Distributionally Sensitive Cost-Benefit Analysis*

Robin Boadway
Queen's University
boadwayr@econ.queensu.ca

Michael Smart
University of Toronto
michael.smart@utoronto.ca

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Abstract

We propose a method for cost-benefit analysis of public policies that identifies potential Pareto improvements when losers from a reform are compensated through income tax changes. Reforms are desirable when they decrease aggregate excess burden in commodity and labor markets. This condition is equivalent to a weighted sum of individual compensating variations, where weights reflect the impacts of the policy reform on individuals' labor incomes, rather than marginal social valuations of transfers to them. We identify cases in which distributional weights are increasing in individuals' marginal income tax rates. We apply the model to analyze child care subsidies.

Keywords: applied welfare economics, weighted surplus, marginal value of public funds

JEL: D6, H2, H4, I3

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1 Introduction

Economic policy reforms frequently result in gains to some individuals and losses to others, rendering the overall impact on society to be ambiguous. The typical approach to cost-benefit analysis (e.g. Harberger, 1971) in such cases is to sum measures of individual gains and losses (or "surplus"). If total surplus from the reform is positive, then gainers could in principle compensate losers through lump-sum transfers to accept the reform (Kaldor, 1939), or losers could not afford to compensate gainers to block the reform (Hicks, 1939), and a potential Pareto improvement is said to exist. The total surplus approach therefore offers the tantalizing possibility that program evaluation can focus on efficiency implications of reforms, leaving distributional considerations for other branches of government. The total surplus approach to cost-benefit analysis is controversial. The compensation envisaged by the Hicks and Kaldor tests is hypothetical, and need not be paid in practice. Moreover, the Hicks and Kaldor tests assume that compensation would be paid through lump-sum transfers, whereas real-world compensation systems would presumably have distortionary costs.[1] These issues have prompted some (e.g. Drèze and Stern, 1987; Boadway, 2016) to advocate a weighted surplus approach, in which individual gains and losses are assigned weights reflecting the presumed social value of transfers to each individual. Recently, the Biden administration has proposed that distributional weighting should become a standard element of applied cost-benefit analysis when distributional impacts of policies are important.[2]

Harberger (1978) rejected the distributionally weighted surplus approach, arguing that it could lead to adoption of highly inefficient reforms mainly to achieve a redistributive goal, when other methods of redistribution could achieve the same at lower cost.[3] Harberger recognized that compensating economic losses from reforms through the income tax system would be costly, and recommended a rough rule-of-thumb in which distributional transfers would receive a weight of 10 percent (p. S115–S116) to reflect this. Hicks himself recognized the importance of distortionary compensation,[4] writing (1939, p. 712) that since

[1]There are other concerns with the Hicks-Kaldor tests as well. Scitovsky (1941) showed that the Hicks and Kaldor criteria can lead to preference cycles. Boadway (1974) showed that a positive sum of compensating variations was necessary but not sufficient for a potential Pareto improvement. In this paper, we abstract from these technical considerations by assuming that producer prices are unchanged with the reforms we study.

[2]Specifically, it contemplates "benefit-cost analysis that applies weights ... to account for the diminishing marginal utility of goods when aggregating those benefits and costs." Office of Management and Budget Draft Circular A-4, April 6, 2023.

[3]As Harberger put it, "Never attribute to an action a benefit that exceeds the alternative cost of achieving the same result."

[4]This view was echoed by Samuelson (1958) who called for analysis of "the problem of 'the feasible optimum' along the lines of Ramsey and Boiteux." (p. 540)

> the best methods of compensation feasible involve some loss in productive efficiency, this loss will have to be taken into account. In practice, it is not unlikely that we shall have to reject on these grounds many measures which would be approved of by the traditional analysis, but which would only be reckoned by that analysis as offering a small gain.

But neither Hicks nor Harberger offered a formal analysis of distortionary compensation nor proposed a specific method of calculating it.

In this paper, we take up the challenge of the Hicks–Harberger perspective, deriving cost-benefit test rules that characterize potential Pareto improvements (PPIs) that are feasible for government. In our framework, gainers and losers from policy reforms are compensated through hypothetical changes to an arbitrary non-linear labor income tax in order to keep each individual's utility fixed at the pre-reform level. This is a feasible version of the compensation test proposed by Kaldor (1939). Distributional impacts of policies therefore matter to the extent that they change the cost of redistribution through the income tax system (Weisbach, 2015). When a reform compensated in this way causes government revenue to rise, then under weak conditions the government may redistribute the additional revenues to obtain a hypothetical Pareto improvement. Thus our framework identifies *feasible PPIs*, extending the Hicks–Kaldor approach to incorporate distortionary costs of compensation explicitly.

In our key result (Proposition 1), we show that a public policy reform is a feasible PPI if and only if it reduces the sum of individual excess burdens in all commodity and labor markets. Excess burden is a standard tool of tax analysis that measures an individual's willingness to pay to abolish a given tax system, net of the revenue it generates for government. Thus excess burden aggregates the benefits and costs of price distortions for consumers and the government. This result demonstrates that policies should be evaluated based on an *unweighted* excess burden criterion, irrespective of the distributional objectives of the government. But, crucially, and consistent with the Hicks–Harberger perspective, our measure of the change in excess burden includes the distortionary effects of the compensation payments themselves. We next examine how our criterion differs from the unweighted total surplus typically calculated by cost-benefit analysts. We characterize the distortion in labor incomes that results from differentially small compensated policy reforms, and offer first-order approximations to the effects of arbitrary large reforms. We show (Proposition 3) that a reform is a feasible PPI when a particular *weighted* sum of individual willingness to pay for the reform is positive. The weights in our formula reflect the effects of the policy reform and its hypothetical compensation on labor incomes and so on tax revenues. In general, our weights do not correspond to the marginal social valuations of income transfers proposed by Drèze and Stern (1987) and others to incorporate distributional concerns into cost-benefit analysis.

To illustrate our framework as simply and as clearly as possible, we study a well-known example from the public economics literature involving reform to consumer taxes and subsidies. This allows us to develop our results using the standard tools of consumer price theory. This tax policy evaluation is a special case. We show how the analysis can be extended to other policy reforms involving changes in prices and quantities that are costly to the government and can make some individuals better off and others worse off. These can include, for example, regulatory changes and public investment expenditures.

To illustrate our method, we simulate a compensated reform to child care subsidies that may affect labor supply, and we calibrate the simulation using estimates of the relevant elasticities from the empirical literature. We show how our weighted surplus criterion leads to specific conclusions about the desirability of universal child care subsidies.

1.1 Related literature

Our approach has antecedents in the applied welfare economics literature. In an important early contribution, Hylland and Zeckhauser (1979) studied policy reforms compensated through a distortionary income tax system, similar to our framework. They considered only policies whose benefits are correlated with income, and separable from labor supply in individual preferences. Under these restrictions on preferences, they showed that feasible PPIs can be identified by their impact on unweighted total surplus – a version of the Samuelson rule for evaluating public projects. Konishi (1995) examines small reforms to commodity tax rates compensated through arbitrary non-linear labor income tax reforms and derives a local version of our Proposition 1. Laroque (2005) showed that when preferences for commodities are weakly separable from labor, then reforms to eliminate distortionary commodity taxes can always be compensated through the income tax. Extending this logic, Kaplow (2006a, 2006b, 2008, 2010, 2020) showed that separability implies arbitrary reforms should be evaluated by their impact on unweighted total surplus. But since separability implies that labor supply does not change with compensated reforms, these contributions simply assume away the distortionary impacts of compensation on labor markets. More recent contributions (e.g. Weisbach and Hemel, 2022) have brought renewed attention to the impact of policies on labor supply and income tax revenues, which is disregarded in the Laroque-Kaplow approach. Our framework generalizes the existing results, showing how distributional impacts can be incorporated into cost-benefit analysis in a simple, operational way.

In an influential recent contribution, Hendren (2020) also proposes a method for evaluating policy changes accompanied by income tax reforms to keep individual utilities unchanged.[5] However, our method differs from his in two main ways. For one, we study discrete rather than differential policy changes. For another, our procedure for evaluating policy changes differs from his. Hendren first identifies the willingness of individuals to pay for the policy change and then augments that willingness to pay by the marginal cost of financing transfers to individuals to cover it. The result is a decision criterion that is a weighted surplus rule, where weights may be construed as the marginal social valuation of lump-sum transfers to each individual, if the initial income tax system is Pareto efficient. In Hendren (2020), the weights assigned to each individual income level are assumed to be fixed, independent of the reform being analyzed. Our framework instead endogenizes the marginal cost of compensating each type and shows how it may change with the reform. This leads to our weighted surplus criterion, where weights reflect distortionary impacts of compensation on tax revenues, rather than presumed social valuations of transfers. Indeed, we identify conditions under which our weights are increasing in the marginal income tax rates facing each income group. It is therefore even possible that our weights increase in incomes for an income tax system exhibiting progressivity of marginal tax rates.

Our framework is related to the "sufficient statistics" approach to policy analysis, developing evaluation rules that depend on fiscal externalities from estimable changes in quantities or elasticities that result from reforms. Some of our results parallel the analysis in Kleven (2021), who adopts a social welfare function approach, decomposing policy impacts into equity and efficiency effects. In contrast to Kleven, we characterize feasible PPIs rather than policies increasing social welfare, and we emphasize that the distortionary costs of compensating losses are integral to policy evaluation. Our approach also has antecedents in the empirical policy evaluation literature. It is not uncommon for applied researchers to estimate the fiscal cost of policy interventions that change labor supply by subtracting an estimate of fiscal externalities from the gross program cost (e.g. Bloom et al., 1997; Baker et al., 2008). Our model provides a formal basis for these calculations as a form of distributionally sensitive cost-benefit analysis, once the additional labor supply impacts of hypothetical compensation are also accounted for. Likewise, more recently, Eisenhauer et al. (2015) has shown how the full distribution of program costs and benefits may be estimated from marginal treatment effects in a Roy self-selection model. Our model shows how the resulting net benefit function can be integrated over income levels in order to arrive at an overall net benefit of the intervention that accounts for the distributive effects of heterogeneity.

The compensation payments in our framework, like in Kaldor (1939), may be hypothetical or real. The cost-benefit analysis framework simply identifies the efficiency gains of PPIs. Whether to pay compensation to generate actual Pareto improvements

[5] Schulz et al. (2022) also derives a generalized compensation principle for reforms that affect productivities and wage rates of individuals and use it to analyze the impact of robotization on the U.S. economy.

or not is a separate distributive choice. That said, the case for efficiency-enhancing reforms may be more unambiguous when, in Hicks's words, they are "freed from distributive complications" through the payment of compensation. In practice, reforms are often not accompanied by explicit compensation (Raskolnikov, 2020). An alternative perspective, due to Harberger and Coate (2000), is that the role of cost-benefit analysis is to recommend against the adoption of inefficient policies, and whether to implement efficient reforms or not requires additional judgments about their distributive consequences.

The plan of the paper is as follows. Section 2 presents our model of consumer price changes compensated through reforms to an arbitrary initial income tax system and derives our unweighted excess burden rule. Section 3 characterizes the distortions in labor income that result from feasible hypothetical compensation, and shows how our rule can be restated as a weighted sum of individual surplus amounts. Section 4 applies the model to analyze child care subsidies. Section 4 shows how our results extend to a more general setting similar to that of Drèze and Stern (1987), in which government policies affect both consumer prices and quantities of public goods that influence consumer welfare in quite general ways. Section 6 concludes.

2 Compensated Policy Evaluation

This section develops an efficiency approach to evaluating policy initiatives that can have distributive consequences. Our procedure provides a measure of the net benefit to all individuals and the government of a policy proposal combined with compensating income tax changes that leave all persons at their pre-reform utility levels. The summation of these compensated net benefits provides an exact measure of the efficiency benefits of the policy-cum-compensation. The approach is closely related to the Kaldor (1939) compensation test which also compensates individuals — albeit hypothetically — using redistributive taxes and transfers to return them to their initial utility level. It differs from the feasible Hicks compensation test proposed by Coate (2000), which was in turn the basis for Hendren (2020). The Hicks test considers whether individuals can be compensated in the pre-reform situation to make them as well off as in the reform outcome. In other words, are there redistributive policies that are Pareto-superior to the policy reform under consideration?

2.1 The setting

The economy in our example is analogous to the one that was introduced by Mirrlees (1970, 1976) to study optimal income and commodity taxes and that has been widely emulated. There is a continuum of individuals of mass one indexed by j who differ in a productivity parameter a_j. Production is linear and a_j is also the pre-tax wage rate.

Type-j individuals supply labor ℓ_j and produce output $y_j = a_j \ell_j$, so aggregate output is $\int y_j dj$. When our analysis applies to an individual of a given type, we suppress the subscript j for simplicity. Output is used to provide a vector of n private goods, denoted $\mathbf{x} = (x_1, \cdots, x_i, \cdots, x_n)$ with fixed producer prices $\mathbf{c} = (c_1, \cdots, c_i, \cdots, c_n)$. Consumer prices are $\mathbf{p} = (p_1, \cdots, p_i, \cdots, p_n)$, where $p_i \gtreqless c_i$ for any given good.

The government has two policy instruments. First, it controls consumer prices through its choice of commodity taxes and subsidies. In a later section we consider a more general interpretation of the government policies $\mathbf{p}$. The government also imposes a non-linear tax on income, $T(y)$, so after-tax income is $y - T(y)$. We assume for simplicity that $T(y)$ is everywhere differentiable. Government revenue requirements are fixed, possibly at zero. Importantly, since our analysis focuses on the possibility of Pareto-improving policy changes, we need not specify government (social) preferences except that they respect the Pareto principle.

Individuals have a common strictly concave utility function of the form $U(\mathbf{x}, \ell) = U(\mathbf{x}, y/a)$. Given government choice of consumer prices $\mathbf{p}$ and the income tax $T(y)$, an individual of a given type chooses goods and income to solve the problem:

$$\max_{\{\mathbf{x},y\}} U(\mathbf{x}, y/a) \quad \text{s.t.} \quad \mathbf{px} = y - T(y) \tag{1}$$

The first-order condition on y is $U_\ell = -\lambda a(1 - T'(y))$, where λ is the Lagrange multiplier. The budget constraint in (1) is non-linear, and we assume that the second-order conditions are satisfied so the first-order conditions define a unique optimum, which we assume to be interior. It will be useful in what follows to depict the solution to problem (1) by linearizing the tax function — and therefore the budget constraint — at the optimum. Let the slope of the budget constraint at the optimum be w and the intercept (virtual income) be n, so that the individual's indirect utility function is

$$V(\mathbf{p}, w, n) = \max_{\{\mathbf{x},\ell\}} \left\{ U(\mathbf{x}, \ell) : \mathbf{px} = w\ell + n \right\} \tag{2}$$

The first-order condition on ℓ, or equivalently y, is by $U_\ell = -\lambda w$. The solutions to (1) and (2) will be the same if the first-order conditions on y and the budget constraints coincide,

$$\begin{aligned} w &= a(1 - T') \\ n &= y - T(y) - w\ell = yT'(y) - T(y) \end{aligned} \tag{3}$$

where the last equality uses $w\ell = (1 - T'(y))y$.

Three properties of the indirect utility function are relevant. First, the same function $V(\mathbf{p}, w, n)$ applies to all a_j types since the type a_j is incorporated in the definition of w by (3). Second, since w is an after-tax wage rate, a policy reform will affect w via its effect on $T'(y)$. Third, the inverse of the indirect utility function $V(\mathbf{p}, w, n)$ is the expenditure function $E(\mathbf{p}, w, u)$, defined by:

$$E(\mathbf{p}, w, u) = \min_{\{\mathbf{x},\ell\}} \left\{ \mathbf{px} - w\ell : U(\mathbf{x}, \ell) = u \right\} \tag{4}$$

whose solutions yield compensated demands for goods and supply of labor,

$$E_i(\mathbf{p}, w, u) = x_i(\mathbf{p}, w, u), \qquad E_w(\mathbf{p}, w, u) = -\ell(\mathbf{p}, w, u) \tag{5}$$

and the individual budget constraint can be written

$$E(p, w, u) = n$$

The above analysis applies to individuals of all types and leads to equilibrium outcomes $\mathbf{x}_j$, y_j, w_j and n_j. In what follows, we assume that income y_j is increasing in productivity-type a_j.[6] We can therefore identify types of individuals by their income in the initial situation. Our approach involves comparing outcomes from an initial tax system with those after commodity taxes have been reformed and all individuals have been compensated by income tax changes to return to their initial utility levels. We denote by superscript '0' initial values, so for a type-a_j person initial values of the endogenous variables are $\mathbf{x}_j^0$, y_j^0, w_j^0 and n_j^0 and of the policy variables are $\mathbf{p}^0$ and $T^0(y_j)$. Final values are denoted by superscript '1' so are written $\mathbf{x}_j^1$, y_j^1, w_j^1, n_j^1, $\mathbf{p}^1$ and $T^1(y_j)$.

We focus on the compensated effects of a commodity tax reform on an individual of some type a_j. We focus on a given individual because the same effects apply to all individuals and because in our model there are no fiscal externalities associated with compensating a given individual on others. We then aggregate results for given individuals to obtain a cost-benefit rule for the economy as a whole. Given the assumption that y_j is increasing in a_j, we identify a person earning y_j^0 in the initial equilibrium as being of productivity type a_j. The following analysis applies to that person. For notational simplicity, we drop the type identifier j from all variables.

Our analysis involves compensating all persons for the price change by reforming the income tax so that they stay at their initial utility level. This entails satisfying the following condition for each individual:

$$u^0 = V(p^0, w^0, n^0) = V(p^1, w^1, n^1) \tag{6}$$

where u^0 is utility before the reform. Using the budget constraint $n^i = E(\mathbf{p}^i, w^i, u^0)$, (6) implies that the compensation for the reform is the compensating variation (CV):

$$n^0 - n^1 = E(\mathbf{p}^0, w^0, u^0) - E(\mathbf{p}^1, w^1, u^0) \equiv CV \tag{7}$$

The CV term in (7) combines the effects of changes in both prices and after-tax wages. Using the compensated demand and supply functions in (5), we can decompose

[6] This is a standard assumption in the optimal income tax literature. It is a sufficient (second-order) condition for satisfying the the incentive compatibility conditions (Mirrlees, 1971). Incentive compatibility is satisfied if individuals do not change their income-earning behavior simply to pretend to be someone else who pays less taxes. The necessary condition is that $\partial V/\partial a_j = -U_l(\mathbf{x}(a_j), y(a_j)/a_j)y/a_j^2$.

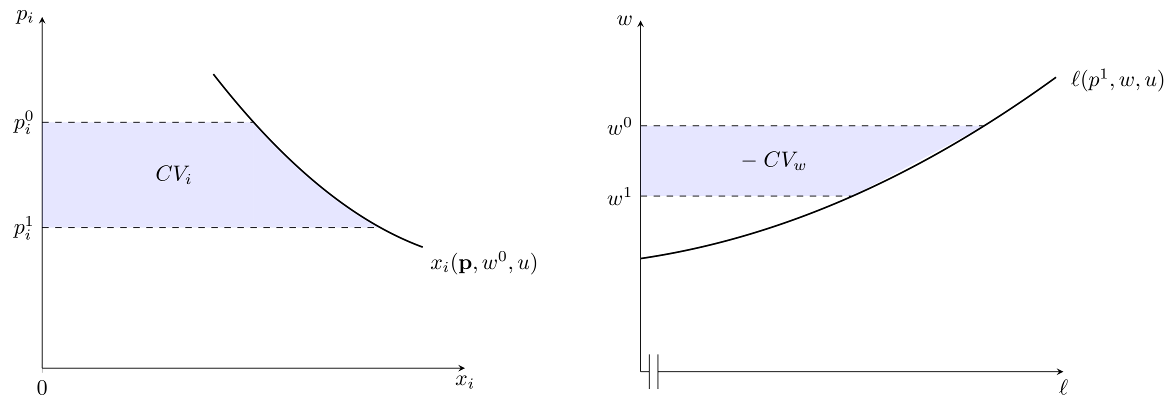


Figure 1: Compensating variation for a compensated reform

Note: The figure depicts the compensating variation for a compensated reform in a single price p_i as the sum of CVs in commodity and labor markets.

the CV into price change effects and after-tax wage change effects as follows:

$$\begin{aligned} \text{CV} &= \int_{\mathbf{p}^1}^{\mathbf{p}^0} \sum_i E_i(\tilde{\mathbf{p}}, w^0, u^0) d\tilde{p}_i + \int_{w^1}^{w^0} E_{\tilde{w}}(\mathbf{p}^1, \tilde{w}, u^0) d\tilde{w} \\ &= \int_{\mathbf{p}^1}^{\mathbf{p}^0} \sum_i x_i(\mathbf{p}, w^0, u^0) d\mathbf{p} - \int_{w^1}^{w^0} \ell(\mathbf{p}^1, w, u^0) dw \end{aligned}$$

This decomposition is depicted in Figure 1. The first term is a line integral representing the changes in CV from a sequence of price changes.[7] Geometrically, the component of the line integral due to a change in p_i is the area to the left of the compensated demand curve for x_i between p_i^1 and p_i^0. Similarly, the second term is the CV from the wage change. It is the area to the left of the compensated labor supply curve between w^0 and w^1. Below we adopt an alternative geometrical interpretation of the change in excess burden.

The CV emerges as the natural way to measure welfare change in our context since we are evaluating policies according to whether a compensated tax reform can make all persons better off than in the initial situation. Equivalently, we consider whether the gainers can compensate the losers through the income tax system so the reform becomes a Pareto improvement over the pre-reform situation. Our approach is parallel to the Kaldor (1939) approach, although unlike Kaldor we assume that compensation can only be done using the distortionary income tax system. Coate (2000) proposes an alternative approach to studying the efficiency of policy reforms. He considers whether a proposed policy reform is Pareto superior to the utility outcomes achieved

[7]There is no path dependency problem with these integrals since the integrands are compensated demand curves. The order in which the prices and wage rates are changed does not affect the value of the CV.

by feasible redistribution policies starting from the initial situation. This will be the case if the losers from the policy change cannot compensate the winners to forgo the change. Measuring utility change by the equivalent variation (EV) — the amount a person would have to be compensated in the initial situation to achieve the utility level after the policy reform — is appropriate in Coate's approach and is equivalent to the feasible Hicks compensation test. Hendren (2020) applies both the Hicks and Kaldor compensation criteria, although in his context there is no difference in magnitude between the CV and EV since he considers only differential policy changes.

2.2 The change in excess burden from a compensated price reform

Our analysis involves using the income tax system to compensate each individual for the change in prices from $\mathbf{p}^0$ to $\mathbf{p}^1$, where we assume the price reforms are the consequence of changes in commodity taxes. To evaluate the net benefit of a compensated tax reform, we must take into account how the tax reform affects both individuals and the government.[8]

A standard way of doing that is by the concept of *excess burden.* The excess burden of a tax system aggregates the benefits to consumers and the government. In our context, the excess burden is the sum of individual compensating variations of the tax system less the revenue cost to the government. We evaluate the excess burden of the tax system in situation $i = 0, 1$ relative to the no-tax situation where $\mathbf{p} = \mathbf{c}$ and $w^i = a$ as follows

$$eb^i = E(\mathbf{p}^i, w^i, u^0) - E(\mathbf{c}, a, u^0) - (\mathbf{p}^i - \mathbf{c})\mathbf{x}^i - (a - w^i)\ell^i \qquad i = 0, 1$$

where we adopt the simplifying notation $\mathbf{x}^i \equiv \mathbf{x}(\mathbf{p}^i, w^i, u^0)$ and $\ell^i \equiv \ell(\mathbf{p}^i, w^i, u^0)$ in what follows where it causes no ambiguity. Note that eb^i is evaluated relative to u^0, the utility level of the initial tax system. This facilitates comparison between the pre- and post-reform situations. The first two terms are the amount of compensation needed to keep the individual at the tax-inclusive utility level, or equivalently, their willingness-to-pay from introducing the tax system $\mathbf{p}^i$. The last two terms are changes in commodity and income tax revenues to the government, the latter reflecting what the equivalent linearized income tax would generate in revenues.[9]

[8] There are three parties involved in the economy: consumers, producers and the government. However, since production is linear, no net benefits or costs accrue to producers, so we need merely tote up net gains to consumers and to the government. Our approach to measuring excess burden is equivalent to that used by Harberger (1978). His Figure 1(b) depicts the net benefit of an excise tax on a single good. It is the loss in consumer surplus (CV) to demanders plus the loss in producer surplus to suppliers less the revenue generated for the government. The result is the standard deadweight loss triangle, which is equivalent to our excess burden discussed in this section.

[9] Note that excess burden is an exact measure of an individual's willingness to pay for removal of

The change in excess burden from the compensated price reform is then:

$$eb^0 - eb^1 = \Big[E(\mathbf{p}^0, w^0, u^0) - E(\mathbf{p}^1, w^1, u^0)\Big] - \Big[(\mathbf{p}^0 - \mathbf{c})\mathbf{x}^0 - (\mathbf{p}^1 - \mathbf{c})x^1\Big] \\ - \Big[(a - w^0)\ell^0 - (a - w^1)\ell^1)\Big] \tag{8}$$

The first term in square brackets is the compensating variation (CV) of the policy change, including both the effect of the change in goods' prices $\mathbf{p}^i$ and of the change in the after-tax wage w^i. It is the amount of compensation the individual requires in the post-reform situation to achieve the utility level of the pre-reform situation. The second term is the change in commodity tax revenues. The third term is the change in income tax revenues that would be generated by the linearized income tax.

The change in excess burden in (8) is for a given individual of type a_j with income y^i_j. Let the distribution of y^i be $F^i(y) = Pr(y^i \leq y)$. We can find the aggregate change in excess burden by summing (8) over all persons:

$$EB^0 - EB^1 = \int eb^0(y)dF^0(y) - \int eb^1(y)dF^1(y)$$

where we adopt the convention of using uppercase letters to refer to aggregate values of variables. If $EB^0 > EB^1$, the policy reform is Pareto-improving in the sense that all persons can be made better off by an income-tax-compensated change in prices $\mathbf{p}$. We say that the policy is efficiency-enhancing. Note that this criterion involves an unweighted sum of changes in excess burden so differs fundamentally from the weighted sum of welfare changes used by Hendren (2020). The difference can be attributed to the way in which distortions involved in the income tax financing of compensation apply. Hendren's criterion weights the sum of CVs or EVs, where the weights incorporate the cost of distortionary finance. In our approach, the summation involves changes in excess burdens rather that CVs, and these excess burdens take into account distortionary financing. Our approach also applies to discrete policy changes, whereas Hendren's methodology is restricted to differential changes.

2.3 Equivalent measures of efficiency changes

Two other measures are equivalent to changes in excess burden. Both of them are intuitive, and both may be more readily implemented. One is the change in government tax revenues resulting from the reform, and the other is the change in the economy's net production.

existing commodity and labor income taxes, which does not rely on a first-order approximation to taxpayer behavior. As such, our framework allows us to arbitrary discrete policy changes.

2.3.1 Change in tax revenues

Using (7), substitute $n^0 - n^1$ for the CV in (8) and use (3) for n^i to obtain after cancelling terms:

$$eb^0 - eb^1 = \left[T^1(y^1) - T^0(y^0)\right] - \left[(\mathbf{p}^0 - \mathbf{c})\mathbf{x}^0 - (\mathbf{p}^1 - \mathbf{c})x^1\right] = g^1 - g^0 \tag{9}$$

where g^i is government tax revenue generated by the individual in situation i ($i = 1, 2$). Aggregating this over all persons yields

$$EB^0 - EB^1 = G^1 - G^0$$

Therefore, aggregate excess burden will fall with the compensated price reform ($EB^0 > EB^1$) if and only if tax revenue rises ($G^1 > G^0$). If $G^1 > G^0$, the government has tax revenues left over after compensating all persons to maintain their initial utility level. These extra revenues can be used to make a Pareto-improvement, for example, by transferring lump sum amounts to some persons through the income tax system.[10]

2.3.2 Change in net production

Suppose instead that we use the definition of $E(\mathbf{p}, w, u)$ from problem (4) to write $E(\mathbf{p}^i, w^i, u^0) = \mathbf{p}^i\mathbf{x}^i - w^i\ell^i$. Substituting this in (8) and using (3), we obtain

$$eb^0 - eb^1 = y^1 - y^0 - \mathbf{c}(\mathbf{x}^1 - \mathbf{x}^0) = \Delta y - \mathbf{c}\Delta\mathbf{x}$$

where $\Delta y - \mathbf{c}\Delta\mathbf{x}$ is this individual's net contribution to production, that is, output less the value of goods consumed at producer prices. Aggregating this over all persons gives:

$$EB^0 - EB^1 = \Delta Y - \mathbf{c}\Delta\mathbf{X}$$

If a policy reduces aggregate excess burden, it will also increase net production. If so, there will be resources left over after all persons have been compensated, and those resources can be used to make some or all persons better off than in the initial situation. In that sense, the policy has improved efficiency.[11]

Thus, we have shown that a policy change involving compensated price changes that reduces aggregate excess burden will also increase aggregate government tax revenue and increase net production in the economy. The following proposition summarizes this.

[10] As Kaplow (2006) notes, lump-sum transfers might violate the government budget constraint if recipients' behavior is discontinuous as would occur, for example, if a small transfer induces a discrete reduction in labor supply. Kaplow assumes that behavior is continuous to avoid this possibility. In these circumstances, a compensated price reform will be Pareto-improving if and only if $G^1 > G^0$.

[11] Note that the equivalence between $G^1 - G^0$ and $\Delta Y - \mathbf{c}\Delta\mathbf{X}$ is a consequence of Walras Law. If in equilibrium, any two of the production constraint, the government budget constraint and individuals' income constraint are satisfied, the third one will be as well.

Proposition 1 *A Pareto-improving compensated price reform is feasible if and only if it reduces excess burden.*

$$G^1 - G^0 = -\left(EB^1 - EB^0\right) \geq 0 \tag{10}$$

The reduction in excess burden is also equivalent to the increase in total production valued at producer prices:

$$-\left(EB^1 - EB^0\right) = (Y^1 - Y^0) - \mathbf{c}(\mathbf{X}^1 - \mathbf{X}^0) \tag{11}$$

Proposition 1 confirms that a compensated price reform is a feasible PPI if and only if it reduces the aggregate excess burden on all goods and labor markets. We return below to characterizing the aggregate excess burden in terms of sufficient statistics.

2.4 Relation to the literature

Proposition 1 is related to a number earlier results that also analyze income-tax compensated policy changes. Hylland and Zeckhauser (1979) considered the evaluation of government expenditure programs whose total monetary benefits were fixed, but whose relative benefits to different individuals were related to incomes. In such a setting, the introduction of an expenditure program could be accompanied by a reform of the income tax such that all persons achieved the same utility level. In these circumstances, the program should be evaluated on efficiency grounds using net monetary benefits without distributional weights. They provided no discussion of how monetary benefits and costs should be measured or the properties of compensating income tax reforms.

Konishi (1995) studied a differential tax reform in the spirit of Corlett and Hague (1953) but in a heterogeneous household economy with non-linear income taxation. Starting with a uniform commodity tax on all goods, he derived conditions under which changing the tax rate on a single good accompanied by an income tax reform to keep all persons at the original utility level would be Pareto-improving. If such a reform is possible (that is, if uniform commodity taxation is not optimal), the direction of reform is that which reduces aggregate excess burden of commodity taxes. This is analogous to Proposition 1, though it only applies to differential changes in a single commodity tax.

Closer to our analysis is Laroque (2005) and Kaplow (2006). They show that an income-tax compensated reform of the commodity tax structure from differentiated to uniform is Pareto-improving if individual preferences are weakly separable in goods and labor.[12] Their result generalizes the classic Atkinson and Stiglitz (1976)

[12]Hellwig (2009) and Boadway and Cuff (2022) establish conditions under which reforms towards uniform taxation can be compensated through reforms to a linear or piecewise linear income tax.

result showing that with weakly separable preferences, if the income tax is optimal, commodity taxes should be uniform. As discussed further below, their result is a special case of Proposition 1 in the sense that their compensated commodity tax reform entails no change in labor supply. That is, with weakly separable preferences, (11) becomes simply $EB^0 - EB^1 = Y^1 - Y^0$.

The results in Proposition 1 suggest that the feasibility of a potential Pareto-improving reform can be judged by using an unweighted aggregate of net benefits, where these can be measured either by the change in excess burdens, the change in tax revenues or the change in net production. This differs from Hendren (2020) who argues for a weighted sum of consumer equivalent or compensating variations. The difference follows from the fact that we have incorporated the distortionary cost of compensation into our net benefit criterion while he has done so sequentially by first finding the compensation required to return individuals to their initial utility level and then calculating the cost to the government of financing that compensation. His approach is facilitated by restricting policies to differentially small ones whereas our approach applies to discrete policy changes.[13]

3 Compensation and labor distortions

Proposition 1 establishes very generally that feasible compensated price reforms can be identified by calculating the unweighted total surplus they generate for consumers net of their fiscal costs, that is, total excess burden. This excess burden rule is conceptually similar to the concept of potential Pareto improvements developed by Hicks (1939) and Kaldor (1939), as adopted in cost-benefit analysis by Harberger (1971, 1978). But, total surplus includes not only the "partial equilibrium" surplus accruing in commodity markets as a direct consequence of the policy reforms. It also includes the change in surplus in labor markets, which in turn reflects the distortionary impacts of compensatory changes in income tax rates resulting from the reform. Labor market distortions resulting from the reform will not in general be proportional at various income levels, and so our cost-benefit criterion (10) is not an unweighted sum of partial equilibrium surpluses in general. In this section and the next, we show how our unweighted excess burdens criterion for a PPI can be expressed as a particular weighted sum of partial equilibrium surpluses, where the weights reflect the efficiency cost of compensation through the hypothetical income tax reform.

To show how labor market distortions should be incorporated into cost-benefit analysis, we first characterize the distribution of post-reform incomes y^1 and of marginal tax rates $T^{1\prime}(y^1)$ that emerge from the simultaneous impact of the partial-

[13] Schulz et al (2022) follow a procedure similar to ours. In their analysis of small policy changes, their criterion for the feasibility of a Pareto-improving change is similar to $G^1 - G^0$. (See their eqs. (11) and (12) in Proposition 1.) They also allow for general equilibrium changes in wage rates.

equilibrium price reforms and from the income-tax compensation itself. Note that the compensation condition (6) must hold at the income levels reported by each type of individual. In the appendix, we differentiate (6) with respect to the pre-reform income to prove this useful result:

Lemma 1 *Assume that $U(\mathbf{x}, \ell)$ is twice continuously differentiable. At every compensated reform policy $(\mathbf{p}^1, T^1(\cdot))$ and income distribution $F^1(\cdot)$,*

$$(1 - T^{1\prime}(y^1))y^1 = \frac{V_n(\mathbf{p}^0, w^0, n^0)}{V_n(\mathbf{p}^1, w^1, n^1)}(1 - T^{0\prime}(y^0))y^0 \tag{12}$$

for almost all y^0.

Equation (12) is a joint condition on post-reform incomes and marginal tax rates that must be satisfied in a compensated reform. To get explicit solutions for the labor income distortion, it is however more convenient to work in the dual price space, characterizing how marginal after-tax wages w change with the reform. To do so, we use the expenditure function $E(\mathbf{p}, w, u)$ defined in (4) to write (12) as

$$\frac{w^1 E_w(\mathbf{p}^1, w^1, u^0)}{E_u(\mathbf{p}^1, w^1, u^0)} = \frac{w^0 E_w(\mathbf{p}^0, w^0, u^0)}{E_u(\mathbf{p}^0, w^0, u^0)} \tag{13}$$

This defines the compensatory marginal after-tax rate, say $w^1 = w(\mathbf{p}^1, u^0)$, that must be paid in the post-reform equilibrium to the agent-type that receives utility u^0 in the status quo. (We suppress the dependence of the function on the initial prices $(\mathbf{p}^0, w^0)$ for notational convenience.) The resulting reported income[14] is then

$$y^*(\mathbf{p}^1, u^0) = -aE_w(\mathbf{p}^1, w(\mathbf{p}^1, u^0), u^0) \tag{14}$$

Equation (14) characterizes the distortion in reported incomes that results from the compensated policy reform, which in turn determines the change in surplus in labor markets which forms part of our general cost-benefit rule (6). But (14) depends on derivatives of the consumer expenditure function on the linearized budget constraint, which is not generally known to the cost-benefit analyst. It is therefore preferable to restate our cost-benefit rule using quantities that are observed or estimable for the researcher. To do so, we now focus on "small" policy reforms $d\mathbf{p} = \mathbf{p}^1 - \mathbf{p}^0$ and calculate differential changes to (6) and (12). Later, we discuss how the effects of "large" policy reforms can be approximated with this approach.

The compensated commodity demand and labor supply functions on the linearized budget constraints are given by (5). We define compensated wage elasticities of

[14]This function generalizes the wage-compensated labor supply function analyzed in Besley and Jewitt (1995) for the case of linear income taxes.

demand and supply in the usual way (Saez, 2001):

$$\varepsilon_{iw} = \frac{wE_{iw}}{E_i} \quad \text{and} \quad \varepsilon_{\ell w} = \frac{wE_{ww}}{E_w}$$

Uncompensated demand and supply functions are $\hat{x}_i(\mathbf{p}, w, n) = E_i(\mathbf{p}, w, V(\mathbf{p}, w, n))$, $i = 1, \ldots, n$, and $\hat{\ell}(\mathbf{p}, w, n) = -E_w(\mathbf{p}, w, V(\mathbf{p}, w, n))$, so that the Slutsky decomposition is

$$\hat{\varepsilon}_{iw} = \varepsilon_{iw} + \eta_i \quad \text{and} \quad \hat{\varepsilon}_{\ell w} = \varepsilon_{\ell w} + \eta_\ell \tag{15}$$

where $\hat{\varepsilon}_{iw}$ and $\hat{\varepsilon}_{\ell w}$ are the uncompensated wage elasticities, and the income effect terms are

$$\eta_i = w\ell\frac{\hat{x}_{in}}{x_i} \quad \text{and} \quad \eta_\ell = w\hat{\ell}_n.$$

With this notation in place, we show in the appendix that differentiating (13) with respect to $\mathbf{p}$ yields:

Lemma 2 *The change in labor income from a small compensated price reform is*

$$dy = -\sum_i \left[\varepsilon_{iw} \; - \; \varepsilon_{\ell w}\omega_i\right] \frac{x_i dp_i}{1-T'} \tag{16}$$

where

$$\omega_i = \frac{\ell}{x_i}\frac{\partial w(\mathbf{p}, u^0)}{\partial p_i} = \frac{\hat{\varepsilon}_{iw}}{1 + \hat{\varepsilon}_{\ell w}} \tag{17}$$

is the required compensation in labor income, per dollar of consumer surplus in commodity market i.

Lemma 2 characterizes the production effects resulting from compensated reforms in terms of estimable compensated and uncompensated elasticities. It shows the potential tradeoffs between the direct effects of taxing or subsidizing commodities complementary or substitutable with labor, and the offsetting effects resulting from the necessary income tax compensation. The effect on output resulting from each component of the reform dp_i is proportional to the marginal surplus $-x_i dp_i$ that it creates, and to the magnitude of the resulting substitution effects in demands, represented by the elasticity expression in brackets in (16). The first term in brackets is the direct effect of the policy reform on labor supply, which is proportional to the compensated wage elasticity of demand ε_{iw}. Thus a reduction $-dp_i > 0$ in the price of a net complement for labor ($\varepsilon_{iw} > 0$) reduces labor supply. At the same time, the reform includes a compensatory reduction in the agent's marginal take-home share $1 - T'$, the effect of which is measured by the second term in brackets. This effect through the income tax channel is proportional to the magnitude of the change in the marginal take-home share (which depends on the uncompensated wage elasticities of x_i and ℓ) and to the compensated wage elasticity of labor supply $\varepsilon_{\ell w}$.

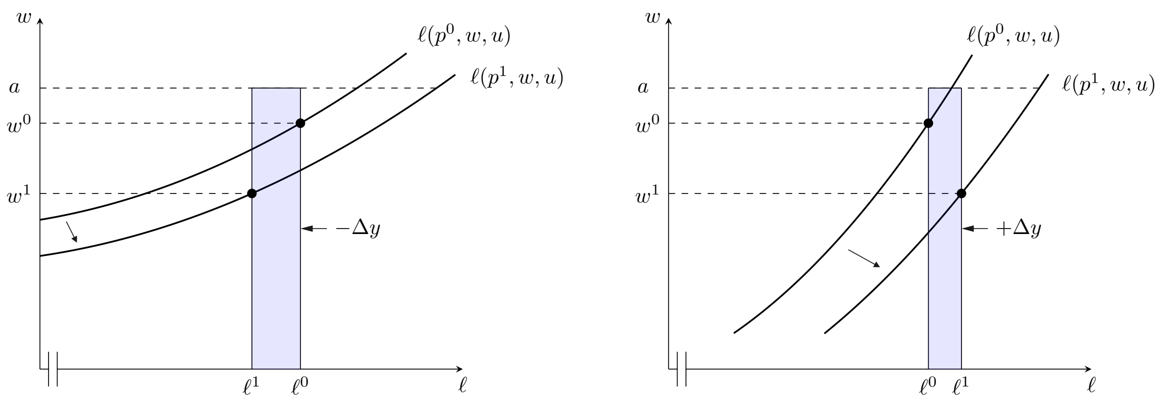


Figure 2: Change in excess burden in the labor market

Note: The figure depicts the labor market response to a subsidy on a product that is a net complement with labor. In the left panel, the labor supply elasticity is high and the excess burden is positive; in the right panel, the labor supply elasticity is small and the excess burden is negative.

On balance, the net effect of such interventions could be positive or negative. Figure 2 depicts the labor market response to a subsidy on a product that is a net complement with labor, for two possible values of the wage elasticity of labor supply $\varepsilon_{\ell w}$. The subsidy shifts the labor supply curve outward from $\ell(\mathbf{p}^0, w, u)$ to $\ell(\mathbf{p}^1, w, u)$. The required compensation through the income tax induces a movement along $\ell(\mathbf{p}^1, w, u)$ from w^0 to w^1. Proposition 1 indicates that the resulting change in excess burden due to labor supply responses equals the change in income Δy, depicted as the shaded regions in Figure 2. In the left panel, $\varepsilon_{\ell w}$ is high and the excess burden is positive; in the right panel, $\varepsilon_{\ell w}$ is small and the excess burden is negative.

To see more clearly the contribution of the elasticity terms to labor market effects, we use the Slutsky decomposition (15) to write (16) as

$$dy = -\sum_i \left[\frac{\varepsilon_{iw}(1+\eta_\ell)}{1+\hat{\varepsilon}_{\ell w}} \;-\; \frac{\varepsilon_{\ell w}\eta_i}{1+\hat{\varepsilon}_{\ell w}}\right] \frac{x_i dp_i}{1-T'}$$

Highly income elastic commodities (those with η_i large) require larger compensatory changes in the income tax schedule for high-income taxpayers, magnifying the distortionary effects of compensation. Highly income elastic commodities should be taxed more (or subsidized less), and so receive a larger weight in our cost-benefit criterion (10).

3.1 Separable preferences

Results in this paper relate the desirability of a reform to its effects on excess burden in commodity and labor markets. Using a framework similar to ours, Laroque (2005) and Kaplow (2006a) showed that a reform to eliminate commodity taxes and subsidies and compensate the change through the income tax system is always feasible – and so Pareto-improving – if labor is weakly separable from commodities in preferences, i.e.

$$U(\mathbf{x}, \ell) = H(\phi(\mathbf{x}), \ell) \tag{18}$$

for some increasing functions $\phi(\cdot)$ and $H(\cdot, \ell)$. This generalized the famous result of Atkinson and Stiglitz (1976) that an optimal non-linear income tax system need not be supplemented by any commodity taxes or subsidies in the separable case. Using the same framework, Kaplow (2006a) showed that arbitrary commodity tax reforms could be evaluated by their impact on unweighted total surplus (or excess burden) in the separable case.

Our results in Proposition 1 and Lemmas 1-2 apply for any smooth utility function, allowing us to strictly generalize the Laroque–Kaplow results. We can therefore examine the appropriate cost-benefit rule in cases where commodity demand and labor supply interact in consumer preferences. We can show that the Laroque–Kaplow results emerge as a special case in our setting:

Proposition 2 *Assume that preferences take the weakly separable form* (18)*. Then a compensated reform in commodity prices:*

(i) leaves reported incomes y^0 unchanged; and

(ii) is feasible if and only if it decreases unweighted excess burdens in commodity markets, i.e.

$$G^1 - G^0 = -\int \mathbf{c} \cdot (\mathbf{x}^1 - \mathbf{x}^0) dF(y^0) \geq 0$$

In the appendix, we prove this using the differential approach of Lemma 2, showing that the elasticity term in brackets in (16) vanishes globally for separable preferences. However, a more direct proof of the proposition originally due to Laroque (2005) is instructive and so is included here.[15]

Let $v(\mathbf{p}, m) = \max\{\phi(x) : \mathbf{p}x \leq m\}$ denote the agent's indirect utility from commodities, given prices and after-tax income m. Because preferences are separable, this function is independent of labor supply. In a compensated reform, $T^1(\cdot)$ may be chosen so that

$$v(\mathbf{p}^1, y - T^1(y)) = v(\mathbf{p}^0, y - T^0(y)) \quad \text{for all } y. \tag{19}$$

[15] It is easily verified that the proof applies to preferences that are pseudo-separable in labor in the sense of Gorman (1976), which strictly generalizes (18).

Faced with the post-reform prices and income tax system, an agent with productivity type a chooses y to maximize

$$H(v(\mathbf{p}^1, y - T^1(y)), y/a) = H(v(\mathbf{p}^0, y - T^0(y)), y/a) \tag{20}$$

Since the agent's utility from any reported income y is unchanged in the compensated reform, it is optimal to choose $y^1(a) = y^0(a)$, i.e. labor income is unchanged. So potential labor market effects of the reform can be ignored and, applying Proposition 1, such a reform is feasible if and only if the reform decreases excess burden in commodity markets.

4 A weighted surplus rule

In the separable case, Proposition 2 shows that the cost-benefit analyst should focus on efficiency effects on commodity markets affected by the price reform, i.e. unweighted changes in excess burdens in commodity markets. There is no need to account for changes in excess burdens in labor markets from income tax reforms. However, Proposition 2 depends crucially on the assumption that labor is separable in consumer preferences, so that compensated reforms induce no changes in labor supply, and therefore no changes in excess burdens on labor markets. To evaluate the desirability of a reform in the general case, Proposition 1 shows that the labor distortions induced by the reform and its compensation must be added to its direct effects on commodity markets. As we show next, when preferences are non-separable, the criterion of Proposition 1 is equivalent to a particular *weighted* sum of individual compensating variations resulting from the reform to commodity prices.

Equation 11 in Proposition 1 implies that a small compensated reform is feasible if and only if it reduces aggregate excess burden; that is, if $dG = dY - \mathbf{c} \cdot d\mathbf{X} \geq 0$. Using the individual budget constraints, this may in turn be expressed as the condition that the aggregate "fiscal externality" from behavioral responses to the reform be positive,

$$dG = \int (T'(y)dy + (\mathbf{p} - \mathbf{c}) \cdot d\mathbf{x})dF(y) \geq 0 \tag{21}$$

which is the differential analogue of (9). The equivalence between marginal excess burden and marginal government revenue from behavioral effects of reforms is a common result in the cost-benefit analysis literature (e.g. Harberger, 1971; Kleven, 2021). Behavioral responses to the reform have no first-order effect on individual welfare (due to the envelope theorem), but they do affect government revenue, which is captured by the usual cost-benefit test. In (21), however, the behavioral responses $(dy, d\mathbf{x})$ also incorporate the effects of the reform on inequality among agents, through the efficiency costs of the compensatory changes to income taxes, which are captured in the general equilibrium behavioral functions $y = -aE_w(\mathbf{p}, w(\mathbf{p}, u^0), u^0)$ and $x_i = E_i(\mathbf{p}, w(\mathbf{p}, u^0), u^0)$.

The behavioral responses $(dy, d\mathbf{x})$ in (21) cannot be directly observed from real-world policy reforms, since they incorporate the effects of hypothetical compensation through the income tax system. To understand the resulting cost-benefit test, it is necessary to relate the responses to underlying structural elasticities. In the appendix, we derive the following "sufficient statistics" formula for our distributionally sensitive cost-benefit test:

Proposition 3 *Consider a small reform* $d\mathbf{p}$ *that induces consumer surplus* $ds_j = -x_j dp_j$ *for an individual with status quo consumption level* x_j *in commodity markets* $j = 1, \ldots, n$. *A compensated reform is feasible if and only if*

$$dG = \sum_j \int \left[\frac{\varepsilon_{jw} - \varepsilon_{\ell w}\omega_j}{1 - T'} + \sum_i \frac{\varepsilon_{ji} - \varepsilon_{\ell i}\omega_j}{1 + \tau_i} \right] (-x_j dp_j) dF \geq 0 \qquad (22)$$

where $\tau_i = (p_i - c_i)/c_i$ *is the status quo ad valorem tax rate.*

Proposition 3 expresses the cost-benefit test criterion as a weighted sum of each individual's partial equilibrium surplus from the reform, $-\sum x_j dp_j$, where the individual weights, the terms in square brackets in (22), depend on estimable compensated and uncompensated elasticities. This expression is complicated in general, because of the interplay of pre-existing distortions τ_j and substitution effects in commodity markets.[16] The weighted surplus rule becomes clearer if we assume that initial commodity tax rates are all equal, $\tau_i = \tau$. In this case, the commodity tax system could be replaced by a scaling of the income tax system to $y - \tilde{T}(y) = (y - T(y))/(1 + \tau)$. We can without loss of generality set $\tau = 0$, and we can evaluate the reform solely by its impact on income tax revenues. This is a simple yet still remarkable result. Corlett and Hague (1953) showed that, beginning from a linear tax on labor income alone in a single-consumer economy, introduction of a small tax on a complement for leisure and reduction in the labor tax would increase consumer welfare. Our approach extends this logic to compensated reforms and redistributive income tax systems.[17]

Applying (22) with $\tau = 0$, we have established:

Corollary 1 *If there are no commodity taxes or subsidies imposed in the status quo allocation, then a compensated reform is feasible if and only if it satisfies the weighted surplus rule*

$$dG = \sum_j \int \underbrace{\frac{T'(y)}{1 - T'(y)}}_{\text{tax factor}} \underbrace{\left(\varepsilon_{jw} - \varepsilon_{\ell w} \frac{\hat{\varepsilon}_{jw}}{1 + \hat{\varepsilon}_{\ell w}} \right)}_{\text{elasticity factor}} \underbrace{(-x_j dp_j)}_{\text{marginal surplus}} dF \geq 0 \qquad (23)$$

[16] For approaches to characterizing changes in excess burden in commodity markets from arbitrary reforms under linear taxes, see Konishi (1995) and Smart (2002).

[17] A similar result was proven in Konishi (1995).

The weighted surplus rule captures the impact of a small reform and its compensation on labor incomes through the elasticity factor, which in turn affects the government budget through the tax factor. These two terms together weight the aggregate partial equilibrium surplus of the reform. The elasticity factor has two components. The first term, ε_{jw}, represents the direct fiscal externality of the policy on income tax revenues. It can be measured directly from the change in income induced by the reform, since[18]

$$-\frac{T'}{1-T'}\varepsilon_{jw}x_j dp_j = T'(y)\frac{\partial y(\mathbf{p}, w, u^0)}{\partial p_j}dp_j$$

is the incremental change in income tax revenues from the direct effect on labor incomes, $\partial y/\partial p_j$. The second term, proportional to $\varepsilon_{\ell w}$, is the fiscal externality resulting from compensation itself, which is absent from the conventional approach to measuring fiscal externalities (e.g. Kleven, 2021). Incorporating this Hicks-Harberger effect into cost-benefit analysis offsets a portion of the fiscal externalities of labor-enhancing reforms, as we show in the application below.

The formula in (22) is valid beginning from any post-reform policy $\mathbf{p}^1$. In principle, the effects of a discrete policy change $\Delta\mathbf{p}$ may therefore be estimated by the first-order approximation $\Delta G \approx \sum \partial G/\partial p_i \Delta p_i$. In practice, since little is known empirically about the derivatives of the relevant elasticities, this will amount to a local approximation at the status quo point $\mathbf{p}^0$.

Observe that distributional weights in (23) do not correspond to the marginal social valuation of transfers to each income group (cf. Hendren, 2020). Indeed, if individual preferences are iso-elastic,[19] the elasticity factor is uniform for all income groups, and the weights in (23) are increasing in marginal income tax rates. Instead of marginal social valuations, weights reflect the fiscal externalities of the reform and its compensation, as conjectured by Weisbach (2015).

5 Application: Child care subsidies

The rise in female labor force participation in recent decades has increased the demand for child care and, with it, created interest among policymakers in subsidy programs to reduce the private cost of child care. Because child care costs are viewed as a significant impediment to labor force participation by secondary earners, there is an enticing possibility that subsidies could increase labor income tax revenues, offsetting some of the fiscal cost of subsidies (Baker et al., 2008) and reducing excess burden in labor markets. At the same time, child care services are consumed disproportionately

[18] This representation uses Slutsky symmetry, i.e. $x_{jw} = -\ell_j$.

[19] Formally, the elasticities in (23) could be independent of income for all y only if preferences were quasi-linear. In general, elasticities do vary with income.

by high-income families. Therefore, any efficiency gains from universal child care subsidy programs through increased labor supply would come at the cost of greater inequality in the incidence of subsidies. Our framework permits us to incorporate this distributive effect that is absent from traditional cost-benefit analysis of child care subsidies, by estimating the additional costs of compensating for it through the income tax.

To illustrate our framework for evaluating these tradeoffs, we consider the effect of a 50 percent universal child care subsidy for Canada. To compensate for the redistributive effects of the reform, we simulate an increase in labor income taxes for secondary earners at each percentile of the family income distribution. Setting tax rates on commodities other than child care to zero, the resulting change in excess burden of Proposition 1 is approximately by

$$\Delta G = \int \left[T'(y)\Delta y + \frac{\sigma}{2}\Delta x_j \right] dF(y) \tag{24}$$

where σ is the subsidy rate on child care expenditures. In (24), the first term reflects the impact of the reform and its compensation on labor incomes, as depicted in Figure 2. The second term reflects the impact of the reform on child care expenditures, for which we employ the usual triangular approximation.

Once again, as in Section 3, our cost-benefit test criterion comprises the direct fiscal externalities of the policy reform, plus the distortionary effects of compensation itself. This is seen more clearly by approximating the quantity changes in (24) as

$$\begin{aligned} \Delta y &= -\left[\frac{\partial y}{\partial p_j} + \frac{\partial y}{\partial w}\frac{\partial w}{\partial p_j} \right] \sigma p_j = \frac{\varepsilon_{jw} - \varepsilon_{\ell w}\omega_j}{1 - T'} \sigma p_j x_j \\ \Delta x_j &= -\left[\frac{\partial x_j}{\partial p_j} + \frac{\partial x_j}{\partial w}\frac{\partial w}{\partial p_j} \right] \sigma p_j = -\frac{\varepsilon_{jj} - \varepsilon_{\ell j}\omega_j}{1 - \sigma} \sigma p_j x_j \end{aligned} \tag{25}$$

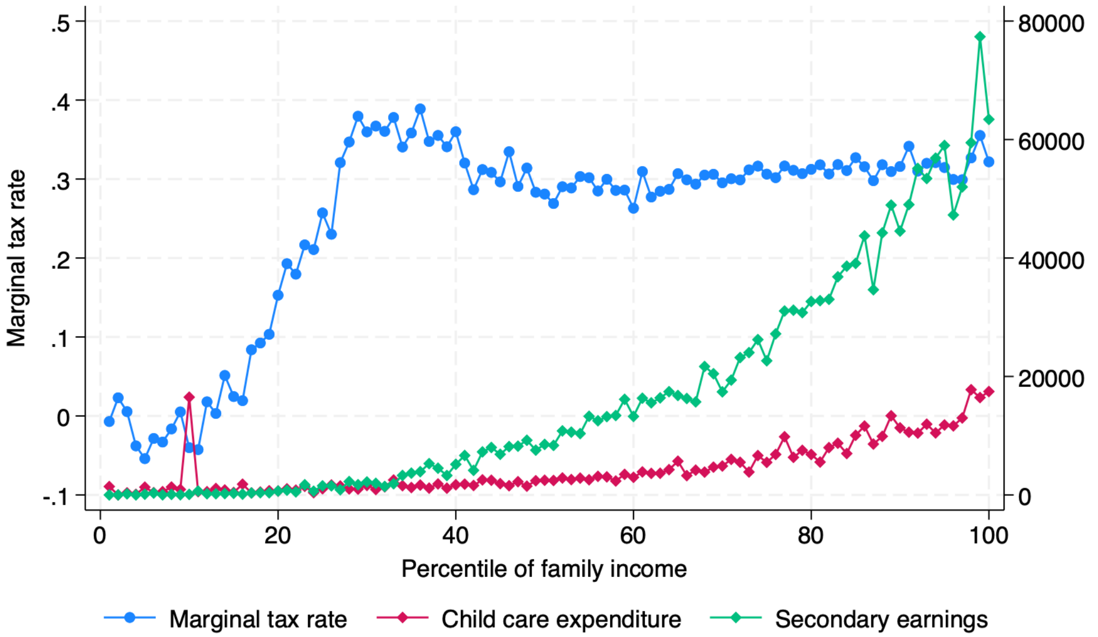
where compensatory changes in after-tax wages are determined from (17).

To calibrate the model, we draw data on incomes and child care expenditures of families with children under the age of six (the age of universal public schooling in Canada) from the Canadian Income Survey, a large pooled cross-sectional survey in the 2012-18 period. The province of Quebec introduced a universal child care program in the 1990s, and we therefore exclude Quebec residents from the simulation.

Figure 3 depicts mean annual secondary earnings,[20] gross child care expenditures,[21] and marginal income tax rates on secondary earners for each percentile of the

[20]Secondary earnings are defined as the lesser earnings of adults in two-parent couples, or earnings of the reference person in single-parent households.

[21]The CIS data report out-of-pocket expenditures on child care. In most Canadian provinces, there are means-tested subsidies available for child care expenses that are paid directly to licensed



Note: Mean values by percentile of family income.

Figure 3: Earnings, child care and taxes

family income distribution.[22] Secondary earnings rise sharply with family income, reflecting the greater proportion of two-earner families at the top of the distribution. Child care expenditures also rise with family income, from approximately \$1100 at the 20th percentile to nearly \$15,000 in the 99th percentile. Thus a universal child care subsidy would be regressive in the sense that a greater proportion of the fiscal benefits would accrue to high-income families. Marginal income tax rates on secondary earners, a key element of our cost-benefit test criterion (24), display a mildly S-shaped pattern. Tax rates peak below the 40th percentile of family income due to the phase-out of means-tested transfers. In particular, Canada has operated a large child tax credit program since 2015, for which benefits are phased out gradually for family incomes above the 20th percentile. Aside from means-tested benefits, the unit of taxation is the individual rather than the family, so that marginal income tax rates on secondary earners are independent of spouses' incomes. Ddue to the individual basis for income taxation and the modest progressivity in marginal tax rates in Canada, secondary marginal tax rates then rise only slowly with family incomes above the

child care providers. To estimate gross pre-subsidy expenditures, we simulate subsidy rates for each family based on incomes and demographics and gross up reported out-of- pocket expenditures by one minus the subsidy rates. See the appendix for details.

[22]We simulate marginal tax rates for each family using the Canadian Tax and Credit Simulator provided by Professor Kevin Milligan.

Table 1: Simulated effects of child care subsidy

	Traditional CBA	Distributionally sensitive CBA	
Income Decile		$\varepsilon_{\ell w}$ =0.4	$\varepsilon_{\ell w}$ =0.8
Lowest	−11	−11	−11
2	−6	−6	−6
3	−2	−5	−7
4	16	4	−4
5	43	22	4
6	80	43	12
7	122	65	17
8	201	103	26
9	292	148	37
Highest	393	203	62
Total	1,125	566	129

Note: The table shows estimated annual total fiscal externalities of a 50% child care subsidy, in millions of real Canadian dollars. The estimated mechanical cost of the subsidy is $5.6 billion. Under the traditional approach to cost-benefit analysis, the aggregate fiscal externality is $1.1 billion. Under the distributionally sensitive approach, it is $566 million assuming a compensated wage elasticity of secondary earnings of $\varepsilon_{\ell w} = 0.4$, and $129 million assuming $\varepsilon_{\ell w} = 0.8$.

median, reaching 32 percent on average in the highest percentile of family incomes

We calibrate the relevant elasticities from estimates in the empirical literature. Examining the 1990s-era introduction of universal child care in Quebec, Baker et al. (2008) estimated the cross-price elasticity of labor supply was $\hat{\varepsilon}_{\ell j} = -0.25$. Research examining other reforms have found labor supply effects of similar or somewhat smaller magnitudes.[23] At the same time, they found that the own-price elasticity of child care services was $\hat{\varepsilon}_{jj} = -0.45$. The greater response of child care use than mothers' labour supply likely reflects substitution from informal care arrangements towards market child care services among existing two-earner families (Havnes and Mogstad, 2011). This substitution induced by the policy is an important component of the excess burden measured in (24).

The remaining key parameters in the simulations are the compensated and uncompensated wage elasticities of secondary earnings. In our base case, we assume the compensated wage elasticity is $\varepsilon_{\ell w} = 0.4$, which is consistent with estimates for mar-

[23]See Morrissey (2017) for a recent survey. For example, Andresen and Havnes (2019) found that a 2002 universal child care subsidy in Norway increased labor force participation of married women by about 50 percent. Havnes and Mogstad (2011) and other researchers, examining earlier reforms, have estimated smaller responses. But the Quebec reform was larger and arguably more salient than most other such reforms.

ried women in western countries reported in Bargain et al. (2014). To estimate the corresponding uncompensated wage elasticity of labor income, we rely on the Slutsky decomposition and the estimates for the U.S. of income effects on labor supply η_ℓ by quartile of income from Golosov et al. (2023). These estimates of wage and income elasticities might be viewed as inconsistent with each other, with the estimates of uncompensated elasticities in Bargain et al. (2014) suggesting either lower income elasticities or higher compensated wage elasticities. To investigate sensitivity of our results to these parameters, we also present results for the case in which $\varepsilon_{\ell w} = 0.8$, which is near the upper end of estimates typically reported in the literature.

Table 1 reports the simulated fiscal externalities of the subsidy for each decile of family total income, using the traditional Hicks-Harberger approach to cost-benefit analysis as well as our own distributionally sensitive approach. Under the traditional approach, fiscal externalities are negligible in the bottom deciles, as additional excess burden in the child care market offsets the relatively small effects on secondary earnings. Fiscal externalities then rise sharply with family income as the effect of the subsidy on earnings rises, reaching \$681 million or \$750 per family in the top quintile. In the aggregate, the traditional fiscal externality is estimated to be \$1.1 billion, or 20 percent of the aggregate mechanical cost of the subsidy, which totals \$5.6 billion. In this sense, child care subsidies strongly pass the traditional cost-benefit test.

But these positive effects of the subsidy come at the cost of greater inequality. An estimated 89 percent of the subsidy accrues to families in the top half of the income distribution, and 52 percent to the top quintile. Compensating these effects through income tax reforms would offset a portion of the positive fiscal externalities of the subsidy, as shown in Proposition 3. Under the baseline assumption that the elasticity of secondary earnings is 0.4, the aggregate fiscal externalities of the compensated reform fall from \$1.1 billion to \$566 million. If the elasticity is 0.8, fiscal externalities are \$129 million, as net benefits of the subsidy are all but eliminated due to the cost of compensating inequality.

Evidently, these quantitative conclusions are specific to the reform being studied, and the assumed impacts on excess burden in commodity and labor markets. The broader message that emerges from the analysis is the relevance of marginal income tax rates in determining distributional weights. For a tax system such as Canada's with a substantial negative income tax component, weights are largest for individuals in the middle and top tails of the income distribution, who generally face the highest marginal tax rates.

6 General policy reforms

The model of Section 2 considered cost-benefit rules for cases where the government sets consumer prices of certain commodities through taxes and subsidies, which is a familiar setting analyzed in much of the previous literature. In fact, our model and our conclusions about cost-benefit rules apply to a much more wider class of policy reforms, encompassing a variety of public goods and regulatory rules as well as corrective taxes for externalities.[24] In this section, we sketch a model of general policy reforms and show how our conclusions apply in this setting.

As before, the planner chooses a policy vector $\mathbf{p}$ and a tax function T, and we consider compensated reforms from $(\mathbf{p}^0, T^0(\cdot))$ to $(\mathbf{p}^1, T^1(\cdot))$. Individuals of productivity type a choose labor income y and a vector of other choices $\mathbf{x}$ to

$$\max_{(\mathbf{x},y)} U(\mathbf{x}, \mathbf{p}, y/a) \quad \text{s.t.} \quad h(\mathbf{p}, \mathbf{x}) \leq y - T(y) \tag{26}$$

Here, the cost to the household of choosing $\mathbf{x}$ when the policy vector is $\mathbf{p}$ is $h(\mathbf{p}, \mathbf{x})$, a twice-differentiable, quasi-convex function. Likewise, let the fiscal cost[25] to the government of policy $\mathbf{p}$ be $f(\mathbf{p}, \mathbf{x})$ when the agent chooses $\mathbf{x}$. The net fiscal contribution of an individual choosing $(\mathbf{x}, y)$ is then

$$g = T(y) - f(\mathbf{p}, \mathbf{x}) \tag{27}$$

This framework incorporates in a very general way consumption public goods p_k for which $\partial U/\partial p_k \neq 0$, as well as public goods which change the marginal cost of private consumption, $\partial^2 h/\partial x_i/\partial p_k \neq 0$. Although we use the term "public good" to describe quantities chosen directly through government policy, our framework can accommodate both publicly-provided private goods, and pure public goods for all individuals consume the same quantity, $x_j = \hat{x}$ for all j. This framework evidently also subsumes the commodity tax-and-subsidy model of Sections 2 and 3 as a special case, with $h(\mathbf{p}, \mathbf{x}) = \mathbf{p} \cdot \mathbf{x}$ as household expenditures on commodities, and $f(\mathbf{p}, \mathbf{x}) = (\mathbf{c} - \mathbf{p}) \cdot \mathbf{x}$ as the fiscal cost of subsidies to consumer purchases.

Let

$$e(\mathbf{p}, \ell, u) = \min_{x} \{h(\mathbf{p}, \mathbf{x}) : U(\mathbf{x}, \mathbf{p}, \ell) \geq u\} \tag{28}$$

be minimum cost of $\mathbf{x}$, given ℓ. Once again, we analyze individuals' choices on the linearized after-tax budget constraint using the value function

$$E(\mathbf{p}, w, u) = \min_{\ell} \{e(\mathbf{p}, \ell, u) - w\ell\} \tag{29}$$

[24]Environmental taxes and "sin" taxes on unhealthy commodities are often criticized for their regressive impacts (Allcott et al., 2019; Conlon et al., 2022).

[25]The fiscal cost of some regulatory policies $\mathbf{x}$ might in principle be zero.

Obviously, $(\mathbf{x}, y)$ attains the maximum in (26) at $u = U(\mathbf{x}, \mathbf{p}, \ell)$ if and only if $\ell = y/a$ attains the minimum of E for (w, n), so that $h - wL = n \iff h = y - T(y)$.

As before, a compensated reform entails choice of income tax parameters (w^1, n^1) for each agent type that satisfy the compensation condition (6). We may define the excess burden of policy $(\mathbf{p}^i, w^i)$ as

$$eb^i = E(\mathbf{p}^i, w^i, u^0) - (a - w^i)\ell(\mathbf{p}^i, w^i, u^0) + f^i \tag{30}$$

which corresponds to our definition of excess burden for the consumer price model, with the fiscal cost of the policy f^i replacing (minus) commodity tax revenues.

Proposition 1 holds for general policy reforms, given the measure of excess burden in (30). That is, a compensated reform is feasible if and only if $G^1 - G^0 = -(EB^1 - EB^0) \geq 0$. While this is an unweighted total surplus criterion, it once again includes the change in surplus in the labor market − which in turn depends on the way in which labor supply is distorted as a result of compensation. The compensated after-tax wage is again a function $w(\mathbf{p}^i, u^0)$ defined in (13). Incorporating general equilibrium responses, the fiscal cost of a policy $\mathbf{p}$ is a function $f^*(\mathbf{p}^i, u^0) = f(\mathbf{p}^i, \mathbf{x}(\mathbf{p}^i, w(\mathbf{p}^i, u^0)), u^0))$.

We may analyze the impact of small reforms using Lemma 1, which is unchanged in this more general setting. Define

$$q_k = -\frac{\partial E(\mathbf{p}, w, u^0)}{\partial p_k} \tag{31}$$

as the individual's marginal willingness to pay for policy p_k.[26] For marginal policy reforms $d\mathbf{p} = (dp_1, \ldots, dp_K)$, the cost-benefit test criterion can be obtained from (10) by differentiating (30) to obtain:

Proposition 4 *A small compensated policy reform* $d\mathbf{p}$ *is feasible if and only if it satisfies the generalized Samuelson rule*

$$dG = \sum_k \int \left[(q_k - f_k^*) + T'(y) \frac{\partial y^*(\mathbf{p}, u^0)}{\partial p_k} \right] dp_k dF \geq 0 \tag{32}$$

This is a generalized Samuelson rule for evaluating public policies. The $q_k - f_k^*$ term in (32) is each individual's marginal valuation of the public policy p_k, net of its fiscal cost. Samuelson (1954) identified Pareto-efficient policies in the first-best case as those for which $\int (q_k - f_k^*) dF = 0$. In (32), the usual Samuelson condition

[26] Lemma 2 holds without change in our more general model, where elasticities are of the shadow prices q_i rather than of demand quantities x_i as in the restricted model of consumer price reforms presented in Section 3. Thus the model tells us that the marginal social value of public policies equals the sum of individual valuations, plus a term representing the policies' impact on labor incomes.

is augmented by an additional term reflecting the effect of the policy on income tax revenues, $\int T' dy dF$, where dy is characterized in Lemma 2. A compensated policy reform is feasible when the sum of the two expressions is positive. The cost-benefit test criterion (32) may be applied to conduct distributionally sensitive cost-benefit analysis of any policy intervention for which heterogeneous net benefits of the policy have been estimated (e.g. Eisenhauer et al., 2015), and the effects of the policy on income tax revenues is known.

When labor supply is separable from policies in preferences, i.e.

$$e(\mathbf{p}, \ell, u) = \hat{e}(\mathbf{p}, \psi(\ell, u)) \tag{33}$$

then we may show, analogous to Proposition 2, that labor supply is unchanged in compensated reforms.[27] The cost-benefit test criterion in (32) then reduces to the Samuelson criterion, i.e. the unweighted total marginal surplus

$$\sum_k \int (q_k - f_k^*) dp_k dF$$

In the separable case, an unweighted total surplus rule should be used to evaluate public goods reforms, independent of distributional concerns.[28]

Our model and main conclusions therefore apply to project evaluation in a general setting in which government policies affect individuals' budgets and utility functions in arbitrary ways. Our model of government policies as bundles of prices and quantities of consumer products complements others' approaches to distributionally sensitive cost-benefit analysis. Hendren (2020) offers a different method of evaluating reforms that lead to changes in lump-sum exogenous income. Likewise, Schulz et al. (2022) develop a cost-benefit test for reforms affecting the distribution of individual wages. In holding the distribution of pre-tax productivity parameters a_j fixed, production in our model is restricted to be additive in individual labor supplies. The effect of reforms and their compensation on pre-tax productivity inequality is a subject left for future research.

[27] A number of versions of the separability restriction have been considered in the previous literature. Thus Hylland and Zeckhauser (1979) showed that a sufficient condition for the unweighted Samuelson rule was that preferences take the form $U(\mathbf{x}, \mathbf{p}, \ell) = F(\mathbf{1} \cdot \mathbf{x} + b(\mathbf{p}, \mathbf{x}), \ell)$ where $\mathbf{1} \cdot \mathbf{x}$ is a composite commodity of private goods, and b is the benefit of the public good in monetary units. Boadway and Keen (1993) and Kaplow (2006b) showed the same for weakly separable utility functions in general, $U(\mathbf{x}, \mathbf{p}, \ell) = F(H(\mathbf{p}, \mathbf{x}), \ell)$. It is easily verified that weak separability of the direct utility function implies (33).

[28] Kaplow (2006b) established the same result for a restricted version of the separability condition (33). Hylland and Zeckhauser (1979) and Shavell (1981) also developed unweighted Samuelson rules in the separable case for evaluating policy reforms that are compensated through the income tax system.

7 Concluding remarks

The quest for an efficiency approach to the evaluation of policies with distributive impacts is longstanding. Hicks (1939) was particularly prescient. In a paper otherwise devoted to an evaluation criterion based on satisfying a hypothetical lump-sum compensation test, he ended by calling for efficiency approaches to evaluation using feasible compensation tests that take into account the loss of efficiency that compensation entails. Harberger (1978) provided a thorough rationale for such an approach and offered some alternatives. Coate (2000) outlined a more formal version of the Hicks proposal involving feasible compensation, albeit without presenting a precise policy evaluation rule.

In evaluating policies for which there can be gainers and losers, we include not only the net benefits of the policy itself to households and the government, but also the net benefits arising from compensating households using the distortionary income tax. We presume the income tax to be the most efficient method available to the government compensate households. We use a particular illustrative example of a tax reform policy, but we indicate its applicability to more general policies involving compensated price changes. The measure of net benefits we use is the excess burden, which includes both the benefits household receive measured by the CV and the revenue costs to the government from both the policy change and the compensation. This measure corresponds with that used by Harberger (1978). We show that the unweighted sum of excess burdens is identical both to the change in government tax revenue and to the change in production in the economy. Importantly, our approach is explicitly developed with large projects in mind.

Our underlying criterion is an unweighted sum of excess benefits of the compensated policy change, where the sum includes excess benefits arising from the compensation itself. We show how this unweighted sum of excess benefits can be transformed into a weighted sum of excess benefits of the policy alone, where the effects of compensation are incorporated into the weights. The weights are based on measurable elasticities so are sufficient statistics, and they are different from the weights derived by Hendren (2020). Our approach can be considered as a formal derivation of cost-benefit rules that are in the spirit of what was proposed by Hicks (1939) and Harberger (1978).

We have derived distributionally sensitive cost-benefit rules for a particular type of policy, that involving price changes resulting from commodity tax changes or regulations. We showed how it could be adapted to other government policies such as public investment projects, as in Hylland and Zeckhauser (1979). It could also be used to evaluate economic shocks, such as wage rate shocks as in Schulz et al. (2022). Policies or exogenous events such as these could have an intertemporal dimension, and our approach could be adopted to take this into account. This might be particularly challenging for longer term policies, like carbon pricing, which have their effects over long multiple generations of persons. Analyzing compensation in this case would be conceptually difficult, not least because of the need for governments to commit to compensation for future generations.

Appendix

A Proofs

Proof of Lemma 1. To economize on notation, we let $z = y^0$ denote the income level reported by an agent of a given type in the status quo economy. Since income is strictly increasing in productivity type under our assumptions, we can write $a(z)$ as the type of agents reporting z in the status quo. Likewise, let $y(z)$ denote the income reported in the post-reform equilibrium by agent type $a(z)$.

Let

$$m^i(y) = y - T^i(y) \qquad i = 0, 1 \tag{A2}$$

denote the after-tax income functions, and observe that by (3)

$$w^i(y) = a{m^i}'(y) \quad \text{and} \quad n^i(y) = m^i(y) - y{m^i}'(y) \tag{A3}$$

The compensation condition (6) therefore holds for $m^1(y)$ if and only if

$$V(\mathbf{p}^1, a(z){m^1}'(y), m^1(y) - y{m^1}'(y)) = V(\mathbf{p}^0, a(z){m^0}'(z), m^0(z) - z{m^0}'(z)) \quad \forall y = y(z) \tag{A4}$$

This is a functional equation in $m^1(\cdot)$ and $y(\cdot)$ that must be satisfied at the compensated post-reform equilibrium. Let

$$\tilde{V}^i(z) = V(\mathbf{p}^i, a(z){m^i}'(y^i(z)), m^i(y^i(z)) - y^i(z){m^i}'(y^i(z))) \tag{A5}$$

denote the utility in equilibrium $i = 0, 1$ of an agent who reports income $y^0(z) = z$ in the status quo $i = 0$ and $y^1(z)$ after the reform. Note that, since $aV_w^i = V_n^i y^i$ (Roy's theorem),

$$\frac{\partial \tilde{V}^i(z)}{\partial z} = V_n(\mathbf{p}^i, w^i, n^i){m^i}'(y^i)y^i \frac{\dot{a}(z)}{a(z)} \tag{A6}$$

where (w^i, n^i) are the parameters of the linearized income tax system at y^i as defined in the main text of the paper.

The compensated income tax system $m^1(z)$ must therefore satisfy

$$\tilde{V}^1(z) = \tilde{V}^0(z) \quad \text{for all } z \in \text{supp } F^0. \tag{A7}$$

Since this is an identity that holds on a set with non-empty interior, the derivatives of the two functions must also be equal for all z in the interior of supp F^0. Applying (A6), it follows that

$${m^1}'(y^1)y^1 = \frac{V_n(\mathbf{p}^0, w^0, n^0)}{V_n(\mathbf{p}^1, w^1, n^1)}{m^0}'(y^0)y^0 \quad \text{for almost all } y^0.$$

Substituting (A2) then yields (12) $\square$

Derivation of (13)**.** Since $V(\mathbf{p}, w, E(\mathbf{p}, w, u)) = u$, we know that $E_u(\mathbf{p}, w, u^0) = 1/V_n(\mathbf{p}, w, n)$. Moreover, $m^i(y) = y - T^i(y)$ implies that $m^{i\prime} = a(1 - T^{i\prime})$. Hotelling's lemma implies that $m^{i\prime} y^i = -aw^i E_w(\mathbf{p}^i, w^i, u^0)$. We can therefore write (12) as

$$\begin{aligned} m^{1\prime}(y^1)y^1 &= \frac{V_n(\mathbf{p}^0, w^0, n^0)}{V_n(\mathbf{p}^1, w^1, n^1)} m^{0\prime}(y^0)y^0 \\ \iff \quad w^1 E_w(\mathbf{p}^1, w^1, u^0) &= \frac{E_u(\mathbf{p}^1, w^1, u^0)}{E_u(\mathbf{p}^0, w^0, u^0)} w^0 E_w(\mathbf{p}^0, w^0, u^0) \end{aligned}$$

Dividing by $E_u(\mathbf{p}^1, w^1, u^0) > 0$ yields (13). $\square$

Proof of Lemma 2. Differentiating (13), the marginal change in the after-tax wage for any small reform $d\mathbf{p}$ is

$$dw = \sum_i \frac{\partial w(\mathbf{p}, u^0)}{\partial p_i} dp_i \tag{A8}$$

where

$$\begin{aligned} \frac{\partial w(\mathbf{p}, u^0)}{\partial p_i} &= -\frac{E_{iw}/E_w - E_{iu}/E_u}{E_{ww}/E_w - E_{wu}/E_u + 1/w} \\ &= \frac{\hat{\varepsilon}_{iw}}{1 + \hat{\varepsilon}_{\ell w}} \frac{x_i}{\ell} \end{aligned} \tag{A9}$$

where we have used the Slutsky decomposition (15) and the definitions of uncompensated demands to write

$$\begin{aligned} \hat{\varepsilon}_{jw} &= -wl \left(\frac{E_{jw}}{E_w E_j} - \frac{E_{ju}}{E_u E_j} \right) \\ \hat{\varepsilon}_{\ell w} &= -wl \left(\frac{E_{ww}}{E_w E_w} - \frac{E_{wu}}{E_u E_w} \right) \end{aligned}$$

The marginal change in labor income is obtained by differentiating $y(\mathbf{p}, w, u^0) = -aE_w(\mathbf{p}, w, u^0)$ with $w = w(\mathbf{p}, u^0)$ to obtain

$$\begin{aligned} dy &= -a \sum_i \left[E_{wi}(\mathbf{p}, w, u^0) + E_{ww}(\mathbf{p}, w, u^0) w_i(\mathbf{p}, u^0) \right] dp_i \\ &= -\frac{a}{w} \sum_i \left[\frac{wE_{iw}}{E_i} - \frac{wE_{ww}}{E_w} \frac{\hat{\varepsilon}_{iw}}{1 + \hat{\varepsilon}_{\ell w}} \right] x_i dp_i \end{aligned} \tag{A10}$$

using Hotelling's lemma, Slutsky symmetry, and (A9). Substituting the elasticity definitions then yields (16). $\square$

Proof of Proposition 2. We prove the result assuming that U is twice continuously differentiable, using the framework of Lemma 2. For a direct proof without assuming differentiability, see the main text of the paper.

Given the separable form of preferences (18), let $e(\mathbf{p}, v) = \min\{px : \phi(x) = v\}$ be the expenditure function for commodity demands, and define $L(v, u)$ as the associated labor supply satisfying $H(v, L(v, u)) = u$. The expenditure function can be written

$$E(\mathbf{p}, w, u) = \min_v e(\mathbf{p}, v) - wL(v, u) = e(\mathbf{p}, v(\mathbf{p}, w, u)) - wL(v(\mathbf{p}, w, u), u)$$

where the first-order condition defining optimal subutility $v(\mathbf{p}, w, u)$ is $e_v = wL_v$. Applying Hotelling's lemma gives compensated demand and supply functions $E_i = e_i(\mathbf{p}, v)$ and $E_w = -L(v, u)$. Differentiating implies

$$\begin{aligned} E_{ww} &= -L_v v_w \qquad E_{ui}/E_{wi} = v_u/v_w \\ E_{uw} &= -L_v v_u - L_u \qquad E_u = -wL_u \end{aligned}$$

Therefore

$$E_{ww}\frac{E_{ui}}{E_{wi}} - E_{uw} + \frac{E_u}{w} = -L_v v_u + L_v v_u + L_u - L_u = 0 \tag{A11}$$

To express (A11) in elasticity form, note that

$$\begin{aligned} 0 &= E_{ww}\frac{E_{ui}}{E_{wi}} + \frac{E_u}{w} - E_{uw} \\ &= \frac{E_u E_i}{w^2 E_{wi}}\left[\frac{wE_{ww}}{E_w} \cdot w\frac{E_{ui}}{E_u}\frac{E_w}{E_i} + \frac{wE_{wi}}{E_i}\left(1 - \frac{wE_{uw}}{E_u}\right)\right] \end{aligned}$$

Substitute the elasticity definitions and Slutsky decomposition (15)

$$\varepsilon_{iw} - \hat{\varepsilon}_{iw} = w\frac{E_{iu}}{E_u}\frac{E_w}{E_i} \quad \varepsilon_{\ell w} - \hat{\varepsilon}_{\ell w} = w\frac{E_{wu}}{E_u}$$

to get

$$0 = \frac{E_u E_i}{w^2 E_{wi}}\left[\varepsilon_{\ell w}(\varepsilon_{iw} - \hat{\varepsilon}_{iw}) + \varepsilon_{iw}(1 - \varepsilon_{\ell w} + \hat{\varepsilon}_{\ell w})\right] \implies \frac{\varepsilon_{iw}}{\hat{\varepsilon}_{iw}} = \frac{\varepsilon_{\ell w}}{1 + \hat{\varepsilon}_{\ell w}}$$

This establishes that uncompensated and compensated wage elasticities are proportional for all commodities i under separability, so that $dy = 0$ in (16) for all compensated reforms $d\mathbf{p}$. $\square$

Derivation of (21). Using (7), the post-reform budget constraint is

$$\mathbf{p} \cdot \mathbf{x} - w\ell = n = n^0 + E(\mathbf{p}, w) - E(\mathbf{p}^0, w^0)$$

Hence, totally differentiating and using Shepherd's lemma,

$$\mathbf{c} \cdot d\mathbf{x} = wd\ell - (\mathbf{p} - \mathbf{c}) \cdot d\mathbf{x} = (1 - T')dy - (\mathbf{p} - \mathbf{c}) \cdot d\mathbf{x} \tag{A12}$$

Proposition 1 shows that $dg = dy - \mathbf{c} \cdot d\mathbf{x}$. Substituting (A12) yields

$$dg = T'dy + (\mathbf{p} - \mathbf{c}) \cdot d\mathbf{x}$$

Integrating this expression over y yields (21). $\square$

Proof of Proposition 3. Differentiating $x_i = E_i(\mathbf{p}, w, u^0)$ and $y = -aE_w(\mathbf{p}, w, u^0)$ with $w = w(\mathbf{p}, u^0)$ yields

$$\begin{aligned} dy &= -a \sum_j \left[E_{wj} + E_{ww} \frac{\partial w}{\partial p_j} \right] dp_j \\ dx_i &= \qquad \sum_j \left[E_{ij} + E_{iw} \frac{\partial w}{\partial p_j} \right] dp_j \end{aligned}$$

Let

$$\omega_j = \frac{\ell}{x_j} \frac{\partial w(\mathbf{p}, u^0)}{\partial p_j} = \frac{\hat{\varepsilon}_{iw}}{1 + \hat{\varepsilon}_{\ell w}}$$

denote the amount by which after-tax labor income $w\ell$ must rise, per dollar of increase in expenditures $x_i dp_i$. Substituting for dy and dx_i,

$$\begin{aligned} dg &= dy - \mathbf{c}d\mathbf{x} \\ &= -\sum_j \left[aE_{wj} - aE_{ww}\omega_j \frac{E_j}{E_w} + \sum_i c_i \left(E_{ij} - E_{iw}\omega_j \frac{E_j}{E_w} \right) \right] dp_j \\ &= -\sum_j \left[\frac{a}{w} \frac{wE_{jw}}{E_j} - \frac{a}{w} \frac{wE_{ww}}{E_w} \omega_j + \sum_i \frac{c_i}{p_i} \left(\frac{p_i E_{ji}}{E_j} - \frac{p_i E_{wi}}{E_w} \omega_j \right) \right] x_j dp_j \end{aligned}$$

Recall that $w/a = 1 - T'$, and let $p_i/c_i = 1 + \tau_i$ where τ_i is the percentage price distortion for commodity i. Then this expression states

$$dg = -\sum_j \left[\frac{\varepsilon_{jw} - \varepsilon_{\ell w}\omega_j}{1 - T'} + \sum_i \frac{\varepsilon_{ji} - \varepsilon_{\ell i}\omega_j}{1 + \tau_i} \right] x_j dp_j \tag{A13}$$

Integrating (A13) over individuals yields (22). $\square$

Proof of Proposition 4. (30) implies that

$$\begin{aligned} deb(\mathbf{p}, w, u^0) &= \sum_k \left[E_k(\mathbf{p}, w, u^0) + f_k^*(p, u^0) - (a - w) \frac{d}{dp_k} \ell(\mathbf{p}, w(\mathbf{p}, u^0), u^0) \right] dp_k \\ &= -\sum_k \left[(q_k - f_k^*) dp_k + \frac{a - w}{a} dy \right] \end{aligned}$$

where we have used $E_w = -\ell$ and $dy = ad\ell$. Noting $w = a(1 - T')$ then yields (32).

Proof that (33) **implies the Samuelson rule.** The first-order condition for labor supply on the linearized budget constraint is

$$w = e_\ell(p, \ell, u^0) = \hat{e}_\psi(p, \psi)\psi_\ell(\ell, u^0) \tag{A14}$$

Hotelling's lemma implies that

$$E_u(p, w, u^0) = e_u(p, \ell, u^0) = \hat{e}_\psi(p, \psi)\psi_u(\ell, u^0) \tag{A15}$$

Substituting (A14)–(A15) into (15) implies

$$\frac{\psi_\ell(\ell^1, u^0)\ell^1}{\psi_u(\ell^1, u^0)} = \frac{\psi_\ell(\ell^0, u^0)\ell^0}{\psi_u(\ell^0, u^0)}$$

The left-hand side of this equation is strictly monotone in ℓ^1, given the assumption that labor income is strictly monotone in wage rates. Therefore, the unique solution has $\ell^1 = \ell^0$. $\square$

Derivation of (24). By zero-degree homogeneity of demands,

$$\sum_i \varepsilon_{ji} + \varepsilon_{jw} = 0$$
$$\sum_i \varepsilon_{\ell i} + \varepsilon_{\ell w} = 0$$

Setting $\tau_i = 0$ for all $i \neq j$,

$$\begin{aligned}\sum_i \frac{\varepsilon_{ji} - \varepsilon_{\ell i}\omega_j}{1 + \tau_i} &= \sum_i \left(\varepsilon_{ji} - \varepsilon_{\ell i}\omega_j\right) - \frac{\tau_j}{1 + \tau_j}\left(\varepsilon_{jj} - \varepsilon_{\ell j}\omega_j\right) \\ &= -\left(\varepsilon_{jw} - \varepsilon_{\ell w}\omega_j\right) - \frac{\tau_j}{1 + \tau_j}\left(\varepsilon_{jj} - \varepsilon_{\ell j}\omega_j\right)\end{aligned}$$

Substituting in (22) yields (24).

B Cost-benefit analysis of child care subsidies

We perform a distributionally sensitive cost-benefit analysis of a simulated 50 percent universal child care subsidy for Canada. Our data are from the annual Canadian Income Survey (CIS) supplement to Statistics Canada's Labour Force Survey. We observe labor earnings, income taxes, and child care expenditures for a repeated cross-section sample of 23,345 families in the 2012–2018 period whose youngest child is under the age of six, the primary users of market child care services in Canada.

To estimate the marginal tax rates on secondary earners, we use the Canadian Tax and Credit Simulator (Milligan, 2021) and the CIS data on individual and family-level taxable incomes and demographic information. Secondary earnings are defined as the lesser earnings of adult family members in two-parent couples, or earnings of the reference person in single-parent households. Our simulated marginal tax rates include the effect of incremental earnings on means-tested tax credits and cash benefit programs at the federal and provincial levels.

In the CIS, respondents are asked to estimate out-of-pocket child care expenditure incurred in the past year in order to job a paid job.[29] In most Canadian provinces, there are means-tested subsidies available for child care expenses that are paid directly to licensed child care providers at subsidy rates that rise to 100 percent for a large families in the bottom quintile of family incomes. To estimate the family's cost of child care at producer prices, we simulate child care subsidy rates for each family based on information on provincial subsidy programs from Macdonald and Friendly (2019) and gross up reported out-of-pocket expenditures by one minus the subsidy rates.

We simulate the effects of the subsidy on earnings and child care expenditures using the first-order approximations in (28), and the first-order approximations to quantity changes in (25). The estimated elasticities of secondary earnings and child care expenditures with respect to the price of child care are those reported in Baker et al. (2008), namely $\hat{\varepsilon}_{jj} = -0.45$ and $\hat{\varepsilon}_{\ell j} = -0.25$. We use a local linear regression to calibrate income elasticities of child care demand to match the pattern of child care expenditure at each percentile of the income distribution, as depicted in Figure 3. To arrive at compensated elasticities from changes in the price of child care, we rely on the Slutsky decomposition in (15).

To model the effects of compensation on labor supply of secondary earners, we draw on estimates of the uncompensated wage elasticity of earnings for married women reported in Bargain et al. (2014) and of income effects on reported incomes in Golosov et al. (2023). We then use (17) to simulate a first-order approximation to the compensating changes in marginal tax rates at each income level. These two

[29] In contrast, information on earnings, income, and taxes paid in the sample are in most cases drawn from administrative data on the family's income tax returns.

sources together suggested a compensated wage elasticity in the range of $[0.4, 0.8]$ are plausible, and we report simulations for the endpoints of this range. The effect of distributional adjustments on the cost-benefit analysis in this case increase monotonically with the value of the compensated wage elasticity.

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