# Testing Superneutrality when Money Growth is Endogenous\*

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A model that tests long-run superneutrality while controlling for endogenous money growth is developed. We find that an exogenous permanent increase in inflation has a positive long-run effect on output for the United States. This finding rejects superneutrality in favor of a Mundell-Tobin effect based on Friedman's famous dictum that a permanent movement in "inflation is always and everywhere a monetary phenomenon." We prove our result is robust to a potentially important source of misspecification by extending an econometric technique to a novel setting. Using similar methods we show a once-popular simpler vector autoregression model that typically found positive output effects in low inflation countries had downward biased estimates due to endogenous money growth. While those estimates were statistically significant about half the time, we argue downward bias may explain why a relatively large number of positive estimates in low inflation countries were insignificant. Our overall conclusion is that Mundell-Tobin effects are more prevalent than was once perceived.

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#### 1. Introduction

The relationship between inflation and output, particularly in the long run, is a significant concern of theorists, empiricists, and policymakers. From an empirical perspective, this long-run relationship has sometimes been addressed with models of permanent shocks to inflation. A widely held view of modern macroeconomics is that permanent changes in the inflation rate are the result of permanent changes in the growth rate of money. We call this the Friedman Principle<sup>1</sup> as it is associated with Milton Friedman famous dictum that "inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output."<sup>2</sup>

If exogenous permanent changes in money growth have no long-run effect on output this is called superneutrality with respect to output. In theory money is sometimes superneutral for nearly all real variables.<sup>3</sup> We will only address whether or not output is superneutral. In contrast, long-run monetary neutrality means that a permanent change in the level of money has no long-run effects on any real variables, and this hypothesis has greater support that does long-run superneutrality.

Economist often prefer to use inflation in place of money growth to empirically address superneutrality. This is because of the debate about the appropriate measure of money to use in empirical studies. Should a broad or a narrow measure be selected? Is a weighted monetary aggregate or an unweighted measure superior?<sup>4</sup> Using an aggregate measure of prices to calculate inflation obviates the need to come to terms with those monetary questions.

One reason testing for superneutrality is important is because this hypothesis is not universally accepted by economic theory.<sup>5</sup> While some theories predict a permanent increase in money growth has no long-run effect on output, there are theories in which output is permanently higher and

<sup>&</sup>lt;sup>1</sup>There are many "principles" that have been derived from Friedman's economic research or his political philosophy, but this is the only one we will make use of in this research.

 $<sup>^2{\</sup>rm This}$  quote can be found in various locations, including Friedman (1970)

<sup>&</sup>lt;sup>3</sup>The notable exception is real money. The demand for money depends on nominal interest rates which are affected by a change in expected inflation which can result from a permanent change in money growth.

<sup>&</sup>lt;sup>4</sup>Unweighted measures are the simple sums central banks produce. A weight of one is provided to anything included in a simple sum aggregate and otherwise a weight of zero is given. A weighted aggregate, such as Divisia measures of Barnett (1978), weighs components in an aggregate by what Friedman and Schwartz (1970, chapter 4) described as each asset's "moneyness".

<sup>&</sup>lt;sup>5</sup>Variation in theoretically predicted effects of money growth have been known for a long time. For example, in a widely cited review article on the subject Orphanides and Solow (1990) begin by quoting Stein (1970): "My main conclusion is that equally plausible models yield fundamentally different results" and then go on to say that "all we have is more reasons for reaching the same conclusion."

others in which output suffers a permanent reduction.

Vector autoregressions (VARs) are commonly used to estimate the effects of shocks, and our particular application estimates the long-run output effect from a permanent inflation shock. A common assumption in that literature is that permanent movements in inflation are themselves exogenous. Based on Friedman's Principle, assuming permanent changes in inflation or in money growth are exogenous would often be equivalent statements.<sup>6</sup> If that assumption is true then empirical work with permanent shocks to inflation may speak directly to whether or not superneutrality describes an economy.

Unfortunately, that exogeneity assumption is sometimes flawed. For a variety of reasons money growth may be endogenous to the state of the economy because of central bank behavior. In that case, a permanent inflation shock would no longer necessarily be equivalent to an exogenous permanent change in money growth, and exogenous shocks are needed to use VAR methods to test long run propositions such as superneutrality. Section 2 delves deeper into sources of endogeneity. Four examples are drawn from central bank experiences. These real-world examples imply permanent inflation shocks are not exogenous and therefore suggest these shocks are unable to be used to test for superneutrality. These examples all seem relevant to experiences of The Federal Reserve Bank of the United States. Previous superneutrality tests find the US is an outlier, compared to other low inflation countries, raising concern that a bias is driving that outcome. Therefore, we develop a model that allows money growth to be endogenous to the real economy to construct exogenous inflation shocks. This model is taken to US data and our test rejects superneutrality in favor of the hypothesis that an exogenous permanent increase in the inflation rate raises output in the long run. A long-run positive output effect - or a permanent decline in the real rate of interest - arising from a permanent increase in the growth rate of money is sometimes called a Mundell-Tobin effect.

Ireland (2007) makes a strong case that one must control for policy's reaction to oil to properly

<sup>&</sup>lt;sup>6</sup>However, a counterexample to equivalence can be derived from the Quantity Equation. To simplify the example, assume velocity growth is unaffected by policy. Given that assumption, if a central bank targets an exogenous rate of inflation and potential output growth changes, then money growth must respond one-for-one to that change to maintain the inflation target. Hence, a policy of insulating inflation from changes in potential output growth would make money growth endogenous to potential output growth. This is another reason one might prefer using inflation rather than money growth in the tests. Alternatively, if policy held growth of money - or growth of nominal GDP - constant in the face of a change in potential output growth inflation would become endogenous.

<sup>&</sup>lt;sup>7</sup>See King and Watson (1997) for an intuitive explanation of why permanent shocks are important. They also developed a novel approach to addressing a variety of neutrality propositions including superneutrality. Unfortunately, there approach is not easily extended to VARs with more than 2 variables.

<sup>&</sup>lt;sup>8</sup>Based on Mundell (1963) and Tobin (1965).

estimate monetary effects. The fourth section of the paper considers the possibility that our model is misspecified because it fails to explicitly include oil prices. Initially we took no stand on the third shock, except that it had no long-run inflation effect. But if we interpret that shock as an oil price shock, then it may affect output and inflation. Consequently, permanent exogenous shocks to inflation are no longer identifiable using the long-run recursive statistical model from Section 3.

In general, parameters from a structure can be mapped into the coefficients from a statistical model. If we can know something about the qualitative nature of certain aspects of structure, we may use that mapping along with empirical evidence to infer other useful information about an economy. Economists are quite willing to make assumptions about the qualitative effects of structural shocks on variables. For example, sign restrictions are often used to identify VAR models.<sup>9</sup> We will make similar qualitative assumptions. But we use them in a completely different way, not for identification and estimation purposes, but instead to interpret estimates from a long-run recursive statistical model.<sup>10</sup> Based on this methodology and a set of very plausible structural assumptions, the paper shows that the US estimates in Section 3 are downward biased if the model is misspecified due to oil prices. Hence, the actual Mundell-Tobin effects are likely to be even larger than our point estimates.

Section 5 re-examines results from some earlier work that attempted to address superneutrality based on bivariate models of inflation and output. Using similar econometric techniques to those in Section 4, we find a similar bias in these models. Specifically, the estimated long-run output effect from a permanent increase in inflation is biased downward because these models failed to control for endogenous money growth. We argue this bias why Bullard and Keating (1995) found with these models that all the low inflation (10.6% or less) countries estimates, except for the US, estimated a positive effect, but most of the estimates are statistically insignificant.

Section 6 summarizes our results and suggests potentially useful future research. The main takeaway is that Mundell-Tobin effects are important and appear more prevalent than previous work seemed to indicate. While existence of these effects will not necessarily inspire the Fed to raise

<sup>&</sup>lt;sup>9</sup> A sampling of important papers on sign restrictions would include: Faust (1998), Uhlig (2005), Mountford and Uhlig (2009), Fry and Pagan (2011), Inoue and Kilian (2013), Baumeister and Hamilton (2015), and Arias et al. (2018)

<sup>&</sup>lt;sup>10</sup>Keating (2013) shows how an econometrician can combine assumptions about specific qualitative features in the structure with empirical findings from a possibly misspecified statistical model to infer additional facts about the structure of an economy.

its inflation target beyond 2%, central banks would be well-advised to re-examine policy in light of models that take persistent real effects of money growth into account.

## 2. Changes in Money Growth may be Endogenous

There are a number of reasons to doubt changes in money growth are exogenous. We examine four examples of endogeneity which are historically relevant. An important implication from each one is that money growth and consequently the rate of inflation rises endogenously in response to adverse real shocks or falls in response to beneficial real shocks. We find this effect in our estimates and later use that implication to reexamine previous VAR findings that have ignored endogeneity.

It has frequently been said that deficit spending is inflationary. In fact, central bankers have sometimes bought government debt to help finance deficit spending.<sup>11</sup> Monetizing debt can make inflation endogenous to real shocks. For example, an adverse aggregate supply shock reduces output which causes revenues to fall and government spending on unemployment benefits and medical expenditures to rise. Consequently, the deficit increases, and so even if only a portion of this debt is monetized that can lead to increases in both money growth and inflation. This mechanism provides one way an adverse real shock may cause a persistently higher rate of inflation and *vice versa*.

The same sort of response to real shock occurs if a central bank conducts policy with an eye towards the original Philips Curve. This explanations is often given for why inflation rates began rising in the second half of the 1960s and reached high levels in the latter 1970s. This so called "The Great Inflation" was put to an end by the Volcker disinflation. Of course, the original Phillips Curve was merely an empirical relationship which had been remarkably stable for a long period of time. This relationship suggested a reliable trade-off whereby policymakers could increase the inflation rate to lower the unemployment rate. During the 1970s real interest rates were often persistently negative, consistent with the view that policymakers were aggressively trying to take advantage of that apparent trade-off.

As inflation and unemployment trended upward together in the 1970s that mindset began to change for a rising share of the economics profession. The work of Friedman and Phelps along with

<sup>&</sup>lt;sup>11</sup>This criticism can still be heard although it was more prevalent in the years before the world's central banks almost uniformly began to reduce inflation rates.

critical extensions by Lucas, Sargent and others argued that prior attempts to connect policy to the empirical Phillips Curve would lead to an equilibrium with greater inflation and no beneficial change to the equilibrium unemployment rate. These models frequently took to heart the natural rate of unemployment and developed models in which that rate was invariant to inflation. However, prior to this change of thought the Fed and other central banks helped ramp up inflation in response to the recession that accompanied adverse real shocks. This played a role in causing stagflation in the 1970s.

Another possible reason inflation trended upward throughout the 1970s is that policymakers were slow to recognize a declining rate of productivity growth. Attempting to maintain output growth at its historical trend after potential growth has fallen naturally induces a rise in inflation. This missperception of the sustainable rate of growth is another example of how an adverse supply shock may cause inflation to rise due to endogenous monetary policy.

Opportunistic disinflation describes how the Fed responded to a resurgence of growth that began in the mid-1990s and lasted for about a decade. The "New Economy" exhibited rapid growth in productivity that for a while was reminiscent of the fast pace of the 1960s. The Fed allowed output to increase, but not at quite the same pace potential output rose, and this induced a decline in the inflation rate. The central bank used a beneficial aggregate supply shock as a means to disinflate without causing a decline in real output. This was a novel approach as previously most economists thought a disinflation would require a recession.

This discussion yields two important ideas. One is that controlling for exogeneity seems to be critical for consistently estimating the long run effect of inflation on output. A second is that each of these explanations for endogenous money growth implies inflation will rise (fall) following an adverse (beneficial) real shock. The sign of this effect may prove useful in interpreting the long-run effects of permanent inflation shocks if the structural VAR is misspecified.

# 3. Testing Superneutrality

This section of the paper consists of four parts. In the first part we present a structure based on long-run economic relationships. The model uses a production function and additional

assumptions to identify exogenous permanent technology shocks. Then exogenous permanent shocks to money growths are identified by controlling inflation rate for the endogenous long-run response of money growth to technology shocks. Output is permitted a long-run response to technology shocks, money growth shocks, and a third type of shock. At this stage we take no strand on this third shock's source.<sup>12</sup>

This model is amenable to structural VAR (SVAR) methods. The second part of this section describes the structural VAR method. Naturally we use an SVAR based on long-run restrictions which accord with our structural model. Our model provides a new method of assessing the empirical relevance of Superneutrality while taking endogenous money growth into account. The third part of this section presents estimates and examines implications of the econometric tests.

In the fourth part of this section we consider whether our results are different than earlier work simply because we used a different measure of real GDP. To do that we estimate a popular bivariate model using chain-weighted real GDP for the United States instead of fixed base year GDP that was at one time the only measure available.

#### 3.1 An Economic Structure

The general form of a dynamic linear economic structure can be written as:

$$\Delta X_t = \theta(L)\varepsilon_t \tag{1}$$

where  $\theta(L) = \theta_0 + \theta_1 L + \theta_2 L^2 + ... = \sum_{j=0}^{\infty} \theta_j L^j$  is the structural moving average representation,  $\varepsilon_t$  are structural shocks, and the number of structural shocks and number of variables are identical. The shocks are assumed to be uncorrelated, and shock variances are normalized to one:  $\mathbb{E}\varepsilon_t\varepsilon_t' = I$ . This normalization to unit variance shocks is for convenience in expressing results but otherwise immaterial. Variables are taken as difference stationary as this is a necessary condition for there to be permanent shocks in a linear time series model with constant parameters. By recursive

<sup>&</sup>lt;sup>12</sup>Later we consider a particular type of shock that is not amenable to the identification assumptions, and we interpret our results under alternative assumptions.

substitution on the structure, one can show that:

$$\lim_{k \to \infty} \frac{\partial X_t}{\partial \varepsilon_{t-k}} = \sum_{j=0}^{\infty} \theta_j = \theta(1) . \tag{2}$$

 $\theta(1)$  represents the cumulative effects of shocks on the level of the variables. We define it as the matrix of long-run structural parameters. All structural restrictions are applied to this matrix.

We develop a trivariate structure that is designed for the purpose of identifying endogenize money growth shocks. Let the production function be of the standard Cobb-Douglas form:

$$\Delta y = \alpha \Delta k + (1 - \alpha)(\Delta n + \lambda^*) \tag{3}$$

written in terms of growth rates of output, labor, and capital along with  $\lambda_t^*$  which is a stochastic shock to technology.<sup>13</sup> Next we assume the capital-to-output ratio shows no long-run trend. This seems empirically valid. Under this assumption capital and output grow at the same rate in the long run:

$$\Delta y = \Delta k \tag{4}$$

Many growth theories predict this occurs in the steady state. Next re-write the technology shock as  $\lambda^* = \alpha_{y/n,\lambda}\lambda$ , where  $\lambda$  as an iid shock with unit variance,  $\alpha_{y/n,\lambda}$  is the standard deviation of technology shocks.

Combining the last 3 equations and simplifying yields:

$$\Delta(y-n) = \alpha_{y/n,\lambda}\lambda . (5)$$

The implication of this equation is that permanent movements in average labor productivity are solely from exogenous technology shocks.<sup>14</sup> All shocks are allowed to affect average labor productivity in the short-run and at business cycles frequencies, but only technology has a long-run effect.

Our next equation assumes that inflation responds to technology shocks because of endogenous

<sup>&</sup>lt;sup>13</sup>This implies that in logarithms multifactor productivity has a unit root. For average labor productivity to be rising over time the technology process must also have positive drift.

<sup>&</sup>lt;sup>14</sup>The same result obtains if technology is learning-by-doing and a linear function (in logarithms) of output per hour, or capital per hour, or both, along with an exogenous technology disturbance.

monetary policy and is also driven by exogenous money growth shocks:

$$\Delta \pi = \alpha_{\pi \lambda} \lambda + \alpha_{\pi \mu} \mu \tag{6}$$

Similar to what was done for technology, the money growth shock is written as the product of its standard deviation,  $\alpha_{\pi\mu}$ , times an iid unit variance shock,  $\mu$ . Our last equation writes output as a function of all three shocks:

$$\Delta y = \alpha_{y\lambda}\lambda + \alpha_{y\mu}\mu + \alpha_{y\tau}\tau \tag{7}$$

We've added  $\tau$  shocks which are iid unit variance disturbances to account for additional factors that in the long run affect real output but not inflation or average labor productivity. At this point we take no stand on the source of such shocks.

If variables are ordered by (y-n,  $\pi$ , y)' and shocks by  $(\lambda, \mu, \tau)'$  the long-run effects matrix for the structure is given as:

$$\theta(1) = \begin{bmatrix} \alpha_{y/n,\lambda} & 0 & 0\\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} & 0\\ \alpha_{y\lambda} & \alpha_{y\mu} & \alpha_{y\tau} \end{bmatrix}$$
(8)

#### 3.2 Econometric Method

This section accomplishes two things. First, it describes how the parameters from a recursive  $\theta(1)$  are estimated using a long-run recursive model.<sup>15</sup> This method is used to estimate the model developed in the previous section. Secondly, we introduce the general relationship between a long-run recursive statistical model and an economic structure that may not be recursive. This relationship is used later in the paper to assess structural implications of VAR results when the long-run recursive statistical model is misspecified.

Assume a finite VAR representation of the data exists:

$$\beta(L)\Delta X_t = e_t \tag{9}$$

with VAR coefficients given as:  $\beta(L) = I - \beta_1 L - \beta_2 L^2 - \dots - \beta_{\kappa} L^{\kappa}$  where  $\kappa$  denotes the number of

<sup>15</sup> Anyone familiar with Hamilton (1994, p 92) proof that recursive models are unique will have already recognized this

lags in the VAR. Dimensions of vectors of residuals,  $e_t$ , and variables,  $X_t$ , are identical. Since the VAR representation is unique, one way to understand how a structure and a statistical model are related is to map each of them into the VAR.

Let the statistical model have the following moving average representation:

$$\Delta X_t = R(L)u_t \tag{10}$$

where  $u_t$  is the vector of shocks with the same dimension as  $X_t$ , and  $R(L) = R_0 + R_1 L + R_2 L^2 + ... = \sum_{j=0}^{\infty} R_j L^j$  is the dynamic effect of each shock. The statistical model is identified by assuming R(1) is a lower triangular matrix and the shocks are contemporaneously uncorrelated and have variances normalized to 1 ( $\mathbb{E}u_t u_t' = I$ ). We also assume each shock is uncorrelated with itself and other shocks at all lags and leads ( $\mathbb{E}u_t u_\tau' \ \forall t \neq \tau$ ), but this is not so much a structural restriction as an assumption that sufficient lags are used in the VAR to make residuals serially uncorrelated. This model is a multivariate extension of the Blanchard and Quah (1989) decomposition. This structural VAR method uses assumptions about the long-run effects of shocks which are consistent with the theory we developed previously.

The long-run impact of the statistical model's shocks on the levels of variables can be found by recursive substitution:

$$\lim_{j \to \infty} \frac{\partial X_t}{\partial u_{t-j}} = \sum_{i=0}^{\infty} R_i = R(1) , \qquad (11)$$

where R(1) is the sum of coefficients in R(L), which represents the cumulative effect of u on X. In general, R(1) may not be identical to  $\theta(1)$ . To see the relationship between the structure and the statistical model we map them both into the VAR. This is done for the statistical model by multiplying equation (10) by  $R_0R(L)^{-1}$  and multiplying equation (1) by  $\theta(0)\theta(L)^{-1}$ , assuming R(L) and  $\theta(L)$  are invertible lag polynomials. These operations yield relationships between shocks and residuals:

$$e_t = R_0 u_t = \theta_0 \varepsilon_t \tag{12}$$

and mappings of structural parameters and statistical model coefficients into the VAR's coefficients:

$$\beta(L) = \theta_0 \theta(L)^{-1} = R_0 R(L)^{-1} . \tag{13}$$

From equation (12) the covariance matrix of residuals is related to short-run parameters in the structure as well as to the short-run coefficients in the statistical model:

$$\Sigma_e = R_0 R_0' = \theta_0 \theta_0' . \tag{14}$$

Given equation (13) the sum of VAR coefficients,  $\beta(1)$ , is related to the long-run parameters in the structure and the long-run coefficients in the statistical model, along with short-run matrices in each case:

$$\beta(1) = \theta_0 \theta(1)^{-1} = R_0 R(1)^{-1} . \tag{15}$$

Eliminating  $R_0$  and  $\theta_0$  from the last two equations and simplifying:

$$\beta(1)^{-1} \Sigma_e \beta(1)'^{-1} = R(1)R(1)' = \theta(1)\theta(1)'. \tag{16}$$

The first equation indicates that R(1) can be estimated by a Cholesky decomposition of  $\beta(1)^{-1}\Sigma_e\beta(1)'^{-1}$ . Typically a 2-step procedure is used. First, estimate the VAR and obtain VAR coefficients and the covariance matrix for residuals. Second formulate the first expression in equation (16) and estimate R(1) with an appropriate Cholesky decomposition.

The second equality in equation (16) indicates how R(1) is related to  $\theta(1)$ . If both are lower triangular matrices then they are equivalent and estimating the long-run recursive model identifies the effects of all the shocks in the structure. But this equation holds even when the structure is not recursive. In that case the exact mapping between structural parameters and the coefficients in the long-run recursive model is obtained from this equation. Under some circumstances one is be able to use these mappings along with information about some of the signs of long-run effects of structural shocks to infer useful information about the underlying structure from the statistical model's estimates. For example, Keating (2013) uses a set of plausible assumptions about the long-run effects of shocks to aggregate demand on the price level and the effects of aggregate supply on output and the price level to infer that in the pre-World War 1 period aggregate demand had a permanent positive effect on output for a group of countries.

If 
$$X_t = ((y-n)_t, \pi_t, y_t)'$$
 our structure is lower triangular, and so  $R(1) = \theta(1)$ . This is due to

the fact that the Cholesky Decomposition is unique for a particular ordering of variables. <sup>16</sup> Thus:

$$\begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} \alpha_{y/n,\lambda} & 0 & 0 \\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} & 0 \\ \alpha_{y\lambda} & \alpha_{y\mu} & \alpha_{y\tau} \end{bmatrix}$$
(17)

and so clearly:

$$\frac{R_{32}}{R_{22}} = \frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}. (18)$$

Hence, this ratio model parameters yields the output effect from a 1 percentage point increase in inflation resulting from an exogenous increase in money growth, precisely what we want to recover from the data. However, we are also very interested in the effects of technology shocks since our exogenous money growth shocks are identified conditionally on the central bank reaction to technology shocks. Specifically, we wish to verify that the technology shock has a permanent negative effect on inflation,  $\frac{R_{21}}{R_{11}} = \frac{\alpha_{\pi\lambda}}{\alpha_{y/n,\lambda}} < 0$ , consistent with our understanding of policy's reaction, and a permanent positive effect on real output,  $\frac{R_{31}}{R_{11}} = \frac{\alpha_{y\lambda}}{\alpha_{y/n,\lambda}} > 000$ , as is generally the case in macroeconomic theory.

#### 3.3 Parameter Estimates and a Test For Superneutrality

Estimation is done using four different sample periods to determine if parameters are relatively stable or if there is evidence of time variation. The first sample in each table is 1960-1992. This starting point is similar to samples used in a number of papers from the earlier literature<sup>17</sup>. The second sample period adds data starting from 1948 to 1960 to the first sample. The third sample adds data from 1992 to 2018 to the first sample period. And the fourth period includes all data from 1948 to 2018. We report estimates of the long-run effects of shocks on variables.

Technology shocks are used to control for central bank policies that make money growth and inflation endogenous to the state of the real economy. Table 1 examines the effect these shocks have on output. In all 4 samples the output effect is large, positive, and statistically significant. These results are consistent with the effect one expects a permanent improvement in technology to have

<sup>&</sup>lt;sup>16</sup>Hamilton (1994) page 92

<sup>&</sup>lt;sup>17</sup>And this is exactly the sample chosen for the US in Bullard and Keating (1995)

on real output.

The effect of a technology shock on inflation is reported in Table 2. A negative effect is found in each sample period, consistent with our previous arguments about how policy responds endogenously to real shocks. For sample periods beginning in 1960 this effect is significant at the 16% level. However, the negative estimates are not significant for each sample starting in 1948. These insignificant inflation results are likely because inflation in the first half of that the 1948-1960 period was remarkably volatile. Intuitively, if inflation becomes more volatile for some reason other than a change in technology, the correlation between TFP and inflation will shrink.

Inflation dynamics from 1948 to 1955 were arguably more extreme than at any other time since World War II for the United States. The year-over-year CPI inflation rate was over 10% in 1948, fell to -2.5% by mid-1949, surged to nearly 10% by March of 1951, before falling, very rapidly for a while, and eventually hit negative rates for most of 1955. These massive swings in inflation are associated with a number of major events. One factor was the Federal Reserve's policy of fixing interest rates at very low levels. This policy, which began during World War II, was designed to lower costs of financing the war, and it ended with the Treasury-Fed Accord of March 1951 which freed the Fed to pursue other objectives. Arbitrarily fixing the nominal interest rate induces indeterminacy in rational expectations models, and increased price volatility would be a likely outcome. <sup>18</sup> A second major event was the Korean War. United States involvement was most intensive from October of 1950 to July of 1953. During that period US defense spending increased at its most rapid post-World War II rate and so was highly stimulative to the economy. However, a third factor was Price Controls which dramatically though temporarily curtailed the war's inflationary pressure midway through the massive increase in military spending. Price Controls were implemented from January of 1951 to February of 1953. Also in that period are two recessions: one from November 1948 to October 1949 and another from July 1953 to May 1954. The latter one stems from a major decline in military spending as major operations in the Korean War came to a close and at roughly the same time price controls were lifted. Each of these dramatic swings from high to low inflation is unrelated to technological change, making it more difficult to estimate the endogenous reaction of money growth to supply shocks.

<sup>&</sup>lt;sup>18</sup>See Woodford (2003) for an in-depth discussion and references to earlier work on indeterminacy of interest rate rules such as Sargent and Wallace (1975).

Our exogenous money growth shocks are obtained after controlling for endogenous policy to the technology shocks that we've identified. Estimates of the long-run output effect from exogenous money growth shocks are found in Table 2. Each one is positive and statistically significant at the 5% level. The estimates range from 0.41 to 0.64. From the confidence bounds one can see that each estimate from the 4 sample periods does not appear to be statistically different from any of the other 3.

Controlling for endogenous money growth yields positive and significant estimates. These estimates contrast with earlier work from various researchers which estimated this effect to be negative and not significant for the United States. These difference suggest that controlling for endogenous money growth is important for estimating the output effect in the US. Consistent with that line of thought is the fact that most of our explanations for endogenous money were drawn from behaviors of the Federal Reserve. These positive estimates now put the US more in line with the other 8 low inflation countries for which Bullard and Keating (1995) estimated a positive long-run effect on output from a permanent inflation shock. Of course, those estimates did not control for endogenous money growth. Section (5) will show that the positive estimates in the bivariate model used by Bullard and Keating (1995) and others imply Mundell-Tobin effects even when money growth is endogenous.

#### 3.4 Does Chain-Weighted GDP Explain why our Results are Different?

Much of the previous work has used a bivariate model and fixed base year measures of real output. This construct is notably inferior to the one we used, which is a Fisher Ideal output measures that the Bureau of Economic Analysis began producing in 1996. These newer measures are also known as chain-weighted real GDP.

Is it possible that our statistically significant positive estimates derive from using chain-weighted GDP while negative estimates that were obtained before resulted from an inferior output measure. To check this we re-estimate the Bullard and Keating (1995) bivariate model using chain-weighted US data. This model is written as:

$$\begin{bmatrix} \Delta \pi_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} R^{\pi P}(L) & R^{\pi T}(L) \\ R^{yP}(L) & R^{yT}(L) \end{bmatrix} \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix}.$$
 (19)

which identifies permanent and transitory shocks to inflation,  $u_t^P$  and  $u_t^T$ , respectively, given that the the long-run parameter matrix is:

$$R(1) = \begin{bmatrix} R_{\pi P} & 0 \\ R_{yP} & R_{yT} \end{bmatrix}. \tag{20}$$

This is a Blanchard and Quah (1989) decomposition, but with permanent and transitory shocks to inflation instead of output. Researchers typically reported estimates of  $\frac{R_{yP}}{R_{\pi P}}$ . Under the assumption that the permanent shock to inflation is an exogenous shock to money growth, these models would be used to test for superneutrality. Of course, we have argued that this view is mistaken and provided estimates to support that this position

Estimating this bivariate model with US data Bullard and Keating (1995) obtain a negative estimate that is statistically insignificant. We've argued that we obtain different results because we account for endogenous money growth and early many early models do not. We re-estimate their bivariate model model but use our chain-weighted output measure in contrast to their fixed base year measure.

Our estimates of the long-run effect on output from a permanent increase in the inflation rate are reported in Table 4. This is done for all 4 sample periods used previously. The first column contains our estimate from the same sample period as Bullard and Keating (1995). The point estimate is positive, which contrasts with their negative point estimate, but it is statistically insignificant. Comparing our bivariate estimates with the US estimate in Bullard and Keating (1995), one sees that our confidence bound includes their point estimate, and their confidence bound includes our point estimate. Thus we conclude use of alternative output series does not explain different results for the United States.

The remainder of Table 4 reports the bivariate model's estimate for the other 3 periods and obtains positive estimates in all case. Interestingly, the other 3 sample periods obtain statistically significant results, with the samples including 1948-1960 data being significant at a higher level. The unusual features in that period that were discussed earlier likely play a role in explaining these differences for the bivariate estimates. We conjecture that endogenous money growth policies were particularly relevant in that 1960-1992 sample period that Bullard and Keating (1995) selected.

### 4. What if Inflation is Endogenous to Energy Market Shocks?

The trivariate model took no stand on the source of the third shock. The estimates are obtained from the assumption that this shock has no long-run effect on average labor productivity or inflation. Energy market shocks are a potentially important additional source of real shocks. Ireland (2007), for example, argues that to properly model monetary policy one must deal with the policy reaction to energy.<sup>19</sup> At one time an economist might handle this by simply inserting the relative price of energy into the VAR as an exogenous variable. But a large body of work now rejects the notion that oil markets are exogenous.<sup>20</sup>

Instead, our approach is to let  $\tau$  be a shock to the relative price of energy. Based on Ireland's argument we change our long-run structural assumptions to allow this shock to also affect inflation due to the Fed's reaction:

$$\Delta \pi = \alpha_{\pi \lambda} \lambda + \alpha_{\pi \mu} \mu + \alpha_{\pi \tau} \tau . \tag{21}$$

Otherwise  $\theta(1)$  remains the same as before.

Now there is one more parameter in  $\theta(1)$  than the number of coefficients in R(1). This makes at least some parameters in a structure under-identified. In spite of not being able to identify all structural parameters from an estimate of R(1), explicit relationships between R(1) coefficients and structural parameters are still obtainable. The effects of the shock to inflation in our statistical model are interpreted in light of this misspecification. To do this we will make reasonable assumptions about the long-run effects of some of the shocks on some of the variables. Based on qualitative assumptions about the sign of specific structural effects we will be able to determine something important about the structure. This is because we will be able to sign the bias obtained by the long-run recursive model.

Given our model of a structure which permits inflation to react to oil prices,  $\theta(1)$  becomes:

<sup>&</sup>lt;sup>19</sup> If energy shocks acted just like technology shocks, our model will have already handled them. But now we will be allowing for another type of supply shock that raises output and lowers inflation but has no long-run effect on average labor productivity.

<sup>&</sup>lt;sup>20</sup>Kilian (2009) is a widely cited example from that literature.

$$\theta(1) = \begin{bmatrix} \alpha_{y/n,\lambda} & 0 & 0 \\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} & \alpha_{\pi\tau} \\ \alpha_{y\lambda} & \alpha_{y\mu} & \alpha_{y\tau} \end{bmatrix}.$$
 (22)

Clearly the structure is no longer long-run recursive and so  $R(1) \neq \theta(1)$ . Insert the previous matrices for R(1) and  $\theta(1)$  into equation (16):

$$\begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} R_{11} & R_{21} & R_{31} \\ 0 & R_{22} & R_{32} \\ 0 & 0 & R_{33} \end{bmatrix} = \begin{bmatrix} \alpha_{y/n,\lambda} & 0 & 0 \\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} & \alpha_{\pi\tau} \\ \alpha_{y\lambda} & \alpha_{y\mu} & \alpha_{y\tau} \end{bmatrix} \begin{bmatrix} \alpha_{y/n,\lambda} & \alpha_{\pi\lambda} & \alpha_{y\lambda} \\ 0 & \alpha_{\pi\mu} & \alpha_{y\mu} \\ 0 & \alpha_{\pi\tau} & \alpha_{y\tau} \end{bmatrix}$$

Then use this expression to solve for each R(1) coefficient as a function of structural parameters. It is easy to show that the first column of R(1) is equal to the first column of  $\theta(1)$ .

$$R_{11} = \alpha_{\nu/\nu} \qquad R_{21} = \alpha_{\pi\lambda} \qquad R_{31} = \alpha_{\nu\lambda} \tag{23}$$

Hence, the structure in Equation (22) allows the particular long-run recursive model to identify long-run effects of the technology shock on each of the variables.<sup>21</sup> However, the second shock conflates effects of the exogenous shocks to oil price and money growth:

$$R_{22} = \sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\tau}^2} \qquad R_{32} = \frac{\alpha_{\pi\mu}\alpha_{y\mu} + \alpha_{\pi\tau}\alpha_{y\tau}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\tau}^2}}$$
(24)

Given our modification to the original structure, the estimates focused on earlier now are inconsistent estimates of the output effect of an exogenous money growth shock.<sup>22</sup>:

$$\frac{R_{32}}{R_{22}} = \frac{\alpha_{\pi\mu}\alpha_{y\mu} + \alpha_{\pi\tau}\alpha_{y\tau}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\tau}^2} \tag{25}$$

<sup>&</sup>lt;sup>21</sup>This is not surprising. The structure is of the class of long-run partially recursive systems. That means  $\theta(1)$  is block recursive, equations within one or more of the blocks have a recursive ordering, and the shocks are uncorrelated. In structures of this form a properly ordered long-run recursive model will identify the effects of shocks to a block of structural equations which is recursive. See Keating (2002).

<sup>&</sup>lt;sup>22</sup>Note the ratio is unaffected by taking the negative or the positive square root solution to  $R_{22}$ .

Dividing numerator and denominator by  $\alpha_{\pi\mu}^2$  rewrites the equation in a useful form:

$$\frac{R_{32}}{R_{22}} = \frac{\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}} + \frac{\alpha_{y\tau}\alpha_{\pi\tau}}{\alpha_{\pi\mu}^2}}{1 + \frac{\alpha_{\pi\tau}^2}{\alpha_{\pi\mu}^2}}$$
(26)

because it isolates  $\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}$  within the expression. This ratio of structural parameters represents the long-run output effect of a one percentage point exogenous increase in money growth, which is what we'd like to estimate. Unfortunately  $\alpha_{\pi\tau} \neq 0$  makes this impossible.

Equation 26 illustrates the two terms that may bias the estimate from the desired ratio of structural parameters. To interpret this equation we assume long-run structural parameters are subject to three inequalities:

S1: 
$$\alpha_{\pi\mu} > 0$$

S2: 
$$\alpha_{y\tau} > 0$$

S3: 
$$\alpha_{\pi\tau} < 0$$
.

Assumption S1 states that a permanent exogenous increase in the growth rate of money permanently raises the inflation rate. This assumption is uncontroversial.<sup>23</sup> S2 indicates an exogenous permanent reduction in energy prices raises output in the long run.<sup>24</sup> Assumption S3 states that a permanent decline in energy prices will result in a permanent decline in the rate of inflation. This assumption comes directly from Ireland (2007).

In light of these structural assumptions we now examine the trivariate model's estimate of the orthogonalized shock to inflation for each of three possible scenarios:  $\alpha_{y\mu} > 0$ ,  $\alpha_{y\mu} = 0$ , and  $\alpha_{y\mu} < 0$ . The first case, when a permanent increase in money growth raise output in the long run is a Mundell-Tobin Effect. When  $\alpha_{y\mu} > 0$  is combined with our 3 structural assumptions the model estimate is biased downward from the structural effect:

$$\frac{R_{32}}{R_{22}} = \frac{\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}} + \frac{\alpha_{y\tau}\alpha_{\pi\tau}}{\alpha_{\pi\mu}^2}}{1 + \frac{\alpha_{\pi\tau}^2}{\alpha_{\pi\mu}^2}} < \frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}.$$
 (27)

<sup>&</sup>lt;sup>23</sup>One might be tempted to assume  $\alpha_{\pi\mu}=1$  based on Friedman's inflation principle. But that restriction would be a mistake, and not because Friedman was wrong! The unit variance for each shock means that each  $\alpha_{vs}$  parameter is the product of the structural effect of shock s on variable v multiplied by the standard deviation of shock s.

<sup>&</sup>lt;sup>24</sup>In Ireland (2007), cost-push shocks are unable to have permanent output effects. That would change if he allowed those shocks to follow a random walk rather than the stationary AR(1) process he assigned to them.

The inequality obtains because if money growth is endogenous the second term in the numerator is negative and the denominator is greater than 1. Two important implications arise from this result. First, this ratio of statistical model parameters may be zero or negative when a Mundell-Tobin effect is operational. Secondly, a positive estimate implies that if money growth is endogenous the Mundell-Tobin effect is expected to be even larger.

The second case is when the economy exhibits superneutrality. Combining  $\alpha_{y\mu}=0$  with our other assumptions implies a negative estimate:

$$\frac{R_{32}}{R_{22}} = \frac{\frac{\alpha_{y\tau}\alpha_{\pi\tau}}{\alpha_{\pi\mu}^2}}{1 + \frac{\alpha_{\pi\tau}^2}{\alpha_{\pi\mu}^2}} < 0.$$
 (28)

Hence, downward bias due to endogenous money growth also occurs in this bivariate statistical model when an economy exhibits superneutrality.

And finally consider what happens when  $\alpha_{y\mu} < 0$ . In this is reverse Mundell-Tobin Effect case:

$$\frac{R_{32}}{R_{22}} = \frac{\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}} + \frac{\alpha_{y\tau}\alpha_{\pi\tau}}{\alpha_{\pi\mu}^2}}{1 + \frac{\alpha_{\pi\tau}^2}{\alpha_{\pi\mu}^2}} < 0$$
(29)

as both terms in the numerator are less than zero and the denominator continues to be positive. Under a reverse Mundell-Tobin effect  $\frac{R_{32}}{R_{22}}$  and  $\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}$  are each less than zero. However, we can't tell if they are equal to one another or if one is smaller than the other. This ambiguity arises because the second term in the numerator makes the estimate even more negative than  $\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}$  while the second term in the denominator pushes the negative number toward zero.

This analysis leads to two conclusions. First, negative estimates are uninformative about the long-run impact of money growth on output when money growth is endogenous. A negative estimate may occur if a Mundell-Tobin effect, or superneutrality, or a reverse Mundell-Tobin effect characterizes the economic structure. Secondly, positive estimates are indicative of a Mundell-Tobin effect whether or not money growth is endogenous to supply shocks. The implication for our trivariate model is that the positive US estimate is probably biased downward and the actual Mundell-Tobin effect is likely to be even larger.

# 5. Endogeneity and the Long-run Parameters in Bivariate Models

Clearly, by not controlling for endogeneity bivariate models that identify permanent and transitory shocks to inflation would be biased. Rather than discard a substantial body of research due to potential misspecification, we will examine the bias in those models to determine if that evidence is informative.

In this case consider a system of equations where  $X_t = (\pi_t, y_t)$  and the structural shocks are given as  $\varepsilon_t = (\mu_t, \lambda_t)$ . In this case the long-run parameter matrix can be specified as:

$$\theta(1) = \begin{bmatrix} \alpha_{\pi\mu} & \alpha_{\pi\lambda} \\ \alpha_{y\mu} & \alpha_{y\lambda} \end{bmatrix}. \tag{30}$$

The way statistical model and and this structure are related comes from inserting into equation (16) and R(1) from equation (20). Then one easily solves for the ratio of estimated parameters that interests us in terms of structural parameters:

$$\frac{R_{yP}}{R_{\pi P}} = \frac{\alpha_{\pi\mu}\alpha_{y\mu} + \alpha_{\pi\lambda}\alpha_{y\lambda}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2} \tag{31}$$

Note that this is almost identical to (26). Next we apply a set of structural assumptions:

S4: 
$$\alpha_{u\lambda} > 0$$

S5: 
$$\alpha_{\pi\lambda} < 0$$
.

We continue to assume S1 and now add S4 and S5. Based on these assumptions it is easy to show every result for  $\frac{R_{32}}{R_{22}}$  derived in the previous section carries over directly to  $\frac{R_{yP}}{R_{\pi P}}$  except  $\tau$  is replaced by  $\lambda$ .

Hence, the implications for this bivariate model are much the same. Negative estimates are uninformative because they may occur when money growth is endogenous, if a Mundell-Tobin effect, superneutrality, or a reverse Mundell-Tobin effect is operational. Secondly, positive estimates are indicative of a Mundell-Tobin effect whether or not money growth is endogenous to supply shocks.

In about half of the low inflation countries for which Bullard and Keating (1995) estimated the bivariate model the long-run output effect from a permanent shock to inflation was positive but statistically insignificant. That paper argued this evidence failed to reject superneutrality. In fact,

9 of the 10 countries with inflation average less than 11% obtained positive estimates. The US was the only one with a negative estimate but it was insignificant. And the 6 countries with inflation over 10% obtained a negative or essentially zero estimate. Random sampling error might explain insignificance, but it is unlikely to have caused almost all the insignificant estimates for low inflation countries to be positive.

Estimates we provide earlier imply that controlling for endogenous money growth was important for the US. Furthermore, 3 low inflation countries - Japan, Ireland and Spain - have positive estimates that are very nearly significant. The downward bias in bivariate models that fail to endogenize money growth may be causing estimates to be insignificant.

#### 5.1 Downward Bias may be Prevalent

The condition for downward bias in the bivariate model's estimate is:

$$\frac{R_{yP}}{R_{\pi P}} < \frac{\alpha_{y\mu}}{\alpha_{\pi\mu}} \quad . \tag{32}$$

Combining this inequality with equation (31) and simplifying, yields a condition on structural parameters under which there is downward bias due to endogenous money growth:

$$\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}} > \frac{\alpha_{y\lambda}}{\alpha_{\pi\lambda}}.\tag{33}$$

Under our assumptions, the right hand side of the inequality is negative. And given that  $\alpha_{\pi\mu} > 0$  the model's estimate of the long-run output effect of a permanent increase in inflation is downward biased even when the economy experiences superneutrality or a reverse Mundell-Tobin effect. If  $\alpha_{y\lambda}$  and  $\alpha_{\pi\lambda}$  are of equal in absolute value, then downward bias occurs whenever  $\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}$ . is greater than -1.0. And if a technology shock has a larger effect (in absolute value) on output than inflation the right side of equation (33) would be even more negative than -1.0. We conclude that the bivariate model's estimate seems to be downward biased over a wide range of possible structures, not just when a Mundell-Tobin effect is germane. This suggests the high inflation countries for which Bullard and Keating (1995) estimated zero parameter values, and possibly even countries with negative estimates, a Mundel-Tobin effect may be operational.

## 6. Concluding comments

Understanding the real effects of inflation has always been an important task in macroeconomic research. The topic is even more relevant now that the Federal Reserve recently expressed willingness to allow inflation to rise temporarily about its inflation target, provided it has been for some time below target. Another reason for renewed interest is that the debate amongst macroeconomists about the benefits and costs of inflation has been rekindled with some economists advocating a higher target rate. In contrast to most related empirical work with VARs, our model endogenizes the way money growth and inflation react to real shocks. We use it to examine US data, in part, because previous research found US results are qualitatively different than findings for many other developed economies with low inflation. Controlling for endogenous money growth we estimate a statistically significant positive long-run output effect of an exogenous increase in inflation. The finding remains when a variety of interesting data samples are used. The paper then shows this positive output effect is robust to misspecification of a very plausible type - the omission of energy price shocks from the model. Also, we re-examine some findings from previous empirical work under this misspecification from endogenous money growth. These papers found relatively weak evidence against superneutrality. We show that the bias from assuming permanent shocks to inflation are exogenous may explain why these models obtained weakened evidence of Mundell-Tobin effects.

Our findings indicate macroeconomic benefits may occur when a central bank allows the inflation rate to be higher. Mundell-Tobin effects appear relevant, particular in low inflation economies. But do these effects raise the optimal target rate for inflation? Much prior work on this question has been based on models which do not permit a substantial Mundell-Tobin effects. The optimal inflation target seems likely to depend on the manner in which Mundell-Tobin effects are modeled as as well as the mechanism through which welfare is adversely affected by inflation. These questions are interesting, potentially important, and topics we plan to address in future work.

Table 1: Long-Run Percentage Change in Output In Response to a Permanent 1 percentage point Increase in Output Per Hour

	1960-1992	1948-1992	1960-2018	1948-2018
Endogenous Fed Model: Quarterly data	1.00** (0.34,1.66)	0.93** (0.37,1.47)	0.63** (0.03,1.22)	0.71** (0.42,0.99)

Endogenous Fed Model: 3 variable model with  $[\Delta ln(y/h), \Delta \pi, \Delta ln(y)]$  allows for the Fed to endogenously respond to changes in productivity

\*\* Zero is excluded form the 90% error band

Table 2: Long-Run Percentage Point Change in Inflation In Response to a Permanent 1 percentage point Increase in Output Per Hour

	1960-1992	1948-1992	1960-2018	1948-2018
Endogenous Fed Model: Quarterly data	$-0.30^*$ (-0.67,0.07)	-0.04 (-0.47,0.39)	$-0.23^*$ (-0.47,0.01)	-0.05 $(-0.35, 0.25)$

Endogenous Fed Model: 3 variable model with  $[\Delta ln(y/h), \Delta \pi, \Delta ln(y)]$  allows for the Fed to endogenously respond to changes in productivity
\*\* Zero is excluded form the 90% error band

Table 3: Long-Run Percentage Change in Output In Response to a Permanent 1 percentage point Exogenous Increase in Inflation

	1960-1992	1948-1992	1960-2018	1948-2018
Endogenous Fed Model: Quarterly data	$0.56^{**}$	0.41**	$0.64^{**}$	0.51**
	(0.06, 0.1.05)	(0.11,0.70)	(0.15,1.15)	(0.21,0.81)

Endogenous Fed Model: 3 variable model with  $[\Delta ln(y/h), \Delta \pi, \Delta ln(y)]$  allows for the Fed to endogenously respond to changes in productivity

<sup>\*</sup> Zero is excluded form the 68% error band

 $<sup>^*</sup>$  Zero is excluded form the 68% error band

 $<sup>^*</sup>$  Zero is excluded form the 90% error band

<sup>\*</sup> Zero is excluded form the 68% error band

Table 4: Long-Run Percentage Change in Output In Response to a Permanent 1 percentage point Increase in Inflation

	1960-1992	1948-1992	1960-2018	1948-2018
Bullard and Keating Model: Quarterly data	0.29 (-0.40,1.03)	0.69** (0.31,1.10)	0.44* (-0.13,1.03)	0.74** (0.40,1.10)

Bullard and Keating Model: 2 variable model with  $[\Delta\pi, \Delta ln(y)]$  \*\* Zero is excluded form the 90% error band \* Zero is excluded form the 68% error band

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