. (2013) also study VSL and WTP in the context of life cycle models with recursive preferences. We contribute to these

papers by incorporating HK, and GPV as well, by characterizing the links between and structurally estimating all four measures with a common database.

### 1.3 Policy relevance: A road safety example

Our integrated value of life framework remains an essentially positive exercise in that we do *not* provide a normative ranking of the various life values. Indeed, this paper fully accords with previous literature that different life valuation methods are not substitutes, but rather complement one another. Which of these four instruments should be relied upon depends on the questions to be addressed. A simple example may illustrate the relevance and applicability of our findings. Consider the case where a dangerous segment on a public road is associated with the death of  $N$  drivers per year. Modifications at cost  $G$  could save  $n \geq 1$  lives. Our integrated approach provides single-step estimates of the WTP, HK, VSL and GPV instruments that can address four different *ex-ante* and *ex-post* policy issues associated with this road safety example.

The first policy question is whether these road modifications are economically justifiable. Our VSL estimate computes the *societal* willingness to pay for a mortality reduction of  $n = 1$  unidentified person and is therefore appropriate for the relevance of spending  $G$  public funds on road safety. The second policy question is whether or not other alternatives (e.g. speeding fines) should complement and/or could be more efficient than road work. Our WTP measure calculates the *individual* marginal rate of substitution between wealth and mortality risk and is therefore applicable to infer the agents' responses to any level of change in death risk exposure. Consider next the case where a life-threatening accident involving driver  $j$  did occur on that particular road segment. If driver  $j$  is alive and maintained on life support, our GPV measure calculates that person's valuation of his own life and can be used to decide whether or not terminal care should be maintained. If the driver  $j$  dies as a result of his accident, both our HK and GPV values can be used by courts in litigation against the state for having maintained an excessive level of mortality risk in public roads. Indeed, the Human Capital value gauges driver  $j$ 's tangible losses associated with lost net income, whereas the GPV provides a measure of  $j$ 's intangible 'loss of life's pleasures' relied upon by courts for hedonic damages calculations. Both the GPV and HK values complement the VSL and are therefore useful to the government for value-at-risk calculations in deciding

whether or not to spend public funds on road adjustment. The four instruments we recover are thus theoretically (common model, assumptions, definitions) and empirically (structural estimation, common data base) consistent with one another. Our approach is also very flexible and can be adapted to a different model of human capital accumulation (e.g. with aging, work/leisure choices, ...) and/or different data bases or stratification of a common data (e.g., general population, tax payers, general, or particular road drivers, ...).

The rest of the paper is organized as follows. We first present and solve our human capital model in Section 2. The associated optimal rules and welfare are used to characterize the implied life valuations in Section 3. Section 4 reviews the empirical strategy. Section 5 presents the structural parameters and life value estimates, while robustness is assessed in Section 6. Concluding remarks are presented in Section 7. All supplementary material, including proofs, is regrouped in the Online Appendix.

## 2 Human Capital Model

### 2.1 Economic environment

#### 2.1.1 Overview

We focus on a continuous-time life cycle model of endogenous human capital (e.g. skills, health) accumulation, subject to exogenous stochastic capital (e.g. unemployment, morbidity) and duration (e.g. mortality) shocks. Capital is valuable because of the additional income it provides. In addition to investment, we characterize optimal dynamic choices in consumption/savings, risky portfolio, and insurance against capital shocks. We feature generalized recursive, rather than VNM preferences, that separate attitudes toward financial risk from those toward inter-temporal substitution. This characteristic not only better reconciles consumption with financial decisions, but crucially ensures that the agent unconditionally prefers life over death. Our market setup is inherently incomplete with three sources of risks (financial, mortality and capital) and only two assets. However, the model can be recast as an equivalent setup with complete markets and heavier discounting.<sup>10</sup>

---

<sup>10</sup>All proofs and additional theoretical results for the current and subsequent sections are regrouped in the Online Appendix.

We review the effects of the model's key theoretical assumptions in the Robustness Section 6 below. These include allowing for direct utilitarian services (Section 6.1.1), endogenous mortality and morbidity risks exposures (Section 6.1.2), incorporating work/leisure decisions (Section 6.1.3), investment vs consumption perspectives on spending on human capital (Section 6.1.4), aging (Section 6.1.5) as well as access to insurance against human capital shocks (Section 6.1.6). Finally, we contrast our model with an alternative popular choice in the human capital literature (Section 6.2).

### 2.1.2 Planning horizon and human capital dynamics

The agent's planning horizon is limited by a stochastic age at death  $T_m$  satisfying:

$$\lim_{h \rightarrow 0} \frac{1}{h} \Pr [T_m \in (t, t + h] \mid T_m > t] = \lambda_m, \quad (1)$$

such that the probability of death by age  $t$  is monotone increasing in the arrival rate  $\lambda_m > 0$ :

$$\Pr(T_m \leq t) = 1 - e^{-\lambda_m t}. \quad (2)$$

Subsequent analysis will focus on changes in mortality risk exposure stemming from permanent changes in death intensity  $\lambda_m$ .<sup>11</sup>

The agent invests at rate  $I_t$  in his human capital  $H_t$  whose law of motion is given by:

$$dH_t = (I_t^\alpha H_t^{1-\alpha} - \delta H_t) dt - \phi H_t dQ_{st}. \quad (3)$$

In this equation, the Cobb-Douglas parameter  $\alpha \in (0, 1)$  captures diminishing returns to investment,  $\delta > 0$  measures the continuous deterministic depreciation of human capital in the absence of investments, and  $dQ_{st}$  is the increment of a Poisson process with constant intensity  $\lambda_s$  whose jumps depreciate the capital stock by a factor  $\phi \in (0, 1)$ .

The law of motion (3) admits alternative interpretations of human capital. If  $H_t$  is associated with skills (e.g. Ben-Porath, 1967; Heckman, 1976), then investment  $I_t$  comprises education and training choices made by the agent whereas  $dQ_{st}$  can be interpreted as stochastic unemployment, or technological obsolescence shocks that depreciate the

---

<sup>11</sup>See also Murphy and Topel (2006) for a similar perspective.

human capital stock. If  $H_t$  is instead associated with the agent's health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), then investment takes place through medical expenses or healthy lifestyle decisions whereas the stochastic depreciation occurs through morbidity shocks.

### 2.1.3 Budget constraint and preferences

The agent's income rate is given by:

$$Y_t = Y(H_t) = y + \beta H_t, \quad (4)$$

and includes both an exogenous base income  $y$  and a positive income gradient  $\beta$  for human capital capturing higher labor income for skilled or healthy individuals. Individuals can trade in a risk-less asset with return  $r$ , as well as in two risky assets to smooth out shocks to consumption: stocks and insurance against human capital depreciation. Financial wealth  $W_t$  evolves according to the dynamic budget constraint:

$$dW_t = (rW_t + Y_t - c_t - I_t) dt + \pi_t \sigma_S (dZ_t + \theta dt) + x_t (dQ_{st} - \lambda_s dt), \quad (5)$$

where  $\sigma_S > 0$  is the volatility of the stock,  $\theta = (\mu - r)/\sigma_S$  is the market price of financial risk and  $Z_t$  is a Brownian motion. In addition to investment  $I_t$ , the agent selects consumption  $c_t$ , the risky portfolio  $\pi_t$  and the number of units  $x_t$  of actuarially-fair depreciation insurance. The latter pays one unit of the numeraire upon the occurrence of a depreciation shock, and can be interpreted as unemployment insurance (if  $H_t$  is associated with skills), or as medical, or disability insurance (if  $H_t$  is associated with health status).

Following Hugonnier et al. (2013) we define the indirect utility of an alive agent as:

$$V(W_t, H_t) = \sup_{(c, \pi, x, I)} U_t, \quad (6a)$$

where preferences are given by

$$U_t = E_t \int_t^{T_m} \left( f(c_\tau, U_\tau) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_\tau} \right) d\tau, \quad (6b)$$

where  $\sigma_t(U) = d\langle Z, U \rangle / dt$  denotes the diffusion of the continuation utility process, and  $f(c, u)$  is the Kreps-Porteus aggregator function defined by:

$$f(c, u) = \frac{\rho u}{1 - 1/\varepsilon} \left( \left( \frac{c - a}{u} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right). \quad (6c)$$

The preference specification in (6) belongs to the stochastic differential utility class proposed by Duffie and Epstein (1992) and is the continuous-time analog of the discrete-time recursive preferences of Epstein and Zin (1989, 1991).<sup>12</sup> It is characterized by a subjective discount rate  $\rho > 0$ , a minimal subsistence consumption level  $a > 0$ , risk-neutrality with respect to both depreciation shocks and death, and disentangles the agent's elasticity of inter-temporal substitution (EIS)  $\varepsilon \geq 0$ , from his constant relative risk aversion with respect to financial risk  $\gamma \geq 0$ . As explained in Hugonnier et al. (2013) and confirmed in Theorem 1 below, the homogeneity properties of our specification implies that any feasible consumption process  $c_t - a \geq 0$  is associated with a positive continuation utility and therefore guarantees preference for living over death:  $V_t \geq V^m \equiv 0$ , where  $V^m$  is the utility at death.

## 2.2 Optimal rules

### 2.2.1 Solving the model

The agent's dynamic problem (6), subject to (3) and (5) can be recast through the Hamilton-Jacobi-Bellman (HJB):

$$\begin{aligned} 0 = & \max_{\{c, \pi, x, I\}} \frac{(\pi \sigma_S)^2}{2} V_{WW} + H \left[ (I/H)^2 - \delta \right] V_H + [rW + \pi \sigma_S \theta - c + y + \beta H - I - x \lambda_s] V_W \\ & + \frac{\rho V(W, H)}{1 - \frac{1}{\varepsilon}} \left[ \left( \frac{c - a}{V(W, H)} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right] - \frac{\gamma (\pi \sigma_S V_W)^2}{2V(W, H)} - \lambda_m V(W, H) \\ & - \lambda_s V(W, H) \left[ 1 - \frac{V(W + x, H(1 - \phi))}{V(W, H)} \right]. \end{aligned}$$

It can also be solved in two separate steps<sup>13</sup> involving:

<sup>12</sup>See also Palacios (2015) for a Human Capital problem with Duffie and Epstein (1992) preferences.

<sup>13</sup>See Bodie et al. (1992); Hugonnier et al. (2013); Palacios (2015); Acemoglu and Autor (2018) for discussion and applications of separability of investment and financial decisions in human capital problems.

1. An *hypothetical* infinitely-lived agent first solves the optimal investment by maximizing the discounted value of the  $H$ -dependent part of net income:

$$P(H_t) = \sup_{I \geq 0} E_t \int_t^\infty \frac{m_\tau}{m_t} (\beta H_\tau - I_\tau) d\tau.$$

where

$$m_t = \exp \left( -rt - \theta Z_t - \frac{1}{2} \theta^2 t \right). \quad (7)$$

is the stochastic discount factor induced by the prices of financial assets. The human wealth  $P(H)$  is then combined with the agent's financial wealth and the present value of his base income stream net of minimal consumption expenditures to obtain the agent's net total wealth as:

$$\begin{aligned} N(W_t, H_t) &= W_t + E_t \int_t^\infty \frac{m_\tau}{m_t} (Y(H_\tau^*) - I_\tau^* - a) d\tau \\ &= W_t + \frac{y - a}{r} + P(H_t) \end{aligned} \quad (8)$$

An important consequence of this characterization is that, due to complete financial markets, both the agent's optimal human capital investment  $I^*$  and his human wealth  $P(H_t)$  can be determined independently of his preferences with respect to time or risk.

2. The finitely-lived agent then selects the remaining policies  $\bar{c}_t = c_t - a$ ,  $\pi_t$  and  $\bar{x}_t = x_t - \phi P(H_t)$  by maximizing utility (6), subject to the law of motion for net total wealth:

$$dN_t = (rN_t - \bar{c}_t)dt + \pi_t \sigma_S (dZ_t + \theta dt) + \bar{x}_t (dQ_{st} - \lambda_s dt).$$

The remaining optimal consumption, portfolio and insurance policies, as well as indirect utility function reinstate a role for preferences and finite lives and are calculated as functions of  $P(H_t)$  and  $N(W_t, H_t)$ .

### 2.2.2 Closed-form solutions

In the context of our parametric model, and under the completeness assumption both optimization steps described earlier can be carried out, leading to the following result.

**Theorem 1** *Assume that the parameters of the model are such that*

$$(r + \delta + \phi\lambda_s)^{\frac{1}{\alpha}} > \beta, \quad (9a)$$

*and denote the Tobin's-q of human capital by  $B > 0$ , the unique solution to:*

$$\beta - (r + \delta + \phi\lambda_s)B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}} = 0, \quad (9b)$$

*subject to:*

$$r + \delta + \phi\lambda_s > (\alpha B)^{\frac{\alpha}{1-\alpha}}. \quad (9c)$$

*Assume further that the marginal propensity to consume out of net total wealth,  $A > 0$  satisfies:*

$$A(\lambda_m) = \varepsilon\rho + (1 - \varepsilon) \left( r - \lambda_m + 0.5\frac{\theta^2}{\gamma} \right), \quad (10a)$$

$$> \max \left( 0, r - \lambda_m + \frac{\theta^2}{\gamma} \right). \quad (10b)$$

*Then,*

1. *the human wealth and net total wealth are given as:*

$$P(H_t) = BH_t \geq 0, \quad (11)$$

$$N(W_t, H_t) = W_t + \frac{y - a}{r} + P(H_t) \geq 0, \quad (12)$$

2. *the indirect utility for the agent's problem is:*

$$V_t = V(W_t, H_t, \lambda_m) = \Theta(\lambda_m)N(W_t, H_t) \geq 0, \quad (13a)$$

$$\Theta(\lambda_m) = \tilde{\rho}A(\lambda_m)^{\frac{1}{1-\varepsilon}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{\varepsilon}{1-\varepsilon}} \quad (13b)$$

and generates the optimal rules:

$$\begin{aligned}
c_t^* &= c(W_t, H_t, \lambda_m) = a + A(\lambda_m)N(W_t, H_t) \geq 0, \\
\pi_t^* &= \pi(W_t, H_t) = (\theta/(\gamma\sigma_S))N(W_t, H_t), \\
x_t^* &= x(H_t) = \phi P(H_t) \geq 0, \\
I_t^* &= I(H_t) = \left( \alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}} \right) P(H_t) \geq 0,
\end{aligned} \tag{14}$$

where any dependence on death intensity  $\lambda_m$  is explicitly stated.

Conditions (9) encompass transversality restrictions for a finite shadow value of human capital, whereas conditions in (10) are required to ensure positive marginal propensity to consume (MPC) out of net wealth  $A > 0$ , as well as for minimal consumption requirements  $c_t > a$ . Restrictions (9) and (10) jointly ensure that the continuation utility  $V_t$  in (13) is finite and that the solutions in (14) are well-defined. The constant  $B$  in (11) can naturally be interpreted as the marginal value (i.e. Tobin's  $Q$ ) associated with human capital. It is implicitly defined in (9) as an increasing function of the income gradient  $\beta$  and a decreasing function of the rate of interest  $r$  and the expected depreciation rate  $\delta + \phi\lambda_s$ .

Three features of the optimal rules are particularly relevant for life valuation. First, the two-step solution method ensures that both human wealth (11) and the net total wealth (12) are independent of the death intensity  $\lambda_m$ . Second and related, the exposure to exogenous death risk  $\lambda_m$  affects welfare only through  $\Theta(\lambda_m)$  in (13b), via its impact on the marginal propensity to consume  $A(\lambda_m)$ . Equation (10) establishes that  $A'(\lambda_m) = \varepsilon - 1 \lesseqgtr 0$ , i.e. this MPC effect is entirely determined by the elasticity of inter-temporal substitution. An increase in death risk  $\lambda_m$  induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume. On the one hand, more discounting requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC). This effect is dominant at low elasticity of inter-temporal substitution  $\varepsilon \in (0, 1)$ . On the other hand, heavier discounting makes future consumption less desirable prompting the agent to shift future towards current consumption (i.e. by increasing the MPC). This *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution  $\varepsilon > 1$ . Observe that, separate  $\varepsilon$  and  $\gamma$  parameters entail that a high EIS can coincide with high risk aversion, a flexibility

that cannot be attained under VNM preferences which impose  $\gamma = 1/\varepsilon$ . Equivalently, whether the agent prefers to live fast and die young or not is independent of his attitudes towards risk.

Third, the welfare in (13) is increasing in both wealth and human capital stock and is decreasing and convex in the death intensity  $\lambda_m$  at all EIS levels since:

$$\Theta'(\lambda_m) = -\tilde{\rho}A(\lambda_m)^{\frac{\varepsilon}{1-\varepsilon}} \leq 0, \quad (15a)$$

$$\Theta''(\lambda_m) = \tilde{\rho}\varepsilon A(\lambda_m)^{\frac{2\varepsilon-1}{1-\varepsilon}} \geq 0. \quad (15b)$$

Hence, whereas the sign of the effects of death risk  $\lambda_m$  on the MPC (10) depends on the EIS, preference for life implies that higher mortality exposure unconditionally reduces the marginal value of net total wealth  $\Theta(\lambda_m)$  in (13b) and therefore lowers welfare  $V_t$  in (13a). Importantly, as shown below in Section 3.3, a decreasing and convex effect of death risk on welfare entails that the willingness to pay to avoid increases in mortality is increasing and concave in death risks.

## 3 Willingness to Pay and Values of Life

### 3.1 Overview

Figure 1 displays the two channels we rely upon to calculate the life valuations implied by the solutions in Theorem 1 for the human capital model of Section 2. First, in node (a), the capital dynamics  $dH_t$  evaluated at the optimal investment  $I_t^*$  yield the optimal path for human capital  $H_t^*$ , and associated net income  $D_t^* = Y(H_t^*) - I_t^*$ . In node (b), this dividend can be capitalized using the model-implied SDF  $m_t$  to obtain the HK value of life. Second, in nodes (c) and (d), Hicksian variational analysis is applied on the indirect utility  $V(W_t, H_t, \lambda_m)$  to compute the willingness to pay  $v_t$  to avoid increases  $\Delta$  in death risk  $\lambda_m$ . The marginal WTP  $v_\Delta$  in (e) yields the VSL whereas we show that the limiting WTP yields the Gunpoint value in node (f).<sup>14</sup>

---

<sup>14</sup>We assume throughout this section that the parameters of the model satisfy the regularity conditions (9) and (10) and abstract from time subscripts whenever possible to alleviate notation.

### 3.2 Human Capital Value of Life

The Human Capital Value of life is the market value of the net cash flow associated with human capital and that is foregone upon death (e.g. Kiker, 1966; Eden, 1972; Conley, 1976; Lewbel, 2003; Huggett and Kaplan, 2013, 2016). In our setting, this net cash flow is the marketed income, minus the money value of investment expenses, where both are evaluated at the optimum:

**Definition 1 (HK value of life)** *The Human Capital value of life is*

$$v_{h,t} = E_t \int_t^{T_m} \frac{m_\tau}{m_t} (Y(H_\tau^*) - I_\tau^*) d\tau, \quad (16)$$

where  $m_t$  is the stochastic discount factor induced by the prices of financial assets,  $I^*$  denotes the agent's optimal human capital investment, and  $H^*$  denotes the corresponding path of his human capital process.

We can substitute investment  $I^*$  from (14) in the law of motion (3) to recover the optimal path for human capital  $H^*$  and corresponding income flow  $Y(H^*)$ . Recall also that the agent's investment opportunity set induces a unique stochastic discount factor  $m_t$  given by (7). Combining both in (16) leads to the following result.

**Proposition 1 (HK)** *The Human Capital value of life solving (16) is:*

$$v_h(H, \lambda_m) = C_0(\lambda_m) \frac{y}{r} + C_1(\lambda_m) P(H) \quad (17)$$

where the constants  $(C_0, C_1) \in [0, 1]^2$  are defined by:

$$C_0(\lambda_m) = \frac{r}{r + \lambda_m}, \quad (18a)$$

$$C_1(\lambda_m) = \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}} + \delta + \lambda_s \phi}{r + \lambda_m - (\alpha B)^{\frac{\alpha}{1-\alpha}} + \delta + \lambda_s \phi}, \quad (18b)$$

and where human wealth  $P(H)$  is given in (11).

Unlike step-1 of the solution method in Section 2.2.1, the discounted present value of net income is computed over a (stochastic) finite horizon  $T_m$  and must be therefore be corrected for mortality exposure  $\lambda_m$ . The first term in (17) is the present value  $y/r$  of the agent's base income  $y = Y(0)$  calculated over an infinite horizon and adjusted for the

exposure to death risk by multiplying with the constant  $C_0 \in [0, 1]$  in (18a). The second term is the present value  $P(H)$  of the net human capital cash flow  $\beta H_t - I^*$  over an infinite horizon and this value is corrected for finite life by multiplying with the constant  $C_1 \in [0, 1]$  in (18b). Both  $C_0(\lambda_m), C_1(\lambda_m)$  are decreasing functions of the death intensity  $\lambda_m$ , consistent with a lower HK value for shorter longevity.

### 3.3 Willingness to pay to avoid a change in death risk

Next, consider an *admissible* change  $\Delta$  in the intensity of death from base level  $\lambda_m$  in (1), i.e. one for which the indirect utility remains well defined when evaluated at the modified death exposure. The analysis of the WTP to avoid imminent death risk in a Gunpoint setting (discussed in Section 3.5) naturally designates the Hicksian Equivalent Variation (EV), rather than Compensating Variation (CV) as the relevant measure of willingness to pay (resp. to accept compensation) to avoid (resp. to forego) detrimental (resp. beneficial) changes in mortality.<sup>15</sup> We use standard variational analysis to define the corresponding Hicksian EV as follows:

**Definition 2 (Hicksian Equivalent Variation)** *Let  $\mathcal{A}$  be the admissible set of permanent changes  $\Delta \geq -\lambda_m$  in death intensity such that the condition (10) of Theorem 1 hold when  $\lambda_m$  is evaluated at  $\lambda_m^* = \lambda_m + \Delta$ . Then the Equivalent Variation to avoid  $\Delta \in \mathcal{A}$  is implicitly given as the solution  $v = v(W, H, \lambda_m, \Delta)$  to:*

$$V(W - v, H; \lambda_m) = V(W, H; \lambda_m^*). \quad (19)$$

where  $V(W, H; \lambda_m)$  is an indirect utility function.

For unfavorable changes  $\Delta > 0$ , the EV (19) indicates a willingness to pay  $v > 0$  to remain at base risk instead of facing higher mortality. For favorable changes  $\Delta < 0$ , the EV is a willingness to accept (WTA) compensation equal to  $-v > 0$  to forego lower risk.

The properties of the willingness to pay  $v$  with respect to the increment in death risk follow directly from those of the indirect utility  $V(W, H; \lambda_m)$ . In particular, we can

---

<sup>15</sup>Whereas paying out the WTP under a gunpoint threat is rational, accepting compensation against certain and instantaneous death when terminal wealth is not bequeathed and life is preferred to death cannot be. Since we abstract from bequests in our benchmark model in Section 2, we therefore adopt the EV, rather than CV perspective and focus on the WTP to avert unfavorable risks in subsequent analysis. For completeness, the extension to CV measures is nonetheless presented in Online Appendix C.1.

substitute  $v(W, H, \lambda_m, \Delta)$  in (19), take derivatives and re-arrange to obtain:

$$v_\Delta = -\frac{V_{\lambda_m}}{V_W}, \quad (20a)$$

$$v_{\Delta\Delta} = \frac{V_{\lambda_m\lambda_m} - V_{WW}(v_\Delta)^2}{-V_W}, \quad (20b)$$

where a subscript denotes a partial derivative. Monotonicity  $V_W \geq 0$  and preference for life over death  $V_{\lambda_m} \leq 0$  therefore induce a willingness to pay  $v$  that is increasing in  $\Delta$ , whereas the diminishing marginal utility of wealth  $V_{WW} \leq 0$  and of survival probability  $V_{\lambda_m\lambda_m} \geq 0$  are sufficient to induce a concave WTP function in mortality risk exposure.

Relying on the indirect utility given in (13) for the human capital problem in Section 2 allows us to solve for the Hicksian variation as follows:

**Proposition 2 (Hicksian EV)** *The Equivalent Variation solving (19) is:*

$$v(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)} \right] N(W, H). \quad (21)$$

*It is increasing and concave in  $\Delta$  with*

$$\inf_{\Delta \in \mathcal{A}} v(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(0)}{\Theta(\lambda_m)} \right] N(W, H) \quad (22a)$$

$$\sup_{\Delta \in \mathcal{A}} v(W, H, \lambda_m, \Delta) = N(W, H). \quad (22b)$$

where net total wealth  $N(W, H)$  is given in (12) and its marginal value  $\Theta(\lambda_m)$  is given in (13b).

The WTP in (21) equals zero if either  $\Delta = 0$  or if the agent's elasticity of intertemporal substitution  $\varepsilon = 1$ . Indeed, for unit elasticity, the MPC  $A$  in (10a), and therefore the marginal utility of net total wealth  $\Theta$  (13b), are both independent from  $\lambda_m$ . Moreover, the properties in (15) established that the indirect utility  $V(W, H; \lambda_m)$  in (13a) is decreasing and convex in the death intensity  $\lambda_m$ . Consequently, the weights  $\Theta(\lambda_m^*)/\Theta(\lambda_m) \in [0, 1]$  for detrimental changes  $\Delta \geq 0$  and the willingness to pay is an unconditionally increasing function of net total wealth  $N(W, H)$ . Combining (15) with (20) confirms a monotone increasing and concave willingness to pay to avoid increases in death risk exposure in (21), consistent with standard economic intuition of diminishing

marginal valuation of additional longevity (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2017).

The lower bound on the WTP in (22a) is obtained by setting  $\Delta = -\lambda_m$  yielding the WTA a compensation in order to forego zero death risk exposure.<sup>16</sup> From equations (10) and (13b) this bound exists and is finite. Equation (22b) further establishes that the willingness to pay is bounded above by net total wealth  $N(W, H)$ . When the elasticity of inter-temporal substitution is larger than one, this upper bound corresponds to the asymptotic WTP. When the EIS is below one, the upper bound corresponds to a maximal admissible WTP satisfying the transversality constraint (10) (see Online Appendix B.3).

### 3.4 Value of a Statistical Life

#### 3.4.1 Theoretical VSL

The VSL is the marginal rate of substitution between life and wealth, evaluated at base risk (e.g. Eeckhoudt and Hammitt, 2004; Murphy and Topel, 2006; Bellavance et al., 2009; Andersson and Treich, 2011; Aldy and Smyth, 2014). Adapted to our setting, the VSL is defined as:

**Definition 3 (VSL)** *The Value of a Statistical Life  $v_s = v_s(W, H; \lambda_m)$  is the negative of the marginal rate of substitution between the probability of death and wealth computed from the indirect utility  $V(W, H; \lambda_m)$  evaluated at base risk:*

$$v_s = -\frac{V_{\lambda_m}(W, H; \lambda_m)}{V_W(W, H; \lambda_m)} \quad (23)$$

where  $V(W, H; \lambda_m)$  is an indirect utility function.

Using Definition 3 and welfare (13), we can calculate the theoretical expression for the VSL for the parametrized model as follows.

**Proposition 3 (VSL)** *The Value of a Statistical Life solving (23) is:*

$$v_s(W, H, \lambda_m) = \frac{1}{A(\lambda_m)} N(W, H), \quad (24)$$

---

<sup>16</sup>See also Eeckhoudt and Hammitt (2004) for a WTP to fully eliminate mortality risk.

where the marginal propensity to consume  $A(\lambda_m)$  is given in (10) and net total wealth  $N(W, H)$  is given in (12).

The Statistical Life value is unconditionally positive, increasing in net worth, and decreasing in the MPC. Hence, both the WTP to avoid admissible detrimental changes (21) and the VSL (24) are unconditionally increasing in wealth and the shadow value of human capital  $BH$ . Observe that since the MPC out of wealth is typically low (e.g. see Carroll, 2001, for a review), and because  $A(\lambda_m)$  is the MPC out of both  $N(W, H)$  and  $W$ , the VSL is expected to be significantly larger than net disposable resources  $N(W, H)$ .

### 3.4.2 Relation with empirical VSL

We can rely on the WTP property (20a) to rewrite the VSL in (23) as a marginal willingness to pay:

$$v_s(W, H; \lambda_m) = \frac{\partial v(W, H; \lambda_m, \Delta)}{\partial \Delta} = \lim_{\Delta \rightarrow 0} \frac{v(W, H; \lambda_m, \Delta)}{\Delta}. \quad (25)$$

Contrasting the theoretical definition of the VSL as a MWTP in (25) with its empirical counterpart reveals the links between the two measures. Indeed, the empirical VSL commonly relied upon in the literature can be expressed as:

$$v_s^e(W, H; \lambda_m, \Delta) = \frac{v(W, H; \lambda_m, \Delta)}{\Delta}, \quad (26)$$

for small increment  $\Delta = 1/n$ , where  $n$  is the size of the population affected by the change. The theoretical measure of the VSL in (25) is the limiting value of its empirical counterpart in (26) when the change  $\Delta \rightarrow 0$  or, equivalently, when population size  $n \rightarrow \infty$ . The importance of the bias between the empirical and theoretical VSL's ( $v_s^e - v_s$ ) will consequently depend on the curvature of the willingness to pay  $v$ , as well as on the size and sign of the change  $\Delta$ , an issue to which we will return shortly.

### 3.4.3 Relation with collective WTP

We can also use our theoretical measure for the individual WTP to compute the collective willingness to pay to save a human life. Given a finite population of agents indexed  $j \in \{1, 2, \dots, n\}$  and a set of social weights  $\boldsymbol{\eta} \in \mathbb{R}_+^n$ , we can assume homogeneous

parameters across agents<sup>17</sup> and exploit the linearity of the WTP function (21) in wealth and human capital to derive the collective WTP as:

$$\sum_{j=1}^n \eta_j v_j(W_j, H_j, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right] \sum_{j=1}^n \eta_j N(W_j, H_j).$$

Imposing identical unit weights  $\eta_j = 1, \forall j$  yields:

$$\sum_{j=1}^n v_j(W_j, H_j, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right] nN(\bar{W}, \bar{H}) = nv(\bar{W}, \bar{H}, \lambda_m, \Delta).$$

Evaluating the latter at  $\Delta = n^{-1}$  yields the empirical VSL (26) measure commonly used in the literature:

$$\sum_{j=1}^n v_j(W_j, H_j, \lambda_m, \Delta) = \frac{v(\bar{W}, \bar{H}, \lambda_m, \Delta)}{\Delta} = v_s^e(\bar{W}, \bar{H}, \lambda_m, \Delta),$$

i.e. under unit weights, the empirical VSL  $v_s^e$  is the collective WTP, corresponding to  $n$  times the individual WTP evaluated at mean wealth and human capital.

## 3.5 Gunpoint Value of Life

### 3.5.1 Theoretical GPV

We next resort to the Gunpoint value as a additional life valuation measure. To do so, we adapt the Hicksian EV in Definition 2 to define the GPV as follows:

**Definition 4 (GPV)** *The Gunpoint value  $v_g$  is the WTP to avoid certain, instantaneous death and is implicitly given as the solution to:*

$$V(W - v_g, H; \lambda_m) = V^m \tag{27}$$

where  $V(W, H; \lambda_m)$  is an indirect utility, and  $V^m$  is the finite utility at certain death.

The willingness to pay  $v_g$  can be interpreted as the maximal amount paid to survive an *ex-ante* unforecastable and *ex-post* credible highwaymen threat. Unlike the HK, the Gunpoint value does not uniquely ascribe the economic worth of an agent to the

---

<sup>17</sup>Parametric homogeneity across agents is a key assumption for identification purposes in our empirical strategy (Section 4).

capitalized net labor income that agent generates. Moreover, the GPV is theoretically computable at any admissible death intensity and applicable in life-or-death situations. As such, it is well suited in end-of-life terminal care decisions where neither the HK, nor the VSL are appropriate (Philipson et al., 2010).

Combining Definition 4 with the indirect utility (13), and noting that  $V^m \equiv 0$  for preferences (6) reveals the following result for the GPV:

**Proposition 4 (GPV)** *The Gunpoint value of life solving (27) is:*

$$v_g(W, H) = N(W, H), \quad (28)$$

where  $N(W, H)$  is the net total wealth in (12).

In the absence of bequest motives, the agent who is forced to evaluate life at gunpoint would be willing to pay the hypothetical (i.e. step-1) value of pledgeable resources. The discussion of net total wealth in (8) establishes that this amount corresponds to his entire financial wealth  $W$ , plus the capitalized value of his net income along the optimal path  $Y(H^*) - I^*$ . However, the previous discussion emphasized that the minimal consumption level  $a$  is required at all periods for subsistence. Its cost therefore cannot be pledged in a highwaymen threat and must be subtracted from the Gunpoint value. Indeed, it can be shown (Hugonnier et al., 2013, Prop. 2) that net total wealth  $N(W, H)$  is equal to:

$$N(W_t, H_t) = E_t \int_t^\infty \frac{m_\tau}{m_t} (c_\tau^* - a) d\tau. \quad (29)$$

To survive, the agent is thus willing to pledge the net present value of his optimal consumption stream (net of unpledgeable minimal subsistence), at which point he becomes indifferent between living and dying. This result can be traced to recursive preferences under which the foregone utility is measured in the same units as the foregone excess consumption. Interestingly, since net total wealth is independent from the agent's other preferences  $(\rho, \varepsilon, \gamma)$  and from the death intensity  $(\lambda_m)$ , so is the GPV. Because death is certain and instantaneous when life is evaluated at gunpoint, the attitudes towards time and risk, as well as the level of exposure to death risk become irrelevant.

### 3.5.2 Relation with other life valuations

Combining (29) with Proposition 1 shows that the difference between the Gunpoint (28) and HK (17) values of life can be expressed as:

$$\begin{aligned} v_g(W_t, H_t) - v_h(H, \lambda_m) &= W_t - \frac{a}{r} + E_t \int_{T_m}^{\infty} \frac{m_{\tau}}{m_t} (Y(H_{\tau}^*) - I_{\tau}^*) d\tau \\ &= W_t - \frac{a}{r} + (1 - C_0) \frac{y}{r} + (1 - C_1) P(H_t) \end{aligned}$$

The first two terms reflect the financial wealth and (capitalized) minimal consumption that affect net total wealth and therefore optimal consumption and welfare, but have no effects on optimal investment and therefore on the optimal path for net income  $Y(H^*) - I^*$ . The third and last terms show the mortality risk adjustments  $(C_0, C_1) \in [0, 1]^2$  on the net cash flow that are present in the HK value but not in the GPV. The Gunpoint value is therefore expected to be larger than the Human Capital value, except in the cases where financial wealth  $W_t$  is low relative to minimal consumption requirements  $a/r$ .

The links between the willingness to pay in (21) and the GPV in (28) are intuitive and follow directly from the properties of the WTP. Indeed, the Gunpoint value corresponds to the admissible upper bound (22b) on the willingness to pay to avoid a change in death risk exposure:

$$v_g(W, H) = \sup_{\Delta \in \mathcal{A}} v(W, H, \lambda_m, \Delta). \quad (30)$$

This upper bound exists and is finite by admissibility, i.e. compliance with transversality restrictions. Moreover, comparing (24) and (28) establishes that:

$$v_g(W, H) = A(\lambda_m) v_s(W, H, \lambda_m). \quad (31)$$

Estimates of the marginal propensity to consume  $A(\lambda_m)$  are typically low, ranging between 2-9% for housing wealth and around 6% for financial wealth (e.g. Carroll et al., 2011, p. 58). Consequently, the predicted gap between the GPV and VSL is positive and large.

To gain further insight on the WTP-VSL-GPV links it is useful to set  $t = 1$  in the probability of death (2) and evaluate for:

$$\mathcal{P} \equiv \Pr(T_m \leq 1) = 1 - e^{-\lambda_m},$$

a monotone increasing function of  $\lambda_m$ . The willingness to pay  $v(\Delta_{\mathcal{P}}) = v(W, H; \mathcal{P}, \Delta_{\mathcal{P}})$  can then be analyzed over changes  $\Delta_{\mathcal{P}} \in [-\mathcal{P}, 1 - \mathcal{P}]$  from base risk  $\mathcal{P}$  and is plotted in Figure 2. This graph emphasizes the central role of the WTP and illustrates why the theoretical VSL is expected to be larger than its empirical counterpart, and both are expected to be much larger than the GPV.

From properties (20), the WTP (solid blue line) is an increasing, concave function of the change in death risk  $\Delta_{\mathcal{P}}$ . The theoretical VSL  $v_s$  in (25) is the marginal willingness to pay, i.e. the slope of the dashed red tangent evaluated at base death risk ( $\Delta_{\mathcal{P}} = 0$ ). It is equivalent to the linear projection corresponding to the total wealth spent to save one person (i.e. when  $\mathcal{P} + \Delta_{\mathcal{P}} = 1.0$ ) and is equal to the distance [a,f]. The empirical VSL  $v_s^e$  in (26) is computed for a small (i.e. infra-marginal) change  $\Delta_{\mathcal{P}}^e > 0$  and is the slope of the dashed-dotted green line; equivalently, it is the linear projection represented by the distance [b,e]. The empirical VSL measure  $v_s^e$  will thus understate its theoretical counterpart  $v_s$  when  $\Delta_{\mathcal{P}}^e \gg 0$  and when the WTP is concave. Moreover, equation (30) establishes that the Gunpoint value corresponds to the admissible upper bound on the WTP, i.e. the limiting WTP when death is certain as represented by the distance [c,d] in Figure 2. A concave WTP entails that a linear extrapolation under either the theoretical, or the empirical VSL will thus overstate the Gunpoint value attributed to one's own life, as confirmed from our discussion of (31).

## 4 Structural estimation

### 4.1 Overview

To estimate the willingness to pay and the three life valuations, we first follow a long tradition associating the agent's human capital to his health (e.g. see the Hicks' lecture by Becker, 2007, for a review). Second, we estimate the technological, preferences and parameters for the model outlined in Section 2 by contrasting the theoretical decisions

in Theorem 1 to their observed counterparts in PSID. Third, the estimated structural parameters can then be combined with observed wealth and health statuses to compute the closed-form expressions for the life valuations in Section 3.<sup>18</sup>

## 4.2 Econometric model

We adopt a cross-sectional perspective to estimate our human capital model and compute the associated life valuations. For identification purposes, we assume that all agents  $j = 1, 2, \dots, n$  take their wealth  $W_j$  and health  $H_j$  statuses as given and:

1. follow the optimal rules in Theorem 1 in selecting consumption  $c_j$ , portfolio  $\pi_j$ , insurance  $x_j$ , and investment  $I_j$ ;
2. share homogeneous preference, technological and distributional parameters, i.e.  $\Theta_j = \Theta \in \mathbb{R}_+^k, \forall j$ .

The nonlinear multivariate econometric model for  $\mathbf{Y}_j = [Y_j, c_j, \pi_j, x_j, I_j]'$  is written as:

$$\mathbf{Y}_j = \mathbf{B}_0(\Theta) + \mathbf{B}_W(\Theta)W_j + \mathbf{B}_H(\Theta)H_j + \mathbf{u}_j, \quad \mathbf{u}_j \sim \text{NID}(\mathbf{0}, \Sigma). \quad (32)$$

The  $5 \times 3$  matrix of reduced-form parameters (RFP)  $\mathbf{B}(\Theta)$  are linked to the preference, technological and distributional parameters in  $\Theta$  by the closed-form expressions (14) and are summarized in panel a of Table 1. Importantly, focusing on the optimal rules conveniently eliminates any endogeneity issue since the allocations in  $\mathbf{Y}_j$  are expressed in feedback-form using the (pre-determined) wealth  $W_j$  and health  $H_j$  state variables. Under the assumption that the  $\mathbf{u}_j$ 's in (32) are (potentially correlated) Gaussian error terms, we can then rely on a non-linear Maximum Likelihood (ML) estimator to estimate a subset:

$$\Theta^e = (y, \beta, \delta, \alpha, \lambda_s, \lambda_m, a, \gamma, \varepsilon)$$

---

<sup>18</sup>Online Appendix D addresses various empirical details associated with the estimation, including cross-sectional identification, panel alternatives with fixed effects, and conversion scaling from household to individual data.

of the structural parameters in  $\Theta = \{\Theta^e, \Theta^c\}$ . Following standard practices, the remaining subset of the structural parameters

$$\Theta^c = (\phi, \mu, r, \sigma_S, \rho,)$$

is calibrated either following a thorough search procedure ( $\phi$ ), or at usual values in the literature (other parameters).

### 4.3 Data

We use a sample of 7949 U.S. individuals obtained from the 2017 wave of the Panel Study of Income Dynamics, (PSID, Institute of Social Research, 2020) and weighted with corresponding individual and family weights (Chang et al., 2019). All nominal variables in per-capita values (i.e., household values divided by household size)<sup>19</sup> and scaled by  $10^{-6}$  for the estimation. The agents' independent and dependent variables are constructed as follows.

We first define financial wealth  $W_j$  as the sum of risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets. We next proxy the health variable  $H_j$  through the respondent's polytomous self-reported health statuses (Poor, Fair, Good, Very Good and Excellent) that are linearly converted to numeric values.<sup>20</sup> On the one hand, French (2005) raises the issue that agents in the PSID may understate their true health in order to justify being out of the labor force. In our case, this could lower our value for  $H_t$  and could bias upwards our estimate of  $\beta$  in the income equation. Both would result in a lower value of human wealth  $P(H_t) = BH_t$ , and consequently lower life values. On the other hand, however, self-assessed morbidity and mortality indicators have been shown to be valid predictors of actual health outcomes, such that this potential bias might not be as acute as feared.<sup>21</sup> Moreover, other approaches, such as specifying unobserved health as a latent variable,

<sup>19</sup>We discuss the relevance and empirical effects of resorting to other equivalence scale (ES) measures as alternatives to per-capita scaling in Online Appendix D.2.

<sup>20</sup>In particular, values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

<sup>21</sup>See in particular Benjamins et al. (2004); Hurd and McGarry (2002); Crossley and Kennedy (2002); Hurd et al. (2001); Hurd and McGarry (1995) for discussions and evidence on validity of self-reported statuses, sickness and longevity indicators.

and estimating its effect on self-reported status, or on other health indicators, as well as on investment have been shown to be valid alternatives in modeling health processes (e.g. Wagstaff, 2002; Ried and Ulrich, 2002). However, our conversion, while admittedly arbitrary, is much simpler to implement and fairly robust to scaling errors.<sup>22</sup>

The dependent variables in  $\mathbf{Y}_j$  are the household income, consumption, portfolios, health insurance and health expenditures. First, we use total family income to calculate  $Y_j$ . Second, a comprehensive measure of consumption is absent in the PSID data. We therefore follow an interpolation approach to infer  $c_j$  from the reported food, utility and transportation expenditures.<sup>23</sup> Third, the risky portfolio  $\pi_j$  is calculated as the share of financial wealth  $W_j$  being held in risky assets. Fourth, health insurance  $x_j$  is measured by spending on health insurance premium. Finally, health investment  $I_j$  are computed using the out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.<sup>24</sup>

Table 2 presents summary statistics for all, elders (65 and more) and adults (age 21–64). The mean age is 45 for all, 73 for elders and 40 for adults. The mean health status is 2.85, between good and very good and predictably declines with age. Mean per-capita financial wealth is low at 62 K\$, highly skewed and over three times higher for elders. A non-negligible share is invested in risky assets. Mean income is also low (23 K\$) and highly skewed, and falls for elders. Spending is mainly in non-durable consumption goods (12.35 K\$) and much lower for health insurance and expenditures (820 \$).

## 5 Estimation results

### 5.1 Structural parameters

Column 1 in Table 3 reports the estimated (standard errors in parentheses) and calibrated (with subscripts <sup>c</sup>) parameters for our benchmark model.<sup>25</sup> Overall, the latter are

---

<sup>22</sup>Indeed, note further that the optimal investment in (14) is proportional to health. Consequently, the health growth determining the optimal capital path is constant and invariant to the scaling in  $H_t$ . We also experimented with nonlinear scaling by replacing the affine with a Box-Cox transformation of  $H_t$  with no significant effects on our results.

<sup>23</sup>See Skinner (1987); Guo (2010) for interpolation details. See also Andreski et al. (2014) for comparison and validation of PSID consumption data with Consumer Expenditures estimates.

<sup>24</sup>We discuss the effects of discrepancies between investment  $I_t$  and out-of-pocket expenses  $O_t$  in Section 6.1.4 below.

<sup>25</sup>Columns 2–6 are analyzed in the Robustness section 6.

precisely estimated and are consistent with other estimates for this type of model (e.g. Hugonnier et al., 2013, 2020).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ( $\alpha = 0.7413$ ). Deterministic depreciation is important ( $\delta = 3.70\%$ ) and morbidity is non-negligible with additional depletion of  $\phi = 1.36\%$ ,<sup>26</sup> and average waiting time between occurrence of  $\lambda_s^{-1} = 10.0$  years. Both elements suggest that health shocks we are capturing are consequential, rather than benign. Second, exposure to mortality risk is also important ( $\lambda_m = 0.0342$ ), corresponding to a remaining expected lifetime of  $\lambda_m^{-1} = 29.2$  years, somewhat lower than observed in the data.<sup>27</sup> Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ( $\beta = 0.0061$ ), as well as an estimated value for base income that is close to poverty thresholds ( $y \times 10^6 = 12.7$  K\$).<sup>28</sup> The financial parameters  $(\mu, \sigma_S, r)$  are calibrated from the observed moments of the S&P500 and 30-days T-Bills historical returns.

The preference parameters in panel d indicate realistic aversion to financial risk ( $\gamma = 2.4579$ ). The estimated minimal consumption level is somewhat larger than base income ( $a \times 10^6 = 13.4$  K\$). As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2020), the elasticity of inter-temporal substitution is close to, but larger than one ( $\varepsilon = 1.0212$ ) and is consistent with a *Live Fast and Die Young* effect whereby a higher risk of death increases the marginal propensity to consume.<sup>29</sup> The null hypothesis of VNM preferences  $H_0 : \gamma = 1/\varepsilon$  is unambiguously rejected in favor of our Non Expected Utility specification.

Finally, panel e reports the composite parameter estimates of interest. The marginal propensity to consume out of wealth  $A = 5.04\%$  is well in line with other estimates (e.g. Carroll et al., 2011). The estimate for the human capital Tobin's- $q$  is  $B = 0.0709$  is consistent with a large share of human capital  $P(H) = BH$  in net total wealth  $N(W, H)$ .<sup>30</sup>

<sup>26</sup>Hugonnier et al. (2013) estimate  $\phi = 1.11\%$  using pooled PSID data from 1999 to 2007.

<sup>27</sup>The remaining life expectancy at age 45 in the US in 2017 was 36.1 years (all), 34.2 (males) and 37.9 (females) (Arias and Xu, 2019).

<sup>28</sup>For example, the 2017 poverty threshold for single-agent households was 12.5 K\$ (U.S. Census Bureau, 2020).

<sup>29</sup>Our elasticity is also close to the calibrated EIS values of  $1/\sigma = 1.25$  used by Córdoba and Ripoll (2017), as well as values of 1.17 for PSID data in Huggett and Kaplan (2016); Vissing-Jorgensen and Attanasio (2003), and of 1.5 in Bansal and Yaron (2004); Palacios (2015).

<sup>30</sup>When evaluated at the mean health and wealth level in Table 2, we estimate an average net total wealth  $N(W, H)$  in (8) of 251 K\$, 81% of which is attributable to human wealth  $P(H)$ .

## 5.2 Estimated valuations

We next compute and report the life valuations calculated at the estimated parameters in Table 3. We rely on a Bootstrap procedure with 500 iterations to evaluate the associated standard errors in order to account for both the parametric uncertainty and data distribution over  $(H, W)$ .

### 5.2.1 Human Capital Value of Life

The HK value of life  $v_h(H)$  given in (17) is reported in Table 4.a. Consistent with predictions, the human capital values are independent from  $W$  and increasing in  $H$ , ranging from 206 K\$ (Poor health) to 358 K\$ (Excellent health), with a mean value of 300 K\$. These figures are realistic and compare advantageously with other HK estimates in the literature and provide a first out-of-sample confirmation that the structural estimates are reasonable.<sup>31</sup>

### 5.2.2 Value of Statistical Life

Table 4.b reports the Statistical Life values in (24) by observed health and wealth statuses. The VSL mean value is 4.98 M\$, with valuations ranging between 1.13 M\$ and 12.92 M\$. These values are well within the ranges usually found in the empirical VSL literature.<sup>32</sup> The concordance of these estimates with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded. Importantly, our theoretically and empirically integrated approach confirms the large VSL–HK gaps identified in the empirical literature.

It is also possible to assess a measure of the marginal vs infra-marginal WTP bias by calculating the empirical VSL measure in (26). Setting  $\Delta = 1/n = 1/7949$  and  $\lambda_{m0}^* = \lambda_m + \Delta$ , we recover an aggregate VSL of 4.97 M\$, which, as expected, is lower, but close to the mean theoretical value of  $v_s(W, H, \lambda_m) = 4.98$  M\$. This result confirms that

---

<sup>31</sup>Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K\$ at age 20, peaking at less than 900 K\$ at age 45 and falling steadily towards zero afterwards.

<sup>32</sup>A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 M\$ (2000 base year, corresponding to 8.6 M\$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 M\$ and 10 M\$. Robinson and Hammitt (2016) report values ranging between 4.2 and 13.7 M\$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 M\$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 M\$ (2006\$), corresponding to 8.8 M\$ in 2016 (U.S. Environmental Protection Agency, 2017). León and Miguel (2017) find VSL amounts ranging from 577K\$ to 924K\$ calculated from a willingness to pay to face death risk in transportation to the international airport in Sierra Leone.

the theoretical and empirical values are close to one another, i.e. the individual MWTP is well approximated by the collective WTP corresponding to the empirical VSL when  $\Delta = 1/n$  is small (i.e. the sample size is large).

The VSL is increasing in both wealth and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Murphy and Topel, 2006; Andersson and Treich, 2011; Robinson and Hammitt, 2016). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks and are thus less willing to pay to attain further improvements (or prevent deteriorations). Since our benchmark model abstracts from endogenous mortality and better health increases net total wealth  $N(W, H)$ , our estimates unambiguously indicate that the former effect is dominant and that improved health raises the VSL.<sup>33</sup>

### 5.2.3 Gunpoint Value

Table 4.c reports the Gunpoint values in (28). The mean GPV is 251 K\$ and the estimates are increasing in both health and wealth and range between 57 K\$ and 651 K\$. The Gunpoint is thus of similar magnitude to the HK value of life and both are much lower than the VSL. Indeed, this finding was already foreseeable from equation (31) indicating that the VSL/GPV ratio is inversely proportional to the marginal propensity to consume. Since our estimates in Table 3, panel e reveal that  $A(\lambda_m) = 5.04\%$ , we identify a VSL that is 19.84 times larger than the GPV.

As mentioned earlier, no equivalent estimates of the Gunpoint value are to be found in the literature. In order to gain perspective, we can compare with the net worth of households, using the more comprehensive measures computed by the U.S. Census Bureau. Accounting for the value of all financial, pension, residential and durables assets net of outstanding debt reveals median (mean) values of 107 K\$ (390 K\$) in 2017 (U.S.

---

<sup>33</sup>Section 6.1.2 below allows for endogenous morbidity and mortality risks exposure. The estimation of that model confirms that the effects on life valuations are moderate and that the VSL remains increasing in health (Hugonnier et al., 2021, Tab. 2, panel b).

Census Bureau, 2021b, Tab. 1, 5) which provides additional evidence that our  $v_h, v_g$  measures are realistic.

#### 5.2.4 Willingness to pay

We emphasized that both the empirical and theoretical VSL will overstate the GPV corresponding to the upper bound on the concave willingness to pay. To help visualize this gap, Figure 3 is the estimated counterpart to Figure 2 and plots the willingness to pay  $v(W, H, \lambda_m, \Delta)$  as a function of  $\Delta$  calculated from (21) at the estimated parameters and relying on the mean wealth and health status.

The strongly concave estimated WTP in Figure 3 is informative as to why the VSL is much larger than the Human Capital and Gunpoint values. Indeed, an agent with average health and wealth statuses is willing to pay 33 K\$ to avoid an increase of  $\Delta = 0.0071$  which shortens his current horizon of 29.9 years by 5 years and would pay 246 K\$ to avoid an increase of  $\Delta = 0.2013$  which lowers expected remaining lifetime by 25 years. This last value is already close to the HK and GPV values of 300 K\$ and 251 K\$, which are both much lower than the VSL of 4.98 M\$. Equivalently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment, as foreshadowed in our discussion of (24) and (31).

## 6 Robustness

### 6.1 Theoretical model assumptions

#### 6.1.1 Other health services

The model assumes that the sole motivation for investing in  $H_t$  relates to its positive effects on marketed income in (4). However, the valuation of human capital can also be made with respect to its non-marketed utilitarian services. Indeed, the model can be adapted for non-workers by first defining  $\tilde{c}_t \equiv c_t - \beta H_t$ , and rewriting the budget

constraint (5) and aggregator (6c) as:

$$dW_t = (rW_t + y - \tilde{c}_t - I_t) dt + \pi_t \sigma_S (dZ_t + \theta dt) + x_t (dQ_{st} - \lambda_s dt), \quad (33a)$$

$$f(\tilde{c}, u, H) = \frac{\rho u}{1 - 1/\varepsilon} \left( \left( \frac{\tilde{c} - a + \beta H}{u} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right) \quad (33b)$$

The agent then selects  $\tilde{c}_t$  and the other controls where income is fixed at  $y$  in (33a), and taking into account the utilitarian benefits of human capital  $\beta H$  in (33b). As shown in Hugonnier et al. (2013, Remark 3), the theoretical results are unaffected under this alternative interpretation. This property is especially useful when applying the model to agents who, for reasons of age, illness, or choice are unable or unwilling to work, e.g. in end-of-life analysis (e.g. Philipson et al., 2010; Hugonnier et al., 2020). In the equivalent setup in (33),  $y$  refers to a fixed (e.g. pension) income flow, while  $\beta H$  captures implicit services (e.g. health marginal benefits associated with consumption and/or leisure).

### 6.1.2 Health effects and aversion for mortality and morbidity risks

A second source of valuable services of health capital concerns its capacity to lower sickness and death risks exposure for healthier agents. These effects can be captured by replacing the constant arrival rates  $\lambda_m, \lambda_s$  in (1) and (3) by health-decreasing Poisson intensities:

$$\begin{aligned} \lambda_m(H_{t-}) &= \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m}, \\ \lambda_s(H_{t-}) &= \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}}, \end{aligned}$$

where  $H_{t-} = \lim_{s \uparrow t} H_s$  is health prior to occurrence of the sickness shock. It may be further argued that the agent is not indifferent to exposure to these risks, but displays separate risk aversions towards mortality ( $\gamma_m$ ) and morbidity ( $\gamma_s$ ). This model is analyzed in further details in Hugonnier et al. (2013). Our model is a restricted case where both endogeneity and source-dependent aversion are abstracted from, i.e.  $\lambda_{k1} = 0$ , and  $\gamma_k = 0$  for  $k = m, s$ .

In a separate technical appendix (Hugonnier et al., 2021), we show how approximate closed-form solution to the agent's optimal rules can be obtained for this more general

case. Overall, our main theoretical conclusions remain valid when adjusted for endogenous death and sickness risk exposures, as well as non-indifference to the source of those risks.

Somewhat unsurprisingly, a structural estimation reveals that adding these additional services from health capital raises life valuations.<sup>34</sup> We conclude that our key findings remain qualitatively robust when accounting for positive health effects on morbidity and mortality as well as source-dependent risk aversion.

### 6.1.3 Incorporating work-leisure decisions

Our model abstracts from work-leisure decisions. Appending the latter does not modify our main framework which can be interpreted as a reduced-form version with embedded optimal work-leisure choices. To see why, consider a modification along standard practices where the agent allocates a unit time endowment between paid work and valuable leisure,  $\ell \in [0, 1]$ , and replace income (4) and preferences (6c) with:

$$Y_t = y + \beta H_t + w(1 - \ell), \quad (34a)$$

$$f(c, u) = \frac{\rho u}{1 - 1/\varepsilon} \left( \left( \frac{c - a + b \ln(\ell)}{u} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right), \quad (34b)$$

where  $w$  is a wage and  $b \in [0, w]$  denotes the strength of the preference for leisure. Online Appendix C.2 formally solves optimal leisure for this modified problem as:

$$\ell^* = \frac{b}{w} \in [0, 1].$$

---

<sup>34</sup> Using 2013-PSID data set for our benchmark (reported in Table 5, column 4) and Hugonnier et al. (2013) reported in Hugonnier et al. (2021, Tab. 2) models yields the following average life values:

Model	Benchm.	Hugonnier et al. (2013)
Year	2013	2013
$v_h$	377.66	493.63
$v_s$	5536.52	8142.57
$v_g$	282.34	460.09

Substituting back into the income (34a) and preferences (34b) yields:

$$Y_t = y^* + \beta H_t$$

$$f(c, u) = \frac{\rho u}{1 - 1/\varepsilon} \left( \left( \frac{c - a^*}{u} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right),$$

where

$$y^* \equiv y + w - b \geq y \quad \text{and} \quad a^* \equiv a - b \ln(b/w) \geq a,$$

which is iso-morphic to our original income (4), and preferences (6c). Equivalently, our specification (1)–(6) can be interpreted as a reduced form embedding optimal work-leisure choices along the lines specified by (34).

#### 6.1.4 Health investment and out-of-pocket expenses

Our empirical strategy assumes a one-to-one relationship between investment  $I_t$  and out-of-pocket medical spending in the PSID data-set. Two reasons suggest why this might not be the case. First, the individual co-payments are only a share of total medical expenses for health-insured agents. Second, this assumption entails that all of out-of-pocket expenditures have beneficial effects on  $H_t$ . However, one may argue that at least part of the uninsured health expenditures, especially with respect to dental or home care is more attributable to consumption, than to actual investment in one's health.

Accounting for such discrepancies between OOP expenditures  $O_t$  and investment  $I_t$  can be implemented by supposing that actual investment is a constant share of out-of-pocket expenses i.e.  $I_t = \psi O_t$  where  $\psi$  is larger (resp. less) than one in the co-payment (resp. consumption) case and is implicitly assumed to be one in the benchmark model. The dynamics for human capital (3) are then replaced by:

$$dH_t = [\Psi O_t^\alpha H_t^{1-\alpha} - \delta H_t] dt - \phi H_t dQ_{st}. \quad (35)$$

where  $\Psi = \psi^\alpha$  is a total factor productivity (TFP) term equal to one in the benchmark model.

Online Appendix C.3 shows that the alternate technology (35) only affects the Tobin's- $q$ ,  $\tilde{B}$  which must satisfy the following equations:

$$(r + \delta + \phi\lambda_s)^{\frac{1}{\alpha}} > \beta, \quad (36a)$$

$$\beta - (r + \delta + \phi\lambda_s)\tilde{B} - (1 - 1/\alpha)(\Psi\alpha\tilde{B})^{\frac{1}{1-\alpha}} = 0, \quad (36b)$$

$$r + \delta + \phi\lambda_s > \Psi^{\frac{1}{1-\alpha}}(\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}}. \quad (36c)$$

When evaluated at our benchmark parameter estimates and at mean health and wealth levels, we find in Online Appendix Table 2 that effects of allowing  $\Psi \neq 1$  on  $\tilde{B}$  are very limited. Consequently, so are the effects on human wealth  $\tilde{P}(H) = \tilde{B}H$ , net total wealth  $\tilde{N}(W, H) = W + (y - a)/r + \tilde{P}(H)$  and consequently on all valuations. We conclude that our results are robust to discrepancies between out-of-pocket expenses and investment.

### 6.1.5 Aging

**Age-dependent parameters** Our closed-form expressions for the willingness to pay and the three life valuations have thus far abstracted from aging processes. The latter can be incorporated although at some computational cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in  $\lambda_{mt}$ ,  $\lambda_{st}$ ,  $\phi_t$ ,  $\delta_t$ , or  $\beta_t$  results in age-dependent MPC and Tobin's- $Q$  that solve the system of ordinary differential equations:

$$\dot{A}_t = A_t^2 - (\varepsilon\rho + (1 - \varepsilon)(r - \lambda_{mt} + \theta^2/(2\gamma))) A_t, \quad (37a)$$

$$\dot{B}_t = (r + \delta_t + \phi_t\lambda_{st})B_t + (1 - 1/\alpha)(\alpha B_t)^{\frac{1}{1-\alpha}} - \beta_t, \quad (37b)$$

subject to appropriate boundary conditions. Allowing for aging and solving these differential equations for  $A_t, B_t$  implies that the solutions for  $C_{0t}, C_{1t}$ , the marginal value  $\Theta_t(\lambda_{mt})$ , as well as the human and total wealth  $P_t(H), N_t(W, H)$  are also age-dependent. All the previous results remain applicable with these time-varying expressions. Such aging processes are particularly suitable for elders who face age-increasing exposures to sickness  $\dot{\lambda}_{st} > 0$ , and death  $\dot{\lambda}_{mt} > 0$ . Appending these processes along the lines suggested by equations (37) is useful to produce realistic life cycle paths for wealth and health (e.g. see St-Amour, 2018, for a survey).

**Stratification by age** An estimation of the model with age-dependent parameters  $\Theta_t$  is beyond the scope of this paper. We can nonetheless verify the realism of the constant parameters assumption and how it affects our valuations by stratifying across the old (65 and over) and the young (less than 65) agents, re-estimating the econometric model (32) and re-calculating the HK, VSL and GPV life values across the two subgroups.

Table 3, presents the estimated deep parameters (with calibrated parameters unchanged) for old (column 2) and young (column 3) sub-samples (the calibrated parameters remain set to the values in Table 3). Somewhat unsurprisingly, being older is associated with faster depreciation in the absence of investment to maintain the health capital ( $\delta$ ), as well as increased exposure to sickness ( $\lambda_s$ ) and mortality ( $\lambda_m$ ) risks. The technological ( $\alpha$ ), income ( $y, \beta$ ) and preferences ( $a, \gamma$ ) remain generally unaffected by aging, except for the elasticity of inter-temporal substitution ( $\varepsilon$ ) which is lower and less than one, suggesting less consumption responsiveness to movements in interest rates and in death risk exposure for elders. In panel e, the shadow value  $B$  is slightly increased, whereas the MPC  $A$  is lower. We conclude that our assumption of age-invariant deep parameters is not at odds with the data. With the exception of predictable increased exposure to morbidity and mortality risks, and decreased responsiveness to interest rates, elders and young agents share similar parameters.

Table 5 gauges the effects of aging on life valuation.<sup>35</sup> In panel a, column 2, the HK value is unsurprisingly lower for elders, a direct consequence of a higher estimated death intensity  $\lambda_m$  lowering the expected duration of the net income flow parameters  $C_0, C_1$  in (18). Conversely, the VSL (panel b) is higher for elders, due to a lower MPC  $A$ , as well as a higher shadow value of health  $B$  that raises the net total wealth  $N(W, H)$ ; the latter also explains why the GPV is higher for elders.

We conclude that our key assumption of age-invariant parameters is not invalidated. The estimated preference, income and technological parameters remain generally comparable across age groups. Other distributional parameters vary with age in a predictable fashion, consistent with higher death and sickness exposure for elders. Whereas these sub-group results are reassuring for our age-invariance assumption, a full treatment of aging along the lines of the dynamic processes (37), or allowing for cohort effects would

---

<sup>35</sup>We restrict the presentation of life values by health sub-groups. The full results stratified by wealth quintiles can be obtained upon request.

be required for more definitive answers on the impact of age. We leave such analysis on the research agenda.

### 6.1.6 Human capital shocks insurance

Our model's solutions are obtained assuming that actuarially-fair insurance against human capital shocks is available. This assumption is essential to compute the net present value of the returns to investment, and consequently the net total wealth that is central to life valuations.

Our empirical implementation associates human capital  $H_t$  to health and insurance premia  $x_t$  to medical insurance coverage. Since our data set is 2017. i.e. after Affordable Care Act (ACA, aka Obamacare) became operational in 2014, the health insurance coverage assumption appears reasonable.<sup>36</sup> Whereas the model cannot be generalized to allow for imperfect insurance markets,<sup>37</sup> we can partially gauge the effects of incomplete coverage via a time variation assessment. In particular, we re-estimate our benchmark model using PSID data for two pre-ACA years, 2013 and 2009, that are associated with higher health uninsurance rate (see footnote 36).

Tables 3 reports the parameter estimates for 2013 (column 4) and 2009 (column 5). Again, our results are generally similar, with some exceptions. We estimate lower values for the Cobb-Douglas  $\alpha$ , and for sickness and death intensities  $\lambda_s, \lambda_m$ . Conversely, depreciation  $\delta$ , income  $y, \beta$ , risk aversion and EIS  $\gamma, \varepsilon$  parameter estimates increase. Whereas the shadow price  $B$  is higher, the MPC  $A$  is unaffected in panel e, indicating that a lower insurance coverage in pre-ACA years is not associated with an increase in precautionary savings. The combination of higher Tobin's- $q$  and lower death intensity results in higher HK, VSL and GPV values in Table 5, columns 4 and 5.

The presence of other confounding factors (e.g. the aftermath of the financial crisis of 2008) imply that such time variation exercises should be taken with caution. Notwithstanding this caveat, we conclude that our key results remain generally stable and/or vary predictably across time periods.

---

<sup>36</sup>The uninsured rate for 2017 was 8.7% for all, and 10.2% for individuals under 65, i.e. before Medicare coverage. In comparison, the pre-ACA uninsured rates were 14.5% (all) and 16.7% (less than 65) in 2013, and 15.1% (all) and 17.2% (less than 65) in 2009 (U.S. Census Bureau, 2021a, Tab. HIC-6 and 9 ACS).

<sup>37</sup>Allowing strictly positive exposure to capital depreciation  $\lambda^s, \phi > 0$  with incomplete coverage  $x_t < \phi P(H_t)$  is tantamount to undiversifiable risks in  $Y(H_t)$  for which optimal strategies are notoriously difficult to compute in closed-form and require numerical approaches.

## 6.2 Alternative human capital model

### 6.2.1 Theoretical framework

Our benchmark model nests other well-known life cycle models of health demand. In particular, the widely-used Grossman (1972); Ehrlich and Chuma (1990) (GEC) framework abstracts from morbidity ( $\phi, \lambda_s = 0$ ) and associated insurance ( $x = 0$ ). It also simplifies preferences by imposing VNM utility ( $\gamma = 1/\varepsilon$ ), without minimal consumption requirements ( $a = 0$ ). In this case, the agent's problem simplifies to:

$$V(W_t, H_t) = \sup_{(c, \pi, I)} U_t, \quad (38a)$$

$$U_t = 1_{\{T_m > t\}} E_t \int_t^{T_m} e^{-\rho\tau} \left( \frac{c_\tau^{1-\gamma}}{1-\gamma} \right) d\tau,$$

subject to:

$$dH_t = [I_t^\alpha H_t^{1-\alpha} - \delta H_t] dt,$$

$$dW_t = [rW_t + Y_t - c_t - I_t] dt + \pi_t \sigma_S [dZ_t + \theta dt], \quad (38b)$$

$$Y_t = y + \beta H_t.$$

Imposing the restrictions ( $\phi, \lambda_s, x, a = 0$  and  $\gamma = 1/\varepsilon$ ) on the solution to our problem yields the indirect utility and optimal rules for the restricted problem (38), as well as the corresponding HK, WTP, VSL and GPV valuations:

$$\tilde{v}_h(H) = C_0 y + \tilde{C}_1 \tilde{P}(H), \quad (39a)$$

$$\tilde{v}(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\tilde{\Theta}(\lambda_m^*)}{\tilde{\Theta}(\lambda_m)} \right] \tilde{N}(W, H), \quad (39b)$$

$$\tilde{v}_s(W, H, \lambda_m) = \frac{1}{\tilde{A}(\lambda_m)} \tilde{N}(W, H), \quad (39c)$$

$$\tilde{v}_g(W, H) = \tilde{N}(W, H). \quad (39d)$$

The expressions for  $C_0, \tilde{C}_1, \tilde{P}(H), \tilde{\Theta}(\lambda_m), \tilde{N}(W, H)$  and  $\tilde{A}(\lambda_m)$  are outlined in Corollaries 1 and 2 in Online Appendix C.4. They reveal that, *ceteris paribus*, both the relevant human, and net wealth measures  $\tilde{P}(H)$  and  $\tilde{N}(W, H)$  are increased by the absence of exogenous morbidity  $\lambda_s = 0$ , which raises the Tobin's- $Q$  to  $\tilde{B} \geq B$ . Moreover, the absence of survival consumption ( $a = 0$ ) further raises  $\tilde{N}(W, H) \geq N(W, H)$ .

Notwithstanding these quantitative differences, the restricted valuations (39) are qualitatively similar to those for the more general model. The HK value  $\tilde{v}_h$  in (39a) remains an affine function of health only. Equation (4b) in Online Appendix C.4 establishes that the marginal value  $\tilde{\Theta}(\lambda_m)$  remains a decreasing, and convex function. Consistent with Figure 2, the WTP  $\tilde{v}$  in (39b) is again an increasing, and concave function in the death risk increment  $\Delta$  and converges to the latter as exposure to death risk increases. It follows that the linear projection bias of the VSL discussed earlier is unconditionally present for the restricted model. The Value of a Statistical Life  $\tilde{v}_s$  in (39c) is also increasing in net total wealth, whereas the Gunpoint Value  $\tilde{v}_g$  in (39d) confirms that all available net worth is spent to survive a highwaymen threat. From a theoretical perspective, we conclude that our main conclusions regarding life valuations remain valid when we consider an alternative model for human capital.

### 6.2.2 Empirical evaluation

The econometric model for the restricted Grossman (1972); Ehrlich and Chuma (1990) framework can be adapted from our benchmark (32) with restrictions outlined in panel (b) of Table 1. With the exception of insurance  $x_j$  which is abstracted from, the empirical strategy for the GEC model is therefore iso-morphic to our benchmark (32). Moreover, it shares most of the theoretical predictions with respect to the values of life and therefore constitutes a natural alternative to our benchmark model. Finally, since this econometric model is a nested case of (32), the identification arguments in Section 4 also apply. We consequently proceed with its estimation, using the same ML estimator and same data set.

The estimated parameters for the restricted model (with calibrated parameters unchanged) are reported in column 6 of Table 3. Overall, the deep parameters remain similar, with some exceptions. First, we estimate a lower Cobb-Douglas parameter  $\alpha$ , as well as a higher depreciation rate  $\delta$  which tends to over-compensates the absence of morbidity risk.<sup>38</sup> We also estimate a higher mortality rate  $\lambda_m$  and risk aversion  $\gamma$ , as well as a lower EIS which is restricted to be the inverse of the risk aversion  $\varepsilon = 1/\gamma$  under VNM preferences. The composite parameters in panel e indicate a significant reduction in the MPC  $A$  and a less pronounced one for the Tobin's- $q$   $B$ .

---

<sup>38</sup>In particular, the depreciation rate for the restricted model  $\delta = 0.0495$  is 30% larger than the deterministic plus expected stochastic depreciation for the benchmark:  $\delta + \phi\lambda_s = 0.0383$ .

The valuations  $\tilde{v}_h$ ,  $\tilde{v}_s$  and  $\tilde{v}_g$  from the GEC model are reported in column 6 of Table 5. First, the higher depreciation  $\delta$ , as well as higher mortality rate result in a lower HK value (202 K\$ vs 300 K\$). Conversely, both the VSL (11.97 K\$ vs 4.98 M\$) and GPV (371 K\$ vs 251 K\$) values are higher. These results confirm our discussion in Section 6.2.1. Indeed, abstracting from sickness risks  $\lambda_s$ , and from minimal consumption  $a$  results in higher net total wealth  $\tilde{N}(W, H) > N(W, H)$  justifying a higher GPV. In addition, our estimation reveals a lower MPC for the GEC model; a larger net total wealth divided by a lower MPC, justifies why we obtain a much larger VSL in (39c) for the restricted model.

We conclude that while the theoretical valuations are qualitatively similar, abstracting from occurrence and insurance against sickness risk, as well as from minimal consumption requirements results in quantitative adjustments for the restricted model that do not overturn our main conclusions. Importantly, our discussion of the estimated parameters in Table 3 revealed that the theoretical restrictions associated with the Grossman (1972); Ehrlich and Chuma (1990) model were individually rejected, thereby validating our benchmark model over the restricted one.

## 7 Conclusion

We contribute to the life valuation literature by providing the first *joint benchmark* estimates of willingness to pay, HK, VSL and Gunpoint values in the context of a theoretically, and empirically integrated approach. First, a flexible life cycle model of human capital accumulation is solved in closed form. Second, its optimal rules, as well as associated indirect utility are combined with Asset Pricing, and Hicksian variational analysis to calculate analytical expressions for the willingness to pay to avoid changes in death risk, as well as the Human Capital, Statistical and Gunpoint values of life. Third, the estimation of the optimal rules using PSID-2017 data provides structural estimates of the four life valuation concepts. Our integrated approach thus allows for theoretically and empirically rigorous estimates of life valuations that are directly linked and comparable to one another. We confirm the large discrepancies with an average HK value of 300 K\$ and a VSL of 4.98 M\$ and show that the Gunpoint value of 251 K\$ is similar to the HK.

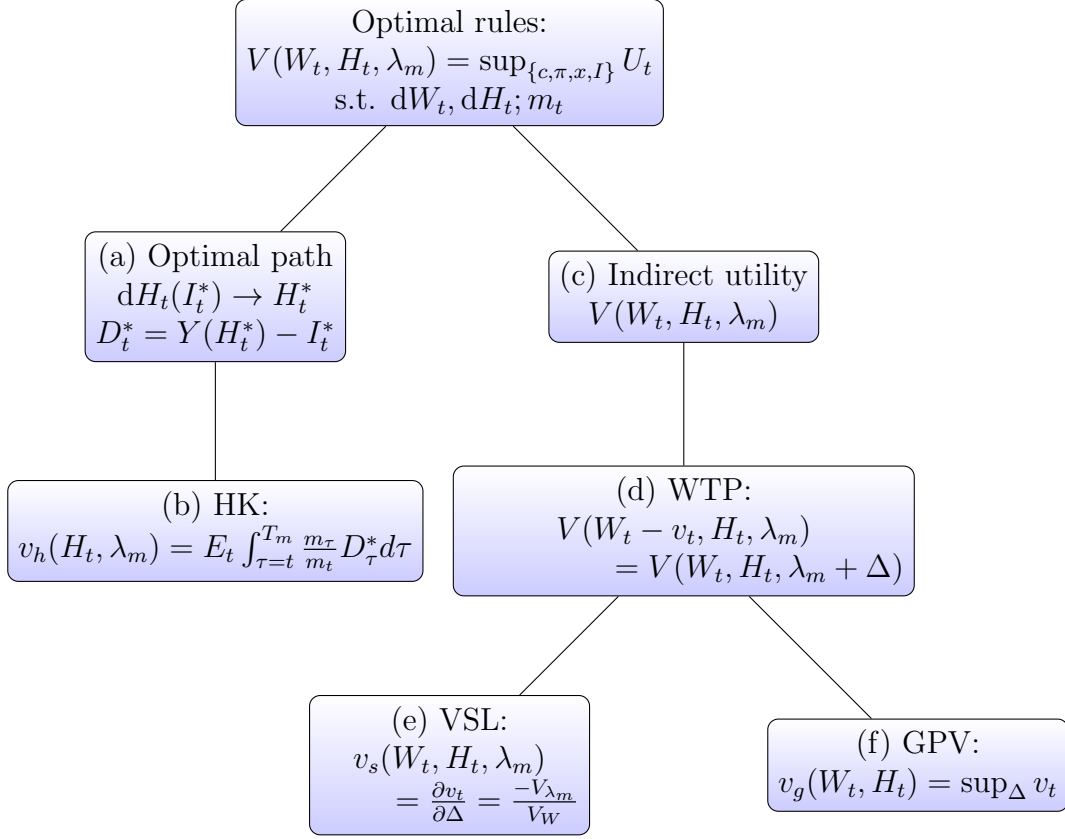
We also show that the much larger VSL is entirely attributable to the strongly concave estimated WTP.

We review and confirm the robustness of our results along many dimensions. First, our theoretical model abstracts from, yet is iso-morphic to one with utilitarian health services and endogenous labor/leisure choices. Second, it is readily adaptable to accommodate endogenous morbidity and mortality risk exposures, discrepancies between out-of-pocket medical expenses and health investment, aging or well-known human capital modeling alternatives. Third, although our setup assumes full insurance coverage against health shocks, we show that empirical results remain similar at periods where coverage actually differs (i.e. pre- and post-ACA samples). Finally, qualitatively similar predictions were derived for the popular GEC model of human capital accumulation. However, since this model is nested in ours, formal testing rejected the corresponding set of restrictions.

The Human Capital, Willingness to pay, Statistical Life and Gunpoint values of life remain specialized tools that are complementary to one another and are applicable in specific contexts. Our encompassing approach provides single-step measurement of all four in fully integrated theoretical and empirical environments. The current COVID-19 situation may highlight the relevance of our results. Whereas our framework remains silent on the high economic costs associated with increased morbidity, it provides integrated measures of those linked with higher mortality risk exposure. Indeed, the WTP identifies reactions to incentives and penalties required by sanitary measures enforcement. Allocation of scarce resources for treatment (e.g. ICU) or prevention (e.g. vaccines) may be inferred through the HK or GPV (for identified beneficiaries) or via the VSL (for unidentified ones). The macro trade-offs linked with shutdown could be gauged through the HK or VSL values. Finally, eventual litigation for COVID death caused by individual, medical or policy negligence can be assessed through the HK or GPV.

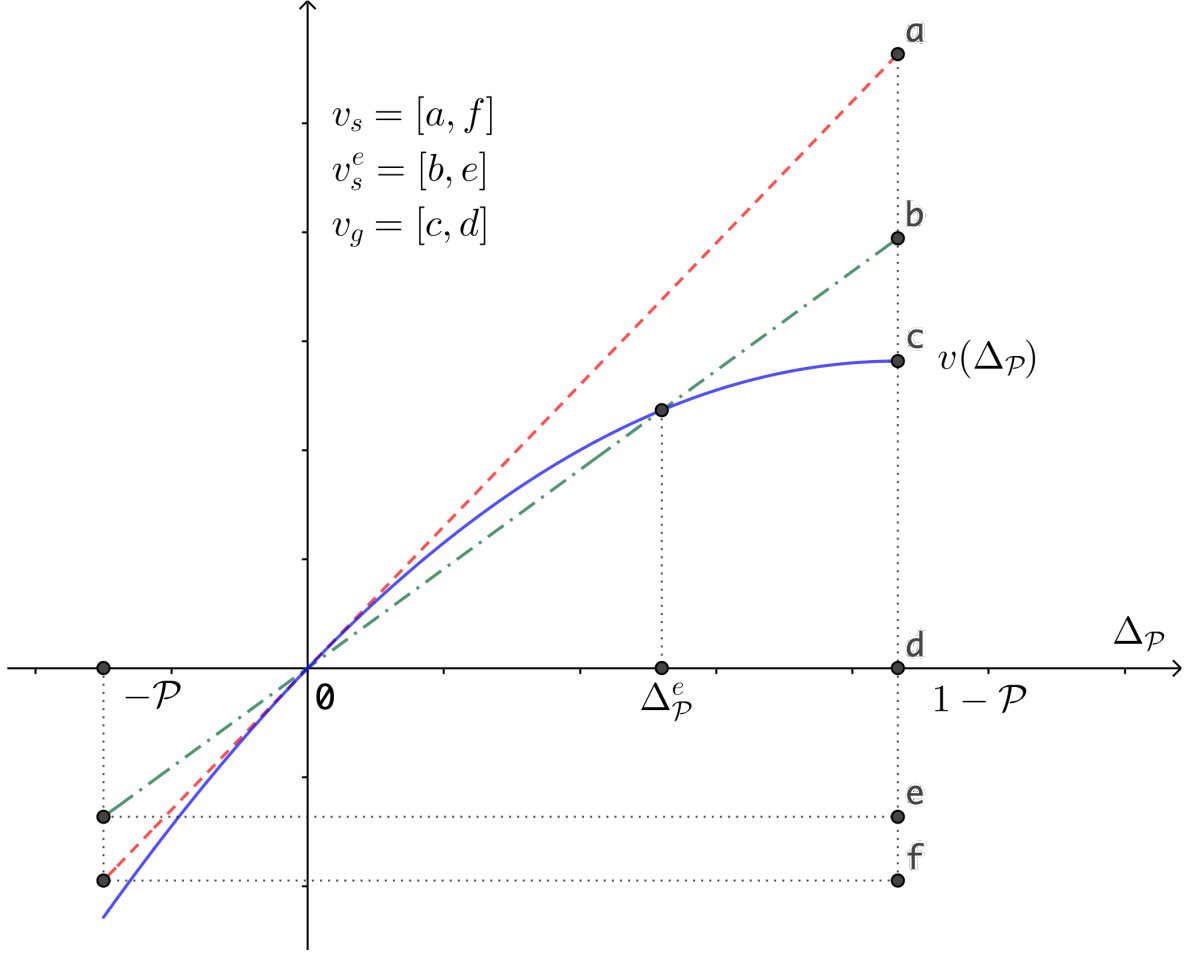
## A Figures

**Figure 1:** Overview of integrated approach to life valuation



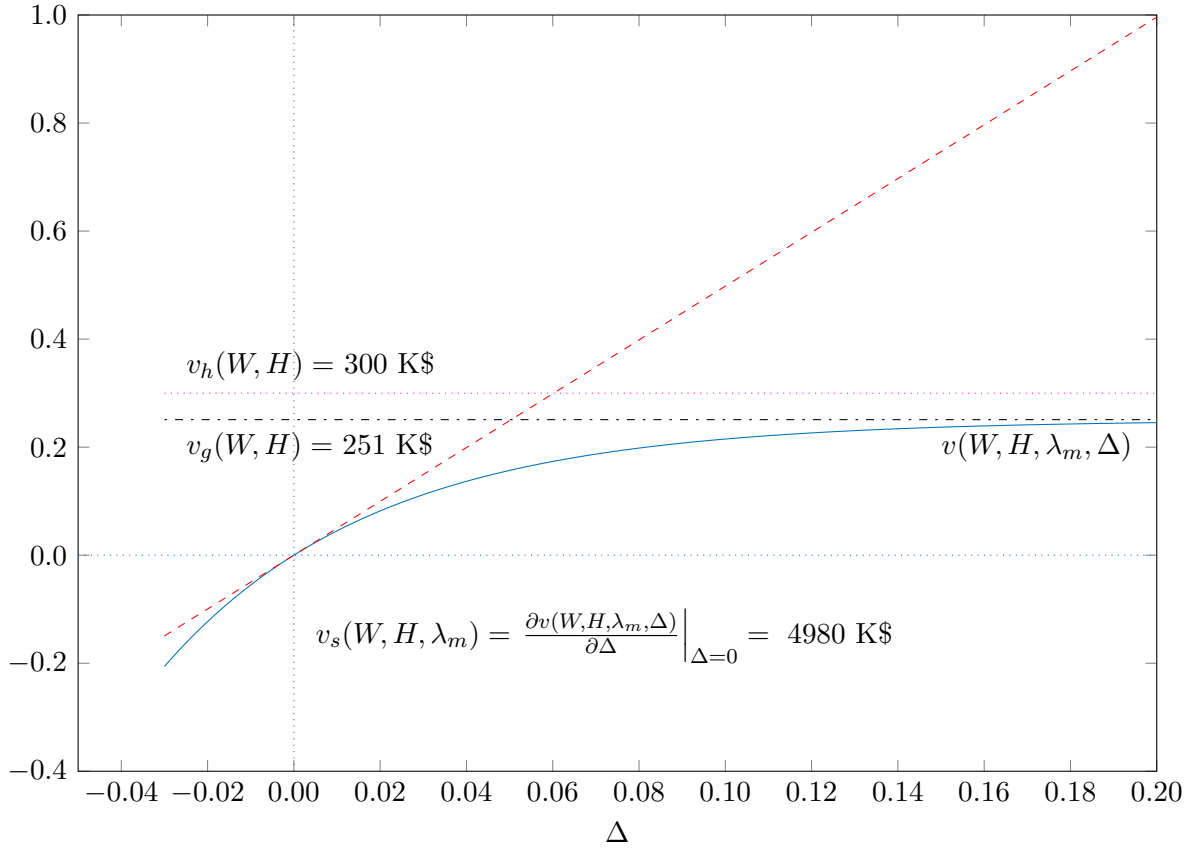
*Notes:* Integrated approach using solutions in Theorem 1 to compute Human Capital (HK)  $v_h$ , Hicksian willingness to pay (WTP)  $v$ , Value of Statistical (VSL)  $v_s$  and Gunpoint value of life (GPV)  $v_g$ .

**Figure 2:** Willingness to pay and life valuations



Notes:  $\Delta P \in [-\mathcal{P}, 1 - \mathcal{P}]$  is change in the probability of death from base exposure  $\mathcal{P} = 1 - e^{-\lambda_m}$ .  $v(\Delta P) = v(W, H; \mathcal{P}, \Delta P)$  is the willingness to pay to avoid  $\Delta P$  is solid blue line.  $v_s = v'(0)$  is the theoretical Value of Statistical Life in (25) is slope of tangent, i.e. dashed red line and equal to distance  $[a, f]$ .  $v_s^e = v(\Delta P^e)/\Delta P^e$  is the empirical Value of Statistical Life in (26) is slope of dashed-dotted green line and equal to distance  $[b, e]$ .  $v_g = \sup_{\Delta P}(v)$  is the Gunpoint value of life in (27) is equal to distance  $[c, d]$ .

**Figure 3:** Estimated WTP, HK, VSL and GPV Values of life (in M\$)



*Notes:* At estimated parameter values, for mean wealth and health levels.  $v(W, H, \lambda_m, \Delta)$  (blue solid line) is the willingness to pay to avoid an increase of  $\Delta$  in exogenous death intensity  $\lambda_m$ ;  $v_h(H, \lambda_m)$  (magenta dashed) is the Human Capital value of life;  $v_g(W, H)$  (black dashed-dotted) is the Gunpoint value of life;  $v_s(W, H, \lambda_m)$  is the Value of statistical life and the slope of the dashed red tangent evaluated at  $\Delta = 0$ .

## B Tables

**Table 1:** Parametric restrictions

(a) Benchmark model			
Eq. $i$	$\mathbf{B}_0^i(\boldsymbol{\Theta})$	$\mathbf{B}_W^i(\boldsymbol{\Theta})$	$\mathbf{B}_H^i(\boldsymbol{\Theta})$
$Y_j$	$y$	0	$\beta$
$c_j$	$a + A(\boldsymbol{\Theta}) \left( \frac{y-a}{r} \right)$	$A(\boldsymbol{\Theta})$	$A(\boldsymbol{\Theta})B(\boldsymbol{\Theta})$
$\pi_j$	$\frac{\theta}{\gamma\sigma_S} \left( \frac{y-a}{r} \right)$	$\frac{\theta}{\gamma\sigma_S}$	$\frac{\theta}{\gamma\sigma_S} B(\boldsymbol{\Theta})$
$x_j$	0	0	$\phi B(\boldsymbol{\Theta})$
$I_j$	0	0	$[\alpha B(\boldsymbol{\Theta})]^{1/(1-\alpha)}$
(b) GEC model			
$Y_j$	$y$	0	$\beta$
$c_j$	$\tilde{A}(\boldsymbol{\Theta}) \left( \frac{y}{r} \right)$	$\tilde{A}(\boldsymbol{\Theta})$	$\tilde{A}(\boldsymbol{\Theta})\tilde{B}(\boldsymbol{\Theta})$
$\pi_j$	$\frac{\theta}{\gamma\sigma_S} \left( \frac{y}{r} \right)$	$\frac{\theta}{\gamma\sigma_S}$	$\frac{\theta}{\gamma\sigma_S} \tilde{B}(\boldsymbol{\Theta})$
$x_j$	—	—	—
$I_j$	0	0	$[\alpha \tilde{B}(\boldsymbol{\Theta})]^{1/(1-\alpha)}$

*Notes:* Parametric restrictions for econometric model (32). (a) Benchmark model expressions for  $B(\boldsymbol{\Theta})$ ,  $A(\boldsymbol{\Theta})$  given in equations (9), (10). (b) GEC model (Grossman, 1972; Ehrlich and Chuma, 1990) expressions for  $\tilde{B}(\boldsymbol{\Theta})$ ,  $\tilde{A}(\boldsymbol{\Theta})$  given in Online Appendix C.4, equations (2), and (3)

**Table 2:** PSID data statistics

	Mean	Std. dev.	Min	Max
a. Age $t$				
- All	45	16	18	99
- Elders	73	7	65	99
- Adults	40	12	18	64
b. Health $H$				
- All	2.85	0.77	1	4
- Elders	2.61	0.83	1	4
- Adults	2.89	0.75	1	4
c. Wealth $W$ (K\$)				
- All	62.38	364.59	0	21250
- Elders	213.75	621.67	0	11400
- Adults	39.75	301.86	0	21250
d. Stock $\pi W$ (K\$)				
- All	36.70	312.10	0	20500
- Elders	130.58	500.08	0	10900
- Adults	22.67	270.36	0	20500
e. Income $Y$ (K\$)				
- All	23.49	34.78	0	850
- Elders	19.20	39.36	0	552
- Adults	24.13	34.00	0	850
f. Consumption $C$ (K\$)				
- All	12.35	13.23	0	337
- Elders	16.46	20.71	0	337
- Adults	11.74	11.58	0	251
g. Insurance $x$ (K\$)				
- All	0.15	0.60	0	17
- Elders	0.25	0.91	0	13
- Adults	0.14	0.54	0	17
h. Health investment $I$ (K\$)				
- All	0.67	2.20	0	88
- Elders	1.60	5.12	0	88
- Adults	0.53	1.21	0	24

*Notes:* Statistics for 2017 PSID data used in estimation. Sample size: All (7949 obs.); Elders ( $t \geq 65$ , 1034 obs.); Young ( $t \leq 64$ , 6915 obs.). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good) and 4.0 (Excellent).

**Table 3:** Estimated and calibrated structural parameters

Model Year/subset	(1) Benchm. 2017	(2) Benchm. 2017, $t \geq 65$	(3) Benchm. 2017, $t < 65$	(4) Benchm. 2013	(5) Benchm. 2009	(6) GEC 2017
a. Law of motion health (3)						
$\alpha$	0.7413 (0.0155)	0.7537 (0.0355)	0.7263 (0.0186)	0.6913 (0.0053)	0.6964 (0.0053)	0.6787 (0.0312)
$\delta$	0.0370 (0.0011)	0.0670 (0.0017)	0.0270 (0.0016)	0.0437 (0.0006)	0.0442 (0.0039)	0.0495 (0.0025)
$\phi^c$	0.0136	0.0136	0.0136	0.0136	0.0136	
b. Sickness (3) and death (1) intensities						
$\lambda_s$	0.1000 (0.0112)	0.1250 (0.0159)	0.0800 (0.0135)	0.0812 (0.0069)	0.0861 (0.0259)	
$\lambda_m$	0.0342 (0.0001)	0.1053 (0.0033)	0.0282 (0.0000)	0.0257 (0.0002)	0.0237 (0.0005)	0.0379 (0.0008)
c. Income (4) and wealth (5)						
$y$	0.0127 (0.0004)	0.0108 (0.0003)	0.0132 (0.0003)	0.0134 (0.0003)	0.0132 (0.0010)	0.0058 (0.0015)
$\beta$	0.0061 (0.0001)	0.0087 (0.0001)	0.0054 (0.0001)	0.0082 (0.0001)	0.0091 (0.0003)	0.0064 (0.0006)
$\mu^c$	0.1080	0.1080	0.1080	0.1080	0.1080	0.1080
$r^c$	0.0480	0.0480	0.0480	0.0480	0.0480	0.0480
$\sigma_S^c$	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
d. Preferences (6)						
$\gamma$	2.4579 (0.0542)	2.3758 (0.0495)	2.7579 (0.0397)	3.1400 (0.0296)	3.2008 (0.0694)	3.4312 (0.0012)
$\varepsilon$	1.0212 (0.0004)	0.8747 (0.0049)	1.1779 (0.0021)	1.0747 (0.0015)	1.2032 (0.0009)	
$a$	0.0134 (0.0007)	0.0118 (0.0003)	0.0138 (0.0004)	0.0138 (0.0004)	0.0147 (0.0007)	
$\rho^c$	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
e. MPC and Tobin's $q$ (10), (9)						
$A$	0.0504 (0.0057)	0.0389 (0.0067)	0.0525 (0.0071)	0.0510 (0.0019)	0.0524 (0.0011)	0.0310 (0.0016)
$B$	0.0709 (0.0084)	0.0748 (0.0015)	0.0717 (0.0027)	0.0884 (0.0086)	0.0982 (0.0077)	0.0659 (0.0076)

*Notes:* Estimated (standard error in parentheses) and calibrated ( $c$ ) structural parameters. Column (1): Econometric model (32), estimated by ML, subject to the parametric restrictions in panel (a) of Table 1 for 2017 data. Columns (2) and (3): Estimated by age sub-groups. Columns (4), (5): Benchmark model for 2013, 2009. Column (6): GEC model (Grossman, 1972; Ehrlich and Chuma, 1990), same econometric model subject to the parametric restrictions in panel (b) of Table 1.

**Table 4:** Estimated values of life (in K\$), benchmark model

Health level	Wealth quintile				
	1	2	3	4	5
a. HK $v_h(W, H, \lambda_m)$ in (17)					
Poor			205.82 (0.63)		
Fair			243.89 (1.09)		
Good			281.96 (1.56)		
Very Good			320.03 (2.03)		
Excellent			358.10 (2.50)		
All			299.52 (1.91)		
b. VSL $v_s(W, H, \lambda_m)$ in (24)					
Poor	1133.74 (14.83)	1136.12 (27.61)	1165.90 (41.01)	1392.54 (54.59)	10176.75 (68.23)
Fair	2189.44 (14.82)	2192.79 (27.59)	2224.82 (41.00)	2457.36 (54.60)	6177.30 (68.23)
Good	3245.13 (15.04)	3248.90 (27.54)	3282.59 (41.06)	3540.56 (54.56)	8141.83 (68.43)
Very Good	4300.82 (20.80)	4304.93 (29.29)	4337.73 (40.81)	4592.25 (54.85)	10442.69 (69.09)
Excellent	5356.52 (112.18)	5360.61 (77.37)	5395.18 (62.98)	5631.50 (83.18)	12921.37 (89.00)
All			4980.38 (49.08)		
c. GPV $v_g(W, H)$ in (28)					
Poor	57.12 (0.03)	57.24 (0.06)	58.74 (0.09)	70.16 (0.12)	512.70 (0.15)
Fair	110.30 (0.03)	110.47 (0.06)	112.09 (0.09)	123.80 (0.12)	311.21 (0.15)
Good	163.49 (0.03)	163.68 (0.06)	165.38 (0.09)	178.37 (0.12)	410.18 (0.15)
Very Good	216.67 (0.05)	216.88 (0.06)	218.53 (0.09)	231.36 (0.12)	526.10 (0.15)
Excellent	269.86 (0.25)	270.07 (0.17)	271.81 (0.14)	283.71 (0.18)	650.97 (0.20)
All			250.91 (2.39)		

*Notes:* Averages of individual values in the PSID sample, computed at estimated parameter values in Table 3, column (1). Bootstrapped standard errors in parentheses (500 replications), corrected for scaling used in estimation.

**Table 5:** Robustness: Estimated life values (in K\$)

Model Year/subset	(1) Benchm. 2017	(2) Benchm. 2017, $t \geq 65$	(3) Benchm. 2017, $t < 65$	(4) Benchm. 2013	(5) Benchm. 2009	(6) GEC 2017
a. HK $v_h(H)$ in (17), and (39a)						
Poor	205.82 (0.63)	109.76 (1.02)	225.67 (1.90)	250.21 (0.68)	261.24 (0.61)	114.52 (1.30)
Fair	243.89 (1.09)	139.25 (1.57)	264.88 (2.78)	301.91 (1.18)	319.65 (1.07)	149.96 (1.29)
Good	281.96 (1.56)	168.74 (2.21)	304.09 (3.78)	353.60 (1.69)	378.06 (1.53)	185.41 (1.42)
Very Good	320.03 (2.03)	198.22 (2.86)	343.30 (4.83)	405.30 (2.20)	436.48 (1.99)	220.86 (1.64)
Excellent	358.10 (2.50)	227.71 (3.53)	382.50 (5.91)	456.99 (2.70)	494.89 (2.45)	256.30 (1.94)
All	299.52 (1.91)	172.13 (2.37)	324.57 (4.18)	377.66 (1.85)	408.56 (1.77)	201.76 (1.59)
b. VSL $v_s(W, H, \lambda_m)$ in (24) and (39c)						
Poor	2178.13 (32.53)	4365.29 (366.53)	1165.79 (106.58)	1859.98 (18.80)	1691.93 (15.56)	7717.35 (91.03)
Fair	2720.43 (39.56)	4608.38 (304.28)	2191.78 (149.10)	3200.64 (53.29)	3093.49 (35.74)	8477.67 (94.84)
Good	4206.53 (42.94)	7992.17 (340.91)	3220.18 (156.93)	4826.12 (48.49)	4794.26 (32.76)	10771.64 (88.60)
Very Good	5802.46 (42.56)	14735.87 (240.82)	4245.13 (150.10)	6381.26 (69.79)	6459.49 (46.45)	13244.07 (94.41)
Excellent	7189.48 (40.47)	16810.38 (179.31)	5272.31 (144.59)	7880.11 (74.87)	7868.81 (56.49)	15377.03 (153.48)
All	4980.38 (49.08)	9972.32 (310.43)	4474.47 (172.04)	5536.52 (38.02)	5620.35 (42.19)	11972.21 (94.65)
c. GPV $v_g(W, H, \lambda_m)$ in (28) and (39d)						
Poor	109.73 (1.59)	169.96 (6.39)	84.56 (5.82)	94.85 (0.88)	88.61 (0.77)	239.27 (2.47)
Fair	137.05 (1.93)	179.42 (6.13)	129.85 (8.14)	163.22 (2.53)	162.02 (1.76)	262.84 (2.58)
Good	211.92 (2.09)	311.16 (7.87)	201.15 (8.56)	246.11 (2.26)	251.10 (1.61)	333.96 (2.41)
Very Good	292.33 (2.07)	573.72 (4.11)	261.78 (8.18)	325.42 (3.25)	338.31 (2.29)	262.34 (2.56)
Excellent	362.20 (1.97)	654.49 (3.05)	340.75 (7.89)	401.86 (3.48)	412.13 (2.78)	476.74 (4.17)
All	250.91 (2.39)	388.26 (6.13)	234.75 (9.37)	282.34 (1.75)	294.36 (2.08)	371.18 (2.57)

*Notes:* Computed at corresponding estimated parameter values in Table 3, columns (1–6). Bootstrapped standard errors in parentheses (500 replications).

## References

- Acemoglu, Daron, and David Autor (2018) ‘Lectures in labor economics.’ Lecture notes, MIT
- Adler, Matthew D., James K. Hammitt, and Nicolas Treich (2014) ‘The social value of mortality risk reduction: VSL versus the social welfare function approach.’ *Journal of Health Economics* 35, 82 – 93
- Aldy, Joseph E., and Seamus J. Smyth (2014) ‘Heterogeneity in the value of life.’ NBER Working Paper 20206, National Bureau of Economic Research, 1050 Massachusetts Avenue Cambridge, MA 02138, June
- Andersson, Henrik, and Nicolas Treich (2011) ‘The value of a statistical life.’ In *A Handbook of Transport Economics*, ed. Andre de Palma, Robin Lindsey, Emile Quinet, and Roger Vickerman (Edward Elgar Publishing Inc.) chapter 17, pp. 396–424
- Andreski, Patricia, Geng Li, Mehmet Zahid Samancioglu, and Robert Schoeni (2014) ‘Estimates of annual consumption expenditures and its major components in the PSID in comparison to the CE.’ *American Economic Review* 104(5), 132 – 135
- Arias, Elizabeth, and Jiaquan Xu (2019) ‘United States life tables, 2017.’ *National Vital Statistics Report* 68(7), 1–65
- Bansal, Ravi, and Amir Yaron (2004) ‘Risks for the long run: A potential resolution of asset pricing puzzles.’ *The Journal of Finance* 59(4), 1481–1509
- Becker, Gary S. (2007) ‘Health as human capital: Synthesis and extensions.’ *Oxford Economic Papers* 59, 379–410. Hicks Lecture delivered at Oxford University
- Becker, Gary S., Tomas J. Philipson, and Rodrigo R. Soares (2005) ‘The quantity and quality of life and the evolution of world inequality.’ *American Economic Review* 95(1), 277–291
- Bellavance, Francois, Georges Dionne, and Martin Lebeau (2009) ‘The value of a statistical life: A meta-analysis with a mixed effects regression model.’ *Journal of Health Economics* 28(2), 444–464

- Ben-Porath, Y. (1967) ‘The production of human capital and the life cycle of earnings.’ *Journal of Political Economy* 75(4), 352 – 365
- Benjamins, Maureen Reindl, Robert A Hummer, Isaac W Eberstein, and Charles B Nam (2004) ‘Self-reported health and adult mortality risk: An analysis of cause-specific mortality.’ *Social Science & Medicine* 59(6), 1297 – 1306
- Bodie, Zvi, Robert C Merton, and William F Samuelson (1992) ‘Labor supply flexibility and portfolio choice in a life cycle model.’ *Journal of Economic Dynamics and Control* 16(3-4), 427–449
- Bommier, Antoine, Danier Harenberg, and François Le Grand (2019) ‘Recursive preferences, the value of life, and household finance.’ Working paper, Available at SSRN <https://ssrn.com/abstract=2867570>, May
- Carroll, Christopher D. (2001) ‘A theory of the consumption function, with and without liquidity constraints.’ *Journal of Economic Perspectives* 15(3), 23 – 45
- Carroll, Christopher D., Misuzu Otsuka, and Jiri Slacalek (2011) ‘How large are housing and financial wealth effects? a new approach.’ *Journal of Money, Credit, and Banking* 43(1), 55 – 79
- Chang, Wen, Raphael Nishimura, Steven G. Heeringa, Katherine McGonagle, and David Johnson (2019) ‘Construction and evaluation of the 2017 longitudinal individual and family weights.’ Technical Report, University of Michigan, Ann Arbor, February
- Conley, Bryan C. (1976) ‘The value of human life in the demand for safety.’ *American Economic Review* 66(1), 45 – 55
- Cook, Philip J., and Daniel A. Graham (1977) ‘The demand for insurance and protection: The case of irreplaceable commodities.’ *Quarterly Journal of Economics* 91(1), 143 – 156
- Córdoba, Juan Carlos, and Marla Ripoll (2017) ‘Risk aversion and the value of life.’ *Review of Economic Studies* 84(4), 1472–1509
- Crossley, Thomas F., and Steven Kennedy (2002) ‘The reliability of self-assessed health status.’ *Journal of Health Economics* 21(3), 643–658

- Doucouliaagos, Hristos, T.D. Stanley, and W. Kip Viscusi (2014) ‘Publication selection and the income elasticity of the value of a statistical life.’ *Journal of Health Economics* 33(0), 67 – 75
- Duffie, Darrell, and Larry G. Epstein (1992) ‘Asset pricing with stochastic differential utility.’ *Review of Financial Studies* 5(3), 411–436
- Eden, Philip (1972) ‘U.S. human capital loss in Southeast Asia.’ *The Journal of Human Resources* 7(3), 384–394
- Eeckhoudt, Louis R., and James K. Hammitt (2004) ‘Does risk aversion increase the value of mortality risk?.’ *Journal of Environmental Economics and Management* 47(1), 13 – 29
- Ehrlich, Isaac, and Hiroyuki Chuma (1990) ‘A model of the demand for longevity and the value of life extension.’ *Journal of Political Economy* 98(4), 761–782
- Epstein, Larry G., and Stanley E. Zin (1989) ‘Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework.’ *Econometrica* 57(4), 937–969
- (1991) ‘Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis.’ *Journal of Political Economy* 99(2), 263–268
- French, Eric (2005) ‘The effects of health, wealth, and wages on labour supply and retirement behaviour.’ *Review of Economic Studies* 72(2), 395–427
- Grossman, Michael (1972) ‘On the concept of health capital and the demand for health.’ *Journal of Political Economy* 80(2), 223–255
- Gruber, Jonathan (2013) ‘A tax-based estimate of the elasticity of intertemporal substitution.’ *Quarterly Journal of Finance* 3(1), 1 – 20
- Guo, Sheng (2010) ‘The superior measure of PSID consumption: An update.’ *Economics Letters* 108(3), 253 – 256
- Hall, Robert E., and Charles I. Jones (2007) ‘The value of life and the rise in health spending.’ *Quarterly Journal of Economics* 122(1), 39–72

- Heckman, James J. (1976) ‘A life-cycle model of earnings, learning, and consumption.’ *Journal of Political Economy* 84(4), S11 – 44
- Hicks, John (1946) *Value and Capital: An Inquiry Into Some Fundamental Principles of Economic Theory*, second ed. (London: Clarendon Press)
- Hofflander, Alfred E. (1966) ‘The human life value: An historical perspective.’ *The Journal of Risk and Insurance* 33(3), 381–391
- Huggett, Mark, and Greg Kaplan (2013) ‘The money value of a man.’ Working Paper 238, Griswold Center for Economic Policy Studies, June
- (2016) ‘How large is the stock component of human capital?’ *Review of Economic Dynamics* 22, 21 – 51
- Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour (2013) ‘Health and (other) asset holdings.’ *The Review of Economic Studies* 80(2), 663–710
- (2020) ‘Closing down the shop: Optimal health and wealth dynamics near the end of life.’ *Health Economics* 29(222), 138–153
- (2021) ‘Technical appendix valuing life as an asset, as a statistic and at gunpoint: Endogenous mortality and morbidity, source-dependent risk aversion.’ <https://people.unil.ch/pascalst-amour/files/2021/02/TechApxHPS13-1.pdf>
- Hurd, Michael D., and Kathleen McGarry (1995) ‘Evaluation of the subjective probabilities of survival in the Health and Retirement Study.’ *Journal of Human Resources* 30(0), s268–s292. Supplement
- (2002) ‘The predictive validity of subjective probabilities of survival.’ *Economic Journal* 112(482), 966–985
- Hurd, Michael D., Daniel McFadden, and Angela Merrill (2001) ‘Predictors of mortality among the elderly.’ In *Themes in the economics of aging*, ed. David A. Wise NBER Conference Report series (Chicago and London: University of Chicago Press) pp. 171–97
- Institute of Social Research (2020) ‘Panel Study of Income Dynamics (PSID).’ <http://psidonline.isr.umich.edu/>. University of Michigan

- Jones, Charles I. (2016) ‘Life and growth.’ *Journal of Political Economy* 124(2), 539 – 578
- Jones, Charles I., and Peter J. Klenow (2016) ‘Beyond gdp? welfare across countries and time.’ *American Economic Review* 106(9), 2426 – 2457
- Jones-Lee, Michael (1974) ‘The value of changes in the probability of death or injury.’ *Journal of Political Economy* 82(4), 835–849
- Kiker, B. F. (1966) ‘The historical roots of the concept of human capital.’ *Journal of Political Economy* 74(5), 481–499
- Landefeld, J. Steven, and Eugene P. Seskin (1982) ‘The economic value of life: Linking theory to practice.’ *American Journal of Public Health* 72, 555–566
- León, Gianmarco, and Edward Miguel (2017) ‘Risky transportation choices and the value of a statistical life.’ *American Economic Journal: Applied Economics* 9(1), 202 – 228
- Lewbel, Arthur (2003) ‘Calculating compensation in cases of wrongful death.’ *Journal of Econometrics* 113(1), 115 – 128
- Mishan, Ezra J. (1971) ‘Evaluation of life and limb: A theoretical approach.’ *Journal of Political Economy* 79(4), 687–705
- Murphy, Kevin M., and Robert H. Topel (2006) ‘The value of health and longevity.’ *Journal of Political Economy* 114(5), 871–904
- Palacios, Miguel (2015) ‘Human capital as an asset class implications from a general equilibrium model.’ *The Review of Financial Studies* 28(4), 978–1023
- Petty, Sir William (1691) *Political arithmetick, or a discourse concerning the extent and values of lands, people, buildings, etc* (London: Robert Caluel)
- Philipson, Tomas J., Gary Becker, Dana Goldman, and Kevin M. Murphy (2010) ‘Terminal care and the value of life near its end.’ Working paper 15649, National Bureau of Economic Research
- Posner, Eric A., and Cass R. Sunstein (2005) ‘Dollars and death.’ *The University of Chicago Law Review* 72(2), 537–598

- Pratt, John W., and Richard J. Zeckhauser (1996) ‘Willingness to pay and the distribution of risk and wealth.’ *Journal of Political Economy* 104(4), 747 – 763
- Ried, Manfred Erbslandvand Walter, and Volker Ulrich (2002) ‘Health, health care, and the environment: Econometric evidence from german micro data.’ In *Econometric Analysis of Health Data*, ed. Andrew Jones and Owen O’Donnell (John Wiley & Sons) chapter 2, pp. 25–36
- Robinson, Lisa A., and James K. Hammitt (2016) ‘Valuing reductions in fatal illness risks: Implications of recent research.’ *Health Economics* 25(8), 1039 – 1052
- Rosen, Sherwin (1988) ‘The value of changes in life expectancy.’ *Journal of Risk and Uncertainty* 1(3), 285–304
- Round, Jeff (2012) ‘Is a qaly still a qaly at the end of life?.’ *Journal of Health Economics* 31(3), 521 – 527
- Shepard, Donald S., and Richard J. Zeckhauser (1984) ‘Survival versus consumption.’ *Management Science* 30(4), 423–439
- Skinner, Jonathan (1987) ‘A superior measure of consumption from the Panel Study of Income Dynamics.’ *Economics Letters* 23(2), 213 – 216
- St-Amour, Pascal (2018) ‘The lifetime dynamics of health and wealth.’ In ‘Oxford Research Encyclopedia of Economics and Finance’ (Oxford University Press)
- Symmons, J. Meyler (1938) ‘The value of life.’ *Economic Journal* 48(192), 744–748
- U.S. Census Bureau (2020) ‘Poverty thresholds.’ <http://www.census.gov/data/tables/time-series/demo/income-poverty/historical-poverty-thresholds.html>
- (2021a) ‘Health insurance historical tables - hhi series.’ <https://www.census.gov/data/tables/time-series/demo/health-insurance/historical-series/hic.html>
- (2021b) ‘Wealth, asset ownership, and debt of households detailed tables: 2017.’ <https://www.census.gov/data/tables/2017/demo/wealth/wealth-asset-ownership.html>

- U.S. Department of Transportation (2016) ‘Revised departmental guidance on valuation of a statistical life in economic analysis.’ <https://www.transportation.gov/office-policy/transportation-policy/revised-departmental-guidance-on-valuation-of-a-statistical-life-in-economic-analysis>
- U.S. Environmental Protection Agency (2017) ‘What value of statistical life does EPA use?’ <https://www.epa.gov/environmental-economics/mortality-risk-valuation>
- Viscusi, W. Kip (2000) ‘Misuses and proper uses of hedonic values of life in legal contexts.’ *Journal of Forensic Economics* 13(2), 111–125
- (2007) ‘The flawed hedonic damages measure of compensation for wrongful death and personal injury.’ *Journal of Forensic Economics* 20(2), 113–135
- Vissing-Jorgensen, Annette, and Orazio P. Attanasio (2003) ‘Stock-market participation, intertemporal substitution, and risk-aversion.’ *American Economic Review* 93(2), 383–391
- Wagstaff, Adam (2002) ‘The demand for health: an empirical reformulation of the grossman model.’ In *Econometric Analysis of Health Data*, ed. Andrew Jones and Owen O’Donnell (John Wiley & Sons) chapter 1, pp. 15–24

Online Appendix to  
“Valuing Life as an Asset, as a Statistic, and at  
Gunpoint”<sup>1</sup>

Julien Hugonnier<sup>1,4,5</sup>, Florian Pelgrin<sup>2,6</sup> and Pascal St-Amour<sup>3,4,6</sup>

<sup>1</sup>École Polytechnique Fédérale de Lausanne

<sup>2</sup>EDHEC Business School

<sup>3</sup>HEC Lausanne, University of Lausanne

<sup>4</sup>Swiss Finance Institute

<sup>5</sup>CEPR

<sup>6</sup>CIRANO

February 23, 2021

# A Contributions to the life valuation literature

This section reviews our paper’s contributions to the Human Capital, Statistical Life and Gunpoint life values, as well as to the theoretical approaches in life valuation. For additional perspective, we provide a summary of the main features of the HK, VSL and GPV models in Table 1.

**Table 1:** HK, VSL and GPV life valuations

	Human Capital (HK)	Value Statistical Life (VSL)	Gunpoint (GPV)
Theory	Asset pricing	Marginal rate of substitution Marginal willingness to pay	Hicksian variation Willingness to pay
Method	$v_{ht}^j = E_t \left\{ \sum_{s=0}^{T_m} m_{t,t+s} D(H_{t+s}^j) \right\}$ $D(H_t^j) = Y(H_t^j) - I_t^j$	$\{v^j(\Delta)\}_{j=1}^n$ for $\Delta = 1/n$ $v_s^e(\Delta) = \sum_{j=1}^n v^j(\Delta) \approx \frac{v(\Delta)}{\Delta}$	$\mathcal{P} = \text{Pr}(\text{Death})$ $V(W - v_g, H, \mathcal{P}) = V(W, H, 1)$
Valued life	Identified	Unidentified (statistical)	Identified
Proxies	- Labor income	- Responses to fines - Wage-fatality nexus, ...	<b>This paper:</b> - Consumption, portfolios, - Health spending, insurance
Applications	Fatality risk pricing/litigation - Occupational - End users	Public safety, health - Transportation - Pollution control	End of life - Terminal care Insur. irrepl. losses Hedonic damages
Values	300K\$–900K\$ (Huggett and Kaplan, 2016) <b>This paper:</b> 399 K\$	4.2M\$–13.7M\$ (Robinson and Hammitt, 2016) <b>This paper:</b> 7.57 M\$	<b>This paper:</b> 393 K\$
Issues	- Non-workers $Y^j$ - Rate of discounting $m_{t,t+s}$ - Endogeneity div./surv. $D^j, T_m$	- Exogeneity death risk $\Delta$ - Agency, non-payers $v^j$ - Linearity/aggreg. prefs. - Stat. vs identified life	- computation $v_g$ - finite values $v_g$ - linearity/concavity $v$

## A.1 Human Capital values of life

As illustrated in the first column of Table 1, the HK model draws from asset pricing theory to compute the economic value of an identified person  $j$  by pricing his expected discounted lifetime net cash-flow stream.<sup>1</sup> That dividend  $D(H^j)$  is the agent’s income  $Y(H^j)$ , net of investment expenses  $I(H^j)$  to maintain his human capital  $H^j$ . Well-known issues include accounting for the distribution of stochastic dividends, defining the appropriate discount factor  $m_{t,t+s}$  which is compatible with the investment opportunity

<sup>1</sup>See Kiker (1966) for historical perspective on HK valuation.

set, as well as the endogeneity of the agent’s income and investment. Moreover, the endogeneity of the duration  $T^m$  of the dividends flow is an issue, with pricing non-income activities (e.g. leisure among the elderly) also associated with HK challenges.

Huggett and Kaplan (2013, 2016) abstract from capital investment  $I^j$  entirely and calculate a HK value by discounting an exogenous income stream  $D(H^j) = Y(H^j)$  using an agent-specific stochastic discount factor  $m_{t,t+s}$  induced by the agent’s optimal consumption and portfolio decisions, i.e. the agent’s IMRS evaluated at the optimal plan. Using estimated distributional parameters for income, and calibrated preferences parameters, they find that the HK value is hump-shaped in the life cycle, peaking at mid-life, and much lower than that implied by (naive) discounting at the risk-free rate  $m_{t,t+s} = (1 + r)^{-s}$ .<sup>2</sup> They attribute the differences to correlation between the agent’s SDF and the income processes, i.e.  $\text{Cov}(m_{t,t+s}, Y_{t+s}) < 0$ , and to corner solutions at the risk-free rate for younger households’ portfolio decisions, that both induce heavier discounting of the dividends flow.

As for Huggett and Kaplan (2013, 2016), we compute the capital value of an income stream. Furthermore, we also rely on recursive preferences to compute optimal consumption and portfolio decisions. However, we focus on the endogenous net dividends stream, where neither income nor investment expenses are exogenously set, but where both are solved in closed form. Moreover, we follow Asset Pricing theory by valuing the human capital dividends stream using the *market-based* and not agent-specific stochastic discount factor, and where the SDF is stemming from the investment opportunity set that is considered in the model in order to guarantee full theoretical consistency. Consequently, the subtraction of investment in our case lowers the capitalized value of the dividends flow whereas the market SDF being orthogonal to the agent’s idiosyncratic net income flow will increase the HK value. Furthermore, Huggett and Kaplan (2013, 2016) also rely on PSID data, but do so to estimate the income forcing process parameters only; preferences parameters are calibrated ex-post to compute the HK value and are not confronted with other variables. In contrast, our empirical approach is much more structurally-oriented; we use PSID data on consumption, portfolio, income, health investment and

---

<sup>2</sup>For high-school workers with low risk aversion ( $\gamma = 4$ ), Huggett and Kaplan (2016, Fig. 4, p. 34) find a HK value of 300K\$ at age 20, 1.1M\$ at age 40 and 500K\$ at age 60. Those values increase to respectively 700K\$, 1.8M\$, and 800K\$ for college graduates. In comparison, the constant risk-free discounting continuously falls from a peak of 2.2M\$ at age 20 to 700K\$ at age 60 for high-school graduates, and from 40M\$ to 1.2M\$ for college graduates.

insurance, combined with health and wealth statuses data to structurally estimate the exogenous forcing processes, human capital technology, distributional and preferences parameters. Finally, our parametrized model is fully adaptable to non-labor valuation since the flow of marketed income related to human capital can also be equivalently recast as non-marketed utilitarian services (see Section 6.1.1 in the paper). We also show how the model can be adapted for explicit modeling of endogenous work/leisure decisions (see Technical Appendix C.2). Both elements are important to calculate HK-inspired valuations for non-working agents.

## A.2 Value of a Statistical Life

**Empirical VSL** The vast VSL literature was initiated by Drèze (1962) and Schelling (1968). In column 2 of Table 1, the Value of a Statistical Life measures a societal marginal rate of substitution between additional life and wealth, also corresponding to its marginal willingness to pay for additional longevity. As a canonical example (e.g. Aldy and Viscusi, 2007), suppose agents  $j = 1, 2, \dots, n$  are individually willing to pay  $v^j = v(\Delta)$  to attain (avert) a small beneficial (detrimental) change  $\Delta = 1/n$  in death risk exposure and satisfying  $v(0) = 0$ . The empirical VSL is the collective willingness to pay  $v_s^e(\Delta) = nv(\Delta) = v(\Delta)/\Delta$ , corresponding to the slope of the WTP function and approximating the MWTP  $v'(\Delta) = \lim_{\Delta \rightarrow 0} v(\Delta)/\Delta$ .

The empirical VSL alternative relies on explicit and implicit evaluations of the Hicksian WTP  $v^j$  for a small reduction  $\Delta$  in fatality risk which is then linearly extrapolated to obtain the value of life. Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.<sup>3</sup> Examples of the latter include responses to prices and fines in the use of life-saving measures such as smoke detectors, speed limitations, or seat belt regulations. The Hedonic Wage (HW) variant of the implicit VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/worker characteristics, the wage elasticity with

---

<sup>3</sup>A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the implicit VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta-meta* analysis of the stated- and revealed-preferences valuations of life.

respect to job fatality risk can be estimated and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008; Shogren and Stamland, 2002).

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. He argues that the assumed exogeneity of the change in fatality risk  $\Delta$  can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. He also argues that agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Ashenfelter further contends that it is unclear whose preferences are involved in the risk/income tradeoff and how well these are understood. For example, if high fatality risk employment attracts workers with low risk aversion and/or high discount rates, then generalizing the wages risk gradient to the entire population could understate the true value of life. An argument related to Ashenfelter's preferences indeterminacy can be made for the HW variant of the VSL. Because wages are an equilibrium outcome, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death.

Our approach addresses many of the issues raised by Ashenfelter (2006). First, we fully allow for endogenous adjustments in the optimal allocations resulting from changes in death risk exposure when we compute the willingness to pay and the VSL. Second, agency issues are absent as the agent bears the entire costs and benefits of changes in mortality. Third, whose preferences are at stake is not an issue as the latter are jointly estimated with the WTP and life valuations by resorting to a widely-used panel of households (PSID). Consequently, these values can safely be considered as representative of the general population. Fourth, labor demand considerations are absent as our partial equilibrium approach takes the return on investment as mortality-risk independent in characterizing the agent's optimal human capital allocations. More fundamentally, we neither rely on the wage/fatality nexus, nor on any other proxy and we make no assumption on the shape of the WTP function but rather derive its properties from the indirect utility function induced by the optimal allocation.

Our results also confirm early conjectures on the pitfalls associated with personalizing unidentified VSL life valuations. Indeed, Pratt and Zeckhauser (1996) argue that concentrating the costs and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e. large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment, thereby reducing the marginal WTP as mortality exposure increases.<sup>4</sup> Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our Gunpoint threat. Their conjecture is warranted in our calculations. We show that the willingness to pay is finite and bounded above by the Gunpoint Value. Diminishing MWTP entails that the latter is much lower than what can be inferred from the VSL.

**Theoretical models of VSL** Hall and Jones (2007) propose a semi-structural measure of life value akin to the Value of a Statistical Life. They adopt a marginal value perspective by equating the VSL to the marginal cost of saving a human life. In their setting, the cost of reducing mortality risk can be imputed by estimating a health production function and by linking health status to death risks. Dividing this marginal cost by the change in death risk yields a VSL-inspired life value. Unlike Hall and Jones (2007) we do not measure the health production function through its effects on mortality, but estimate the technology through the measurable effects of investment on future health status. Indeed, mortality is treated exogenously in our baseline model. Moreover, our fully structural approach does not indirectly evaluate the marginal value of life via its marginal cost, but rather directly through the individual willingness to pay to avoid changes in death risks.

Finally, we share similarities with Murphy and Topel (2006) who resort to a life cycle model with direct utilitarian services of health to study life valuations. In particular, both continuous-time approaches study permanent changes in Poisson death intensity, under

---

<sup>4</sup>Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay \$1 million to reduce the associated mortality risk by 10%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.

perfect markets assumption, and both identify the VSL as a marginal rate of substitution between longevity and wealth. Moreover, both emphasize the key role of the elasticity of inter-temporal substitution in generating diminishing marginal values. However, contrary to Murphy and Topel (2006), our human capital (i.e. health) is endogenously determined in a stochastic environment, whereas we abstract from leisure (see however online Appendix C.2 for an extension). The associated VSL, as well as other life measures, are all increasing in health, rather than health-independent. Importantly, whereas Murphy and Topel (2006) posit an arbitrary process for consumption (see eq. (19), p. 885) and restrict their analysis to hand-to-mouth in their calibration, we solve for optimal consumption, portfolio, insurance, and health expenditures. This allows us to analyze and structurally estimate all life valuations – including the HK, WTP and GPV that are abstracted from in Murphy and Topel (2006) – through the prism of the indirect utility function.

### A.3 Gunpoint value of life

In column 3 of Table 1, a Gunpoint value measures the maximal amount  $v^g$  an agent is willing to pay to remain at current death probability  $\mathcal{P} \in (0, 1)$ , rather than face instantaneous and certain death, i.e  $\mathcal{P} \equiv 1$ . Early references to a Gunpoint value include Jones-Lee (1974) who analyzes the Hicksian Compensating Variation (CV) for changes in the probability of dying in a static setting. The extreme case where the latter tends to one corresponds to a willingness to accept compensation for imminent death. Jones-Lee (1974) shows that this WTA exists and is finite when the least upper bound on the utility at death (e.g. from bequeathed wealth) is large relative to reference expected utility. Our analysis abstracts from bequests and normalizes utility at death to zero, so that the Hicksian Equivalent Variation (EV), i.e. the WTP to avoid death is the appropriate Gunpoint measure and we show formally that it corresponds to the least upper bound on the WTP.

Other early references include Cook and Graham (1977) who study the demand for insurance against irreplaceable losses, defined as one where personal valuation considerations dominate market ones, i.e as having no readily identifiable market-provided replacement in the case of loss (e.g. a family pet, health, a spouse's, or a child's life). The willingness to pay to avoid this loss is defined as the *Ransom value*. If the ransom is a normal good (i.e. is increasing in wealth), Cook and Graham (1977) show that the

state-dependent marginal utility of wealth, conditional on loss, is less than that of wealth minus ransom, conditional on no loss. The agent consequently optimally under-insures at actuarially fair contracts. Under sufficiently large wealth effects on ransom, the agent does not insure against the loss of the irreplaceable good, but against the associated wealth loss. For example, he then selects a life insurance against a spouse's death corresponding to foregone income (plus eventual burial expenses) that has clear analogs to the HK value. Finally, they show that the MRS between wealth and death (corresponding to the VSL) is necessarily larger than the Ransom value.

Eeckhoudt and Hammitt (2004) rely on this framework to focus on the impact of risk aversion on four measures of life value: the VSL, the WTP to fully eliminate death risk (i.e.  $\mathcal{P} > 0 \rightarrow \mathcal{P}^* = 0$ ), or to partially lower it (i.e.  $\mathcal{P} > 0 \rightarrow \mathcal{P}^* < \mathcal{P}_0$ ) and the WTP to eliminate the certainty of death (i.e.  $\mathcal{P} = 1 \rightarrow \mathcal{P}^* = 0$ ). The latter corresponds to Cook and Graham (1977)'s Ransom value where the irreplaceable good is one's own life. In the special case where both the utility and marginal utility of wealth at death are zero (e.g. in the absence of bequest value), they confirm that the Ransom value is the agent's wealth and is independent of attitudes toward risk.

The Ransom value of Cook and Graham (1977); Eeckhoudt and Hammitt (2004) is clearly related to the Gunpoint value as both depend on Hicksian WTP to avoid certain death in gauging a person's own value. The main difference is that we do not rely on a generic utility, but instead we base our analysis on the indirect utility associated to a dynamic human capital problem to characterize the WTP, VSL and GPV. This approach allows us to encompass the HK value as well, to link the different measures and to fully identify the role of preferences, distributional and technological parameters on life valuation.

Implicit references to a GPV are also found in the context of end-of-life care. For example, Philipson, Becker, Goldman and Murphy (2010) contend that "[the VSL] is often prefaced with claiming that it is not how much people are willing to pay *to avoid having a gun put to their head* (presumably one's wealth). However, terminal care decisions are often exactly of that nature" (Philipson et al., 2010, p. 2, emphasis added). We confirm their conjecture that financial wealth is entirely pledged in a highwaymen threat, however we show that so is the agent's human wealth. Since our application associates the latter to health, we thus provide explicit adjustment for an agent's health

status in his life valuations in the spirit of the Quality-Adjusted Life Years (QALY, e.g. Round, 2012). One could also argue that HK measures are inappropriate in terminal care situations where agents are unable to work. The GPV we propose handles such case by equivalently associating the value of health capital to the utilitarian services it can provide (see Section 6.1.1 in the paper).

Murphy and Topel (2006) also implicitly refer to a Gunpoint value in their parametrized analysis of the value of a life year (i.e. utility and net savings at given age). Indeed, commenting on a key variable in their Value of Statistical Life Year (VSLY) analysis, they write that “[t]he ratio  $z_0/z$  asks how much of current composite consumption individuals would sacrifice before they would rather be dead” (p. 885). However, a closer analysis reveals that this ratio, which they calibrate between 5-20% of composite consumption, rather corresponds to a minimal consumption ratio in their non-homothetic VNM preferences. Whereas we show that the Gunpoint value i.e. the total wealth that leaves the agent indifferent between life and death corresponds to the expected discounted value of the *lifetime* consumption stream, and is therefore much larger than minimal consumption.

Finally, in addition to tangible costs, such as the HK values of lost net earnings, or the deceased’s medical and funeral expenses, wrongful death litigation courts can also award compensation for intangible losses. The latter include survivors’ pain and suffering from loss of the deceased’s companionship (e.g. Peebles and Harris, 2015; Lewbel, 2003), as well as compensation for Hedonic Damages representing the value of the deceased’s ‘lost life pleasures’ (see Posner and Sunstein, 2005; Karns, 1990; Smith, 1988, for discussions of legal aspects). Viscusi (2007, 2000); Raymond (1999) provide critical assessments of the erroneous association of Hedonic Damages with the VSL measures. Indeed, the latter better gauges a societal willingness to pay to save someone rather than a person’s own valuation of his life. The HK life value is also inadequate as an utilitarian flow valuation in that it computes the market value of an agent’s net income stream. In contrast, our GPV assesses the WTP that leaves the agent indifferent between life and death and is thus a direct measure of the monetary equivalent of life’s continuation utility. We innovate by computing an integrated Gunpoint value that has so far proved elusive, and that accords with and complement the VSL and HK measures.

## A.4 Theoretical models of life value

**Integrated models** Other researchers have offered encompassing approaches to life valuations. Jones-Lee (1974) proposes a static VNM framework, albeit without human capital considerations, and which focuses on the utility of wealth when alive and at death to analyze the WTP's properties. Marginal WTP for small changes in death risk yield the VSL whereas a Gunpoint-equivalent life value is studied through the willingness to accept compensation for certain death. Conley (1976); Shepard and Zeckhauser (1984); Rosen (1988) analyze Human Capital and Statistical Life values in a life cycle model with perfect and imperfect capital markets. These studies emphasize the role of the EIS and conclude that the VSL is much larger than the HK under reasonable assumptions. Our main contribution to these analyses are that we calculate and structurally estimate closed-form solutions to a much richer parametrized encompassing framework. In particular, we provide WTP, HK, VSL and GPV solutions under non-expected utility settings with endogenous stochastic human capital accumulation. These formulas are estimated under the full set of theoretical restrictions with a common data base.

**Role of preferences** Córdoba and Ripoll (2017) concur with us on the relevance of recursive preferences for life valuation. In particular, they emphasize the importance of disentangling attitudes towards risk, from those towards time. This separation allows for non-indifference with respect to the timing of the resolution of survival uncertainty, and guarantees preference for life over death, even at high risk aversion levels. They also contend that more realistic curvature of the willingness to pay for survival can only be attained by allowing non-linear effects of death probabilities on utility that are abstracted from under VNM preferences. Both their discussion and their calibration emphasize a preference for late, rather than early, resolution of death uncertainty, as well as a diminishing marginal willingness to pay for additional longevity (Córdoba and Ripoll, 2017, Sec. 2.2, and Tab. 1).

Despite these similarities, the parametrized model of Córdoba and Ripoll (2017) is however different from ours. Their closest analog in their Section 3.2 is set in discrete (rather than continuous) time, and lets the agent select consumption only. It fully abstracts from our analysis of endogenous human capital accumulation, stochastic capital depreciation, risky portfolio and insurance choices and their main solutions for life values

are characterized for hand-to-mouth consumers only. Moreover they emphasize mortality risk aversion as key determinant of life values in an homothetic recursive preferences specification. In our setting, the agent is risk-neutral with respect to mortality risk, so that the elasticity of inter-temporal substitution is the main driver of mortality preferences, and we allow for non-homotheticity in a recursive utility setting by introducing minimal consumption requirements. Finally, whereas they obtain closed-form solutions for the VSL, they do not explicitly compute the WTP,<sup>5</sup> and fully abstract from both HK and GPV.

Bommier et al. (2019) also analyze the implications for life valuation of life cycle models of consumption and portfolio choices with recursive preferences. However, important differences remain between the two approaches. Indeed, Bommier et al. (2019) neither allow for human capital and insurance decisions, nor do they analytically solve their model, and therefore do not formally characterize how structural parameters and state variables affect a broad set of life valuations. More specifically, whereas we rely on the explicit solutions for the optimal human capital dynamics and the indirect utility to analyze the HK, WTP, VSL and GPV, Bommier et al. (2019) use the (unsolved) marginal utility of consumption and of death risk to discuss the implications for the VSL only. Moreover, Bommier et al. (2019) calibrate their model to fit the empirical VSL estimates, and *ex-post* assess the resulting life cycle paths of consumption, financial market participation and portfolio. Conversely, we structurally estimate the model by relying on a wide set of cross-equations theoretical restrictions in a multivariate econometric setting to fit the observed financial and human capital decisions, and *then* proceed to gauge the empirical implications for the life valuations.

We borrow from Hugonnier et al. (2013) for specifying and solving our human capital model. We consider a restricted case of that setup along two dimensions. First, while retaining their non-expected utility setting, we simplify our specification of preferences by abstracting from source-dependent risk aversion. Second, while maintaining stochastic sickness and death shocks, we abstract from self-insurance against these risks. Whereas our model is admittedly less general, one benefit is that our optimal rules are characterized in closed form, rather than as approximate solutions. Furthermore, our solutions are much

---

<sup>5</sup>More precisely, the WTP in their setup is simply the VSL times the change in death probability (see the equation before eq. (16)). Instead, we compute the WTP from Hicksian variational analysis and rely on its marginal and limiting properties to characterize the VSL and Gunpoint values.

more tractable, allowing us to pinpoint more clearly how distributional, preferences and technological parameters affect the values of life. More fundamentally, the focus of the two papers is much different. Indeed, the main emphasis of Hugonnier et al. (2013) is on the separation between financial and health-related choices, rather than on the value of life. Whereas they do consider the value of an additional year of longevity (Hugonnier et al., 2013, Tab. 6), they completely abstract from the HK, VSL, WTP, and Gunpoint values for which we provide and estimate analytical solutions. Finally, in a separate technical appendix (Hugonnier et al., 2021) we show that the main theoretical conclusions for that generalized model are maintained and that the empirical life values are of the same order of magnitude.

## B Proofs

### B.1 Theorem 1

The benchmark human capital model of Section 2 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death and depreciation intensities are constant at  $\lambda_m, \lambda_s$  (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e.  $\gamma_s = \gamma_m = 0$ ). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1, Theorem 1) yields the optimal solution in (14). ■

### B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital  $P(H)$  from

$$\begin{aligned} P(H) &= E_t \int_t^\infty \frac{m_\tau}{m_t} [\beta H_\tau^* - I_\tau^*] d\tau, \\ &= BH. \end{aligned}$$

Straightforward calculations adapt this result to a stochastic horizon  $T^m$  and include the fixed income component  $y$  in income (4). ■

### B.3 Proposition 2

Combining the Hicksian EV (19) with the indirect utility (13a) and using the linearity of the net total wealth in (12) reveals that the WTP  $v$  solves:

$$\begin{aligned} \Theta(\lambda_m^*) N(W, H) &= \Theta(\lambda_m) N(W - v, H) \\ &= \Theta(\lambda_m) [N(W, H) - v] \end{aligned}$$

where we have set  $\lambda_m^* = \lambda_m + \Delta$ . The WTP  $v = v(W, H, \lambda_m, \Delta)$  is solved directly as in (21).

Next, by the properties of the marginal value of net total wealth,  $\Theta(\lambda_m^*)$  in (15) is monotone decreasing and convex in  $\Delta$ . It follows directly from (20) that the WTP

$$v(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)} \right] N(W, H)$$

is monotone increasing and concave in  $\Delta$ .

The lower bound follows directly from evaluating finite and admissible  $A(\lambda_m^*), \Theta(\lambda_m^*)$  at  $\lambda_m^* = 0$  in (21). To compute the upper bound, two cases must be considered:

1. For  $0 < \varepsilon < 1$ , the MPC in (10) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\lambda_m^* = \lambda_m + \Delta < \bar{\lambda}_m = \left( \frac{\varepsilon}{1 - \varepsilon} \right) \rho + \left( r + \frac{\theta^2}{2\gamma} \right).$$

Admissibility  $\mathcal{A}$  therefore requires  $\Delta < \bar{\Delta} = \bar{\lambda}_m - \lambda_m$  for the transversality conditions (10) to be verified. The supremum of the WTP is then  $v(W, H, \lambda_m, \bar{\Delta}) = N(W, H)$ .

2. For  $\varepsilon > 1$ , the MPC is monotone increasing and transversality is always verified. Consequently, the WTP is well-defined over the domain  $\Delta \geq -\lambda_m$ . It follows that:

$$\begin{aligned} \lim_{\Delta \rightarrow \infty} \Theta(\lambda_m + \Delta) &= 0 \\ \lim_{\Delta \rightarrow \infty} v(W, H, \lambda_m, \Delta) &= N(W, H) \end{aligned}$$

i.e. the willingness to pay asymptotically converges to net total wealth as stated in (22b).

■

## B.4 Proposition 3

By the VSL definition (23) and the properties of the Poisson death process (11):

$$v_s = \frac{-V_{\lambda_m}(W, H, \lambda_m)}{V_W(W, H, \lambda_m)}$$

From the properties of the welfare function (13a), we have that  $V_{\lambda_m} = \Theta'(\lambda_m)N(W, H)$ , whereas  $V_W = \Theta(\lambda_m)$ . Substituting for  $\Theta$  in (13b) yields the VSL in (24). ■

## B.5 Proposition 4

Combining the Hicksian EV (27) with the indirect utility (13a) and the net total wealth in (12) reveals that the WTP  $v$  solves:

$$\begin{aligned} V^m \equiv 0 &= \Theta(\lambda_m)N(W - v_g, H) \\ &= \Theta(\lambda_m) [N(W, H) - v_g] \end{aligned}$$

Solving for  $v_g$  reveals that it is as stated in (28). Because net total wealth is independent of the preference parameters  $(\varepsilon, \gamma, \rho)$ , so is the Gunpoint Value. ■

## C Other theoretical results

### C.1 Hicksian Compensating Variation

For those instances where appropriate, we can also rely on a similar reasoning to define the Hicksian Compensating Variation as follows:

$$V(W - v^c, H; \lambda_m^*) = V(W, H; \lambda_m)$$

which can be solved as

$$\begin{aligned} v^c(W, H, \lambda_m, \Delta) &= \left[ 1 - \frac{\Theta(\lambda_m)}{\Theta(\lambda_m^*)} \right] N(W, H), \\ &= \frac{-\Theta(\lambda_m)}{\Theta(\lambda_m^*)} v(W, H, \lambda_m, \Delta). \end{aligned}$$

Since  $\Theta'(\lambda_m) < 0$ , it follows that  $0 < v^c < -v$  for  $\Delta < 0$  and  $0 < v < -v^c$  for admissible  $\Delta > 0$ , i.e. the WTP to attain a beneficial or avert a detrimental change in death risk is always less than the corresponding WTA to forego a favorable or accept an unfavorable change in mortality, consistent with standard Hicksian variational analysis (e.g. Smith and Keeney, 2005; Hammitt, 2008).

### C.2 Labor-leisure choices

Denote by  $V(W, H)$  the value function and by  $V_i$  its derivatives with respect to  $i = H, W$ . The Hamilton-Jacobi-Bellman (2.2.1) corresponding to the Human Capital model of Section 2.1 can be modified to allow for optimal work-leisure choices in (34) as follows:

$$\begin{aligned} 0 = & \max_{\{c, \pi, x, I, \ell\}} \frac{(\pi \sigma_S)^2}{2} V_{WW} + H [(I/H)^2 - \delta] V_H \\ & + [rW + \pi \sigma_S \theta - c + y + \beta H + w(1 - \ell) - I - x \lambda_s] V_W \\ & + \frac{\rho V(W, H)}{1 - \frac{1}{\varepsilon}} \left[ \left( \frac{c - a + b \ln(\ell)}{V(W, H)} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right] \\ & - \frac{\gamma (\pi \sigma_S V_W)^2}{2V(W, H)} - \lambda_m V(W, H) - \lambda_s V(W, H) \left[ 1 - \frac{V(W + x, H(1 - \phi))}{V(W, H)} \right]. \end{aligned} \tag{1}$$

Under general separation principles (e.g. Basak, 1999), we can solve for optimal leisure  $\ell^*$  in a first step, substitute back into the HJB, and solve for the other optimal controls

in a second step. In particular, the first-step FOC's for consumption and leisure are respectively given as:

$$V_W = \rho V(W, H)^{\frac{1}{\varepsilon}} (c - a + b \ln(\ell))^{\frac{-1}{\varepsilon}}$$

$$V_W w = \rho V(W, H)^{\frac{1}{\varepsilon}} (c - a + b \ln(\ell))^{\frac{-1}{\varepsilon}} \frac{b}{\ell}$$

dividing one by the other solves for optimal leisure as a constant share of the time endowment given by:

$$\ell^* = \frac{b}{w} \in [0, 1]$$

under the restriction that  $0 \leq b \leq w$ . For the second step, substituting back  $\ell^*$  into the HJB (1) reveals that the latter then becomes:

$$0 = \max_{\{c, \pi, x, I\}} \frac{(\pi \sigma_S)^2}{2} V_{WW} + H [(I/H)^2 - \delta] V_H$$

$$+ [rW + \pi \sigma_S \theta - c + y^* + \beta H - I - x \lambda_s] V_W$$

$$+ \frac{\rho V(W, H)}{1 - \frac{1}{\varepsilon}} \left[ \left( \frac{c - a^*}{V(W, H)} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right]$$

$$- \frac{\gamma (\pi \sigma_S V_W)^2}{2V(W, H)} - \lambda_m V(W, H) - \lambda_s V(W, H) \left[ 1 - \frac{V(W + x, H(1 - \phi))}{V(W, H)} \right],$$

where

$$y^* \equiv y + (w - b) \geq y, \quad a^* \equiv a - b \ln(b/w) \geq a,$$

which is iso-morphic to the HJB (2.2.1) for the original problem. Consequently, the solutions in Theorem 1 remain valid, with  $(a, y)$  replaced by  $(a^*, y^*)$ . ■

### C.3 Health investment and out-of-pocket expenses

We consider the case of discrepancies between OOP,  $O_t$  and investment,  $I_t = \psi O_t$ , where  $\psi < 1$  captures non-investment (e.g. consumption) components in health expenses,  $\psi > 1$  captures co-payment rates for insured agents, and our benchmark model imposes  $\psi = 1$ .

For the modified human capital dynamics:

$$dH_t = [\Psi O_t^\alpha H_t^{1-\alpha} - \delta H_t] dt - \phi H_t dQ_{st}, \quad \Psi = \psi^\alpha$$

the Hamilton-Jacobian-Bellman (HJB) corresponding to our problem is:

$$r\tilde{P} = \beta H + \lambda_s [\tilde{P}_H(1 - \phi)H - \tilde{P}(H)] + \max_{\{O\}} \left\{ \left[ \Psi \left( \frac{O}{H} \right)^\alpha - \delta \right] H\tilde{P}_H - O \right\}.$$

Solving the FOC, and using candidate solution for the human wealth  $\tilde{P}(H) = \tilde{B}H$  reveals that out-of-pocket expenditures  $O$  are proportional to health:

$$O = \left( \Psi \alpha \tilde{B} \right)^{\frac{1}{1-\alpha}} H$$

Substituting back into the HJB shows that the shadow price  $\tilde{B}$  must satisfy:

$$0 = g(\tilde{B}) = \beta - (r + \delta + \phi \lambda_s) \tilde{B} - (1 - 1/\alpha)(\Psi \alpha \tilde{B})^{\frac{1}{1-\alpha}}$$

subject to  $g'(\tilde{B}) < 0$  and relevant transversality condition (see Hugonnier et al., 2013, for details). As for the benchmark model, the expressions for net total wealth are obtained with the modified human wealth:

$$\tilde{N}(W, H) = W + \frac{y - a}{r} + \tilde{P}(H)$$

with the expression for the valuations remaining valid and using  $\tilde{N}, \tilde{P}$ . ■

When evaluated at our benchmark parameter estimates and at the mean health and wealth levels, we find minimal effects on the Tobin's- $q$ , human wealth and net total wealth in Table 2. This allows us to conclude that the effects on the main determinants of life valuations are limited at best.

## C.4 Life valuations for the GEC model

Our framework nests the Grossman (1972); Ehrlich and Chuma (1990) model by imposing  $\lambda_s = \phi = x = 0$  as well as  $a = 0$  and  $\gamma = 1/\varepsilon$ . Adapting our results reveals the following result.

**Table 2:** Health investment and OOP

$\Psi$	$\tilde{B}$	$\tilde{P}(H)$	$\tilde{N}(W, H)$
0.5000	0.0708	201.8595	250.4472
0.7500	0.0709	202.2337	250.8215
1.0000	0.0709	202.3212	250.9089
1.2500	0.0710	202.4957	251.0834
1.5000	0.0711	202.7688	251.3566

*Notes:* At estimated parameters for benchmark model in Table 3, column 1, at mean health and wealth levels. Benchmark model assumes  $\Psi = 1$ .

**Corollary 1** *Assume that the parameters of the model are such that*

$$\beta < (r + \delta)^{\frac{1}{\alpha}}, \quad (2a)$$

*and denote the Tobin's-q of human capital by  $\tilde{B} > 0$ , the unique solution to:*

$$\beta - (r + \delta)\tilde{B} - (1 - 1/\alpha)(\alpha\tilde{B})^{\frac{1}{1-\alpha}} = 0, \quad (2b)$$

*subject to*

$$(\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} < r + \delta. \quad (2c)$$

*Assume further that the marginal propensity to consume out of net total wealth,  $\tilde{A} > 0$  satisfies:*

$$\tilde{A}(\lambda_m) = \frac{\rho}{\gamma} + \left( \frac{\gamma - 1}{\gamma} \right) \left( r - \lambda_m + 0.5 \frac{\theta^2}{\gamma} \right), \quad (3a)$$

$$> \max \left( 0, r - \lambda_m + \frac{\theta^2}{\gamma} \right). \quad (3b)$$

*Then,*

1. the human wealth and net total wealth are given as:

$$\begin{aligned}\tilde{P}(H_t) &= \tilde{B}H_t \geq 0, \\ \tilde{B}(W_t, H_t) &= W_t + \frac{y}{r} + \tilde{B}(H_t) \geq 0,\end{aligned}$$

2. the indirect utility for the agent's problem is:

$$V_t = V(W_t, H_t, \lambda_m) = \tilde{\Theta}(\lambda_m)N(W_t, H_t) \geq 0, \quad (4a)$$

$$\tilde{\Theta}(\lambda_m) = \tilde{\rho}\tilde{A}(\lambda_m)^{\frac{\gamma}{\gamma-1}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{1}{1-\gamma}} \quad (4b)$$

and generates the optimal rules:

$$\begin{aligned}\tilde{c}_t &= \tilde{A}(\lambda_m)\tilde{N}(W_t, H_t) \geq 0, \\ \tilde{\pi}_t(\theta/(\gamma\sigma_S))\tilde{N}(W_t, H_t), \\ \tilde{I}_t &= \left(\alpha^{\frac{1}{1-\alpha}}\tilde{B}^{\frac{\alpha}{1-\alpha}}\right)\tilde{P}(H_t) \geq 0,\end{aligned}$$

where any dependence on death intensity  $\lambda_m$  is explicitly stated.

**Corollary 2 (Life valuation for restricted model)** *The HK, WTP, VSL, and GPV corresponding to the Grossman (1972); Ehrlich and Chuma (1990) model in (38) are given by:*

$$\begin{aligned}\tilde{v}_h(H) &= C_0y + \tilde{C}_1\tilde{P}(H), \\ \tilde{v}(W, H, \lambda_m, \Delta) &= \left[1 - \frac{\tilde{\Theta}(\lambda_m^*)}{\tilde{\Theta}(\lambda_m)}\right]\tilde{N}(W, H), \\ \tilde{v}_s(W, H, \lambda_m) &= \frac{1}{\tilde{A}(\lambda_m)}\tilde{N}(W, H), \\ \tilde{v}_g(W, H) &= \tilde{N}(W, H),\end{aligned}$$

with the constants:

$$\begin{aligned}C_0 &= \frac{1}{r + \lambda_m}, \\ \tilde{C}_1 &= \frac{r - (\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} + \delta}{r + \lambda_m - (\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} + \delta},\end{aligned}$$

and where the modified expressions for marginal value  $\tilde{\Theta}(\lambda_m)$ , and human  $\tilde{P}(H)$ , and net total wealth  $\tilde{N}(W, H)$  are given in Corollary 1.

The proofs of Corollaries 1 and 2 follow directly from imposing the restrictions  $(\phi, \lambda_s, x, a = 0$  and  $\gamma = 1/\varepsilon)$  in the closed-form solutions for our benchmark model and are therefore omitted.

## D Empirical details

### D.1 Identification strategy

#### D.1.1 Cross-sectional identification

The econometric model (32) reveals that there exists a subset of cross-equation restrictions that prevent using a (single) reduced-form estimation of  $\mathbf{B}(\Theta)$ , followed by a just-identified contrast estimator (e.g. minimum distance estimator) of  $\Theta$ . In addition, the structural parameters are further constrained by the nonlinear inequalities in our model (Discussed in Online Appendix D.1.3 below).

A large subset of the deep parameters  $\Theta^e \subset \Theta$  are thus theoretically identified from the cross-equation restrictions governing  $\mathbf{B}(\Theta)$  in Table 1, combined with the nonlinear implicit equality constraint defining the Tobin's- $q$  in (9b) and the nonlinear inequality constraints (9a), (9c) as well as (10b). Towards that purpose, we first follow standard practices in the Asset Pricing and Life Cycle literature by calibrating the returns parameters  $(\mu, r, \sigma_S, \theta)$  and discount rate  $(\rho)$  at usual values. Finally, we also calibrate the capital shock parameter  $\phi$  following a thorough search procedure, such that the remaining estimable parameters are:

$$\Theta^e = (y, \beta, \delta, \alpha, \lambda_s, \lambda_m, a, \gamma, \varepsilon)$$

With these elements in mind, the theoretical restrictions (9) and (10) imply that the composite parameters are linked to  $\Theta^e$  as follows:

$$\begin{aligned} B(\Theta^e) &= B(\beta, \delta, \alpha, \lambda_s) \\ A(\Theta^e) &= A(\varepsilon, \gamma, \lambda_m). \end{aligned} \tag{6}$$

Next, the ten non-zero reduced-form parameters  $\mathbf{B}(\Theta)$  in (32), combined with composite restrictions (6) show that the parameters in  $\Theta^e$  are theoretically identifiable from

the RFP's as follows:

Estim. struct. param. $\Theta^e$	Identif. from RFP $\mathbf{B}$
$y, a$	$B_0^Y, B_0^c, B_0^\pi$
$\beta, \delta, \alpha, \lambda_s$	$B_H^Y, B_H^c, B_H^\pi, B_H^x, B_H^I$
$\gamma$	$B_0^c, B_W^c, B_H^c, B_0^\pi, B_W^\pi, B_H^\pi$
$\varepsilon, \lambda_m$	$B_0^c, B_W^c, B_H^c$

(7)

Indeed, contrasting the number of  $\mathbf{B}$  terms and  $\Theta^e$  in (7) shows that the rank condition is satisfied and there might exist (at least) one solution to the nonlinear estimation method. As a heuristic argument (i.e., without cross-equation restrictions and nonlinear constraints), using the mapping between the structural parameters and the  $\mathbf{B}$  terms in Table 1, it follows that the income equation  $Y_j$  identifies  $y$  and  $\beta$ , the consumption equation  $c_j$  identifies  $\varepsilon$ ,  $\lambda_m$  and  $\gamma$ , the nonlinear parameter functions of the health variable in the insurance  $x_j$ , portfolio  $\pi_j$ , and investment  $I_j$  equations identify  $\delta$ ,  $\lambda_s$ ,  $\alpha$ , and the constant term of the portfolio equation identifies  $a$ . Nevertheless, as to be expected, it does not guarantee the global identification of  $\Theta^e$ . To circumvent this issue, we first assess the flatness of the likelihood function in each dimensions of the parametric space, and then rely on Neural Network methods to select starting values by putting more weight in those regions of  $\mathbb{R}_+^k$  with steeper gradients.

### D.1.2 Panel with fixed effects alternative

Instead of identifying and estimating the structural parameters of interest using a cross-sectional perspective, an alternative might be to combine both the cross-section and time dimension, and thus consider a panel regression. Notably, the nonlinear multivariate econometric model can be appended to include (unobserved) individual heterogeneity, and especially individual fixed effects. However, taking the presence of (nonlinear) intercepts in the consumption, portfolio and income equations, one key issue is the standard dummy variable trap or perfect multicollinearity engendered by the Within transformation. Indeed, exploiting the (individual) Within variability would lead to drop out  $\mathbf{B}_0(\Theta)$  and thus results in a loss of identification and information for the structural parameters (e.g., the base income or the minimal consumption level) that belongs to

$\mathbf{B}_0(\Theta)$ . Furthermore, given the non-separability of the vector of structural parameters, standard adding-up constraints (e.g., the sum of the individual effects for consumption is zero) would not solve the identification issue.<sup>6</sup> In this respect, our estimation strategy only exploits the cross-sectional predictions of the optimal rules (and the income equation) and still remains fully consistent with our theory.

### D.1.3 Nonlinear inequality constraints

Our econometric model (32) can be written as a constrained regression problem with nonlinear equality and inequality constraints. Define  $\Theta^e$  (resp.  $\Theta^c$ ) as the vector of estimated (resp. calibrated) structural parameters in  $\Theta = (\Theta^e, \Theta^c)$  and let  $B$  and  $A$  be the composite Tobin's- $q$  and MPC parameters characterized by (9), and (10). For any objective function  $S_n$  associated with a sample of size  $n$ , the estimation procedure is:

$$\begin{aligned} \max_{B, A; \Theta^e \subset \Theta} S_n(\Theta) \quad \text{s.t.} \\ g_1(\Theta) \geq 0, \\ g_2(B, A, \Theta) = 0, \\ g_3(B, A, \Theta) \geq 0, \end{aligned} \tag{8}$$

where  $g_1$  is a vector of nonlinear inequality constraints capturing sign restrictions on  $\Theta$ ,  $g_2$  is a vector of nonlinear equality constraint(s) associated with (9b) and (10a), and  $g_3$  is a vector of nonlinear inequality constraints (9c) and (10b). It is worth noting that  $S_n$  can be the objective function corresponding to Maximum Likelihood estimation, asymptotic Least Squares estimation, M-estimation or the Generalized Method of Moments estimations (see Gourieroux and Monfort (1995a, ch. 10) and Gourieroux and Monfort (1995b, ch. 21)). Since  $g_2$  implicitly defines  $B = B(\Theta)$  and  $A = A(\Theta)$ , the estimation problem (8) can equivalently be recast as:

$$\begin{aligned} \max_{\Theta^e \subset \Theta} \tilde{S}_n(\Theta) \quad \text{s.t.} \\ m(\Theta) \geq 0, \end{aligned} \tag{9}$$

---

<sup>6</sup>At the same time, a time-varying specification (through age-varying structural parameters) will allow for identification and estimation without resorting to further (arbitrary) identifying restrictions. We leave this issue for future research.

where  $m(\Theta) = [g_1(\Theta), g_3(B(\Theta), A(\Theta), \Theta)]' \in \mathbb{R}^r$ .

Because of the presence of the (nonlinear) inequality constraints  $m(\Theta)$ , one key issue is whether or not  $n^{1/2} (\hat{\Theta}_n^e - \Theta_0^e)$  and consequently  $n^{1/2} (\hat{B}_n - B_0)$  and  $n^{1/2} (\hat{A}_n - A_0)$ , where  $\Theta_0^e$  are the true unknown parameters, are asymptotically normal. More generally, one cannot expect to get an explicit expression of the distribution of the estimator (e.g. Wang, 1996; Gouriéroux and Monfort, 1995a,b). At the same time, the existence and strong convergence of the estimator does not depend on the presence of the nonlinear inequality (and equality) constraints. More specifically, it requires that the observations are independent (in our context), the parameter space being compact, the true parameters  $\Theta_0^e$  being identifiable, the log-likelihood function being continuous w.r.t.  $\Theta^e$ , the existence of  $\mathbb{E}_0[\tilde{S}_n(\Theta)]$  under the null of  $\Theta_0^e$ , the uniform convergence of  $S_n$  (see Gouriéroux and Monfort, 1995b, ch. 7), and that the Jacobian associated to the nonlinear constraints be of full row rank.<sup>7</sup>

The Lagrangean associated with the constrained problem (9) is then given by:

$$\max_{\Theta^e \in \Theta} \mathcal{L}_n(\Theta, \lambda) = \tilde{S}_n(\Theta) + \sum_{j=1}^r \lambda_j m_j(\Theta), \quad (10)$$

where the  $\lambda_j$ 's terms denote the Kuhn-Tucker multipliers. The solutions  $(\hat{\Theta}_n^e, \hat{\lambda}_n)$  to the Lagrangean problem (10) must satisfy the following first-order, sign, and exclusion restrictions:

$$\begin{aligned} \frac{\partial \tilde{S}_n(\Theta)}{\partial \Theta^e} \bigg|_{\hat{\Theta}_n^e, \hat{\lambda}_n} + \sum_{j=1}^r \lambda_j \frac{\partial m_j(\Theta)}{\partial \Theta^e} \bigg|_{\hat{\Theta}_n^e, \hat{\lambda}_n} &= 0, \\ m_j(\Theta), \text{ and } \lambda_j \big|_{\hat{\Theta}_n^e, \hat{\lambda}_n} &\geq 0, \\ \lambda_j m_j(\Theta) \big|_{\hat{\Theta}_n^e, \hat{\lambda}_n} &= 0, \end{aligned}$$

for  $j = 1, 2, \dots, r$ . The solutions  $(\hat{\Theta}_n^e, \hat{\lambda}_n)$  are associated with the restriction on the composite parameters,  $\hat{B}_n = B(\hat{\Theta}_n^e, \Theta^e)$ , and  $\hat{A}_n = A(\hat{\Theta}_n^e, \Theta^e)$ . Given these elements, two situations might arise:

---

<sup>7</sup>Notably uniform convergence is insured if the interior of  $\Theta^e$  is non-empty and  $\Theta_0^e$  belongs to the interior of  $\Theta^e$ . In addition, the Jacobian condition is insured by evaluating the rank of this matrix at the ML estimate of  $\Theta^e$ . Finally, due to the nonlinear constraints and model, there are no general conditions for global identification.

- If  $\Theta_0^e$  belongs to the interior, i.e. is such that  $m_j(\Theta)|_{\Theta_0^e} > 0$ , for  $j = 1, \dots, r$ , then  $\hat{\Theta}_n^e$  is asymptotically equivalent to the unconstrained estimator (in the absence of inequality constraints) defined by:

$$\max_{\Theta^e \subset \Theta} \tilde{S}_n(\Theta)$$

with associated composite parameters  $\hat{B}_n = B(\hat{\Theta}_n^e, \Theta^c)$ , and  $\hat{A}_n = A(\hat{\Theta}_n^e, \Theta^c)$ . Consequently  $\hat{\Theta}_n^e, \hat{B}_n, \hat{A}_n$  are *asymptotically normally distributed*.

- If  $\Theta_0^e$  belongs to the boundary, i.e. is such that  $m_j(\Theta)|_{\Theta_0^e} = 0$ , for  $j = 1, \dots, r$ , then the asymptotic distribution does not have a closed-form solution.<sup>8</sup>

In practice, we proceed with an ex-post verification, i.e.

1. Estimate  $\hat{\Theta}^e$  in the unconstrained equation:

$$\max_{\Theta^e \subset \Theta} \mathcal{L}_n(\Theta, \lambda) = \tilde{S}_n(\Theta),$$

2. Check that the inequality restrictions  $m(\Theta)|_{\hat{\Theta}^e} \geq 0$  are verified at the unconstrained estimate.

## D.2 Effects of equivalence scaling

Our PSID data procedure described in Section 4.3 of the paper scales the resources (financial wealth  $W_t$ , income  $Y_t$ ) and dependent variables (consumption  $C_t$ , health investment  $I_t$  and insurance  $x_t$ , risky asset holdings  $\pi_t$ ) by the number of household members to obtain per-capita variables. The respondent's self-reported health status  $H_t$  is agent-specific, and does not require scaling.

Other equivalence scaling (ES) approaches, such as square root of household size, OECD and modified OECD ES are also available. Their main purpose is to correct for potential economies of scale in household, especially for determining available resources (e.g. an additional child does not necessarily entail proportional expenses). The literature reveals that there is absence of consensus as to which ES measure to use (e.g. OECD, 2013, ch. 8 for discussion). In particular, ES that are appropriate for stock (e.g. wealth)

---

<sup>8</sup>As an application, see Wang (1996)

may not be adequate for flow (e.g. income) and those for resources are not necessarily applicable for expenses. Moreover, ES that are relevant for richer households are not necessarily useful for poorer ones.

In the absence of clear consensus, and because the scale economies arguments are less apparent for health-related expenses (out-of-pocket, insurance) in our estimation, we have selected our simpler per-capita scaling instead of alternative ES approaches. For completeness, we have nonetheless re-estimated our benchmark model, for the 2017 PSID sample, using the square-root and modified OECD ES methods, where the differences in scaling are illustrated in Table 3.

Overall, our estimated parameters in Table 4 remain very robust to the choice of ES. Indeed, contrasting our benchmark (column 1) with the square-root (column 2) or modified OECD (column 3) reveals minimal effects of scaling in almost all instances. In particular, the structural parameters in (10) are unaffected by the choice of ES, such that the MPC  $A(\lambda_m)$  (panel e), and importantly the marginal value  $\Theta(\lambda_m)$  in (13b) remain unchanged. One exception is the larger health loading  $\beta$  in the income equation (4). This is unsurprising since household income is scaled by a lower factor under alternative ES, while the health variable is unscaled. A direct consequence of a higher  $\beta$  in (9) is to raise the Tobin's  $q, B$  in panel e. Consequently, so are human wealth  $P(H) = BH$  and net total wealth  $N(W, H) = W + (y - a)/r + P(H)$ , where the latter also increases due to a higher financial wealth  $W$  from the alternative scaling. The net effects are to raise the life values in Table 5, where, as expected, the impact is modest for agents in poor health/wealth and more potent for others. Overall, we conclude that the effects of alternative ES measure is predictable, and that in the absence of clear consensus, our per-capita scaling remains warranted.

**Table 3:** Alternative equivalence scaling approaches

Nb. Adults	Nb. Child(ren)	Per-capita	Square root	Modified OECD
1	0	1.0	1.0	1.0
2	0	2.0	1.4	1.5
2	1	3.0	1.7	1.8
2	2	4.0	2.0	2.1
2	3	5.0	2.2	2.4
5	0	5.0	2.2	3.0

*Notes:* Source OECD (2013, Tab. 8.1).

**Table 4:** Scaling robustness: Estimated and calibrated structural parameters

Model	(1)	(2)	(3)
Year	Benchm.	Benchm.	Benchm.
Scaling	2017	2017	2017
	Per-capita	Square root	Modif. OECD
a. Law of motion health (3)			
$\alpha$	0.7413 (0.0155)	0.7592 (0.0201)	0.7643 (0.0243)
$\delta$	0.0370 (0.0011)	0.0384 (0.0023)	0.0390 (0.0011)
$\phi^c$	0.0136	0.0136	0.0136
b. Sickness (3) and death (1) intensities			
$\lambda_s$	0.1000 (0.0112)	0.0980 (0.0185)	0.0966 (0.0213)
$\lambda_m$	0.0342 (0.0001)	0.0373 (0.0098)	0.0365 (0.0001)
c. Income (4) and wealth (5)			
$y$	0.0127 (0.0004)	0.0122 (0.0004)	0.0123 (0.0004)
$\beta$	0.0061 (0.0001)	0.0092 (0.0001)	0.0088 (0.0001)
$\mu^c$	0.1080	0.1080	0.1080
$r^c$	0.0480	0.0480	0.0480
$\sigma_S^c$	0.2000	0.2000	0.2000
d. Preferences (6)			
$\gamma$	2.4579 (0.0542)	2.2579 (0.0210)	2.2579 (0.0214)
$\varepsilon$	1.0212 (0.0004)	1.0264 (0.0002)	1.0712 (0.0006)
$a$	0.0134 (0.0007)	0.0139 (0.0006)	0.0140 (0.0006)
$\rho^c$	0.0500	0.0500	0.0500
e. MPC and Tobin's $q$ (10), (9)			
$A$	0.0504 (0.0057)	0.0505 (0.0008)	0.0513 (0.0004)
$B$	0.0709 (0.0084)	0.1053 (0.0019)	0.0998 (0.0006)

*Notes:* Estimated (standard error in parentheses) and calibrated ( $c$ ) structural parameters. Column (1): Econometric model (32), estimated by ML, subject to the parametric restrictions in panel (a) of Table 1 for 2017 data. Columns (2): Benchmark model with square root on household size equivalence scaling for 2017. Columns (3): Benchmark model with modified OECD rule on household size equivalence scaling for 2017.

**Table 5:** Scaling robustness: Estimated life values (in K\$)

Model	(1)	(2)	(3)
Year	Benchm.	Benchm.	Benchm.
Scaling	2017	2017	2017
	Per-capita	Square root	Modif. OECD
a. HK $v_h(H)$ in (17)			
Poor	205.82 (0.63)	217.23 (1.26)	216.54 (2.37)
Fair	243.89 (1.09)	272.55 (2.21)	269.41 (4.15)
Good	281.96 (1.56)	327.88 (3.16)	322.29 (5.93)
Very Good	320.03 (2.03)	383.20 (4.11)	375.17 (7.71)
Excellent	358.10 (2.50)	438.52 (5.06)	428.04 (9.48)
All	299.52 (1.91)	353.92 (3.83)	347.18 (3.66)
b. VSL $v_s(W, H, \lambda_m)$ in (24)			
Poor	2178.13 (32.53)	2701.62 (102.46)	2501.34 (186.38)
Fair	2720.43 (39.56)	3619.34 (61.17)	3352.69 (110.80)
Good	4206.53 (42.94)	5847.01 (98.01)	5428.31 (178.58)
Very Good	5802.46 (42.56)	8082.04 (120.04)	7521.75 (216.30)
Excellent	7189.48 (40.47)	10273.67 (185.48)	9554.52 (336.24)
All	4980.38 (49.08)	6940.49 (118.69)	6451.44 (109.50)
c. GPV $v_g(W, H, \lambda_m)$ in (28)			
Poor	109.73 (1.59)	136.46 (5.07)	128.38 (9.40)
Fair	137.05 (1.93)	182.82 (3.03)	172.08 (5.59)
Good	211.92 (2.09)	295.34 (4.85)	278.61 (9.01)
Very Good	292.33 (2.07)	408.24 (5.94)	386.05 (10.91)
Excellent	362.20 (1.97)	518.94 (9.18)	490.39 (16.96)
All	250.91 (2.39)	350.58 (5.87)	331.12 (5.52)

*Notes:* Computed at corresponding estimated parameter values in Table 4, columns (1–3). Bootstrapped standard errors in parentheses (500 replications).

## References

- Aldy, Joseph E., and W. Kip Viscusi (2007) ‘Age differences in the value of statistical life: Revealed preference evidence.’ Discussion paper 07-05, Resources for the Future, April
- Aldy, Joseph E., and W. Kip Viscusi (2008) ‘Adjusting the value of a statistical life for age and cohort effects.’ *Review of Economics and Statistics* 90(3), 573–581
- Ashenfelter, Orley (2006) ‘Measuring the value of a statistical life: Problems and prospects.’ *Economic Journal* 116(510), C10–C23. Conference papers
- Basak, Suleyman (1999) ‘On the fluctuations in consumption and market returns in the presence of labor and human capital: An equilibrium analysis.’ *Journal of Economic Dynamics and Control* 23(7), 1029 – 1064
- Bellavance, Francois, Georges Dionne, and Martin Lebeau (2009) ‘The value of a statistical life: A meta-analysis with a mixed effects regression model.’ *Journal of Health Economics* 28(2), 444–464
- Bommier, Antoine, Danier Harenberg, and François Le Grand (2019) ‘Recursive preferences, the value of life, and household finance.’ Working paper, Available at SSRN <https://ssrn.com/abstract=2867570>, May
- Conley, Bryan C. (1976) ‘The value of human life in the demand for safety.’ *American Economic Review* 66(1), 45 – 55
- Cook, Philip J., and Daniel A. Graham (1977) ‘The demand for insurance and protection: The case of irreplaceable commodities.’ *Quarterly Journal of Economics* 91(1), 143 – 156
- Córdoba, Juan Carlos, and Marla Ripoll (2017) ‘Risk aversion and the value of life.’ *Review of Economic Studies* 84(4), 1472–1509
- Doucouliafos, Hristos, T.D. Stanley, and W. Kip Viscusi (2014) ‘Publication selection and the income elasticity of the value of a statistical life.’ *Journal of Health Economics* 33(0), 67 – 75

- Drèze, Jacques H. (1962) ‘L’utilité sociale d’une vie humaine.’ *Revue Française de Recherche Opérationnelle* 23(6), 93–118
- Eeckhoudt, Louis R., and James K. Hammitt (2004) ‘Does risk aversion increase the value of mortality risk?.’ *Journal of Environmental Economics and Management* 47(1), 13 – 29
- Ehrlich, Isaac, and Hiroyuki Chuma (1990) ‘A model of the demand for longevity and the value of life extension.’ *Journal of Political Economy* 98(4), 761–782
- Gourieroux, Christian, and Alain Monfort (1995a) *Statistics and Econometric Models*, vol. 1: General Concepts, Estimation, Prediction and Algorithms (Cambridge University Press)
- (1995b) *Statistics and Econometric Models*, vol. 2: Testing, Confidence Regions, Model Selection and Asymptotic Theory (Cambridge University Press)
- Grossman, Michael (1972) ‘On the concept of health capital and the demand for health.’ *Journal of Political Economy* 80(2), 223–255
- Hall, Robert E., and Charles I. Jones (2007) ‘The value of life and the rise in health spending.’ *Quarterly Journal of Economics* 122(1), 39–72
- Hammitt, James K. (2008) ‘Risk in perspective.’ Technical Report 16, Harvard Center for Risk Analysis, March
- Hammitt, James K., and Nicolas Treich (2007) ‘Statistical vs. identified lives in benefit-cost analysis.’ *Journal of Risk and Uncertainty* 35(1), 45 – 66
- Huggett, Mark, and Greg Kaplan (2013) ‘The money value of a man.’ Working Paper 238, Griswold Center for Economic Policy Studies, June
- (2016) ‘How large is the stock component of human capital?’ *Review of Economic Dynamics* 22, 21 – 51
- Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour (2013) ‘Health and (other) asset holdings.’ *The Review of Economic Studies* 80(2), 663–710

- (2021) ‘Technical appendix valuing life as an asset, as a statistic and at gunpoint: Endogenous mortality and morbidity, source-dependent risk aversion.’ <https://people.unil.ch/pascalst-amour/files/2021/02/TechApxHPS13-1.pdf>
- Jones-Lee, Michael (1974) ‘The value of changes in the probability of death or injury.’ *Journal of Political Economy* 82(4), 835–849
- Karns, Jack E. (1990) ‘Economics, ethics, and tort remedies: The emerging concept of hedonic value.’ *Journal of Business Ethics* 9(9), 707–713
- Kiker, B. F. (1966) ‘The historical roots of the concept of human capital.’ *Journal of Political Economy* 74(5), 481–499
- Lewbel, Arthur (2003) ‘Calculating compensation in cases of wrongful death.’ *Journal of Econometrics* 113(1), 115 – 128
- Murphy, Kevin M., and Robert H. Topel (2006) ‘The value of health and longevity.’ *Journal of Political Economy* 114(5), 871–904
- OECD (2013) *OECD Framework for Statistics on the Distribution of Household Income, Consumption and Wealth* (Paris)
- Peeples, Ralph, and Catherine T. Harris (2015) ‘What is a life worth in North Carolina: A look at wrongful-death awards,.’ *Campbell Law Review* 37(3), 497–518
- Philipson, Tomas J., Gary Becker, Dana Goldman, and Kevin M. Murphy (2010) ‘Terminal care and the value of life near its end.’ Working paper 15649, National Bureau of Economic Research
- Posner, Eric A., and Cass R. Sunstein (2005) ‘Dollars and death.’ *The University of Chicago Law Review* 72(2), 537–598
- Pratt, John W., and Richard J. Zeckhauser (1996) ‘Willingness to pay and the distribution of risk and wealth.’ *Journal of Political Economy* 104(4), 747 – 763
- Raymond, Richard (1999) ‘The use or abuse of hedonic value-of-life estimates in personal injury and death cases.’ *Journal of Legal Economics* 9(3), 69–96

- Robinson, Lisa A., and James K. Hammitt (2016) ‘Valuing reductions in fatal illness risks: Implications of recent research.’ *Health Economics* 25(8), 1039 – 1052
- Rosen, Sherwin (1988) ‘The value of changes in life expectancy.’ *Journal of Risk and Uncertainty* 1(3), 285–304
- Round, Jeff (2012) ‘Is a qaly still a qaly at the end of life?.’ *Journal of Health Economics* 31(3), 521 – 527
- Schelling, Thomas C. (1968) ‘The life you save may be your own.’ In *Problems in Public Expenditure Analysis: Papers Presented at a Conference of Experts Held Sept. 15-16, 1966*, ed. Samuel B. Chase (Washington DC: Brookings Institution) pp. 127–162
- Shepard, Donald S., and Richard J. Zeckhauser (1984) ‘Survival versus consumption.’ *Management Science* 30(4), 423–439
- Shogren, Jason F., and Tommy Stamland (2002) ‘Skill and the value of life.’ *Journal of Political Economy* 110(5), 1168 – 1173
- Smith, James E., and Ralph Keeney (2005) ‘Your money or your life: A prescriptive model for health, safety, and consumption decisions.’ *Management Science* 51, 1309–1325
- Smith, Stanley V. (1988) ‘Hedonic damages in wrongful death cases.’ *American Bar Association* 1, 70–73
- Sunstein, Cass R. (2013) ‘The value of a statistical life: Some clarifications and puzzles.’ *Journal of Benefit-Cost Analysis* 4(2), 237 – 261
- Viscusi, W. Kip (2000) ‘Misuses and proper uses of hedonic values of life in legal contexts.’ *Journal of Forensic Economics* 13(2), 111–125
- (2007) ‘The flawed hedonic damages measure of compensation for wrongful death and personal injury.’ *Journal of Forensic Economics* 20(2), 113–135
- Viscusi, W. Kip (2010) ‘The heterogeneity of the value of statistical life: Introduction and overview.’ *Journal of Risk and Uncertainty* 40(1), 1 – 13
- Wang, Jinde (1996) ‘Asymptotics of least-squares estimators for constrained nonlinear regressions.’ *The Annals of Statistics* 2(3), 1316–1326