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Childhood, Well-Being and Fairness*

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Abstract

This paper examines the design of optimal family policies when children have unequal needs (in terms of material goods and parental time), leading to heterogeneous preferences, and have parents with unequal degrees of altruism. We examine the issue of interpersonal well-being comparisons between children by means of consumption-equivalent and time-equivalent indexes, and show that the conditions of existence of these equivalents - as well as their rankings across children - differ across the metric used. We also examine well-being comparisons across parents who differ in their children's preferences and in their altruism. Then, we derive the constrained egalitarian social optimum (where only children's well-being levels are equalized) and the double egalitarian social optimum (where both children's and parents' well-being are equalized). It is shown that the optimal allocation and the optimal family policy vary with the metric used for the measurement of children's well-being.

Keywords: childhood, well-being, heterogeneity, interpersonal comparisons, family policies.

JEL classification codes: D13, I31, I38, H31.

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1 Introduction

As Marx (1875) underlined, abstract equality in front of the Law does not imply real equality across citizens. The gap between legal equality and real-life inequalities appears at the earliest point of life - that is, at the time of birth -, and has various causes, such as unequal needs and unequal endowments. Although children are all equal in front of the Law, they are born in family environments more or less favorable for their personal development. Family environments were shown to have a significant impact on children's health (Barazzetta et al 2021, Costi et al 2025), on children's well-being (Rees 2019, Cherry et al 2020, Gross-Manos et al 2025), on children's educational outcomes (Menta et al 2023), and on various aspects of children's lives as adults, including well-being at the adult age (Frijters et al 2014, Layard et al 2014, Fleche et al 2021).

At the normative level, it is widely acknowledged that children cannot be taken as personally responsible for the characteristics of the family environments where they are born. Hence, from the perspective of fairness, there is a strong normative intuition justifying public policies that abolish well-being inequalities between children due to unequal family environments. The intuition goes as follows. Since Rawls (1971), social justice is often defined as a requirement of fairness. In their microeconomic 'translation' of philosophical intuitions about fairness, Fleurbaey (2008) and Fleurbaey and Maniquet (2010) argued that fairness requires neutralizing the effect of pure circumstances (i.e., factors on which individuals have no control) on individual well-being outcomes. This requirement was formalized by means of the Principle of Compensation, which states that well-being inequalities due to circumstances should be abolished by the State. When applied to well-being inequalities during childhood, the Principle of Compensation provides foundations for family policies neutralizing well-being inequalities between children resulting from unequal family environments.[1]

However, the goal of making all persons equally well-off during their childhood - despite facing unequal family environments - is difficult to define. The reason is that *heterogeneity across children*, which motivates policy intervention, is at the same time at the origin of deep measurement problems. How could one equalize children's well-being when children differ on dimensions (such as needs) that make their well-being levels hardly comparable? Given that children's well-being is nested in (altruistic) parents's well-being, similar questions arise on the side of parents. How could one compare the well-being of altruistic parents having children whose well-being levels are hardly comparable? What would be a fair family policy when families are heterogeneous on dimensions affecting children's - and, hence, altruistic parents's - well-being?

The purpose of this paper is to reexamine the design of optimal family policies - and, in particular, of optimal family allowances - in an economy with heterogeneous children born in (more or less) favorable family environments.

[1] The same argument does not apply to policies aimed at correcting well-being inequalities later on in life, because these inequalities are a product of both family circumstances and individual effort decisions. This paper distinguishes between parental well-being inequalities due either to circumstances (children's needs) or to responsibility variables (parental altruism).

Our analysis will examine how family allowances can be structured to fully offset child well-being inequalities arising from the family environments where children are born. The present paper will reexamine the design of family allowances from the perspective of distribution/compensation across children, and will thus leave aside other issues, such as externalities or public good provision.

While there exist many sources of heterogeneity across children born in different families, our analysis will focus on a framework where children differ along two key dimensions. On the one hand, we will consider a model where children have *needs* that vary across families.[2] We will assume that some children have stronger needs for parental attention/time, whereas other children have stronger needs for material consumption, leading to heterogeneous preferences across children.[3] The relation between heterogeneity in children's needs and heterogeneity in children's preferences can be justified as follows. Since an unsatisfied need is a source of well-being deprivation, two children with unequal needs do not necessarily enjoy the same well-being level at equal (consumption, time) baskets. Hence, heterogeneity in children's preferences can be used as a theoretical shortcut for the modeling of heterogeneity in children's needs across families.[4] On the other hand, our model will also capture another key source of heterogeneity during childhood: children are born in families where parents exhibit unequal degrees of *altruism*, some parents being more sensitive to their children's well-being in comparison to other parents. This source of heterogeneity is distinct from the previous one: parents are responsible for their own preferences, but not (necessarily) for the needs - and the preferences - of their children. But the degree of parental altruism can also affect well-being during childhood, because more altruistic parents will make more efforts to satisfy the needs of their children in comparison to less altruistic parents.

Due to heterogeneity in children's preferences and in parental altruism, the laissez-faire equilibrium of our economy exhibits, without surprise, significant inequalities among children and among parents. The goal of this paper is then to examine how one could design a family policy that would restore fairness, that is, a family policy that could free all children from the burden of family circumstances, and make all persons in childhood equal in terms of well-being achievements. Beyond the equalization of children's well-being, fairness may also require to reduce well-being inequalities between parents if those inequalities are due to circumstances for them (such as their children's needs), but not their degree of altruism (for which they are responsible).

The design of such fair family policy requires, as a preliminary step, to be able to compare well-being levels across children and across parents. In order

[2] On the existence and the measurement of heterogeneity in children's needs, see Katz and Courtney (2015), Jaudes et al (2016), Rosanbalm et al (2016), Fuad et al (2021) and Wilmes and Andresen (2023).

[3] Note that inequalities in children's needs may stem from various sources - genetic, cultural, or social. We do not make any assumption about these underlying causes. Instead, we focus solely on the well-being implications of unequal needs during childhood.

[4] This paper will not examine the issue of the existence of preferences for children, but will take their existence as given. That assumption is a theoretical shortcut used to examine the implications of the unsatisfaction of children's needs.

to study the problem of interpersonal well-being comparison across children, this paper will rely on two distinct well-being indexes, which are both based on children's preferences, but rely on *different metrics for the measurement of children's well-being*. On the one hand, we will rely on the *consumption-equivalent index*, which is a preferences-based index of well-being that relies on the consumption metric (see Fleurbaey 2016). The child's consumption-equivalent is defined as the hypothetical children's material consumption, which, combined with a parental time investment of reference, would make the child as well off as he is currently with his current material consumption and the current parental time investment. On the other hand, we will also quantify children's well-being while relying on another well-being index: the *time-equivalent index*. The child's time-equivalent is defined as the hypothetical children's parental time investment, which, combined with a material consumption of reference, would make the child as well off as he is currently with his current material consumption and the current parental time investment. The time-equivalent index is, like the consumption-equivalent, based on children's preferences, but differs in terms of the metric used for the measurement of children's well-being.[5] Relying on these two distinct metrics for the measurement of children's well-being will allow us to analyze how the solution adopted for the problem of interpersonal well-being comparisons across children will affect the design of optimal family policies.

Anticipating on our results, we first examine the issue of interpersonal well-being comparison between children by means of equivalent consumption and equivalent time indexes of well-being, and show that the conditions of existence of equivalents - as well as their rankings across types - differ across the metric used. We also study the comparison of well-being across parents who differ in terms of their children's needs and in terms of altruism. Then, we derive the constrained egalitarian optimum (where only children's well-being levels are equalized) and the double egalitarian optimum (where both children's and parents' well-being levels are equalized) and study their decentralization. It is shown that *the design of the optimal family policy varies with the postulated metric used for the measurement of children's well-being*. In other words, the particular way in which the problem of well-being comparison across children is solved influences the design of the optimal family policy. The reason is that each metric used for well-being measurement leads to a distinct quantification of the advantages or disadvantages of children in comparison to other children. Another result is that the form of the double egalitarian social optimum depends also crucially on whether parental altruism is regarded as normatively relevant for the measurement of parental well-being. The intuition is that the measure of the (dis)advantages of parents is sensitive to (not) laundering parental preferences from their altruistic component.

As such, this paper casts an original light on the design of family policies, and on the normative foundations underlying these policies in the presence of

[5] For an earlier study of time-equivalents in the specific context of unequal lifetimes, see Onder et al (2025).

heterogeneity across both children and parents. Whereas the existing literature (which relies massively on the utilitarian social objective) assumes perfect comparability of well-being across children and across parents, and focuses on the optimal public intervention in that context, the present work relaxes that perfect comparability assumption, and explores how the theoretical tools of the consumption-equivalent and the time-equivalent indexes can be used to reconstruct a comparability of well-being across children and across parents.

Our findings reveal that the design of optimal family policies is not robust to how the problem of interpersonal well-being comparison across children is solved. Moreover, since children's well-being is nested in parental well-being (because of parental altruism), the resolution of the problem of interpersonal well-being comparisons across children also affects how one can make parental well-being levels comparable, and, hence, the optimal fiscal treatment of parents.

This paper is related to several branches of the literature. First, this paper is related to the public economics literature dedicated to the study of optimal family policies, such as Cigno (1983), Balestrino et al (2002), Cremer et al (2006, 2008), Pestieau and Ponthiere (2013) and Bargain and Doni (2018). The contribution of the present paper lies in its emphasis on heterogeneity in preferences across children and across parents. Second, this paper is also linked to the literature in welfare economics dealing with the problem of interpersonal well-being comparisons by means of consumption-equivalent indexes (Fleurbaey and Maniquet 2013, Fleurbaey 2016, Fleurbaey and Maniquet 2017, Decancq and Schokkaert 2016, Decancq et al 2017, Decancq et al 2019, Fleurbaey and Maniquet 2019, Leroux et al 2022, Fleurbaey and Ponthiere 2023, Da Costa et al 2024) and by means of time-equivalent indexes (Onder et al 2025). The specificity of this paper with respect to that literature is to focus on the issue of well-being comparisons across children. Third, this paper is also related to the literature in normative economics concerned with the issue of justice as compensation (Fleurbaey and Maniquet 2006, 2010, 2013, Fleurbaey 2008, Leroux et al 2022). The contribution of this paper with respect to that literature lies on its emphasis on family policies, and, in particular, on the compensation of children for being born in more or less favorable family environments.

This paper is organized as follows. Section 2 presents the model. Section 3 studies the comparison of well-being across children having different preferences (due to different needs) and born in families where parents have unequal degrees of altruism towards their children. Then, Section 4 studies the comparison of well-being across parents. Section 5 describes two social optima: the constrained egalitarian optimum, where the well-being levels of all children are equalized; and, the double egalitarian optimum, where all children are made equally well-off, and all parents are also made equally well-off. These optima are derived when measuring children's well-being either by means of consumption-equivalent indexes, or by means of time-equivalent indexes. The decentralization of these social optima is also compared across the different metrics used for the measurement of children's well-being. Section 6 illustrates numerically how the chosen metric for the measurement of children's well-being affects the design of the optimal family policy. Section 7 concludes.

2 The model

The population of this economy is composed of families including one child and one parent, who interact on a period whose length is normalized to unity.[6] There are four types of families $i = \{1, 2, 3, 4\}$. Families differ across two dimensions (Table 1).

On the one hand, families differ regarding children's *needs*. Some children have stronger needs in terms of goods, whereas other children have stronger needs in terms of parental time. The non-satisfaction of needs generates a well-being loss. For simplicity, we assume that children's heterogeneity in needs can be modeled as children's heterogeneity in preferences, by means of a single preference parameter $0 \leq \beta_i \leq 1$, which captures the utility weight assigned by children to parental time (with respect to the weight $1 - \beta_i$ assigned to goods). We assume that β_i equals either $\breve{\beta}$ or $\bar{\beta}$, with $\breve{\beta} < \bar{\beta}$.

On the other hand, families differ also regarding the degree of *altruism* of parents towards their children $\alpha_i > 0$, which can be either high, and equal to $\bar{\alpha}$, or low, and equal to $\breve{\alpha} < \bar{\alpha}$.

	parental altruism	
Child needs in time	high $\alpha = \bar{\alpha}$	low $\alpha = \breve{\alpha}$
low $\beta = \breve{\beta}$	type 1 ($\beta_1 = \breve{\beta}$, $\alpha_1 = \bar{\alpha}$)	type 2 ($\beta_2 = \breve{\beta}$, $\alpha_2 = \breve{\alpha}$)
high $\beta = \bar{\beta}$	type 3 ($\beta_3 = \bar{\beta}$, $\alpha_3 = \bar{\alpha}$)	type 4 ($\beta_4 = \bar{\beta}$, $\alpha_4 = \breve{\alpha}$)

Table 1. Four types of families.

Children preferences. The preferences of a child of type $i \in \{1, 2, 3, 4\}$ are represented by the following utility function:

$$(1 - \beta_i)u(g_i) + \beta_i v(T_i) \tag{1}$$

where $u(\cdot)$ is increasing, g_i denotes the material resources dedicated to the child, $v(\cdot)$ is increasing and concave, and T_i is the amount of parental time devoted to children. We assume $0 \leq T_i \leq 1$.

Assuming the existence of children's preferences does not constitute a weak assumption. We use that assumption as a 'theoretical shortcut', which allows us to study how the lack of parental altruism causes well-being losses for children that vary across children, depending on the (implicit) underlying heterogeneity in children's needs in terms of material goods g_i and of parental time T_i.

Parental preferences.

The preferences of a parent of type $i \in \{1, 2, 3, 4\}$ are represented by the following utility function:

$$U(C_i) + \alpha_i[(1 - \beta_i)u(g_i) + \beta_i v(T_i)] \tag{2}$$

where $U(\cdot)$ is increasing and concave, $C_i \equiv w(1 - T_i) - g_i$ is the consumption of the parent, where $w > 0$ is the hourly wage. Thanks to altruism, the

[6] For simplicity, we leave aside the issue of childlessness (see Gobbi 2013, Baudin et al 2015, Leroux et al 2022).

child's well-being is nested in his parent's well-being. Thus, the heterogeneity of children's preferences implies a kind of 'nested heterogeneity': parents with identical altruism degrees have different preferences when their children's preferences differ.

Laissez-faire equilibrium As a benchmark, let us first consider the economy at the laissez-faire, that is, in the absence of any State intervention. A parent of type i chooses to allocate material resources g_i and time T_i to his child in such a way as to maximize his own utility, subject to his resource constraint and subject to the well-being of his child:

$$\max_{g_i,T_i} U(w(1-T_i)-g_i)+\alpha_i[(1-\beta_i)u\left(g_i\right)+\beta_i v(T_i)]$$

The first-order conditions (FOCs) for an interior optimum are:

$$-wU'(w(1-T_i)-g_i)+\alpha_i\beta_i v'(T_i) = 0 \tag{3}$$

$$-U'(w(1-T_i)-g_i)+\alpha_i(1-\beta_i)u'(g_i) = 0 \tag{4}$$

Substituting (4) into (3), we obtain, after simplifications:

$$w(1-\beta_i)u'(g_i)=\beta_i v'(T_i) \tag{5}$$

From the above FOCs, we obtain the following results:

Proposition 1. *At the laissez-faire (interior) equilibrium,*

- *the allocation of material goods to children is:*

$$g_1 > g_2, g_3 > g_4 > 0,$$

- *the allocation of parental time to children is:*

$$T_3 > T_1, T_4 > T_2 > 0.$$

Proof. Applying the Cramer's rule on FOCs (3) and (4), we can prove that $dT_i/d\alpha_i > 0$. Using equation (5), it implies that $dg_i/d\alpha_i > 0$. Applying also the Cramer's rule on FOCs, we can show that $dT_i/d\beta_i > 0$ and $dg_i/d\beta_i < 0$. ■

Children of less altruistic parents (types 2 and 4) benefit, *ceteris paribus*, from fewer parental resources (in terms of goods and time) than the children of more altruistic parents (types 1 and 3). This leads to well-being deprivation for the children of less altruistic parents.

Due to heterogeneity in children's preferences, it is hard to compare the situations of children in terms of well-being. Children's heterogeneity in preferences makes it difficult to quantify the extent of well-being deprivation from which children of less altruistic parents suffer. As a consequence, one cannot identify who, among the children of less altruistic parents, is the worst-off. More precisely, it is hard to know whether the worst-off children are the children of less altruistic parents who have stronger needs in terms of material resources (i.e., type 2), or, on the contrary, those who have stronger needs in terms of parental time (i.e., type 4). Section 3 examines the problem of interpersonal comparison of well-being among children.

3 Well-being comparisons across children

In order to compare the well-being levels of children having distinct preferences, this section considers two distinct ways of solving the problem of interpersonal well-being comparisons: first, the construction of consumption-equivalent indexes; second, the construction of time-equivalent indexes. These two indexes are preference-based measures of well-being (see Fleurbaey 2016, Onder et al 2025). But these measures differ regarding the *metric* used to quantify well-being. While the consumption-equivalent relies on the standard material goods metric, the time-equivalent relies on the time metric.

Consumption equivalents The consumption-equivalent $\hat{g}_i$ is defined as the hypothetical level of consumption that, when combined with a parental time endowment of reference $\bar{T}$, would make the child indifferent relative to his current situation. For the four types of children, the consumption equivalents are defined as follows:

$$\left(1-\breve{\beta}\right) u(\hat{g}_1) + \breve{\beta} v(\bar{T}) = \left(1-\breve{\beta}\right) u(g_1) + \breve{\beta} v(T_1) \quad (6)$$

$$\left(1-\breve{\beta}\right) u(\hat{g}_2) + \breve{\beta} v(\bar{T}) = \left(1-\breve{\beta}\right) u(g_2) + \breve{\beta} v(T_2) \quad (7)$$

$$\left(1-\bar{\beta}\right) u(\hat{g}_3) + \bar{\beta} v(\bar{T}) = \left(1-\bar{\beta}\right) u(g_3) + \bar{\beta} v(T_3) \quad (8)$$

$$\left(1-\bar{\beta}\right) u(\hat{g}_4) + \bar{\beta} v(\bar{T}) = \left(1-\bar{\beta}\right) u(g_4) + \bar{\beta} v(T_4) \quad (9)$$

where the right-hand side of these equalities report the laissez-faire utility levels of children with different needs and with different degrees of parental altruism. The reference parental time $\bar{T}$ is defined as the parental time at which the comparison of well-being across children can rely entirely on the amount of material goods consumed by children, despite their distinct preferences.

Before comparing the levels of consumption-equivalents at the laissez-faire, it is worth examining the conditions under which these indexes exist.

Lemma 1. *For a given child's basket (g_i, T_i) and a given reference parental time $\bar{T}$, the conditions necessary and sufficient for the existence of a (finite positive) consumption-equivalent $\hat{g}_i$ are:*

$$\lim_{\hat{g}_i \to 0} \left(1-\beta_i\right) u(\hat{g}_i) + \beta_i v(\bar{T}) \leq \left(1-\beta_i\right) u(g_i) + \beta_i v(T_i)$$

$$\lim_{\hat{g}_i \to +\infty} \left(1-\beta_i\right) u(\hat{g}_i) + \beta_i v(\bar{T}) \geq \left(1-\beta_i\right) u(g_i) + \beta_i v(T_i)$$

Proof. This lemma can be proved by contradiction. If the first condition were not satisfied, it would be the case that there exists no non-negative child's consumption that would make the child as well off as he is under the basket (g_i, T_i): in all cases the child enjoying the reference parental time and zero consumption would be strictly better off than under the basket (g_i, T_i). Thus a finite positive consumption-equivalent would not exist. Alternatively, if the second condition were not satisfied, it would be the case that there exists no finite child's consumption that would make the child indifferent to the basket (g_i, T_i): in all cases (even with infinite consumption), the child would be strictly worse off with the reference parental time than under the basket (g_i, T_i). Thus a finite positive consumption-equivalent would not exist. ■

The conditions of Lemma 1 guarantee the existence of a finite positive consumption-equivalent, $\hat{g}_i$. When they prevail, a finite positive $\hat{g}_i$ does not only exist, but is also *unique*, because of the monotonicity of $u(\cdot)$ and $v(\cdot)$.

Having studied the existence of a (finite positive) consumption-equivalent index for a child, let us now use that index as a measure of children's well-being. In order to compute $\hat{g}_i$ for each type of children, one must select a reference level for parental time $\bar{T}$. $\bar{T}$ has a precise economic significance: it corresponds to the level of parental time such that, for any pair of children having that amount of parental time, one can be certain that the child benefiting from a higher level of material resources is better off than the child who benefits from a lower level of material resource, *regardless of children's preferences.*

Based on this definition, we believe that the reference parental time $\bar{T}$ should take a low level. The intuition goes as follows. When parental time is low (so that only basic time needs are satisfied), children's deprivation is substantial, no matter what children's preferences are. Hence, it is likely that children benefiting from a higher amount of material resources from their parents will be better off than children receiving a lower amount.

Following that intuition, we will, throughout this paper, compute children's consumption-equivalents $\hat{g}_i$ while assuming that the reference parental time $\bar{T}$ takes its minimum level observed at the laissez-faire, that is, is equal to T_2.

Proposition 2. *At the laissez-faire, when the reference parental time $\bar{T} = T_2$, children's consumption-equivalents $\hat{g}_i$ are all strictly positive, and, if finite, satisfy:*

$$\begin{aligned} \hat{g}_1 &> \hat{g}_2 \text{ and } \hat{g}_3 > \hat{g}_4 \\ \hat{g}_1 &\gtrless \hat{g}_3 \text{ and } \hat{g}_1 \gtrless \hat{g}_4 \\ \hat{g}_2 &\gtrless \hat{g}_3 \text{ and } \hat{g}_2 \gtrless \hat{g}_4 \end{aligned}$$

Proof. From $\bar{T} = T_2 > 0$, we have trivially that $\hat{g}_2 = g_2 > 0$. Given that $T_1, T_3, T_4 > T_2 > 0$, we have $\hat{g}_1 > g_1, \hat{g}_3 > g_3$ and $\hat{g}_4 > g_4$. The rankings are then obtained from the laissez-faire ranking of g_i: $g_1 > g_2$ implies that $\hat{g}_1 > g_1 > \hat{g}_2 = g_2$. Moreover, since $g_3 > g_4$ and $T_3 > T_4$, we have that $\hat{g}_3 > g_3 > \hat{g}_4 > g_4$. Other rankings are indeterminate. ■

Proposition 2 yields only a partial ranking of children's well-being. It shows that, according to the consumption-equivalent indexes, children of types 2 and 4 are among the worst-off, since they have the same needs as children of types 1 and 3 respectively, but less altruistic parents.

But beyond these (partial) ranking, one cannot say more about well-being comparisons between children without making additional assumptions. The reason lies in the heterogeneity of children's preferences. Because of that heterogeneity, one cannot exclude that, at the laissez-faire equilibrium, children of type 1 may, despite more altruistic parents, be worse-off than children of type 4. The same uncertainty arises when comparing children of types 2 and 3.

Time equivalents The time-equivalent $\hat{T}_i$ is defined as the hypothetical parental time level, which, combined with a child's consumption level of reference $\bar{g}$, would make the child indifferent relative to his current situation. For the four types of children, the time-equivalents are defined as follows:

$$\left(1-\breve{\beta}\right)u(\bar{g})+\breve{\beta}v(\hat{T}_1) = \left(1-\breve{\beta}\right)u(g_1)+\breve{\beta}v(T_1) \tag{10}$$

$$\left(1-\breve{\beta}\right)u(\bar{g})+\breve{\beta}v(\hat{T}_2) = \left(1-\breve{\beta}\right)u(g_2)+\breve{\beta}v(T_2) \tag{11}$$

$$\left(1-\bar{\beta}\right)u(\bar{g})+\bar{\beta}v(\hat{T}_3) = \left(1-\bar{\beta}\right)u(g_3)+\bar{\beta}v(T_3) \tag{12}$$

$$\left(1-\bar{\beta}\right)u(\bar{g})+\bar{\beta}v(\hat{T}_4) = \left(1-\bar{\beta}\right)u(g_4)+\bar{\beta}v(T_4) \tag{13}$$

where again, on the right-hand side of these equalities, we report children utility levels obtained at the laissez-faire.

This alternative index of well-being measurement relies on a reference level for the children's material goods $\bar{g}$. This reference level is defined as the level of children's material goods at which the well-being comparison across children can only rely on parental times, even if children have distinct preferences.

Given that the time-equivalent index and the consumption-equivalent index rely on the *same* indifference map, i.e., on the *same* preferences, it is tempting to believe that the chosen metric for well-being measurement has no impact on the conditions of existence of each index of well-being measurement. However, as we shall now see, the conditions necessary and sufficient for the existence of an interior time-equivalent index differ from the conditions that are necessary and sufficient for the existence of an interior consumption-equivalent index.[7]

Lemma 2. *For a given child's basket (g_i, T_i) and a given reference child's consumption $\bar{g}$, the conditions necessary and sufficient for existence of an (interior) time-equivalent $\hat{T}_i$ are:*

$$\lim_{\hat{T}_i\to 0}\left(1-\beta_i\right)u(\bar{g})+\beta_i v(\hat{T}_i) \leq \left(1-\beta_i\right)u(g_i)+\beta_i v(T_i)$$

$$\lim_{\hat{T}_i\to 1}\left(1-\beta_i\right)u(\bar{g})+\beta_i v(\hat{T}_i) \geq \left(1-\beta_i\right)u(g_i)+\beta_i v(T_i)$$

Proof. The proof is identical to the proof of Lemma 1. ■

Although $\hat{g}_i$ and $\hat{T}_i$ rely on the same preferences (i.e., the same indifference map), the existence conditions for the two (interior) well-being indexes are not the same, because of the distinct metrics used for well-being measurement (goods *versus* time). The intuition is that, although each well-being index is computed by 'contracting' a two-dimensional space into a line (by moving along

[7] Note that, due to the monotonicity of $u(\cdot)$ and $v(\cdot)$, the existence conditions for $\hat{T}_i$ imply also the uniqueness of $\hat{T}_i$.

indifference curves up or down towards the postulated reference level), the conditions under which a basket can be assigned to a point along that line differ across the two 'contractions' of the space that are carried out.

The computation of time-equivalent indexes for children requires the selection of a level of reference for parental material resources $\bar{g}$. This reference level is such that, for any pair of children enjoying $\bar{g}$, one can be certain that the child who enjoys more parental time is strictly better off than the child who enjoys less parental time, no matter what the children's preferences are.

Using the same rationale as above, we posit that the reference level for parental material resources should take a low level. Indeed, when the common level of $\bar{g}$ is low (so that only basic material needs are satisfied), one can expect that the child who enjoys more parental time is better off than the child who enjoys less parental time. Following this rationale, we will, throughout this paper, fix $\bar{g}$ to the minimum level of g_i at the laissez-faire, that is, to g_4.

Proposition 3. *At the laissez-faire, when the reference parental material resource $\bar{g} = g_4$, children's time-equivalents $\hat{T}_i$ are all strictly positive, and, if interior, satisfy:*

$$\begin{aligned} \hat{T}_1 &> \hat{T}_2 \text{ and } \hat{T}_3 > \hat{T}_4 \\ \hat{T}_1 &\gtrless \hat{T}_3 \text{ and } \hat{T}_1 \gtrless \hat{T}_4 \\ \hat{T}_2 &\gtrless \hat{T}_3 \text{ and } \hat{T}_2 \gtrless \hat{T}_4 \end{aligned}$$

Proof. From $\bar{g} = g_4 > 0$, we have trivially that $\hat{T}_4 = T_4 > 0$. Given that $T_1, T_2, T_3 > T_4 > 0$, we have $\hat{T}_1 > T_1, \hat{T}_3 > T_3$ and $\hat{T}_4 > T_4$. The rankings are then obtained from the laissez-faire ranking of T_i: $T_3 > T_4$ implies that $\hat{T}_3 > T_3 > \hat{T}_4 = T_4$. Moreover, since $g_1 > g_2$ and $T_1 > T_2$, we have that $\hat{T}_1 > T_1 > \hat{T}_2 > T_2$. ■

As under consumption-equivalent indexes (Proposition 2), it is the case, under time-equivalent indexes, that children of less altruistic parents (types 2 and 4) are among the worst-off children, and that children of more altruistic parents (types 1 and 3) are among the best-off children.

However, the measurement of well-being may vary a lot across consumption-equivalent and time-equivalent indexes.[8] In particular, the ranking between two children in terms of well-being may vary, depending on whether one relies on consumption-equivalent indexes or, alternatively, on time-equivalent indexes. To illustrate that point, Figure 1 compares two children having different preferences and enjoying, respectively, baskets (g, T) and (g', T'), under either the consumption-equivalent $\hat{g}_i$ (constructed under the reference $\bar{T} = T_2$) or the time-equivalent $\hat{T}_i$ (under the reference $\bar{g} = g_4$). The ranking between these two children in terms of well-being differs across well-being measures. When well-being is measured by means of consumption-equivalents, we have: $\hat{g} > \hat{g}'$,

[8] A first difference is that the metric for well-being measurement is different: whereas the consumption-equivalent index quantifies well-being in terms of monetary amounts measured in the positive orthant, with $\hat{g}_i \in]0, +\infty]$, the time-equivalent index measures well-being in terms of time, and, hence, within the unit interval, with $\hat{T}_i \in]0, 1]$.

whereas, when well-being is measured by means of time-equivalents, we have: $\hat{T} < \hat{T}'$. Hence whether one child is regarded as better-off or worse-off than another child varies with the postulated well-being index.

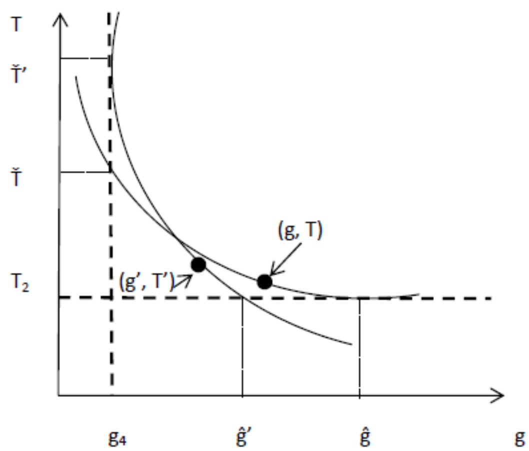


Figure 1: Well-being comparisons across children with consumption-equivalents and time-equivalents.

Figure 1 illustrates that the comparison of well-being across children having different needs - and, hence, different preferences - depends on how well-being is measured, either by means of a consumption-equivalent, or by means of a time-equivalent. Accordingly, the selected metric used for well-being measurement definitely matters for the comparison of well-being across children.

4 Well-being comparisons across parents

The comparison of well-being across parents is complex, because parents differ in their preferences (degrees of altruism α_i) and, also, in the needs of their children (parameters β_i), which affect the well-being of their children for given parental choices (g_i, T_i). Given that children's well-being is *nested* in parental well-being, the heterogeneity of children's preferences creates heterogeneity in parental preferences even if parental altruism were the same.

To make the well-being of parents comparable despite these differences, one can construct parental consumption-equivalent indexes: one looks for the hypothetical parental consumption that, combined with reference levels for children's parental time investment and children's material resources, would make the parent indifferent with respect to his actual situation.

However, an additional difficulty arises here: *the treatment of parental altruism*. This difficulty is known as the issue of the 'laundering' of preferences. The question concerns the normative relevancy of including altruism in the measurement of well-being and for the design of optimal policies. Two approaches can be adopted: either parental preferences are taken as they are and not 'laundered' from their altruistic component, or parental preferences are 'laundered' from their altruistic component. Given that the literature provides mixed views about this issue, this section considers the two approaches.[9]

Let us first measure the well-being of parents without removing the altruistic component of parental well-being. For the four types of parents, fixing the reference parental time to $\bar{T}$ as well as the reference children consumption to $\bar{g}$, parental consumption-equivalents $\hat{C}_i$ are defined as follows:

$$\begin{aligned} U(\hat{C}_1) + \alpha_1[(1-\beta_1)u(\bar{g}) + \beta_1 v(\bar{T})] &= U((1-T_1)w - g_1) + \alpha_1[(1-\beta_1)u(g_1) + \beta_1 v(T_1)] \\ U((\hat{C}_2) + \alpha_2[(1-\beta_2)u(\bar{g}) + \beta_2 v(\bar{T})] &= U((1-T_2)w - g_2) + \alpha_2[(1-\beta_2)u(g_2) + \beta_2 v(T_2)] \\ U(\hat{C}_3) + \alpha_3[(1-\beta_3)u(\bar{g}) + \beta_3 v(\bar{T})] &= U((1-T_3)w - g_3) + \alpha_3[(1-\beta_3)u(g_3) + \beta_3 v(T_3)] \\ U(\hat{C}_4) + \alpha_4[(1-\beta_4)u(\bar{g}) + \beta_4 v(\bar{T})] &= U((1-T_4)w - g_4) + \alpha_4[(1-\beta_4)u(g_4) + \beta_4 v(T_4)] \end{aligned}$$

$\hat{C}_i$ is defined as the hypothetical parental consumption level that would, provided parental time is fixed to $\bar{T}$ and children consumption is fixed to $\bar{g}$, make the parent indifferent with respect to his current situation (right-hand side of the above equations).

The conditions necessary and sufficient for the existence of the parental consumption-equivalent under a reference levels $\bar{T}$ and $\bar{g}$ are the following.

Lemma 3. *For a given parental basket (g_i, T_i) and a given reference parental time $\bar{T}$ and reference children consumption $\bar{g}$, the conditions necessary and sufficient for the existence of a (finite positive) parental consumption-equivalent $\hat{C}_i$ in the absence of laundering of parental preferences are:*

$$\begin{aligned} \lim_{\hat{C}_i \to 0} U(\hat{C}_i) + \alpha_i[(1-\beta_i)u(\bar{g}) + \beta_i v(\bar{T})] &\leq U((1-T_i)w - g_i) + \alpha_i[(1-\beta_i)u(g_i) + \beta_i v(T_i) \\ \lim_{\hat{C}_i \to +\infty} U(\hat{C}_i) + \alpha_i[(1-\beta_i)u(\bar{g}) + \beta_i v(\bar{T})] &\geq U((1-T_i)w - g_i) + \alpha_i[(1-\beta_i)u(g_i) + \beta_i v(T_i) \end{aligned}$$

Proof. The proof is identical to the proof of Lemma 1. ■

In the absence of laundering of parental preferences, more altruistic parents have, to some extent, an advantage in comparison to less altruistic ones: they obtain more well-being from the well-being of their children. In other words, the mere act of caring for their children's well-being generates greater utility for highly altruistic parents (types 1 and 3) than for their less altruistic counterparts (types 2 and 4). As a result, less altruistic parents experience a form of well-being deprivation. The parental consumption-equivalents defined above capture this well-being loss associated with lower altruism towards children,

[9] Hammond (1987) and Mirrlees (2007) argue in favor of the laundering of preferences, in order to avoid the double counting of children's well-being. On the contrary, Kaplow (1995, 2008) and Farhi and Werning (2013) argue against the laundering of preferences.

while balancing it against the material deprivation that more altruistic parents incur through reduced consumption.

However, one may question the existence of parental well-being deprivation as a consequence of their lack of altruism. Indeed, one may argue that, when measuring parental well-being, one should concentrate only on the self-oriented part of well-being. From that perspective, the capacity of parents to derive well-being from children's well-being would then be regarded as irrelevant. This leads us to the alternative approach: the laundering of parental preferences.

Excluding the altruistic component of parental preferences requires to set $\alpha_i = 0$ for all types of parents. As a consequence, parental consumption-equivalent indexes $\hat{C}_i$ are now defined as:

$$\begin{aligned}
U(\hat{C}_1) &= U((1-T_1)w - g_1) \\
U(\hat{C}_2) &= U((1-T_2)w - g_2) \\
U(\hat{C}_3) &= U((1-T_3)w - g_3) \\
U(\hat{C}_4) &= U((1-T_4)w - g_4)
\end{aligned}$$

In that situation, the parental consumption-equivalent is simply equal to the level of parental consumption. This guarantees the existence of parental consumption-equivalents since parental consumption is always non-negative and finite, unlike what prevailed in the absence of laundering.

Let us now use parental consumption-equivalent indexes $\hat{C}_i$ to compare parental well-being at the laissez-faire equilibrium. For the sake of consistency with the construction of well-being indexes for children, we suppose that both the reference parental time and the reference level of children consumption are set to the same reference levels as previously, i.e. $\bar{T} = T_2$ and $\bar{g} = g_4$.

Proposition 4. *Consider the comparison of parental well-being at the laissez-faire by means of parental consumption-equivalents. Suppose $\bar{T} = T_2$ and $\bar{g} = g_4$.*

- *Under the laundering of parental preferences, parental consumption equivalents at the laissez-faire are ranked as follows:*

$$\begin{aligned}
\hat{C}_2 &> \hat{C}_1 \text{ and } \hat{C}_4 > \hat{C}_3 \\
\hat{C}_1 &\gtrless \hat{C}_3,\ \hat{C}_1 \gtrless \hat{C}_4 \text{ and } \hat{C}_2 \gtrless \hat{C}_3
\end{aligned}$$

- *Without the laundering of parental preferences, the ranking of parental consumption-equivalents at the laissez-faire is ambiguous:*

$$\hat{C}_1 \gtrless \hat{C}_2,\ \hat{C}_1 \gtrless \hat{C}_3,\ \hat{C}_1 \gtrless \hat{C}_4,\ \hat{C}_2 \gtrless \hat{C}_3,\ \hat{C}_2 \gtrless \hat{C}_4 \text{ and } \hat{C}_3 \gtrless \hat{C}_4$$

Proof. See the Appendix. ■

Proposition 4 shows that the interpersonal well-being comparison between parents depends crucially on whether the altruistic component of parental preferences is regarded as relevant for well-being measurement. Under the laundering of parental preferences, when preferences of children are the same, less altruistic parents are regarded as strictly better off than more altruistic parents. This result comes from the fact that more altruistic parents make more efforts, in time and material goods, to raise their children at the expense of their own consumption. As a consequence, and given that the effects of these larger efforts on their children's well-being are not taken into account under the laundering approach, more altruistic parents are worse-off, *ceteris paribus*, than less altruistic parents. Hence, from an egalitarian perspective, priority should be given to more altruistic parents, who make more sacrifices for their children. However, this result may not hold once there is no laundering of parental preferences. The reason is that the effects of parental efforts on children's well-being are then incorporated in the well-being indexes of parents (through children's utility), and may outweigh the lower levels of the self-oriented well-being component achieved by more altruistic parents in comparison to less altruistic parents.

5 Egalitarian social optima

In the economy studied in this paper, the definition of the social optimum is made complex by the existence of several sources of heterogeneity, which are *nested* in each other: children's heterogeneity in preferences affects, through parental altruism, parents' preferences (which also differ in the degree of altruism). To deal with heterogeneity, Sections 3 and 4 examined interpersonal well-being comparisons across children facing unequal needs and unequal degrees of parental altruism, as well as interpersonal well-being comparisons across parents whose children have distinct preferences, and who are more or less altruistic.

While Sections 3 and 4, by focusing on individual well-being measurement, described the informational basis relevant for the definition of social well-being, these sections did not address the question of the *aggregation* of these well-being levels into a social welfare objective. To address that issue, the relevant concept is the social welfare function (SWF), which aggregates and weights the well-being levels of all persons. The SWF incorporates, within the economic calculus, all value judgements about the 'just society'.

This paper will rely on egalitarian SWFs, which we justify as follows. Following Fleurbaey (2008) and Fleurbaey and Maniquet (2010), fairness requires to distinguish between *circumstances*, over which the individual has no control, and *choice variables*, for which the individual can be regarded as responsible. From a normative perspective, it can be argued that well-being inequalities due to circumstances are unfair, because individuals could not have done otherwise. This intuition underpins the Principle of Compensation, which states that well-being inequalities due to circumstances should be abolished by the State. However, well-being inequalities due to variables chosen by individuals should not be regarded as unfair, because individuals should be held responsible for their choices. This intuition is captured by the Principle of Liberal Reward, which states that well-being inequalities due to choice variables should be left unchanged. Interestingly, it is often the case that the choices made by individuals differ depending on the circumstances they face, so that the two principles cannot be both satisfied (Fleurbaey and Maniquet 2010). One must then choose between the Principle of Compensation and the Principle of Liberal Reward.

Children's preferences (related to their needs) and the degree of altruism of their parents can be viewed as pure circumstances faced by children. Hence, one should not accept that children's well-being varies because of unequal needs or unequal parental altruism, for which they bear no responsibility. This motivates the consideration of *egalitarian social optima* that equalize well-being across all children, regardless of their needs or family background.

On the side of parents, well-being is a mix of circumstances (children's preferences) and choice variables (their own altruism). It is not always possible to satisfy both the Principle of Compensation and the Principle of Liberal Reward. Under the laundering of parental preferences, one can both satisfy the Principle of Compensation (i.e., equalize the well-being of all children) and allow for unequal well-being among parents based on unequal degrees of altruism, in line with the Principle of Liberal Reward. However, without the laundering of parental preferences, circumstances (children's preferences) affect parental well-being for a given degree of parental altruism, so that it is difficult to equalize all children's well-being without affecting well-being inequalities among parents having unequal altruism.

Accounting for these difficulties, this paper will consider two distinct social optima, which both satisfy the Principle of Compensation from the perspective of children's well-being (and thus, compensate children for the circumstances they face), but which differ in the treatment of parental well-being inequalities:

- The *constrained egalitarian optimum*: the allocation of material and time resources that maximizes the minimum level of children's well-being (in line with the Principle of Compensation applied to children), whereas the same (arbitrary) level of consumption is given to all parents (which may still lead to parental well-being inequalities depending on their degree of altruism, in line with the Principle of Liberal Reward).

- The *double egalitarian optimum*: the allocation of material and time resources that maximizes the minimum level of children's well-being (in line with the Principle of Compensation applied to children), while at the same time imposing the constraint that all parents enjoy the same well-being (in line with the Principle of Compensation applied to parents).

5.1 The constrained egalitarian optimum

The constrained egalitarian social optimum maximizes the well-being of the worst-off of all children, subject to all parents receiving the same consumption level $\bar{C} > 0$.[10] In order to examine the robustness of the optimal family policy to how the problem of interpersonal well-being comparisons between

[10] It is necessary to set $\bar{C} > 0$, as otherwise the maximization of children's well-being would lead to leaving no resources to parents ('tyranny of children').

children (discussed in Section 3) is solved, this section will first study the constrained egalitarian optimum when children's well-being is measured by means of consumption-equivalents, and, second, reexamine that social planning problem when children's well-being is measured by means of time-equivalents.

For the sake of simplicity, we will, in the remainder of this section, assume that children's preferences are quasi-linear.

Assumption 1 (quasi-linear preferences for children)**.** $u'(g_i) = 1$.

The main virtue of this assumption is to simplify the comparison of egalitarian optima based on consumption-equivalent or time-equivalent indexes of well-being. Of course, this assumption has also some costs. In particular, it oversimplifies the decentralization of the social optimum. This is the reason why, when considering the numerical illustration in Section 6, we relax Assumption 1 and assume instead that $u(.)$ takes a logarithmic form.

5.1.1 Social optimum under consumption-equivalents

The constrained social planning problem can be written as:

$$\max_{\substack{g_1,g_2,g_3,g_4 \\ T_1,T_2,T_3,T_4}} \min\{\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4\}$$

$$\text{s.t.} \qquad \sum_{i=1}^{4} w(1-T_i) = 4\bar{C} + \sum_{i=1}^{4} g_i \tag{14}$$

This problem is solved in the Appendix. Proposition 5 summarizes our results.

Proposition 5. *Assume that children's well-being is measured by consumption-equivalents $\hat{g}_i$ with $\bar{T} = T_2$. Under Assumption 1, the constrained egalitarian optimum $\{g_i^*, T_i^*\}\, \forall i \in \{1,2,3,4\}$ is such that*

$$\begin{aligned} g_1^* &= g_2^* > g_3^* = g_4^* \\ T_1^* &= T_2^* < T_3^* = T_4^* \end{aligned}$$

All children are made equally well-off: $\hat{g}_i^* = \hat{g}_j^*\ \forall i \neq j$. Under $C_i^* = \bar{C}\ \forall i$, parental well-being is such that :

> *under laundering of parental preferences, $\hat{C}_1^* = \hat{C}_2^* = \hat{C}_3^* = \hat{C}_4^* = \bar{C}$,*
> *without laundering of parental preferences, $\hat{C}_4^* < \hat{C}_2^*, \hat{C}_3^* < \hat{C}_1^*$.*

Proof. See the Appendix. ■

At this social optimum, children are fully compensated for the more or less adverse circumstances they face, i.e. both their heterogeneous preferences (reflecting needs beyond their control) and their parent's altruism, for which they are likewise not responsible. At the allocation of Proposition 5, all children, independently of their parents' preferences, achieve the same level of well-being (measured by means of consumption-equivalents). However, the well-being levels of parents are not necessarily equalized at the constrained egalitarian optimum. Under laundering of parental preferences, parents are all equally well-off at that social optimum. But in the absence of laundering of parental preferences, more altruistic parents are, *ceteris paribus*, better-off than less altruistic parents.[11]

The decentralization of the constrained egalitarian optimum can be achieved as follows.

Proposition 6. *The constrained egalitarian optimum when children's well-being is measured by means of consumption-equivalents $\hat{g}_i$ with $\bar{T} = T_2$, can be decentralized as follows.*

- *The State provides the optimal child allowance g_i^* to each child of type i by means of (non-exchangeable) vouchers.*
- *The State imposes a lump-sum tax $L_i^* = w(1 - T_i^*) - \bar{C}$ to each parent of type i, so as to equalize all parental consumptions to $\bar{C}$.*

Proof. See the Appendix. ■

The constrained egalitarian optimum can be implemented by directly allocating the optimal amount of consumption goods g_i^* to children—through non-exchangeable vouchers giving access to in-kind material resources or school materials—and by providing lump-sum transfers L_i^* to parents so as to reach the target level of parental consumption $\bar{C}$.[12] In Appendix 9.4, it is shown that the first-best constrained egalitarian optimum is incentive-compatible, that is, it can be implemented under asymmetric information.

5.1.2 Social optimum under time-equivalents

Let us now explore the robustness of our findings to the measurement of children's well-being. For that purpose, this subsection derives the constrained egalitarian optimum when children's well-being is measured by means of time-equivalent indexes, under reference consumption $\bar{g}$ set to g_4.

The constrained egalitarian social planning problem is now:

$$\max_{\substack{g_1,g_2,g_3,g_4\\T_1,T_2,T_3,T_4}} \min\left\{\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4\right\}$$

$$\text{s.t.} \qquad \sum_{i=1}^{4} w(1 - T_i) = 4\bar{C} + \sum_{i=1}^{4} g_i \tag{15}$$

That alternative social planning problem is solved in the Appendix and the following proposition summarizes our results.

[11] Indeed, all children are, for given children's preferences, treated equally at the constrained egalitarian optimum, so that more altruistic parents derive, from that same treatment of their children, a higher well-being than less altruistic ones, other things remaining equal.

[12] Note that, under quasi-linear children's preferences, a tax on labor earnings is not required to decentralize the optimal parental time investment in children T_i, since the laissez-faire first-order condition for T_i already coincides with the optimal trade-off. See Appendix 9.3.

Proposition 7. *Assume that children's well-being is measured by time-equivalents with* $\bar{g} = g_4$. *Under Assumption 1, the constrained egalitarian optimum* $\left\{g_i^+, T_i^+\right\}$ $\forall i \in \{1, 2, 3, 4\}$ *is such that*

$$\begin{aligned} g_1^+ &= g_2^+ \gtrless g_3^+ = g_4^+ \\ T_1^+ &= T_2^+ < T_3^+ = T_4^+ \end{aligned}$$

All children are made equally well-off: $\hat{T}_i^+ = \hat{T}_j^+$ $\forall i \neq j$. *Under* $C_i^* = \bar{C}$ $\forall i$, *parental well-being is such that :*

> *under laundering of parental preferences,* $\hat{C}_1^+ = \hat{C}_2^+ = \hat{C}_3^+ = \hat{C}_4^+ = \bar{C}$,
> *without laundering of parental preferences,* $\hat{C}_2^+ < \hat{C}_1^+, \hat{C}_4^+ < \hat{C}_3^+$.

Proof. See the Appendix. ■

The constrained egalitarian optimum under time-equivalents looks, from a purely qualitative perspective, close to the social optimum under consumption-equivalents. Under both social optima, the well-being levels of all children are equalized, independently of their preferences, and no matter how altruistic their parents are. Moreover, the constrained egalitarian optimum also equalizes all parental well-being levels under the laundering of parental preferences, whereas in the absence of laundering, more altruistic parents are, *ceteris paribus*, better off than less altruistic parents.[13]

Having stressed this convergence in qualitative terms, some differences remain. A major difference is that the ranking of children's material consumptions is no longer unambiguous. Whereas children with more intense time needs received fewer material resources when the egalitarian optimum was computed on the basis of consumption-equivalent indexes, this is not necessarily the case when the egalitarian optimum is based on time-equivalent indexes. Proposition 8 compares the two constrained egalitarian social optima.

Proposition 8. *The comparison of the constrained egalitarian optimum based on children's consumption-equivalents (***) with the constrained egalitarian optimum based on children's time-equivalents (*$^+$*) yields the following results:*

- *parental time-investments are exactly equal in the two social optima:*

$$T_i^* = T_i^+ \forall i \in \{1, 2, 3, 4\}$$

- *children's consumptions differ in the two social optima:*

$$g_i^* \gtrless g_i^+ \forall i \in \{1, 2, 3, 4\}$$

[13] Note that, using a proof similar to the one in Appendix 9.4, it can be shown that the first-best constrained egalitarian optimum when children's well-being is measured by means of time-equivalents can be implemented under asymmetric information, as it is the case for the first-best constrained egalitarian optimum when children's well-being is measured by means of consumption-equivalent indexes.

- *in the special case when all children have the same preferences (i.e. $\breve{\beta} = \bar{\beta}$), the two social optima are exactly the same:*

$$T_i^* = T_i^+ = T^{*+} \text{ and } g_i^* = g_i^+ = g^{+*} \forall i \in \{1, 2, 3, 4\}$$

Proof. See the Appendix. ■

Whether one measures children's well-being by means of consumption-equivalents or time-equivalents, the constrained egalitarian social optimum involves exactly the same levels of parental time-investments. The reason is that, in the two social planning problems, the conditions for the optimal allocation of parental time between labor and children are exactly the same. However, children's optimal material consumptions are generally not equal across the two social optima. There is an exception: in the special case where all children have the same preferences, the two constrained social optima are quantitatively the same (even when parental preferences differ).

Thus, the optimal egalitarian allocation depends on the chosen metric for children's well-being measurement. The intuition is as follows. The total amount of material resources available for children's consumption is the same across the two social planning problems. However, the chosen metric for well-being measurement influences well-being rankings and the form of the optimal compensation. Consequently, the equalization of the consumption-equivalent indexes across children cannot be achieved with the same allocation of material resources as the one equalizing all time-equivalent indexes.

Similarly to Proposition 6, the constrained egalitarian optimum under time-equivalents can be decentralized by means of (non-exchangeable) vouchers, so as to implement the optimal amount of material consumption g_i^+ to each child of type i. Additionally, the State should impose a lump-sum tax L_i^+ to each parent of type i, so as to equalize all parental consumptions to $\bar{C}$. The two decentralization problems rely on qualitatively similar policy instruments. However, because the levels of material consumption vary depending on whether children's well-being is measured in terms of consumption- or time-equivalents, the corresponding optimal policy also differs. As a result, the optimal design of family policies is not robust to the chosen metric of children's well-being. The key intuition is that this metric shapes how children's relative advantages and disadvantages are assessed, and therefore determines the size of the policy instruments required to restore fairness.

5.2 The double egalitarian optimum

In this section, we consider an alternative social optimum - the double egalitarian optimum -, where not only children's well-being levels are equalized, but, also, all parental well-being levels. The justification is that parents are not responsible for the preferences of their children. Hence, even though they can be regarded as responsible for their own preferences (i.e. their altruism), there is some support for equalizing all parents's well-being levels.

When studying the double egalitarian social optimum, we will, as above, rely on two alternative metrics for the measurement of children's well-being. Moreover, in each case, we will distinguish between the cases where there is either laundering or no laundering of parental preferences.

5.2.1 Social optimum under consumption-equivalents

The double egalitarian social planning problem can be written as follows:

$$\begin{aligned} &\max_{\substack{C_1,C_2,C_3,C_4\\ g_1,g_2,g_3,g_4\\ T_1,T_2,T_3,T_4}} \min\{\hat{g}_1,\hat{g}_2,\hat{g}_3,\hat{g}_4\} \\ \text{s.t.}\quad & \sum_{i=1}^{4} w(1-T_i) = \sum_{i=1}^{4} C_i + \sum_{i=1}^{4} g_i \\ \text{s.t.}\quad & \hat{C}_1 = \hat{C}_2 = \hat{C}_3 = \hat{C}_4 = \tilde{C} \end{aligned} \tag{16}$$

where $\tilde{C} > 0$ is a reference parental well-being level, at which all parental well-being levels will be equalized.[14]

Depending on whether there is laundering of parental preferences or not, parental consumption-equivalents $\hat{C}_i$ are defined differently. Under laundering of parental preferences, parental consumption-equivalents are:

$$\begin{aligned} U(\hat{C}_1) &= U(C_1^{**}) \\ U(\hat{C}_2) &= U(C_2^{**}) \\ U(\hat{C}_3) &= U(C_3^{**}) \\ U(\hat{C}_4) &= U(C_4^{**}) \end{aligned}$$

where the superscript ** refers to the double egalitarian social optimum.

In the absence of laundering of parental preferences, and assuming $\bar{T} = T_2$ and $\bar{g} = g_4$, parental consumption-equivalents are:

$$U(\hat{C}_1) = U(C_1^{**}) + \bar{\alpha}\left[(1-\breve{\beta})\,(g_1^{**} - g_4) + \breve{\beta}(v(T_1^{**}) - v(T_2))\right] \tag{17}$$

$$U(\hat{C}_2) = U(C_2^{**}) + \breve{\alpha}\left[(1-\breve{\beta})\,(g_2^{**} - g_4) + \breve{\beta}(v(T_2^{**}) - v(T_2))\right] \tag{18}$$

$$U(\hat{C}_3) = U(C_3^{**}) + \bar{\alpha}\left[(1-\bar{\beta})\,(g_3^{**} - g_4) + \bar{\beta}(v(T_3^{**}) - v(T_2))\right] \tag{19}$$

$$U(\hat{C}_4) = U(C_4^{**}) + \breve{\alpha}\left[(1-\bar{\beta})\,(g_4^{**} - g_4) + \bar{\beta}(v(T_4^{**}) - v(T_2))\right] \tag{20}$$

These two social planning problems are solved in the Appendix. Proposition 9 summarizes our results.

Proposition 9. *Assume that children's well-being is measured by consumption-equivalents $\hat{g}_i$ with $\bar{T} = T_2$. Assume that parent's well-being is measured by*

[14] Imposing a positive reference parental well-being $\tilde{C} > 0$ avoids the problem of the 'tyranny of children'. Otherwise, it could be socially optimal to fully sacrifice parental well-being in the name of maximizing children's well-being.

consumption-equivalents $\hat{C}_i$ *with* $\bar{T} = T_2$ *and* $\bar{g} = g_4$. *Under Assumption 1, the double egalitarian optimum* $\{C_i^{**}, g_i^{**}, T_i^{**}\}$ $\forall i \in \{1, 2, 3, 4\}$ *is such that:*

$$\begin{aligned} g_1^{**} &= g_2^{**} > g_3^{**} = g_4^{**}, \\ T_1^{**} &= T_2^{**} < T_3^{**} = T_4^{**}. \end{aligned}$$

Under laundering of parental preferences,

$$C_1^{**} = C_2^{**} = C_3^{**} = C_4^{**} = \tilde{C}$$

In the absence of laundering of parental preferences

$$C_1^{**} < C_2^{**}, C_3^{**} < C_4^{**},$$

All children are made equally well-off: $\hat{g}_i^{**} = \hat{g}_j^{**}$ $\forall i \neq j$. *All parents are made equally well-off:* $\hat{C}_i^{**} = \hat{C}_j^{**}$ $\forall i \neq j$.

Proof. See the Appendix. ■

At the double egalitarian optimum, all children are equally well-off, despite having distinct preferences (due to different needs), and despite being born in families exhibiting unequal degrees of parental altruism. Whereas that result was already achieved under the constrained egalitarian social optimum, the double egalitarian optimum exhibits another feature: the equalization of the well-being levels of parents. This feature is also a key difference with respect to the laissez-faire equilibrium. But the implications of the equalization of parental well-being levels vary depending on whether there is laundering of parental preferences. Without laundering, less altruistic parents receive, at the double egalitarian optimum, more consumption than more altruistic parents *ceteris paribus*. However, under the laundering of parental preferences, all parents receive, at the double egalitarian optimum, the same consumption.

Like in the previous constrained egalitarian optimum, the double egalitarian optimum under consumption-equivalents can be decentralized using (non-exchangeable) vouchers, so as to directly implement the optimal amount of material consumption g_i^{**}. Additionally, the State should impose a lump-sum tax L_i^{**} on each parent of type i, so as to reach the optimal levels of parental consumptions C_i^{**}, which now may differ across types when there is no laundering of parental preferences. But an important difference is that, as we show in Appendix 9.8, the first-best double egalitarian optimum is not always implementable under asymmetric information, unlike the first-best constrained egalitarian optimum.[15]

[15] To be precise, the first-best double egalitarian optimum is, under the laundering of parental preferences, always implementable under asymmetric information. However, this is not the case when there is no laundering of parental preferences (see Appendix 9.8).

5.2.2 Social optimum under time-equivalents

When children's well-being is measured by means of time-equivalent indexes, the double egalitarian social planning problem becomes:

$$\max_{\substack{C_1,C_2,C_3,C_4\\ g_1,g_2,g_3,g_4\\ T_1,T_2,T_3,T_4}} \min\left\{\hat{T}_1, \hat{T}_2, \hat{T}_3, \hat{T}_4\right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{4} w(1-T_i) = \sum_{i=1}^{4} C_i + \sum_{i=1}^{4} g_i$$

$$\text{s.t.} \quad \hat{C}_1 = \hat{C}_2 = \hat{C}_3 = \hat{C}_4 = \tilde{C} \tag{21}$$

where the parental consumption-equivalents $\hat{C}_i$ admit two distinct definitions, depending on whether or not there is laundering of parental preferences (see *supra*). Proposition 10 summarizes our results.

Proposition 10. *Assume that children's well-being is measured by time-equivalents $\hat{T}_i$ with $\bar{g} = g_4$. Assume that parents's well-being is measured by means of consumption-equivalents $\hat{C}_i$ with $\bar{g} = g_4$ and $\bar{T} = T_2$. Under Assumption 1, the double egalitarian optimum $\left\{C_i^{++}, g_i^{++}, T_i^{++}\right\}$ $\forall i \in \{1,2,3,4\}$ is such that:*

$$\begin{aligned} g_1^{++} &= g_2^{++} \gtrless g_3^{++} = g_4^{++}, \\ T_1^{++} &= T_2^{++} < T_3^{++} = T_4^{++}. \end{aligned}$$

Under laundering of parental preferences,

$$C_1^{++} = C_2^{++} = C_3^{++} = C_4^{++} = \tilde{C}$$

In the absence of laundering of parental preferences,

$$C_3^{++} < C_4^{++}, C_1^{++} < C_2^{++}.$$

All children are made equally well-off: $\hat{T}_i^{++} = \hat{T}_j^{++}$ $\forall i \neq j$. All parents are made equally well-off: $\hat{C}_i^{++} = \hat{C}_j^{++}$ $\forall i \neq j$.

Proof. See the Appendix. ■

When based on children's time-equivalents, the double egalitarian social optimum shares some key qualitative features with the double egalitarian social optimum based on children's consumption-equivalents: under both optima, all children are equally well-off, and all parents are equally well-off. However, the two allocations are clearly distinct, as shown in Proposition 11.

Proposition 11. *The comparison of the double egalitarian optimum based on children's consumption-equivalents (**) with the double egalitarian optimum based on children's time-equivalents ($^{++}$) yields the following results:*

- *parental time-investments are exactly equal in the two social optima:*

$$T_i^{**} = T_i^{++} \ \forall i \in \{1,2,3,4\}$$

- *children's consumptions differ in the two social optima:*

$$g_i^{**} \gtrless g_i^{++} \ \forall i \in \{1,2,3,4\}$$

- *parents's consumptions differ in the two social optima:*

under preference laundering, $C_i^{**} = C_i^{++} = \tilde{C} \ \forall i \in \{1,2,3,4\}$

in the absence of preference laundering, $C_i^{**} \gtrless C_i^{++} \ \forall i \in \{1,2,3,4\}$

Proof. See the Appendix. ■

Exactly as in the comparison of constrained egalitarian social optima, the two double egalitarian optima share the same optimal parental investments, but differ in terms of children's consumption. Note that in addition, parental consumptions may now also differ across the two optima, depending on whether there is preference laundering.

Like for the previous optima, this double egalitarian optimum based on time-equivalents can be decentralized using the same policy instruments as in the previous sections: (non-exchangeable) vouchers so as to implement the optimal amount of material consumption g_i^{++}, together with lump-sum taxes L_i^{++} on each parent of type i so as to obtain the optimal levels of parental consumptions to C_i^{++}.

In sum, moving from the constrained egalitarian optimum to the double egalitarian optimum does not affect the type of policy instruments required to decentralize the optimum, but only their optimal levels. These levels also depend crucially on the chosen metric for measuring children's well-being: either consumption goods (under the consumption-equivalent index) or parental time (under the time-equivalent index). The precise way in which the problem of interpersonal well-being comparisons between children is solved translates into different levels of the same set of policy instruments.

6 Numerical illustration

To illustrate how the optimal family policies vary with the metric used for the measurement of children's well-being, this section will assign numerical values to the structural parameters and compute the constrained egalitarian optimum and the double egalitarian optimum under consumption-equivalent and time-equivalent indexes. Then, we will compare the levels of the different fiscal instruments that decentralize the social optima under study.

The four types of families, differing in children's needs and in parental altruism, are calibrated as follows (Table 2).

	parental altruism	
Child needs in time	$\bar{\alpha} = 0.8$	$\breve{\alpha} = 0.5$
$\breve{\beta} = 0.5$	type 1 ($\beta_1 = 0.5$, $\alpha_1 = 0.8$)	type 2 ($\beta_2 = 0.5$, $\alpha_2 = 0.5$)
$\bar{\beta} = 0.8$	type 3 ($\beta_3 = 0.8$, $\alpha_3 = 0.8$)	type 4 ($\beta_4 = 0.8$, $\alpha_4 = 0.5$)

Table 2. The numerical illustration: four types of families.

Children's utility functions and parents' utility functions take the following logarithmic form:

$$\begin{aligned} u(g_i) &= \log(g_i) \\ v(T_i) &= \log(T_i) \\ U(C_i) &= \log(C_i) \end{aligned}$$

As detailed in Appendix 9.11, if one assumed instead that $u(g_i)$ is linear (as in Section 5), the numerical exercise would lead to no labour earnings taxation. Hence, for the sake of realism, we assume, in this numerical part, that $u(g_i)$ is increasing and concave.

According to Eurostat, the median gross hourly earnings were 14.9 euros in the European Union (2022).[16] Hence, we will fix $w = 15$.

Considering the laissez-faire equilibrium, we can see that there exist substantial well-being inequalities between all children, whether children's well-being is measured either by means of a consumption-equivalent index (Figure 2, under $\bar{T} = T_2$), or by means of a time-equivalent index (Figure 3, under $\bar{g} = g_4$). Importantly, the hierarchy of children's types in terms of measured well-being is not invariant to the chosen index for well-being measurement. Whereas children living in families of type 3 (high time needs with high parental altruism) exhibit the highest well-being under consumption-equivalents, this is not the case under time-equivalents, where children of type 1 (low time needs with high parental altruism) exhibit higher well-being levels. The identification of the worst-off children varies also with the chosen metric for well-being measurement. Whereas the worst-off children belong to families of type 2 (low time needs with low parental altruism) under the consumption-equivalent index, the worst-off children belong to families of type 4 (high time needs with low parental altruism) under the time-equivalent index. This illustrates the results of Propositions 2 and 3.

Turning now to parental well-being, we can see that, at the laissez-faire, the ranking of parents in terms of well-being (measured by the consumption-equivalent index) varies depending on whether there is laundering of preferences. In the absence of laundering, the worst-off parents are parents of type 4 (high children time needs and low parental altruism), whereas under laundering, the worst-off parents are types 1 and 3 (with high altruism), as shown on Figures 4 and 5. Those findings illustrate the results of Proposition 4.

[16] Source: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Earnings_statistics

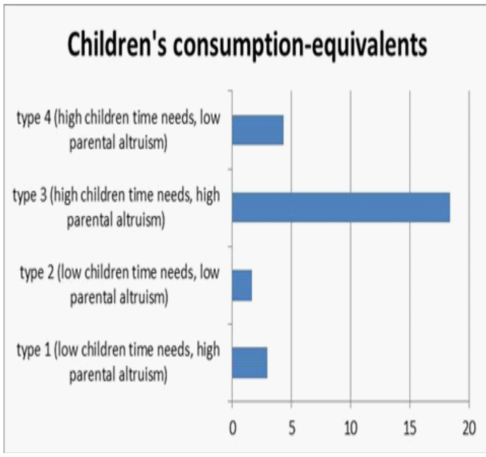


Figure 2. Children's consumption equivalent indexes at the laissez-faire.

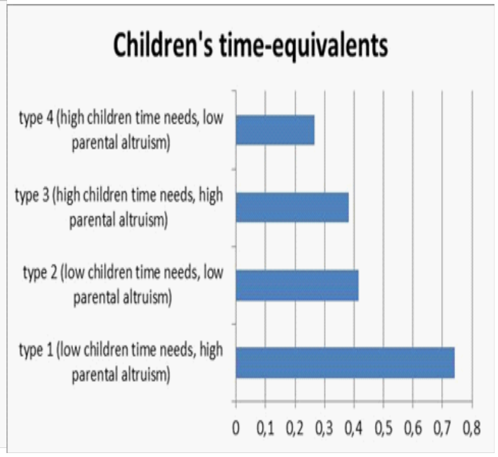


Figure 3. Children's time equivalent indexes at the laissez-faire.

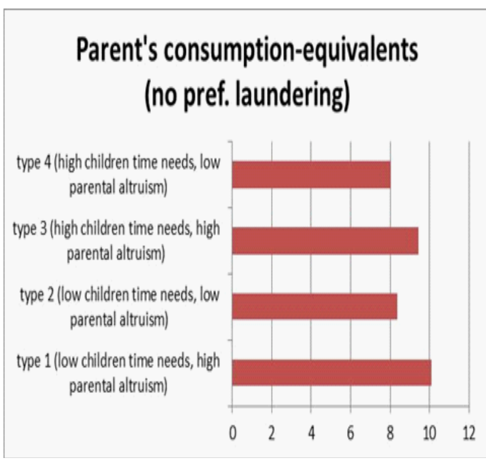


Figure 4. Parents' consumption-equivalent indexes at the laissez-faire (no preference laundering).

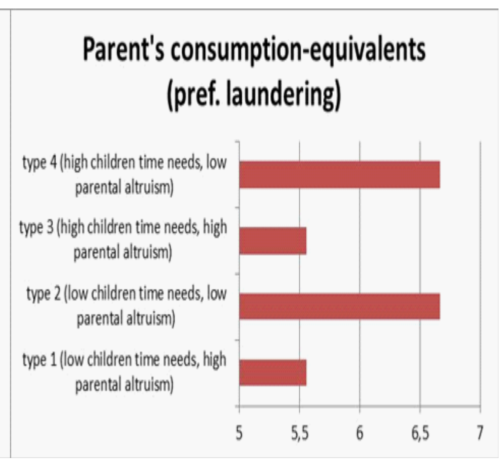


Figure 5. Parents' consumption-equivalent indexes at the laissez-faire (preference laundering).

In Appendix 9.11, Table A1 reports the optimal values for $(g_i, T_i, C_i)\, \forall i = \{1, 2, 3, 4\}$ as well as the values of the policy instruments $(\tau_i, L_i)\, \forall i = \{1, 2, 3, 4\}$ decentralizing the different social optima derived in the theoretical part of the paper. Under the constrained egalitarian optimum, we set $\bar{C} = 1$. For the double egalitarian optimum, we set $\tilde{C} = 1$.

The following figures report the optimal levels of each policy instrument under the different optima. Figure 6 compares the optimal levels of the direct children allowance across the four types of families, under the six different decentralized optima. A first observation is that, under all decentralized optima, families where children have the same needs are always treated in a similar way with respect to that policy instrument.

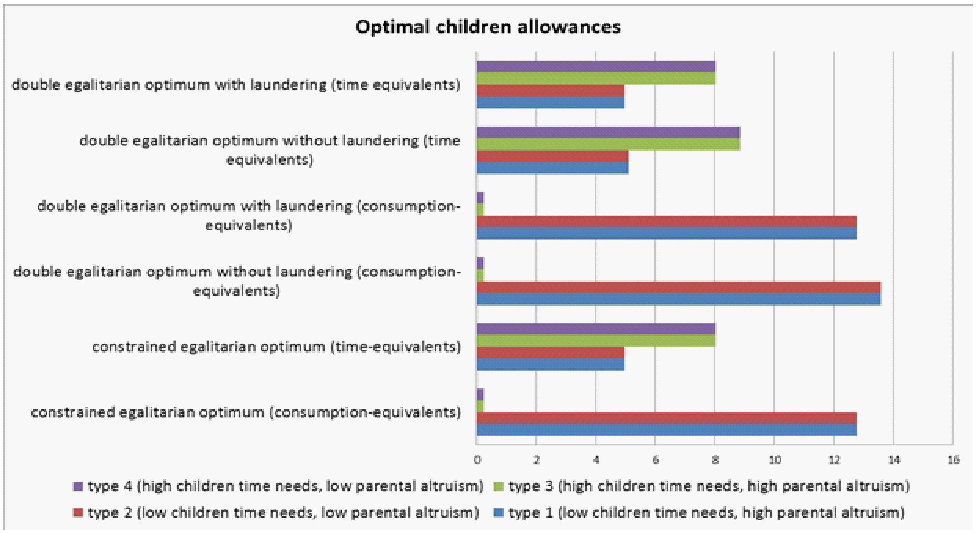
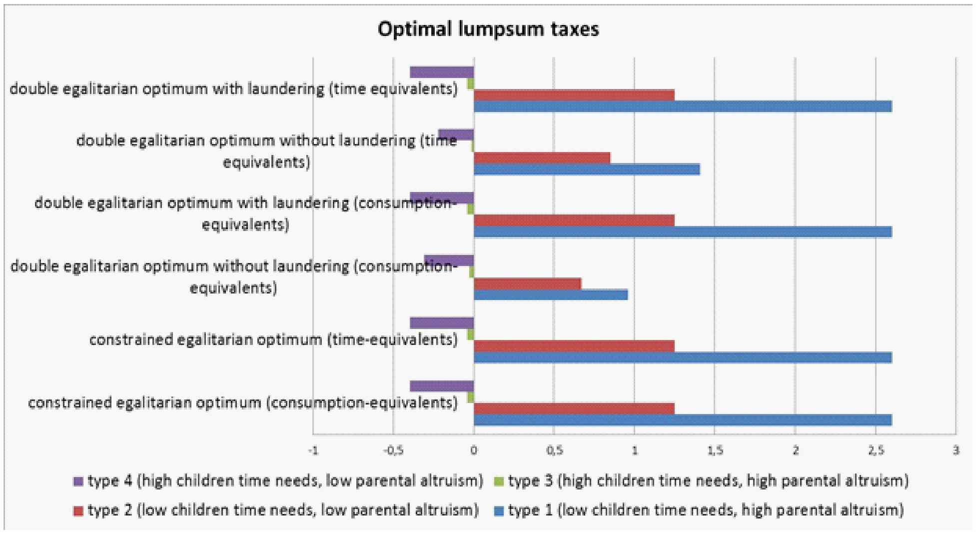


Figure 6. Optimal children allowances under the six decentralization problems.

However, the level of the optimal children allowance varies depending on the chosen metric for the measurement of children's well-being. Under the constrained problem, the optimal children allowance is much larger for children with low time needs than for children with high time needs when children's well-being is measured by means of consumption-equivalent indexes. The opposite prevails once children's well-being is measured by means of time-equivalent indexes. Under the double egalitarian problem, the ranking between the optimal children allowances across family types varies also with the chosen metric for the measurement of children's well-being, but does not change when assuming parental preferences laundering or not.

Let us now consider lump-sum taxes (Figure 7). The optimal lump-sum taxes exhibit robustness to the decentralization of the different optima: in all six cases, parents of types 1 and 2 (whose children have lower time needs) are taxed, in comparison to parents of types 3 and 4 (whose children have higher time needs) who instead obtain a lump-sum transfer. This result is robust to the chosen metric for the measurement of children's well-being.



Optimal lumpsum taxes under the six decentralization problems.

Finally, as shown in Figure 8, the ranking of the type-specific taxes on labour earnings is robust to the optimum considered, with $\tau_1 < \tau_2 < \tau_3 < \tau_4$. High-altruism parents whose children need low parental time, pay the lowest tax on labour earnings whereas, at the other end of the ranking, low-altruism parents whose children have high time needs pay the highest labour earnings tax rate. The underlying intuition is to more strongly discourage the labor supply of parents whose children require greater parental time investment, as well as that of parents with lower levels of altruism.

Figure 8. Optimal labour earnings tax under the six decentralization problems.

All in all, this numerical exploration illustrates that, although the rankings of type-specific lump-sum taxes and labour earnings taxes are robust across the six decentralized optima, the optimal level of children allowances varies with the chosen metric for well-being measurement.

7 Concluding remarks

It is a truism to say that, although all children are equal in front of the Law, they are not treated equally, because they are born in various family environments, which are more or less favorable to their flourishing as children (and, later on, as adults). It is also widely acknowledged that children cannot be held responsible for well-being inequalities due to the more or less favorable circumstances faced during their infancy. Hence, there is a strong normative support for family policies that would neutralize well-being inequalities due to family circumstances, in order to make all children enjoy their childhood equally.

But despite these two - hardly questionable - claims, the design of the optimal family policy remains a complex issue. The reason why the design of such a policy is complex lies in the roots of the problem: the presence of heterogeneity across children. Heterogeneity in children's needs translates itself into heterogeneity in children's preferences, making the interpersonal comparison of their well-being difficult. Moreover, the fact that parental well-being depends, through altruism, on their children's (heterogeneous) preferences suffices to make parental well-being hard to compare across parents. Furthermore, the fact that parents differ in their degree of altruism towards their children is also a source of heterogeneity. Consequently, it is not trivial to compare the well-being across children and across parents, making the design of a fair family policy difficult.

The present paper proposed a path to overcome those difficulties, by using equivalent-consumption and equivalent-time indexes in order to construct comparable measures of well-being across children, and, also, across parents. We then used these comparable measures of well-being to characterize two social optima that do justice to the intuition of compensating children for the unequal circumstances they face during their childhood: on the one hand, the constrained egalitarian optimum, which equalizes all children's well-being levels while giving equal consumption to all parents; on the other hand, the double egalitarian optimum, which equalizes all children's well-being levels, as well as all parents' well-being levels. We examined how these social optima can be decentralized in a first-best setting.

Our main result is that the design of the optimal family policy is not invariant to the metric used for the measurement of well-being across children. Depending on whether one uses consumption-equivalent indexes or time-equivalent indexes, the derived policies required for the decentralization of the social optima differ. The underlying intuition is that different metrics for well-being measurement leads to distinct quantifications of the (dis)advantages of children with respect to other children, and, also, of the (dis)advantages of parents with respect to other parents. Hence, the family policy that yields a fair allocation of resources is necessarily sensitive to how these advantages and disadvantages are measured.

Another important implication of our results concerns the discrepancy between existing family policies and the fair ones. Several differences should be underlined. First, actual family policies are generally limited to monetary family allowances, whereas the decentralization of egalitarian social optima would also require, in addition to monetary lump-sum transfers, type-specific (non-exchangeable) vouchers (giving rise to in-kind direct transfers to children), to best satisfy children's heterogeneous needs despite unequal degrees of parental altruism. Our results suggest also that the taxation of labour earnings should be designed so as to induce parents to allocate the socially optimal amount of time to their children, whose level differs from the laissez-faire. Moreover, the optimal levels of in-kind transfers depend on how one makes children's well-being levels comparable, that is, on how heterogeneity is taken into account in the social calculus underlying the design of family policies. This aspect is in sharp contrast with existing family policies, which do not rely on solid normative foundations regarding the treatment of heterogeneity across families.

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9 Appendix

9.1 Proof of Proposition 4

After simplifications, consumption-equivalents without laundering of parental preferences are defined as follows:

$$\begin{aligned}
U(\hat{C}_1) &= U((1-T_1)w - g_1) + \alpha_1 \left[(1-\breve{\beta})\left[u\left(g_1\right) - u\left(g_4\right)\right] + \breve{\beta}\left[v(T_1) - v(T_2)\right]\right] \\
U(\hat{C}_2) &= U((1-T_2)w - g_2) + \alpha_2(1-\breve{\beta})\left[u\left(g_2\right) - u\left(g_4\right)\right] \\
U(\hat{C}_3) &= U((1-T_3)w - g_3) + \alpha_3\left[(1-\bar{\beta})\left[u\left(g_3\right) - u\left(g_4\right)\right] + \bar{\beta}\left[v(T_3) - v(T_2)\right]\right] \\
U(\hat{C}_4) &= U((1-T_4)w - g_4) + \alpha_4\bar{\beta}\left[v(T_4) - v(T_2)\right]
\end{aligned}$$

At the laissez-faire equilibrium, we have: $g_1 > g_2, g_3 > g_4 > 0$ and $T_3 > T_1, T_4 > T_2 > 0$. We also have $\alpha_1 = \alpha_3 > \alpha_2 = \alpha_4$. From those inequalities, we have: $U((1-T_2)w - g_2) > U((1-T_1)w - g_1)$ as well as $U((1-T_4)w - g_4) > U((1-T_3)w - g_3)$. However, those inequalities do not allow us to know more about the ranking of consumption-equivalents. Hence we have:

$$\hat{C}_1 \gtrless \hat{C}_2, \hat{C}_1 \gtrless \hat{C}_3, \hat{C}_1 \gtrless \hat{C}_4, \hat{C}_2 \gtrless \hat{C}_3, \hat{C}_2 \gtrless \hat{C}_4 \text{ and } \hat{C}_3 \gtrless \hat{C}_4$$

Under the laundering of parental preferences, parents's consumption-equivalents are:

$$\begin{aligned}
U(\hat{C}_1) &= U((1-T_1)w - g_1) \\
U(\hat{C}_2) &= U((1-T_2)w - g_2) \\
U(\hat{C}_3) &= U((1-T_3)w - g_3) \\
U(\hat{C}_4) &= U((1-T_4)w - g_4)
\end{aligned}$$

Given that, at the laissez-faire, we have: $g_1 > g_2, g_3 > g_4 > 0$ and $T_3 > T_1, T_4 > T_2 > 0$, it follows that:

$$\begin{aligned}
\hat{C}_2 &> \hat{C}_1 \text{ and } \hat{C}_4 > \hat{C}_3, \\
\hat{C}_1 &\gtrless \hat{C}_3,\ \hat{C}_1 \gtrless \hat{C}_4 \text{ and } \hat{C}_2 \gtrless \hat{C}_3.
\end{aligned}$$

9.2 Proof of Proposition 5

The objective function of the constrained social planning problem written in (14) is not differentiable. But the problem can be rewritten as the maximization of the consumption-equivalent of children of type 2 (assuming this type is the worst-off), subject to egalitarian constraints specifying the equalities of all children's consumption-equivalents, as well as subject to the above resource constraint.

Under Assumption 1, the Lagrangian of this problem can be written as:

$$\begin{aligned}
\mathcal{L} &= g_2 + \lambda_1[g_1 + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1) - v(T_2)] - g_2] \\
&+ \lambda_3[g_3 + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3) - v(T_2)] - g_2] \\
&+ \lambda_4[g_4 + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_4) - v(T_2)] - g_2] \\
&+ \mu[\sum_{i=1}^{4} w(1-T_i) - 4\bar{C} - \sum_{i=1}^{4} g_i]
\end{aligned}$$

where we replaced for $\bar{T} = T_2$ in the definitions of $\hat{g}_i$ (see equations 6-9). The first-order conditions of this problem are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial g_2} &= 1 - (\lambda_1 + \lambda_3 + \lambda_4) - \mu = 0 \\
\frac{\partial \mathcal{L}}{\partial T_2} &= \frac{\breve{\beta}}{1-\breve{\beta}} v'(T_2)(1 - (\lambda_1 + \lambda_3 + \lambda_4)) - \mu w = 0 \\
\frac{\partial \mathcal{L}}{\partial g_i} &= \lambda_i - \mu = 0 \, \forall i = \{1,3,4\} \\
\frac{\partial \mathcal{L}}{\partial T_i} &= \lambda_i \frac{\beta_i}{1-\beta_i} v'(T_i) - \mu w = 0 \, \forall i = 1,3,4
\end{aligned}$$

Rearranging these conditions, we obtain the following trade-off for T_i:

$$\frac{\beta_i}{1-\beta_i} v'(T_i) = w \, \forall \, i = \{1,2,3,4\}. \tag{22}$$

which implies that $T_3^* = T_4^* > T_1^* = T_2^*$. From the binding egalitarian constraints, we can then deduce :

$$\begin{aligned}
\hat{g}_1 = \hat{g}_2 \iff g_1^* + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1^*) - v(T_2)] &= g_2^* + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_2^*) - v(T_2)] \implies g_1^* = g_2^*, \\
\hat{g}_3 = \hat{g}_4 \iff g_3^* + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3^*) - v(T_2)] &= g_4^* + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_4^*) - v(T_2)] \implies g_3^* = g_4^*, \\
\hat{g}_1 = \hat{g}_3 \iff g_1^* + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1^*) - v(T_2)] &= g_3^* + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3^*) - v(T_2)] \implies g_1^* > g_3^*,
\end{aligned}$$

leading to the following complete ranking:

$$g_1^* = g_2^* > g_3^* = g_4^*.$$

Let us now look at the well-being of parents at the optimal allocation. It will depend on whether parents' preferences are laundered from altruism or not. If preferences are laundered from the altruism component, parental consumption equivalents $\hat{C}_i^*$ are all equal to the constrained level of parental consumption $\bar{C}$, since $U(\hat{C}_i^*) = U(\bar{C}) \forall i$. Hence, the well-being of parents is mechanically equalized. If preferences are not laundered, parental consumption-equivalents are:

$$
\begin{aligned}
U(\hat{C}_1) &= U(\bar{C}) + \bar{\alpha}\left[(1-\breve{\beta})(g_1^* - g_4) + \breve{\beta}(v(T_1^*) - v(T_2))\right] \\
U(\hat{C}_2) &= U(\bar{C}) + \breve{\alpha}\left[(1-\breve{\beta})(g_2^* - g_4) + \breve{\beta}(v(T_2^*) - v(T_2))\right] \\
U(\hat{C}_3) &= U(\bar{C}) + \bar{\alpha}\left[(1-\bar{\beta})(g_3^* - g_4) + \bar{\beta}(v(T_3^*) - v(T_2))\right] \\
U(\hat{C}_4) &= U(\bar{C}) + \breve{\alpha}\left[(1-\bar{\beta})(g_4^* - g_4) + \bar{\beta}(v(T_4^*) - v(T_2))\right]
\end{aligned}
$$

where reference levels $\bar{T}$ and $\bar{g}$ are set respectively to T_2 and g_4. Recall that $\bar{\alpha} > \breve{\alpha}$. Since $g_1^* = g_2^*$ and $T_1^* = T_2^*$, we have that $\hat{C}_1^* > \hat{C}_2^*$. Since $g_3^* = g_4^*$ and $T_3^* = T_4^*$, $\hat{C}_3^* > \hat{C}_4^*$.

Note that parents' consumption-equivalents can be rewritten as a function of children consumption equivalents as follows:

$$U(\hat{C}_i) = U(C_i) + \alpha_i(\hat{g}_i - \bar{g})(1 - \beta_i) \,\forall i \tag{23}$$

Under the full equalization of the $\hat{g}_i$s, and since $C_i = \bar{C} \,\forall i$, we further obtain that $\hat{C}_2^* > \hat{C}_4^*$ and $\hat{C}_1^* > \hat{C}_3^*$. This completes the proof.

9.3 Proof of Proposition 6

Under quasi-linear children's preferences, the laissez-faire condition (5) simplifies to

$$\frac{\beta_i}{1-\beta_i} v'(T_i) = w$$

which is identical the optimal trade-off for parental time investment in children (eq. 22). This implies that the laissez-faire level of parental time is identical to its optimal level and therefore, that a tax on labour earnings is not necessary.

The optimum can thus be decentralized by directly providing material consumption g_i^* to parents (and children). In order to ensure that consumption of every parent is equal to $\bar{C}$, we also introduce lump-sum tax, L_i. This optimal level of this tax takes the simple following form

$$L_i^* = w(1 - T_i^*) - \bar{C} \,\forall i \in \{1, 2, 3, 4\}.$$

9.4 Second-best constrained egalitarian optimum

To examine whether the first-best constrained egalitarian optimum can be implemented under asymmetric information, let us first consider the social optimum when children's well-being is measured by means of consumption-equivalent indexes.

The utility of parents i is written as

$$U(C_i) + \alpha_i[(1-\beta_i)u(g_i) + \beta_i v(T_i)]$$

According to Proposition 5 (constrained egalitarian optimum under consumption-equivalents), since $C_i = \bar{C}\ \forall i$, the only reason why a parent of type i may want to mimic a parent of type j is because the utility obtained by their children (second part in the equation above) is higher. In the following, we prove that this can never happen.

A parent of type 3 (equivalently of type 4) is better off with his allocation rather than the one designed for a type 1 (equivalently of a type 2) if and only if[17]

$$(1-\bar{\beta})u(g_3) + \bar{\beta}v(T_3) \geq (1-\bar{\beta})u(g_1) + \bar{\beta}v(T_1)$$

Subtracting $-\bar{\beta}v(\bar{T})$ on both sides, we obtain

$$(1-\bar{\beta})u(\hat{g}_3) \geq (1-\bar{\beta})u(g_1) + \bar{\beta}v(T_1) - \bar{\beta}v(\bar{T})$$

Rearranging the RHS, we can further obtain

$$\begin{aligned} & (1-\bar{\beta})u(\hat{g}_1) \geq (1-\breve{\beta})u(\hat{g}_1) + (\breve{\beta}-\bar{\beta})u(g_1) + (\breve{\beta}-\bar{\beta})[v(\bar{T}) - v(T_1)] \\ \iff \quad & u(\hat{g}_1) \leq u(g_1) + [v(\bar{T}) - v(T_1)] \end{aligned}$$

since $\hat{g}_1 = \hat{g}_3$. Replacing for the definition of $\hat{g}_1$ on the LHS, we further obtain

$$\frac{\breve{\beta}}{1-\breve{\beta}}(v(T_1) - v(\bar{T})) \leq [v(\bar{T}) - v(T_1)]$$

which leads after some simplifications to

$$0 \leq v(\bar{T}) - v(T_1)$$

In the paper, we have assumed $\bar{T} = T_2$ with T_2 evaluated at the laissez-faire. Also, $T_1^* = T_2^*$ under the constrained egalitarian optimum (see Proposition 5). We have also shown that the laissez-faire level of time needs equals that of the optimum (see Proof 9.3), under quasi-linearity of the child's utility function. This leads to $v(\bar{T}) = v(T_2) = v(T_2^*) = v(T_1^*)$, so that the above inequality is binding.

Hence, the child (and the parent) of type 3 (equiv. of type 4) gets the same utility whether the parent claims to be a type 1 (equiv. a type 2) or tells the truth. This leads to the conclusion that there is no mimicking incentive and that the optimal allocation under the constrained egalitarian optimum is also implementable under asymmetric information.

[17] For notational simplicity we drop the superscript $*$ in the calculations below but it refers to the optimal levels of g_i and T_i.

9.5 Proof of Proposition 7

The objective of the constrained social planning problem (15) is not differentiable. But it can still be rewritten as the maximization of the time-equivalent of a type-2 child (we assume here that the worst-off child is a type 2), subject to egalitarian constraints specifying the equalities of all children's time-equivalents, as well as subject to the above resource constraint.

The Lagrangian of this alternative egalitarian problem is:

$$\begin{aligned}
\mathcal{L} &= v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4) \\
&+ \lambda_1[v(T_1) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_1 - g_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \lambda_3[v(T_3) + \frac{1-\bar{\beta}}{\bar{\beta}}(g_3 - g_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \lambda_4[v(T_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \mu[\sum_{i=1}^{4} w(1-T_i) - 4\bar{C} - \sum_{i=1}^{4} g_i]
\end{aligned}$$

The first-order conditions of this problem are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial T_2} &= v'(T_2)(1-\lambda_1-\lambda_3-\lambda_4) - \mu w = 0 \\
\frac{\partial \mathcal{L}}{\partial g_2} &= \frac{1-\breve{\beta}}{\breve{\beta}}(1-\lambda_1-\lambda_3-\lambda_4) - \mu = 0 \\
\frac{\partial \mathcal{L}}{\partial T_i} &= \lambda_i \frac{1-\beta_i}{\beta_i} v'(T_i) - \mu w = 0 \, \forall i = \{1,3,4\} \\
\frac{\partial \mathcal{L}}{\partial g_i} &= \lambda_i \frac{1-\beta_i}{\beta_i} - \mu = 0 \, \forall i = 1,3,4
\end{aligned}$$

Exactly as for the consumption-equivalents, we obtain that T_i^+ is defined by equation (22) implying that

$$T_1^+ = T_2^+ < T_3^+ = T_4^+.$$

From the binding egalitarian constraints, we can then deduce :

$$\begin{aligned}
\hat{T}_1 &= \hat{T}_2 \Longleftrightarrow \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^+ - g_4\right) + v(T_1^+) = \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_2^+ - g_4\right) + v(T_2^+) \implies g_1^+ = g_2^+, \\
\hat{T}_3 &= \hat{T}_4 \Longleftrightarrow \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(g_3^+ - g_4\right) + v(T_3^+) = \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(g_4^+ - g_4\right) + v(T_4^+) \implies g_3^+ = g_4^+, \\
\hat{T}_1 &= \hat{T}_3 \Longleftrightarrow \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^+ - g_4\right) + v(T_1^+) = \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(g_3^+ - g_4\right) + v(T_3^+) \implies g_1^+ \gtrless g_3^+,
\end{aligned}$$

leading to the following incomplete ranking:

$$g_1^+ = g_2^+ \gtrless g_3^+ = g_4^+.$$

Consider now parental well-being. Under laundering of parental preferences, we have that all parents have the same consumption-equivalent, equal to $\bar{C}$. If preferences are not laundered, parental consumption-equivalents are:

$$\begin{aligned}
U(\hat{C}_1) &= U(\bar{C}) + \alpha_1 \left[(1-\breve{\beta}) \left(g_1^+ - g_4 \right) + \breve{\beta}(v(T_1^+) - v(T_2)) \right] \\
U(\hat{C}_2) &= U(\bar{C}) + \alpha_2 \left[(1-\breve{\beta}) \left(g_2^+ - g_4 \right) + \breve{\beta}(v(T_2^+) - v(T_2)) \right] \\
U(\hat{C}_3) &= U(\bar{C}) + \alpha_3 \left[(1-\bar{\beta}) \left(g_3^+ - g_4 \right) + \bar{\beta}(v(T_3^+) - v(T_2)) \right] \\
U(\hat{C}_4) &= U(\bar{C}) + \alpha_4 \left[(1-\bar{\beta}) \left(g_4^+ - g_4 \right) + \bar{\beta}(v(T_4^+) - v(T_2)) \right]
\end{aligned}$$

where reference levels $\bar{T}$ and $\bar{g}$ are set respectively to T_2 and g_4. Recall that $\bar{\alpha} > \breve{\alpha}$. Since $g_1^+ = g_2^+$ and $T_1^+ = T_2^+$, while we have that $\hat{C}_1^+ > \hat{C}_2^+$. Since $g_3^+ = g_4^+$ and $T_3^+ = T_4^+$, we have that $\hat{C}_3^+ > \hat{C}_4^+$.

Note that parents' consumption-equivalents can be rewritten as a function of children time equivalents as follows:

$$U(\hat{C}_i) = U(C_i) + \alpha_i \beta_i [v(\hat{T}_i) - v(\bar{T})] \, \forall i \tag{24}$$

Under the full equalization of the $\hat{T}_i$s, the constraint that $C_i = \bar{C} \, \forall i$ further implies that $\hat{C}_1^{++} < \hat{C}_3^{++}$ and $\hat{C}_2^{++} < \hat{C}_4^{++}$. This completes the proof.

9.6 Proof of Proposition 8

When comparing the two social planning problems, the optimal levels of T_i^* and T_i^+ are defined by the same equation (22). This implies that $T_i^* = T_i^+ \forall \, i = \{1, 2, 3, 4\}$.

Given the resource constraint of the economy, these equalities imply that:

$$\sum_{i=1}^{4} g_i^* = \sum_{i=1}^{4} g_i^+ = G \tag{25}$$

where $G = \sum_i w(1 - T_i^+) = \sum_i w(1 - T_i^*)$. Equivalently, the total amount of children's consumption is the same at the two social optima.

Let us now use the equations describing the equalities of consumption-equivalents and time-equivalents for types 1 and 3. In the first social planning problem, the condition is given by:

$$\hat{g}_1 = \hat{g}_3 \iff g_1^* + \frac{\breve{\beta}}{1-\breve{\beta}} [v(T_1^*) - v(T_2)] = g_3^* + \frac{\bar{\beta}}{1-\bar{\beta}} [v(T_3^*) - v(T_2)]$$

and in the second problem, the condition is given by:

$$\hat{T}_1 = \hat{T}_3 \iff \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}} \left(g_1^+ - g_4 \right) + v(T_1^+) = \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}} \left(g_3^+ - g_4 \right) + v(T_3^+)$$

Denote $T_1^* = T_1^+ = T^A < T_3^* = T_3^+ = T^B$. We have:

$$\hat{g}_1 = \hat{g}_3 \iff g_1^* + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T^A) - v(T_2)] = g_3^* + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T^B) - v(T_2)]$$

and in the second problem

$$\hat{T}_1 = \hat{T}_3 \iff \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^+ - g_4\right) + v(T^A) = \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(g_3^+ - g_4\right) + v(T^B)$$

Note that because of the budget constraint (25) and other equalities $g_1^* = g_2^*$, $g_3^* = g_4^*$, $g_1^+ = g_2^+$ and $g_3^+ = g_4^+$, we have: $2g_1^* + 2g_3^* = G = 2g_1^+ + 2g_3^+$.

Hence,

$$g_3^* = \frac{G}{2} - g_1^* \text{ and } g_3^+ = \frac{G}{2} - g_1^+$$

Substituting for these expressions in the above two conditions yield:

$$g_1^* + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T^A) - v(T_2)] = \frac{G}{2} - g_1^* + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T^B) - v(T_2)]$$

and, in the second problem,

$$\frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^+ - g_4\right) + v(T^A) = \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(\frac{G}{2} - g_1^+ - g_4\right) + v(T^B)$$

which simplifies to

$$g_1^* = \frac{\frac{G}{2} + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T^B) - v(T_2)] - \frac{\breve{\beta}}{1-\breve{\beta}}[v(T^A) - v(T_2)]}{2} \tag{26}$$

$$g_1^+ = \frac{\frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}\left(\frac{G}{2} - g_4\right) + v(T^B) - v(T^A) + g_4\frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}}{\frac{\left(1-\breve{\beta}\right)}{\breve{\beta}} + \frac{\left(1-\bar{\beta}\right)}{\bar{\beta}}} \tag{27}$$

In general, we have thus $g_1^+ \neq g_1^*$, leading to $g_3^+ \neq g_3^*$.

There is one exception: when all children's preferences are the same. In that case, $\bar{\beta} = \breve{\beta} = \beta$ and $T^A = T^B = T$ and the two expressions become:

$$g_1^* = \frac{\frac{G}{2} + \frac{\beta}{1-\beta}[v(T) - v(T_2)] - \frac{\beta}{1-\beta}[v(T) - v(T_2)]}{2} = \frac{G}{4}$$

and

$$g_1^+ = \frac{\frac{(1-\beta)}{\beta}\left(\frac{G}{2} - g_4\right) + v(T) - v(T) + g_4\frac{(1-\beta)}{\beta}}{\frac{(1-\beta)}{\beta} + \frac{(1-\beta)}{\beta}} = \frac{G}{4}$$

This equality yields the equivalence between the two constrained social optima in the absence of heterogeneity on children's preferences (despite inequalities in parental preferences).

9.7 Proof of Proposition 9

The double egalitarian social planning problem in its general form is written as in problem (16).

Consider first the case where there is no laundering of preferences. We then have that parental consumption-equivalents are defined by equations (17) - (20). The objective is not differentiable, but the problem can be rewritten as the maximization of the consumption-equivalent of the worst-off children (e.g. of type 2), subject to the egalitarian constraints specifying the equalities of all children's consumption-equivalents and of all parent's consumption-equivalents as well as to the resource constraint of the economy.

Under Assumption 1, the Lagrangian can be written as follows:

$$\begin{aligned}
\mathcal{L} &= g_2 + \lambda_1[g_1 + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1) - v(T_2)] - g_2] \\
&+ \lambda_3[g_3 + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3) - v(T_2)] - g_2] \\
&+ \lambda_4[g_4 + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_4) - v(T_2)] - g_2] \\
&+ \mu[\sum_{i=1}^{4} w(1-T_i) - \sum_{i=1}^{4} C_i - \sum_{i=1}^{4} g_i] \\
& \quad +\psi_1\left[\hat{C}_1 - \hat{C}_2\right] + \psi_3\left[\hat{C}_1 - \hat{C}_3\right] + \psi_4\left[\hat{C}_1 - \hat{C}_4\right] + \psi_5\left[\hat{C}_1 - \tilde{C}\right]
\end{aligned}$$

where we replaced for $\bar{T} = T_2$ in the definitions of $\hat{g}_i$ (see equations 6-9).

Rearranging the first-order conditions of this problem and under Assumption 1, we obtain that T_i^{**} is defined by equation (22) implying that $T_3^{**} = T_4^{**} > T_1^{**} = T_2^{**}$.

From the binding egalitarian constraints of children, we can then deduce :

$$\begin{aligned}
\hat{g}_1 = \hat{g}_2 \iff g_1^{**} + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1^{**}) - v(T_2)] &= g_2^{**} + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_2^{**}) - v(T_2)] \implies g_1^{**} = g_2^{**}, \\
\hat{g}_3 = \hat{g}_4 \iff g_3^{**} + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3^{**}) - v(T_2)] &= g_4^{**} + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_4^{**}) - v(T_2)] \implies g_3^{*} = g_4^{**}, \\
\hat{g}_1 = \hat{g}_3 \iff g_1^{**} + \frac{\breve{\beta}}{1-\breve{\beta}}[v(T_1^{**}) - v(T_2)] &= g_3^{**} + \frac{\bar{\beta}}{1-\bar{\beta}}[v(T_3^{**}) - v(T_2)] \implies g_1^{**} > g_3^{**},
\end{aligned}$$

leading to the following complete ranking:

$$g_1^{**} = g_2^{**} > g_3^{**} = g_4^{**}.$$

Let us now rank parental consumptions. Parental consumption equivalents as defined above should be equalized so that $\alpha_1 = \alpha_3 = \bar{\alpha} > \alpha_2 = \alpha_4 = \breve{\alpha}$ together with $g_1^{**} = g_2^{**}$ and $T_1^{**} = T_2^{**}$ leads to $C_2^{**} > C_1^{**}$. Similarly, $g_3^{**} = g_4^{**}$ and $T_3^{**} = T_4^{**}$ leads to $C_3^{**} > C_4^{**}$. From equation (23), the full equalization of the $\hat{g}_i$s together with the constraint that the $\hat{C}_i$s should also be equalized imply that $C_2^{**} < C_4^{**}$ and $C_1^{**} < C_3^{**}$.

Consider now the case where there is laundering of parental preferences. In that case, the problem is the same as above, except that the equalization of parental consumption-equivalents is achieved when

$$C_1^{**} = C_2^{**} = C_3^{**} = C_4^{**} = C^{**} = \tilde{C}.$$

9.8 Second-best double egalitarian optimum

Under the laundering of parental preferences, $C_i^{**} = \tilde{C}\,\forall i$ (see Proposition 9). Using the same reasoning as in Appendix 9.4 for the comparison of children's utilities under possible mimicking as well as the fact that parental consumptions are identical, we can conclude that there is no reason for a parent to claim of being of another type.

Without the laundering of preferences, we have at the optimum that $C_4^{**} > C_3^{**}, C_2^{**} > C_1^{**}$. As before, the utility of children is the same whether they wish to mimic or not, so that the only reason why parents would like to mimic is related to differences in parental consumptions. In that case, every parent would have an interest in claiming to be of type 4 (with the highest consumption), unlike under the constrained egalitarian optimum (where all parental consumptions are equalized). Thus it is here impossible to satisfy the egalitarian constraint and incentive constraint simultaneously. In that case, the only way to avoid mimicking is to provide the same parental consumption to any parent of type i, thus leaving some inequalities in terms of consumption equivalents: $\hat{C}_4 < \hat{C}_2, \hat{C}_3 < \hat{C}_1$ (exactly as in the constrained egalitarian optimum without laundering). Hence, the first-best double egalitarian optimum cannot be implemented under asymmetric information, unlike the first-best constrained egalitarian optimum.

9.9 Proof of Proposition 10

The objective of the double egalitarian problem (21) is not differentiable. But the problem can be rewritten as the maximization of the time-equivalent of the worst-off child subject to egalitarian constraints specifying the equalities of all children's time-equivalents, to the resource constraint of the economy. We assume as before that the worst-off child is a type 2. The Lagrangian of this alternative egalitarian problem is:

$$\begin{aligned}
\mathcal{L} &= v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4) \\
&+ \lambda_1[v(T_1) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_1 - g_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \lambda_3[v(T_3) + \frac{1-\bar{\beta}}{\bar{\beta}}(g_3 - g_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \lambda_4[v(T_4) - (v(T_2) + \frac{1-\breve{\beta}}{\breve{\beta}}(g_2 - g_4))] \\
&+ \mu[\sum_{i=1}^{4} w(1-T_i) - \sum_{i=1}^{4} C_i - \sum_{i=1}^{4} g_i] \\
&\quad + \psi_1\left[\hat{C}_1 - \hat{C}_2\right] + \psi_3\left[\hat{C}_1 - \hat{C}_3\right] + \psi_4\left[\hat{C}_1 - \hat{C}_4\right] + \psi_5\left[\hat{C}_1 - \tilde{C}\right]
\end{aligned}$$

where we replaced for $\bar{g} = g_4$ in the definitions of $\hat{T}_i$ (see equations (10) - (13)).

We again obtain that T_i^+ is defined by equation (22) implying that $T_1^{++} = T_2^{++} < T_3^{++} = T_4^{++}$.

From the binding egalitarian constraints, we can then deduce :

$$\begin{aligned}
\hat{T}_1 &= \hat{T}_2 \iff \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^{++} - g_4\right) + v(T_1^{++}) = \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_2^{++} - g_4\right) + v(T_2^{++}) \implies g_1^{++} = g_2^{++}, \\
\hat{T}_3 &= \hat{T}_4 \iff \frac{(1-\bar{\beta})}{\bar{\beta}}\left(g_3^{++} - g_4\right) + v(T_3^{++}) = \frac{(1-\bar{\beta})}{\bar{\beta}}\left(g_4^{++} - g_4\right) + v(T_4^{++}) \implies g_3^{++} = g_4^{++}, \\
\hat{T}_1 &= \hat{T}_3 \iff \frac{\left(1-\breve{\beta}\right)}{\breve{\beta}}\left(g_1^{++} - g_4\right) + v(T_1^{++}) = \frac{(1-\bar{\beta})}{\bar{\beta}}\left(g_3^{++} - g_4\right) + v(T_3^{++}) \implies g_1^{++} \gtrless g_3^{++},
\end{aligned}$$

leading to the following incomplete ranking:

$$g_1^{++} = g_2^{++} \gtrless g_3^{++} = g_4^{++}.$$

Let us now rank parental consumptions. Parental consumption-equivalents are equalized so that $\alpha_1 = \alpha_3 = \bar{\alpha} > \alpha_2 = \alpha_4 = \breve{\alpha}$ together with $g_1^{++} = g_2^{++}$ and $T_1^{++} = T_2^{++}$, leads to $C_2^{++} > C_1^{++}$. Similarly, $g_3^{++} = g_4^{++}$ and $T_3^{++} = T_4^{++}$ leads to $C_4^{++} > C_3^{++}$.

Finally, let us consider the ranking of parents' consumption. Under the full equalization of the $\hat{T}_i$s, the equalization of the $\hat{C}_i$s together with equation (24) then implies that $C_3^{++} < C_1^{++}$ and $C_4^{++} < C_2^{++}$.

Consider now the case where there is laundering of parental preferences. In that case, the problem is the same as above, except that the equalization of parental consumption-equivalents is achieved when

$$C_1^{++} = C_2^{++} = C_3^{++} = C_4^{++} = C^{++} = \tilde{C}$$

9.10 Proof of Proposition 11

When comparing the two social planning problems, the optimal levels of T_i^{**} and T_i^{++} are defined by the same equation (22). This implies that $T_i^{**} = T_i^{++} \forall\, i = \{1, 2, 3, 4\}$.

However, these are the only choice variables that take the same levels in the two double egalitarian problems. Using the same argument as in the proof of Proposition 8, we obtain that $g_i^{**} \neq g_i^{++}$ so that optimal children consumptions differ across the two social optima.

Finally, it is obvious that under preference laundering, $C_i^{**} = C_i^{++} = \tilde{C}\ \forall i \in \{1, 2, 3, 4\}$. Without preference laundering, there is no reason why these should be equal and, generally $C_i^{**} \neq C_i^{++}$.

9.11 Numerical exercise

In the numerical section, we relax Assumption 1 and assume instead that $u(.)$ is increasing and concave. In that case, the laissez-faire condition for T_i is different from the optimal one and one needs to reintroduce labour taxation. We denote by τ_i, the rate of the labour tax.

The other instruments remain identical to those of Proposition 6. We assume that material consumption of children g_i^*, is directly provided to parents (and children). In order to ensure that parents' consumptions are also equal to their optimal levels, we introduce lump-sum taxes, L_i. The individual decentralized problem writes as follows

$$\max_{T_i} U(w(1 - T_i)(1 - \tau_i) - g_i - L_i) + \alpha_i[(1 - \beta_i)u(g_i) + \beta_i v(T_i)]$$

The first-order condition with respect to T_i takes the following form write:

$$-w(1 - \tau_i)U'(C_i) + \alpha_i \beta_i v'(T_i) = 0$$

where C_i is the optimal level of consumption of a parent with type i, and is different depending on the optimum considered.

From the above FOC, the optimal level of the labour tax rate can be rearranged as follows:

$$\tau_i^* = 1 - \alpha_i \beta_i \frac{v'(T_i)}{wU'(C_i)}$$

Replacing with the condition for T_i^*, equation (22), we obtain[18]

$$\tau_i^* = 1 - \frac{\alpha_i(1 - \beta_i)}{U'(C_i)}$$

Note that when parental consumption is constrained and equal to $\bar{C}$, we obtain the following ranking

$$\tau_4^* > \tau_3^*,\ \tau_2^* > \tau_1^*.$$

[18] Note that if we replace $U'(C_i)$ by its expression in eq. (4), and assume that $u'(.) = 1$, $\tau_i = 0 \forall i$. We are thus back to the results obtained in the main part of the paper.

after replacing for the values of β_i and α_i.

We also define the lump-sum tax as $L_i = w(1-\tau_i)(1-T_i) - C_i$, where T_i is the optimal level of parental time and is invariant to the optima considered but where C_i takes different values under the different optima.

In the next table, we find the optimal levels of $(C_i, g_i, T_i)\,\forall i \in \{1,2,3,4\}$. The optimal levels of T_i are obtained using directly eq. (22).

The other choice variables $(C_i, g_i)\,\forall i \in \{1,2,3,4\}$ are obtained using the egalitarian constraints for children and the resource constraint of the economy, setting $\bar{C} = 1$ under the constrained egalitarian with and without preference laundering (second and third columns in the next table).

Note that except for the direct allocation of material consumptions to children (i.e. the g_is), the decentralization of the constrained optimum under consumption-equivalents and time-equivalents are identical and independent of the laundering or not of preferences, since the T_is and the levels of $\bar{C}$ are the same in the 2 specifications.

Finally, the decentralization of the double egalitarian optimum (under both consumption and time equivalents) requires to set a level on $\tilde{C}$, which is the well-being level at which all parental well-being consumption-indexes are equalized. We set $\tilde{C} = 1$. The results of our simulations are summarized in the following table.

	L F	**Cons. CE**	**Cons. TE**	**Unc. CE w/o**	**Unc. CE with**	**Unc.TE w/o**	**Unc.TE with**
g_1	2.22	12.77	4.97	13.58	12.77	5.09	4.97
g_2	1.67	12.77	4.97	13.58	12.77	5.09	4.97
g_3	0.89	0.23	8.03	0.25	0.23	8.84	8.03
g_4	0.67	0.23	8.03	0.25	0.23	8.84	8.03
T_1	0.22	0.1	0.1	0.1	0.1	0.1	0.1
T_2	0.17	0.1	0.1	0.1	0.1	0.1	0.1
T_3	0.36	0.4	0.4	0.4	0.4	0.4	0.4
T_4	0.27	0.4	0.4	0.4	0.4	0.4	0.4
C_1	5.56	$\bar{C} = 1$	$\bar{C} = 1$	0.37	$\tilde{C} = 1$	0.54	$\tilde{C} = 1$
C_2	6.67	$\bar{C} = 1$	$\bar{C} = 1$	0.53	$\tilde{C} = 1$	0.68	$\tilde{C} = 1$
C_3	5.56	$\bar{C} = 1$	$\bar{C} = 1$	0.67	$\tilde{C} = 1$	0.38	$\tilde{C} = 1$
C_4	6.67	$\bar{C} = 1$	$\bar{C} = 1$	0.78	$\tilde{C} = 1$	0.54	$\tilde{C} = 1$
τ_1	-	0.6	0.6	0.85	0.6	0.78	0.6
τ_2	-	0.75	0.75	0.87	0.75	0.83	0.75
τ_3	-	0.84	0.84	0.89	0.84	0.94	0.84
τ_4	-	0.9	0.9	0.92	0.9	0.95	0.9
L_1	-	2.6	2.6	0.96	2.6	1.41	2.6
L_2	-	1.25	1.25	0.67	1.25	0.85	1.25
L_3	-	-0.04	-0.04	-0.027	-0.04	-0.015	-0.04
L_4	-	-0.4	-0.4	-0.31	-0.4	-0.22	-0.4

Table 1: Numerical values of $(g_i, T_i, C_i, \tau_i, L_i)$ under the laissez-faire and the different social optima.